Advanced Gas Dynamics
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Lecture - 31
Example Problem using Thin Airfoil Theory

So, let us continue from the last lecture we will finish the Thin Airfoil Theory. So, so where we have stopped was that getting an expression for the zero normal boundary condition on the streamline right. So, essentially if I were to do this again.

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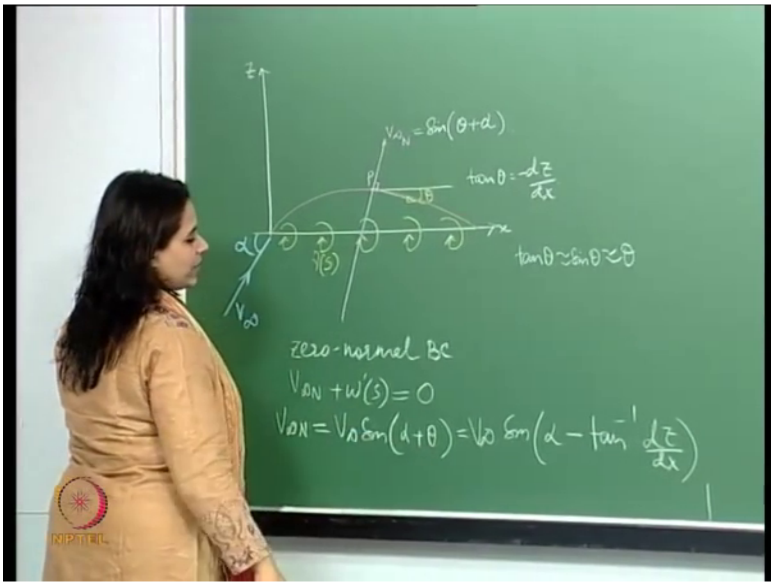

That and this is the camber line that we are looking at right. And of course this is the camber line that we are looking at when I say a here; so for a thin airfoil.

So, basically I am distributing my vortices on the x axis right, but I am so far as find the zero normal boundary condition on the streamline. So, say at this particular point P. So, this is my stuff and so essentially the for thin airfoil right. So, we need to find a an expression for this, such that we still in with the we satisfy the boundary equation on the you know camber line and here the camber line is steal is still the streamline.

So, the chord line is not the stream line, the streamline is still the camber line which means who I still have a flow which goes that way. So, therefore, I need to calculate the zero normal boundary condition on the streamline; so having done that right. So, we said that, at this when we said the zero normal boundary condition. So, totally in this velocity gives you the vorticity distribution here ok.

So, and this basically we found out we calculated this as if you see over here right. So, and this is basically the flow, this is basically this flow and this is the angle of attack of a right. So, therefore, this V infinity N it came out to be sin of theta towards this right and we do so found out that of course, you see from here if you look from here right. So, if you look at this right: therefore, if I am going to right this, ok.

So, what I get here is that now V infinity now V this if I were right this and therefore, I can also right this as theta. So, this is what we get out here. Now if we consider you know this to be a small perturbation problem, then that if theta is small then I can write tan theta right. So, from here if you see, theta is stand inverse this. So, therefore, here theta I can just simply write this as this theta I can simply write this as d z by dx.

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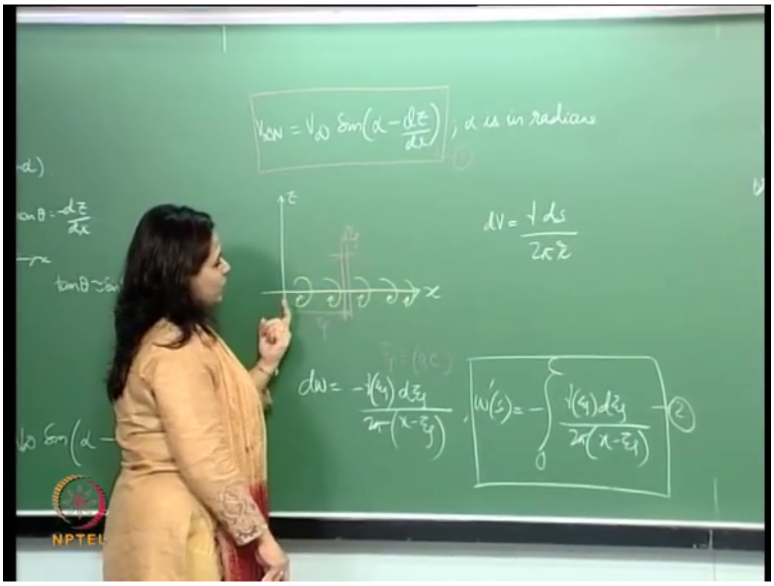

So, basically therefore, I will write my normal component in this right where right. So, this is V infinity sin of alpha minus dz by dx and the alpha is not radius. Therefore, we get this. Now let us get the normal component of the in this velocity right. So, if I say look at this right. So, again and I have essentially a vorticity distribution like this right.

So, now if I have something like this, let me take this let me consider at this distance to be z let say we will consider this to be z and this element here to be dz understand. So, basically z goes from say goes along x. So, you can say that z is essentially goes from 0 to c on the chord length right. So, at a particular c for take a small element of dz over here. Therefore, now how we will we calculate I mean what is the we need to get an expression for this component out here ok in this velocity due to this distribution.

So, if you remember this is velocity distributed this is a standard expression. So, any standard work should give you this if you do not remember. Therefore, in this particular case the where we would write this here. So, the total say velocity distribution in this case would be the gamma is distributed over this say z into dz just going by this expression here. So, dz which is dx over here 2 pi and r is in distance between any point and the location of this 2 pi. So, it would be x minus z ok.

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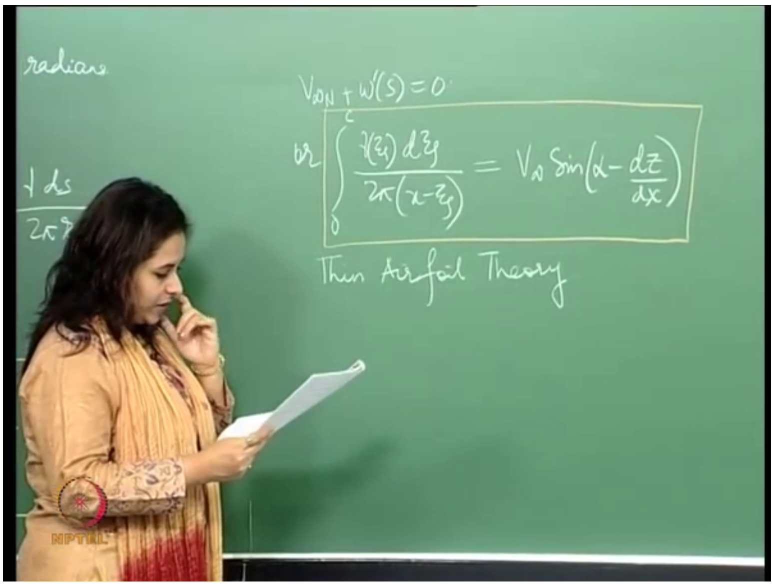

So, if I do that this is dw and the total velocity distribution therefore,. So, the entire velocity distribution over this a for would be. So, I will right that basically here. So, here I will write this as and what we basically need to do this goes from the z goes from 0 to c is not it. So, this 0 goes from 0 to c and let us write this as let store this up and call this as a this ok.

Therefore, what we have here these were. So, if I write this down now the expressions for this. So, basically this is a and this is equal to. So this is nothing but the. So, this is essentially the statement. So, I am from the boxes out. So, this is essentially the statement of the thin airfoil theory which is nothing but the imposition of the zero normal boundary condition where a camber line is streamline and in this particular case we are considering the airfoil to be seen enough. So, that we distributed distribute the vortices on the cord line instead of the camber line ok.

So, basically instead of saying ds which is a curve line which is take dz you know this makes a competition little easier. So, that is how we take care of basically we can of linearizing using a linearize theory to this. So, this is our thin airfoil theory. Now let us go back therefore, to the problem that we were trying to solve where instead I have to solve this go head and see how we will use this linearize theory to get the coefficient of lifton drag right for the flat plate. So, if I am sort of you remind you what we are going to do. So, essentially we said we had a flat plate the problem of chord lines of certain cord line c, which is which is at an angle of attack of alpha in a supersonic you know a free stream. So, use the linearize theory to calculate you know the CL and cd.

So, basically what you know when we use a linearized theory what you know by now we should be understanding that when we use a linearized theory and add a correction to it, then we should get an expression for CL and CD in the supersonic stream. Because we are talking about compressible flow, when you saying supersonic free stream, whereas you saying use linearize theory which would not be you know compressible. So, how would be instead of go about here ok.

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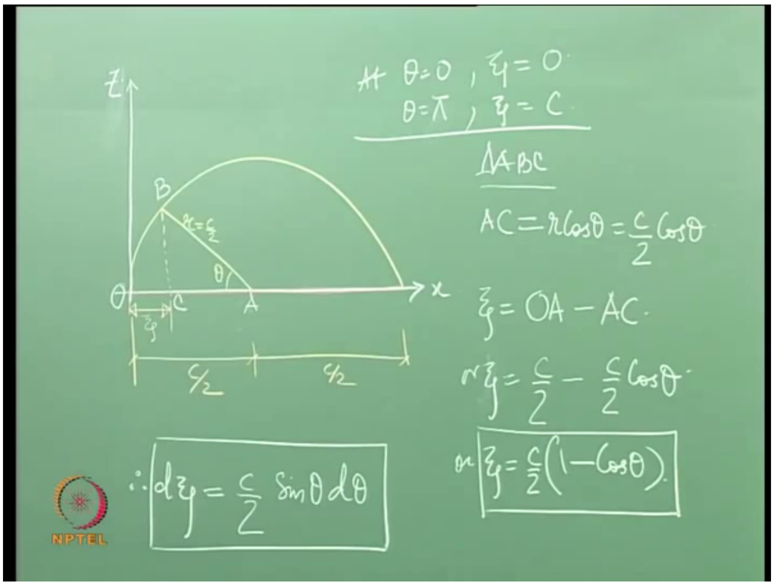

So, now little bit of geometry here, little bittle geometry. So, if we start with this problem now let say let us call this see as we have done this. So, see this is the origin. Now if I use spherical coordinates here how would that look like what is; that means? So, if I say use a spherical coordinates here if I you know do this. So, say you know I have this. So, I have this radial line, I have this radial line here.

Now, this is my airfoil. So, basically if what I am saying is if I have my airfoil over here which in this case is the flat plate, this or here is my flat plate and this is the flat plate of you know line a chord c right which is what we have here; so in this particular case. So, if I take a spherical coordinate system. So, then this is my one radial direction which in this particular case is equal to c by 2 right. So, I basically taking the centre of this and drawing a curve you know a circle which is you have circle rather; so of radius c by 2 ok.

So, in that case now if I say drop this perpendicular from here. So, then now this, this distance is basically z. So, this distance is c. So, you can say at this particular point out here, here the x location of this point is z if I do this. So, from here we can see basically let us say this is say this is c right if you. So, take this triangle A B C if you take this ac AC is r cos theta which is c by 2 cos of theta.

So, therefore, z z is equal to this z which is essentially OC is OA minus AC right and or OA out here look at this another radial line. So, that is c by 2 minus AC is c by 2 cos of theta or z is equal to c by 2 1 minus cos theta. So, this is the relationship. So, this z coordinate if I write in terms of the chord length and the this angle this. So, therefore, from here the z would be what right and of course, a also if you can see from the geometry here that if I write it here. So, at theta equal to 0 z is equal to 0 and at theta equal to c z is equal to sorry theta equal to pi z is equal to c. So, this is just geometry is the talking about geometry if I were to use this coordinate system ok.

Now, in this particular case what we have is a flat plate. So, the camber line out here is a straight line, this is the straight line which is having an angle of attack of alpha which would mean what. Let us come here and say look at this, let us you know instead of draw this picture out here. So, what we have if you look at this in this particular case. So, we have a camber line like this, which is making which is certain angle of attack alpha here ok.

Now, instead of this what we have here let just erase all of this; so what we have right now let me draw it here and showing you sort of the comparison.

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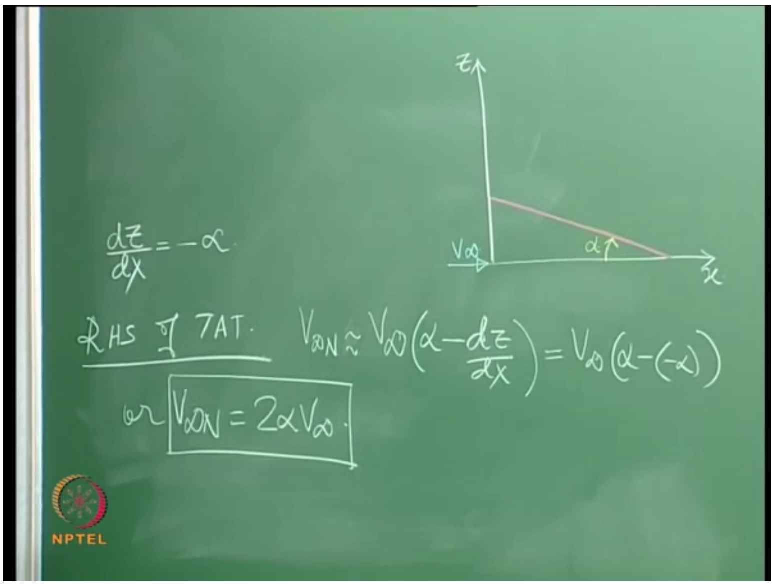

So what we have here now if you look at this, what we have here say. So, this is a flat plate which is making an angle alpha incase the same thing if you keep that horizontal along this and give it an axis or you and basically. So, what we are saying is in this particular case of course, is that we have a free stream which comes like this, this is my free stream ok.

So, you see here this actually is now the streamline and compare that with this over here. So, this is my streamline again this is 0 to c the chord length is 0 to c. Therefore, if you look at this over here d z dx is equal to what? This is the slope which is minus alpha is not it. So, this is d z dx over here. Therefore, the right end side if you look at this, this is the thin airfoil theory. So, this is the right hand side actually this is the right hand side of this equation the thin airfoil theory; which is V infinity sin of alpha minus d z dx and using small if this angle is more enough then we can just write as V infinity into alpha minus d z dx. Therefore, writing that write inside using the right hand side of the statement of thin airfoil theory ok.

So, there the way we would write that is essentially is not it and in this particular case d z by dx is minus alpha. So, what we have over here is or this is the right hand side of it. So, the next thing of course, basically we are you know you know just applying the thin airfoil theory to this particular case ok.

Let us look at this I guess. So, we have a z which we are you know this is our geometry now this is our geometry, where our z is this and our d z is this is not it. If I write that in terms of spherical coordinates, then we can basically let the integral go from 0 to pi ok.

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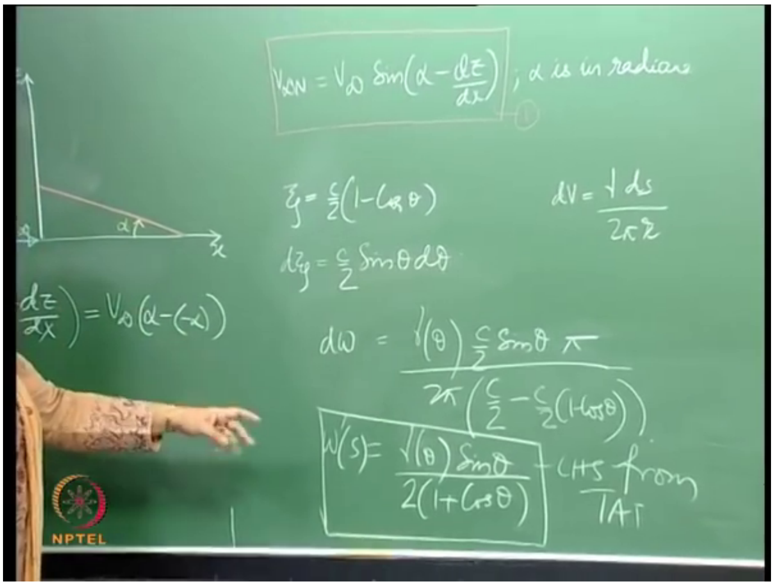

So, using I am going to use that and write this down here first. So, z is equal to c by 2 and dz is equal to c by 2. So, in this particular case therefore. So, what we have over here is that d w, ok.

So, in this case therefore, it becomes what. So, this gamma I will write the in terms of theta and ds is dz here. So, which is c by 2 sin theta d theta by 2 pi. So, r you know that total is c by 2, c by 2 minus z which is. So, this is for the total derivative. So, this comes out to be let us do this here. So, c by 2 this would go right. So, what we would have over here. So, what we essentially would get over here is gamma of theta, sin theta sin and this d theta now we are basically going for this thing going for the entire plate. So, then you do that and then you get this in to 1 plus cos theta right is that, ok.

So, this is the left hand side of it. So, if I were to. So, therefore, I can say in this particular case this is my left hand side, this is the left hand side from the thin airfoil theory. So, this is the totaling this velocity from the vorticity distribution and we have got an expression for entire normal component of the velocity for this flat plate. So, now, let us use this, having basically just applied the thin airfoil theory here let us use this now ok.

So, we are we have done this, now let us go head and calculate what we set out do initially. So, what we will do now. So, let us now you know get an expression therefore, equate these two to get an expression for this gamma of theta over here, ok.

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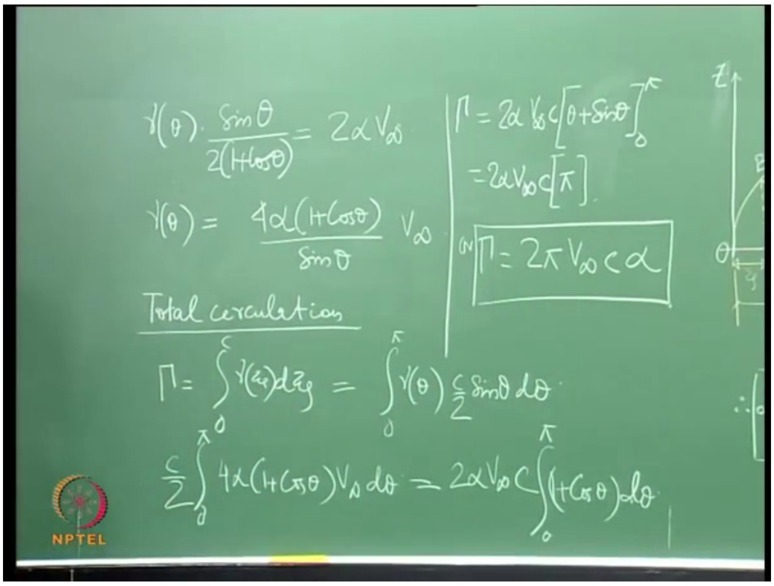

So, now what we have over here is that gamma of theta into. So, this is the. So, this is the left hand side and the right hand side, this is essentially the thin airfoil theory apply to this particular problem. So, Linearize theory apply to this particular problem.

So, when I do that. So, what I get? I get basically if you look here; so an expression for gamma of theta which is equal to 4 for alpha to 1 plus cos theta by sin theta into V infinity. So, this is the expression for gamma of theta this is the expression for gamma of theta. So, this is the distribution. So, therefore, total circulation, when we calculate the total circulation then we will be able to calculate the total lifting total drag ok.

So, total circulation therefore, is essentially gamma which is. So, total circulation if you do this. So, in this particular case of course, we will write this to it we will use this theta coordinates we will go from 0 to pi. So, we will use gamma of theta and dz has we have use before. So, that is c by 2 sin theta d theta right this is the total circulation we will bring in this expression of gamma of theta into here and when I finish the let we do this. So, if I do this let us do this here. So, this becomes c by 2 0 to pi, let see gamma of theta is four alpha 1 plus cos theta, this sin theta this sin theta cancels out ok.

Then we have V infinity and d theta. So, let us take four alpha and V infinity out. So, what we get here is 2 alpha V infinity C 2 alpha V infinity C into 0 to pi 1 plus cos theta d theta. So, this is what we gets if I were instead of completed over here itself if I use this place bare me. Therefore, my total circulation here, it becomes what two alpha V infinity c, what I get here is sin theta sin theta sorry here. So, forgot the theta we will intimating here; so theta plus sin theta 0 to pi right which becomes 2 alpha V infinity C.

So, we get pi and pi plus sin pi is 0 minus 0 minus you know sin zero is zero. So, we essentially get this. So, or total circulation is 2 pi V infinity c alpha which makes sense actually here you remember what we got earlier is just 2 pi alpha for (Refer Time: 32:04) it. So, the total circulation is 2 pi V infinity C into alpha right. Now we will calculate the CL and CD to do that let us say we will come over here.

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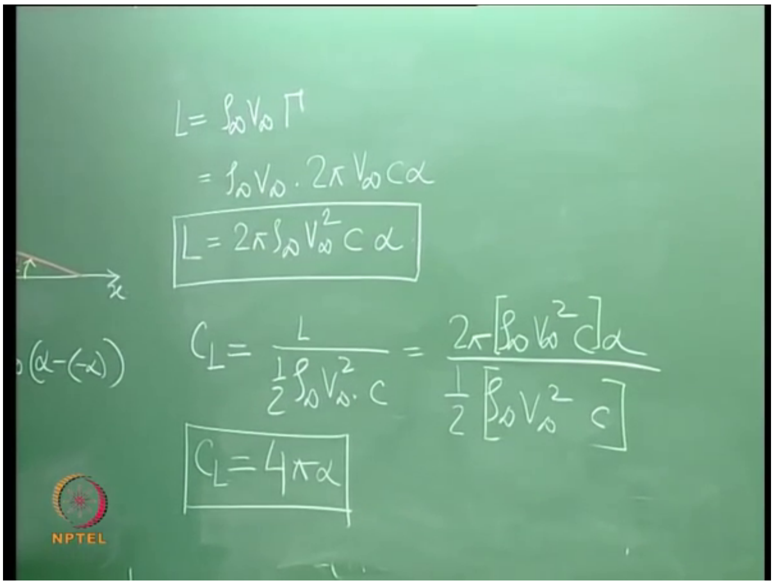

Let say. So, what we will do here now is that now lift per unit span therefore, is given as right this is the lift per unit span. So, once if I would I get that. So, I can write this as rho infinity V infinity into this equation which we have just derived which is 2 pi V infinity c alpha if I do that. So, then what I get here is 2 pi rho infinity V infinity square C alpha 2 pi rho infinity V infinity square C.

So, this is my this is my lift and therefore CL becomes half rho infinity square V infinity square into small in this particular case it is a 2 d airfil. So, just c right even a span. So, if I do that let us get this from here. So, what we get is 2 pi rho infinity V infinity square C alpha half rho infinity V infinity square into C. So, what you can see from here what we get is that 4 pi rho infinity square V infinity square of this thing cancels out. So, essentially if we look at this these box cancel out. So, what we get is 4 pi alpha ok.

So, the CL out here using the linearizing theory is four pi alpha and. So, this is our CL right. So, then have we will calculate the CD and cr for the mach number here which was supersonic mach number you will just see me n infinity right we need to calculate that now for this particular case, now what we will do here in this particular case. So, let us go head and use our small perturbation theory and or linearization theory to connect the two. So, we have found out an expression for a using the linearized theory thin airfoil theory, ok.

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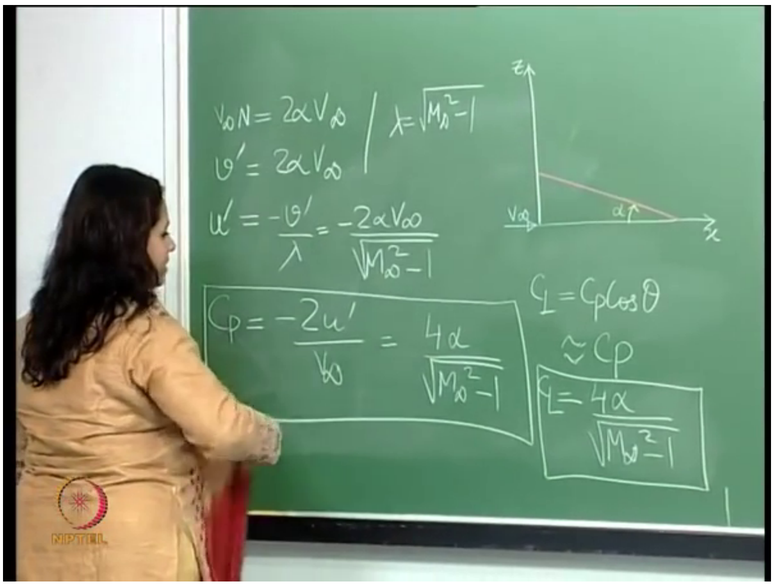

So, now here the normal component is 2 alpha V infinity we have seen this. So, the normal component out here is 2 alpha V infinity. So, if I use a perturbation theory here this normal component is nothing but v dash which is this and in if you remember right you know when we use the correction in this particular case since this is our supersonic flow we will use this (Refer Time: 36:58) over here, then our u component becomes this. So, this refer what we get is. So, if you do that now it is easy. So, we get coefficient of pressure which is this is something we have done using a small perturbation using perturbation theory we know we have done this so many times.

So, if I do this. So, therefore, from here from here therefore, this becomes this is 4 alpha by this. Now, if you look at this, this is the coefficient of pressure in this particular case. So, now CL, CL is the lift. So, which is Cp you know cos theta. So, if you look at this particular case of here.

So, then this becomes what? This becomes us theta tends to pi we go over. So, basically what we are doing here is if we integrate this right the entire length of the airfoil which we will which basically going from 0 to pi. Therefore, theta tends to pi it is 1 right. So, therefore, this becomes essentially Cp. So, therefore, our CL we for the supersonic case out here is 4 alpha. So, 4 alpha using this and again of course, CD, CD we take the other component of the Cp. So, let me write that over here if I write that over here.

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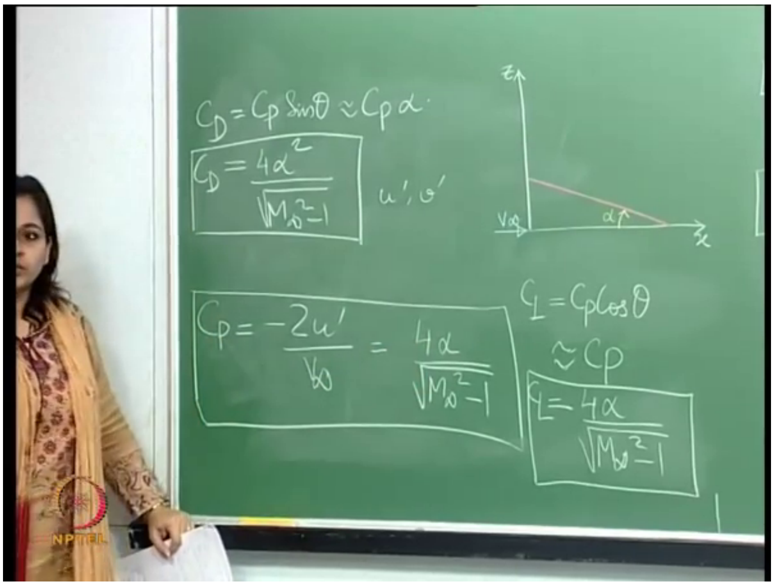

That CD of course, is Cp sin theta. So, again this in this particular becomes Cp. So, sin theta it tends theta. So, this becomes this particular case alpha.

So, therefore, in here; so therefore, CD becomes four alpha square by. So now, you can clearly see the difference essentially the difference between the thin airfoil theory. And if we use the perturbation theory and a correction to get CL and CD for the same airfoil; so I use the same airfoil theory and I got the CL as 4 pi alpha over here and instead of here first when I started with this, I got an a expression using the if I use a perturbation theory perturbation theory here which is again a linearize theory and use the normal component of the perturbation velocity and that would be the normal component V alpha and infinity N we use that and then we got an expression for u dash and v dash which is the perturbation velocities. In using that we were able to get basically Cp we are able to write it in Cp using the correction for the supersonic flow ok.

So, and of course, using Cp we coefficient of pressure we write the CL and the CD for this particular case, I can you can and you can see the difference. So, when I use the thin airfoil theory is basically this whereas, this I can use for the supersonic flow. So, you can see the correction here. So, essentially you can say the correction would be pi by M infinity square minus 1 if you basically if you look at the relationship here. So, I can say write this as incompressible in case if I write the incompressible here. So, if I were to do that then in this particular case ok.

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So, our CL supersonic is basically pi by we can use that lambda also we use the lambda right pi by lambda into CL 0. So, this is the correction we use in this particular case. So, I think we will stop here and pickup some things next time on.

Thank you.