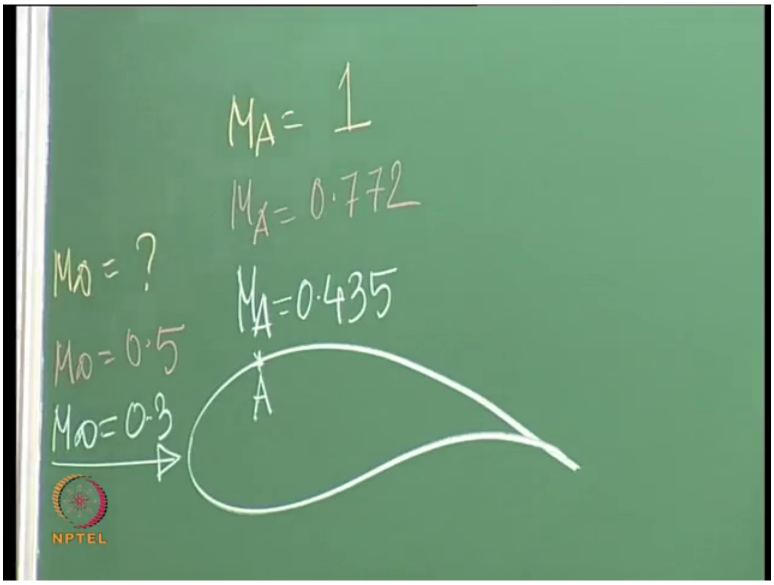
Advanced Gas Dynamics
Dr. Rinku Mukherjee
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 30
Critical Mach Number and Thin Airfoil Theory

So let us, what we will do, start with today is, the critical Mach number. Now what do we mean by that.

(Refer Slide Time: 00:28)

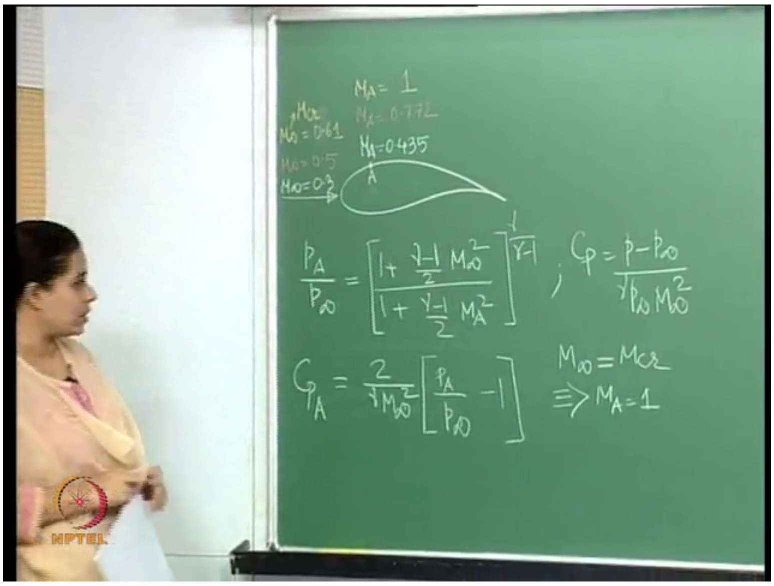


Now consider this surface right, and say we have say a Mach number, incoming Mach number say 0.3 right, and at this particular point, at this particular location, say A on the surface of this air foil, we have minimum pressure, and hence maximum Mach number, maximum speed, and this, for this particular case say is.

So, we have an incoming Mach number for this particular Mach number, at this point A, at this location A we have minimum pressure and maximum Mach number which is 0.435. Now let us say we increase this Mach number, we increase this Mach number to, say 0.5 right. Then the corresponding Mach number at this point becomes say 0.772.

So, now if we sort of do a reverse calculation; now if we say that the Mach number here, is 1. If the Mach number is 1, then what is the corresponding incoming Mach number? So, what we are doing here is, first we said that we have a location here, which is minimum pressure maximum Mach number right. So, we have an incoming Mach number which is 0.3, corresponding Mach number here is 0.435. We increase this Mach number to 5. So, the corresponding MA becomes 0.72. Now we say that, now in that case what is the Mach number to induce, or to have a Mach number at this location as 1, to have sonic region at 1, what should be the corresponding Mach number? This is essentially. Well in this particular case this value is say 1.

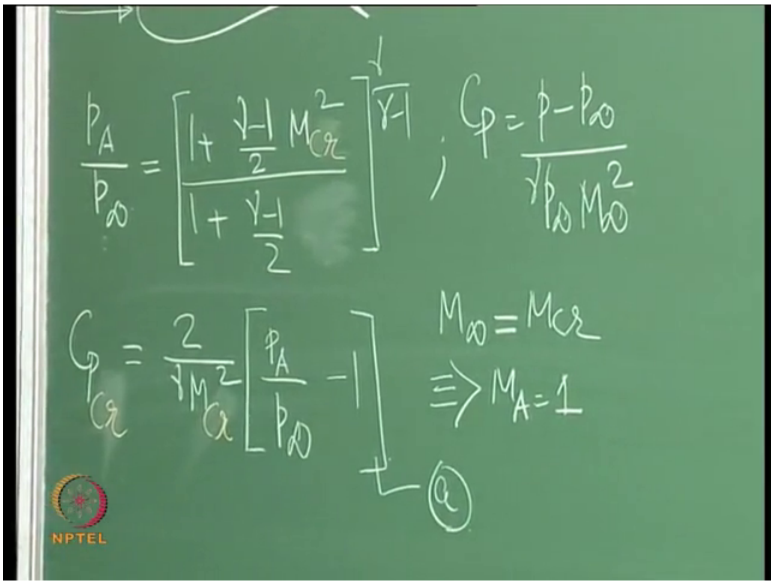
(Refer Slide Time: 03:21)



And this Mach number is also called the critical Mach number. So, basically by definition this is the Mach number, Mach number at which sonic flow is encountered first by the air foil. So, now, having sort of, you know driven this down here now. If you remember this, you know the isentropic relationships. So, P A the pressure at A, and this is the free stream would be given by. And also if you remember the coefficient of pressure, is basically. And also therefore, we let us write coefficient of pressure, at the location A, I can write this as basically this. So, let me just write that as P A by P infinity right minus 1. I can write this, and P A is again you can see this you can write in terms of the Mach number.

Therefore, if this is the critical mach. if A then becomes you know, if you consider the Mach number, as the critical Mach number right, then MA becomes one. So, when I say that. So, if you know. So, basically what you are saying is that when this free stream Mach number becomes the critical Mach number, at that point MA which is the Mach number at the point A becomes sonic or 1. So, when Mach, the free stream Mach number is the critical Mach number, this implies that Mach number at the point A is 1. So, if I introduce that here, if I introduce that here. So, let us do that. So, then P A essentially becomes.

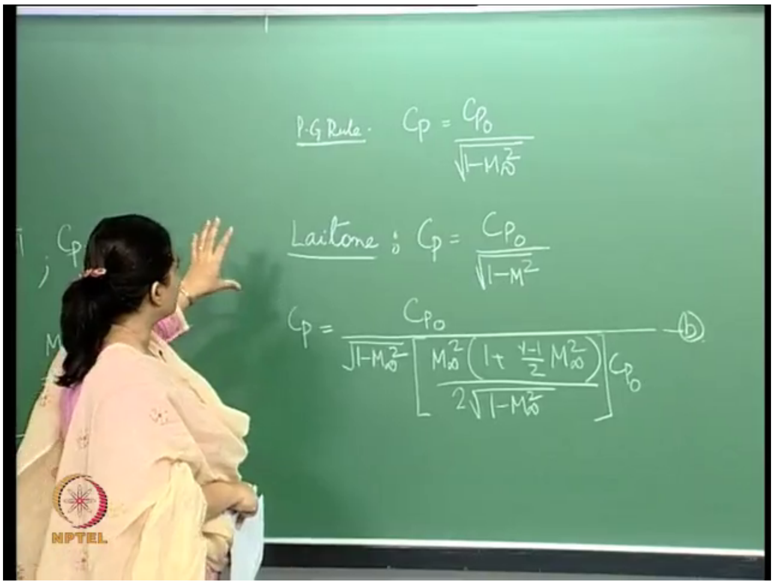
(Refer Slide Time: 06:26)



This is the critical Mach number. This goes off. So, correspondingly we have this. So, then also I can say at this point C P A or this is the critical, critical coefficient of pressure, again this also becomes critical, so this. So, let us call this as, let us call this expression as. So, this is what we, the basically what we mean by the critical Mach number, and the corresponding critical coefficient of pressure.

So, now, what we sort of you know did last, you know the last lecture, was the, first of all the Prandtl Glauert rule right, Prandtl Glauert rule which gave us a connection between the compressible flow coefficient of pressure, and the incompressible flow coefficient of pressure.

(Refer Slide Time: 07:53)

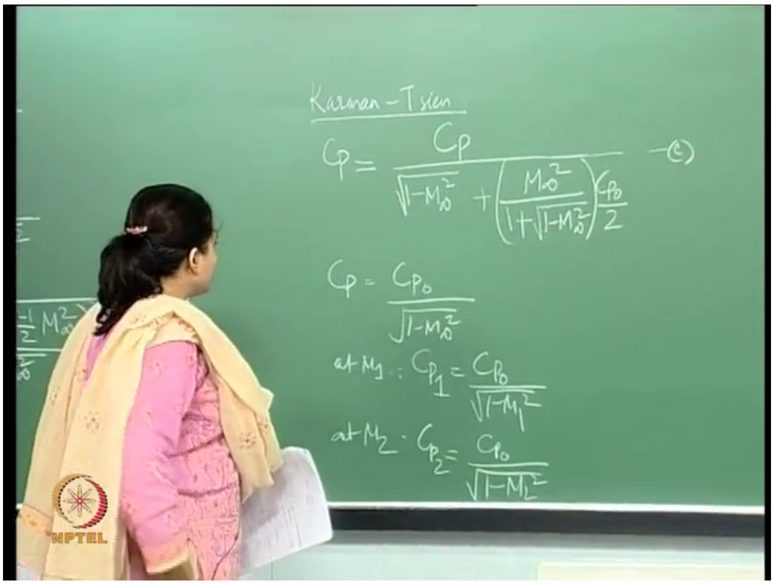


So, the Prandtl Glauert rule said that this is the C P right, this is the. So, Prandtl Glauert rule basically what we see over here, is that this is the incompressible coefficient of pressure, and this is. So, with some corrections to it, we get the coefficient of pressure for the compressible flow right.

Now, we know similar corrections, now the thing is that this the coefficient of pressure, coefficient of pressure with a little, with some corrections involved, can be then used for, with suitable corrections actually, can be used for compressible flows right. So, I will just write down two of such corrections suggested, and we will see how these look like. So, this you know Laitone suggested this, where he considers that the local Mach number right. If you see this, basically he considers the local Mach number, not the free stream.

Now, I think the greater question here is, why do we need these corrections at all. We will just talk about that in a bit, let us get over with this and we will discuss this. So, this is a correction. So, instead of just the free stream Mach number, he uses the local Mach number here. And if you know I can relate that using isentropic relationships to the free stream Mach number, and then I get a, you know this looks like, if I do that, if I write that in terms of the free stream Mach number, it looks like this . So, this is the correction which is suggested by Laitone, and there is yet another one, which is suggested by Karman and Tsein.

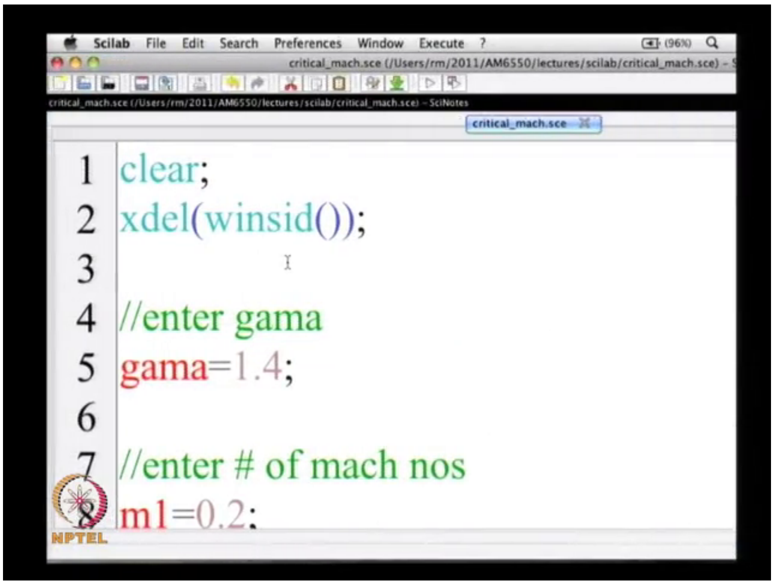
(Refer Slide Time: 10:49)



So, let us see that, and that looks like this. So, basically so what we have over here is that this is the Prandtl Glauert rule right, and these are the two corrections which is suggested basically two corrections suggested by Laitone and Karman and Tsein. Now it is important to understand here, that if you remember when we started right, when we started doing linearized, you know linearized theory, and we took small perturbations. We got what was the acoustic equations right, we got the acoustics equation, in which basically we got a wave right, which was traveling at the speed of sound.

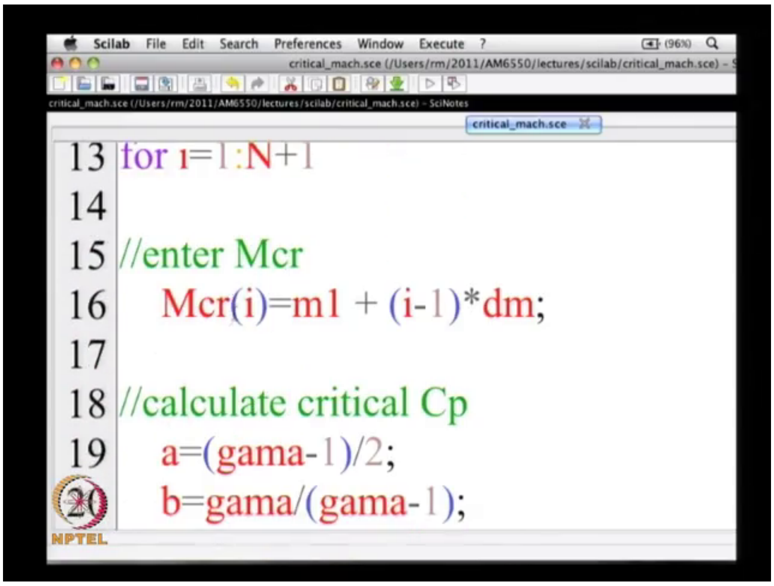
And then we said we will not consider small perturbation. We use a method of characteristics, and then we. So, basically we try to solve exact equations, and then what we got the each locally the velocity is different, which is what it should be right. So, therefore, it is important to consider the local Mach number. It is important to understand or to find out the local properties; so in this case which is the Mach number. So, therefore, these corrections are important. So, now, if I use this, if I use this and. So, what we will do is, you know say do a plot of this. Now this with a suitable correction, using either of these the Prandtl Glauert rule, Laitone or Karman Tsein, we will see that we will guess a more or less usable coefficient of pressure for the compressible flows.

(Refer Slide Time: 13:44)



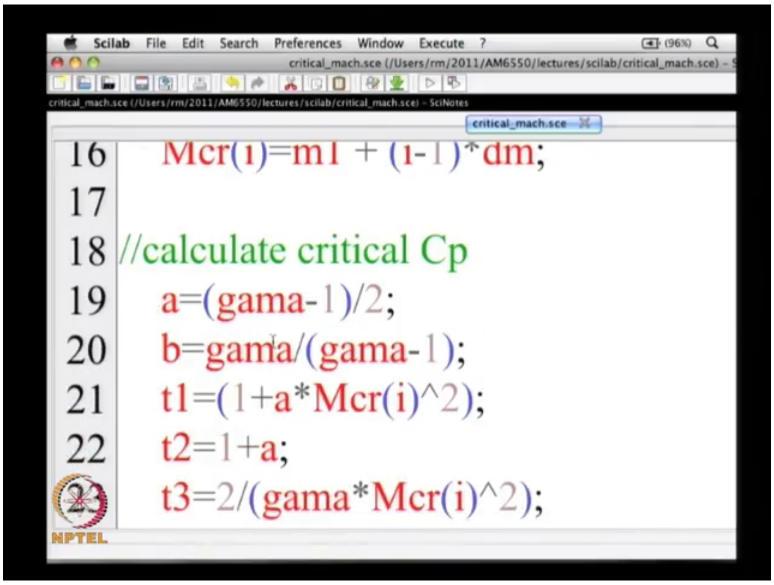
What I have done here, is again I have used essentially Scilab. So, having used that let me show you this. So, what I will do is, I will plot this and then I will show you; so basically for a series of subsonic Mach numbers.

(Refer Slide Time: 14:10)



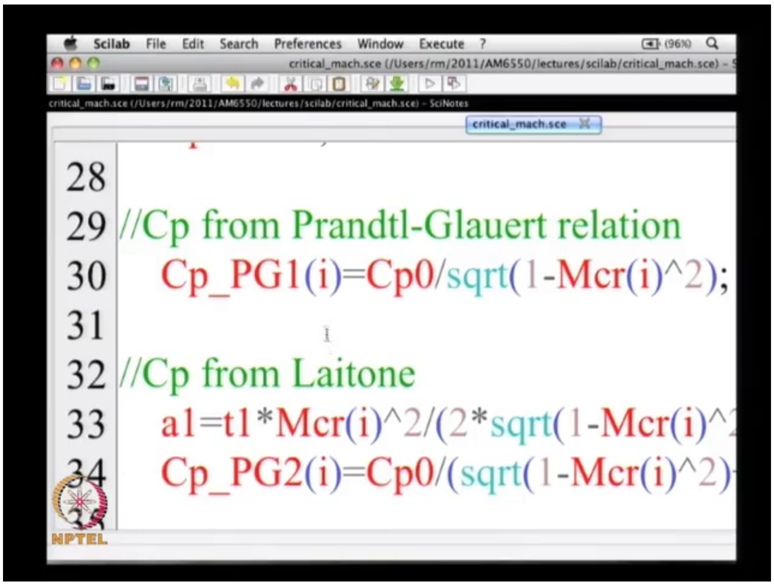
So, now, if you see this is the critical Mach number etcetera.

(Refer Slide Time: 14:14)



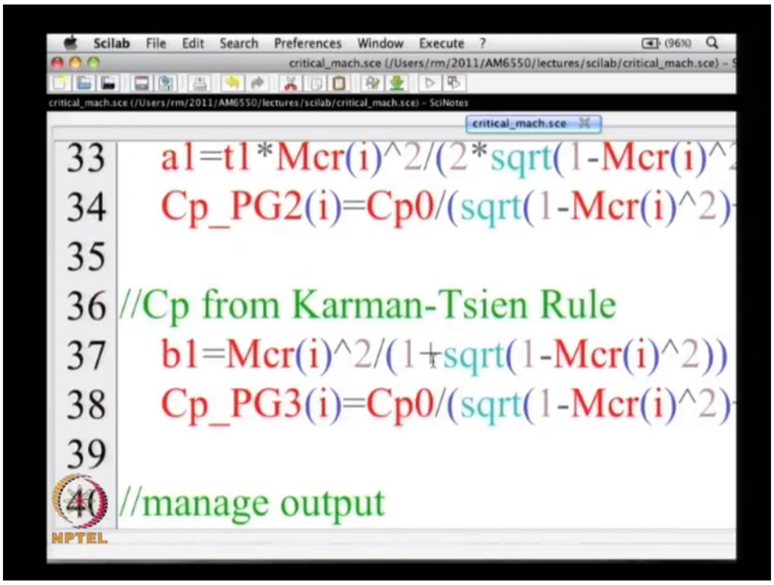
So, then we calculate the corresponding critical coefficient of pressure. So, here basically I calculated using the equation, which I have just developed right and so doing that.

(Refer Slide Time: 14:30)



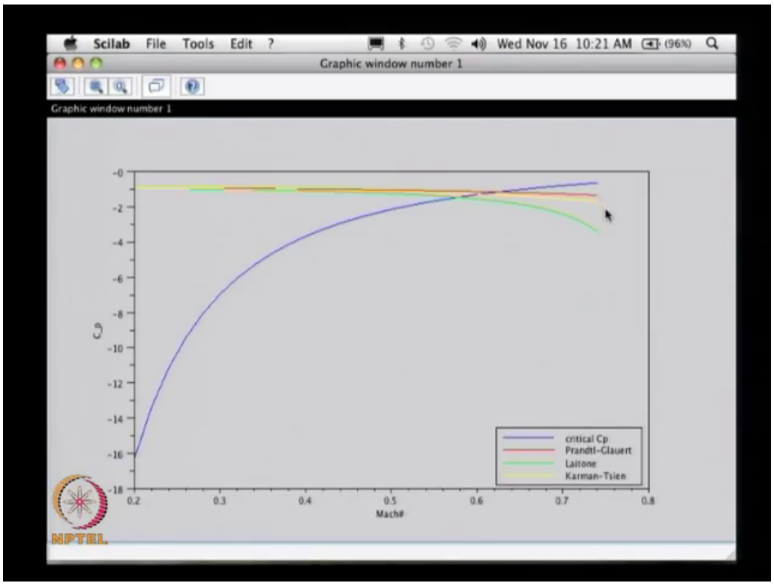
So, this is the Prandtl Glauert relationship. This is the, this is the C P from corrections using from Laitone and C P from Karman Tsein rule.

(Refer Slide Time: 14:39)



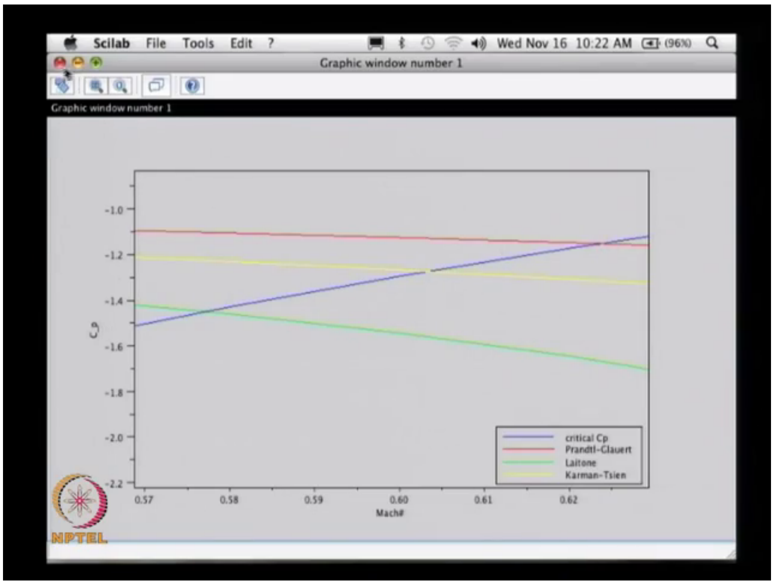
So, having done that, let us look at the result for this. Let us look at the result for this. So, if I do this.

(Refer Slide Time: 14:55)



Now let us look here. So, this is basically the critical coefficient of pressure, the blue line. If you look at this, the blue line here, is the critical coefficient of pressure, which I just calculated, and the red line here is doing the same with the Prandtl Glauert equation. The yellow line is, using the corrections of Karman Tsein, and the green is using that of laitone. So, now, if I say zoom in to this area, if I say zoom in to this area say.

(Refer Slide Time: 15:38)

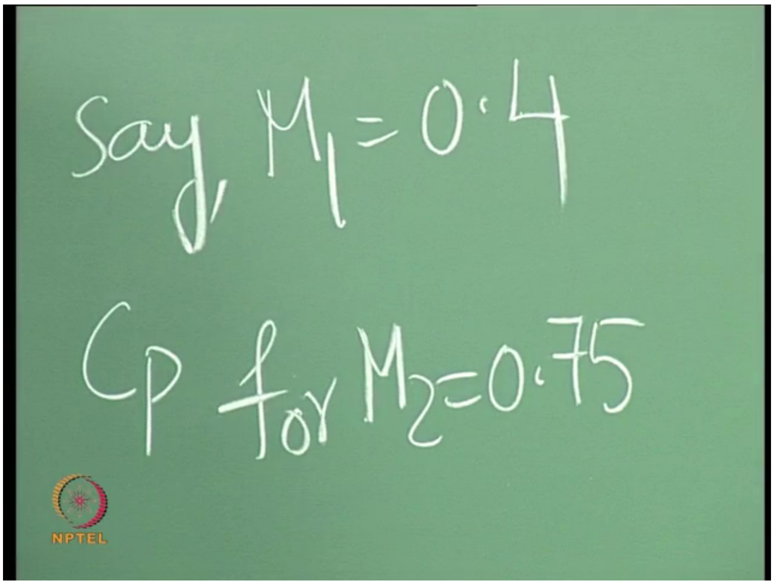


So, any of these intersection points: so this is basically the critical coefficient of pressure incompressible. So, with the suitable corrections, we get usable you know critical coefficient of pressure. So, that will be given by the intersection of these curves right, as you can see from here. So, that is a basically a good way of using the, you know incompressible, you know incompressible coefficients of pressure Mach number etcetera to the supersonic, to the compressible flows.

Now, also the Prandtl Glauert rule, then the Prandtl Glauert rule also has interesting sa,y just look at say a use of this. This is the Prandtl Glauert rule right. Now say for example, so we have basically. So, say we are considering two Mach numbers, say we are considering two particular Mach numbers over the same air foil. So, basically for the same air foil, we will have the same in incompressible coefficient of pressure. So, this does not change, but the incoming Mach number changes, which means say at M 1.

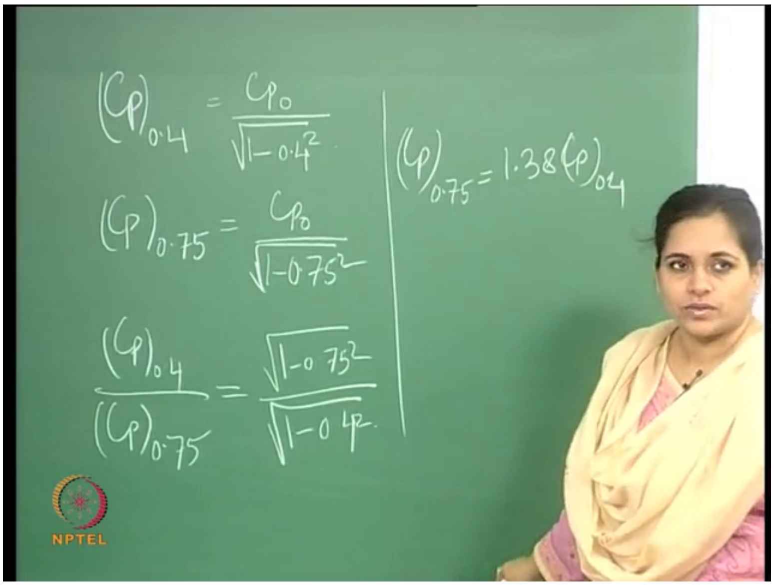
So, let us call that as C P 1, the corresponding coefficient of pressure is, the incompressible coefficient of pressure does not change. So, we have this, so here. So, let us say this, and similarly say at you know. We take another Mach number which is this, the corresponding coefficient of pressure will be 1 minus M 2 square.

(Refer Slide Time: 18:21)



Now, say if I were to write in terms of, say you know M 1 is equal to 4, M 1 is equal to 4, then C P for, say M 2 equal to 0.75.

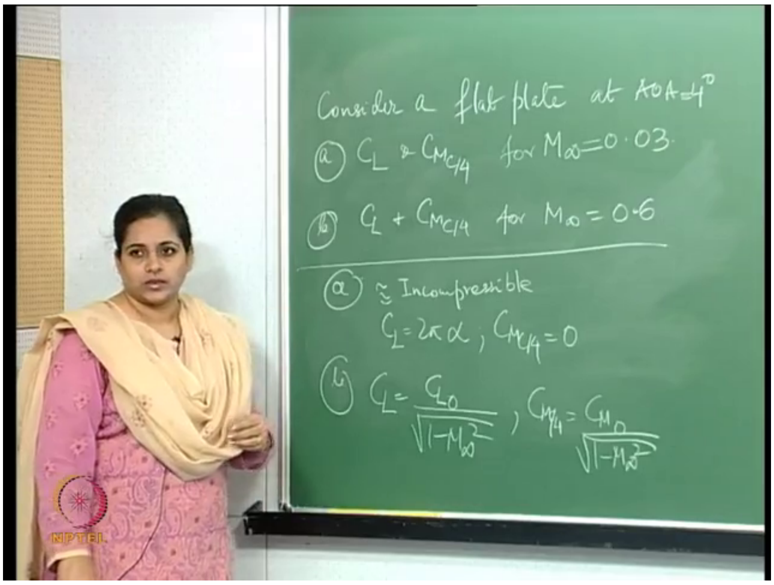
(Refer Slide Time: 18:51)



is that C P for say 0.4 is equal to C P naught by 1 minus 0.4 square, and C P at 0.75 is C P naught by 1 minus 0.75 square. So, what do we do over here? So, therefore, let us divide the whole thing right. Therefore, say C P 0.4 C P 0.75 is equal to

So this is it. Or we what we can write from here is that C P at 0.75. So, we can basically use the Prandtl Glauert rule in a manner like this. So, let us sort of look at you know, look at a relevant problem, and see what we can do. What I did actually here, is that we basically have two Mach numbers 0.75 and 0.4, and you can, you know you can connect the two. You can see, you can actually each of these coefficients of pressure individually can be expressed in terms of the in compressible pressure coefficient right, and if either one, and then also, if either one is known you can calculate the other, in terms of each other as well. So, that is what I am trying to show over here. So, let us look at a problem. Let us look at a problem say. So, now, say we have.

(Refer Slide Time: 21:21)

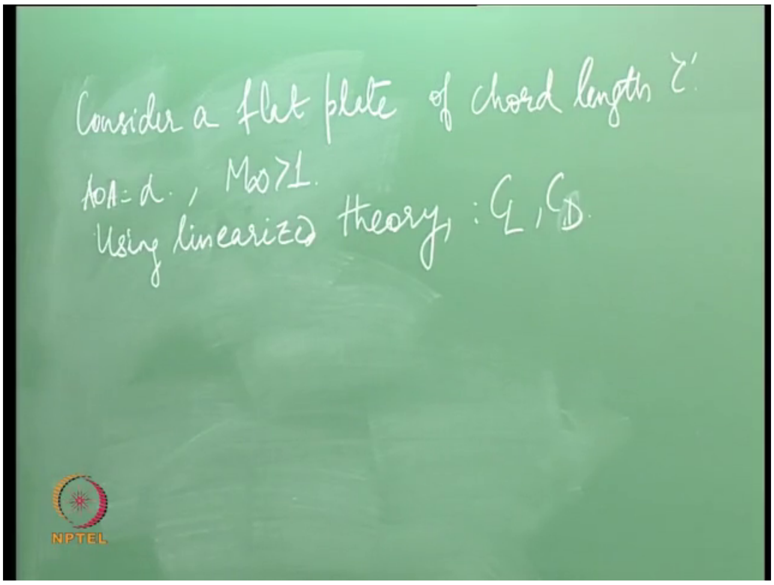


So, consider right, a flat plate at an angle of attack of 4 degrees right. So, then first job is calculate the C L, and the quarter chord coefficient of pitching moment, for free stream Mach of 0.03, and the second case is that. So, you know calculate C L and C M C by 4 for Mach number 0.6. So, basically we have a flat plate, we have a flat plate, we have a flat plate, which is making an angle of attack of 4 degrees. So, we want to calculate the C L and C M for a Mach number of 0.03, and the and at Mach number 0.6, how do we go about this.

Well, Mach number 0.03 we can regard that as incompressible, so this particular case. So, A we can say this is incompressible right, we can say this is incompressible right, and then if that is so, then your C L becomes 2 pi alpha, where alpha is the angle of attack. This is a readymade solution and flat plate for a chord, it is 0. if this is, this is it. Then we calculate the C L and C M for a Mach number 0.6. How will we do this? If you remember what we did last time, a Prandtl Glauert rule, it gives us a relationship between the C L and C M of the compressible and incompressible flows as well right. So, if you remember. So, what we had was C L was incompressible C L right, and right.

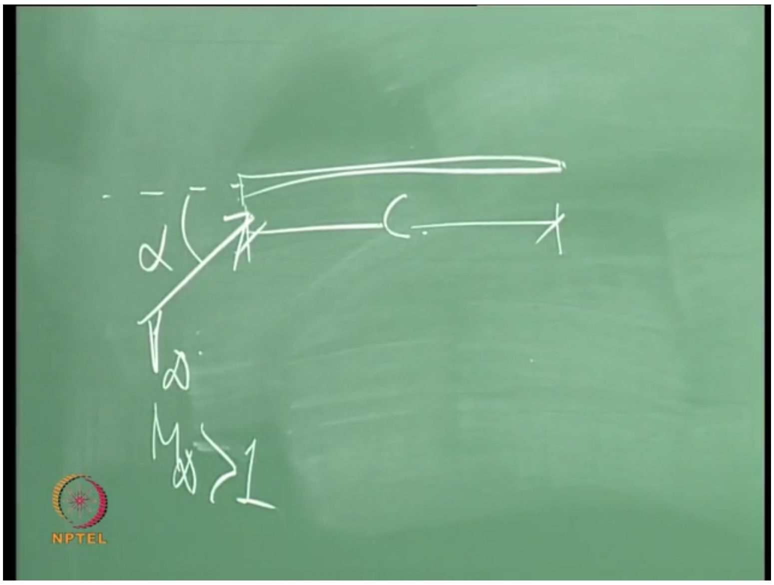
So, we have a. So, this is basically the M, the free stream Mach number here is 0.6. So, we are considering a compressible flow here, but what we do is, we have found out C L and C M in for incompressible. So, just using this particular correction which is a Prandtl Glauert rule, we shall be able to calculate C L and C M C by 4 all right. So, basically that is a problem. Let us look at another problem, let us look at another problem, and which goes like this, which goes like this.

(Refer Slide Time: 25:46)



Now, if you. So, basically consider a flat plate of chord length C right, and angle of attack is alpha supersonic free stream, which is Mach number is this thing. So, using linearized theory, derive C L and C D. So, derive C L and C D; so coefficient of lift, and coefficient of drag. So, we basically have a flat plate of certain chord length.

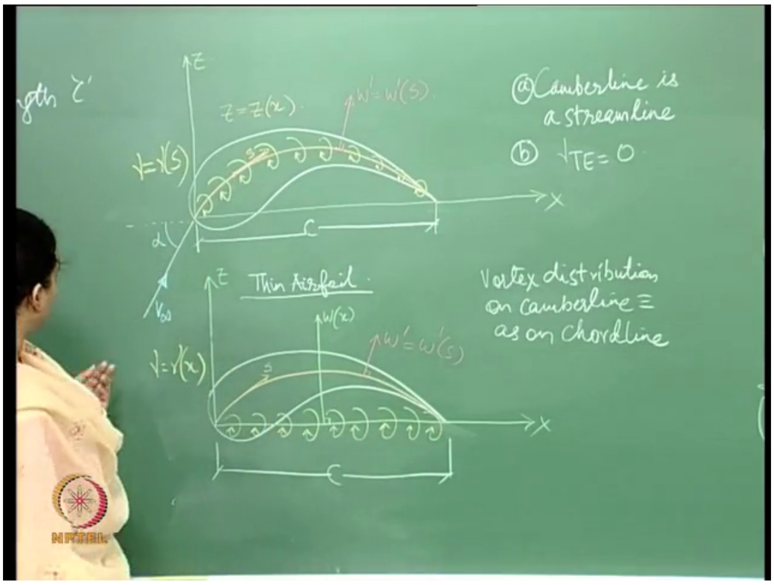
(Refer Slide Time: 27:01)



Basically you have a flat plate. This is the chord length. It is an angle of attack at. It is an angle of attack of alpha, and a supersonic flow is impinging on this, so we have a free stream Mach number is supersonic. So, using linearized theory, we reach and calculate C L and C D.

So, now, the way I will proceed for this is, doing a. You know just doing a very brief review or an interaction to something called thin air foil theory, which is incompressible flow over a thin air foil, which is essentially, is thin air foil theory. So, let me start with that. We will do the treatise, and they will come back and solve this problem all right. So, now, what exactly is this thin air foil theory? So, now, I am going to draw this in a slightly exaggerated manner, so you can see.

(Refer Slide Time: 28:37)



So, let me draw an air foil. So, I do that, and then I draw the camber line of this. I draw the camber line. This is the camber line, which is essentially the midpoint between the upper and the lower surfaces, and then what I am going to do, is draw an axis system. Let that be that, and let that be this, that.

So, let us call it that, and therefore, here basically, you mean you could have, you know you can have a free stream basically. It is making say an angle of attack, this. So, now, numerically in order to calculate lift, we basically have a vorticity distribution, over this effort. So, the way we will go about that, is let us just say, we have a distribution of vortices which looks like this we using this camber line, and distributing these vortices over here right. We distribute these vortices, and now this is essentially a, we can, we will just write this, we this is. Now this surface, this surface let us just say, this is S. Now and this, the vortices distributed over this surface, this this curved surface which is the camber out here. So, therefore, my vorticity distribution, if I were to.

Clean this up a little bit. So, then I what I will say is, my vorticity distribution is, over this surface S; so gamma is basically strength of this distribution, and let us just say the curve, curve out here, this curve is basically given by this, is not it. And also here, let us just say the, at any say, you know a particular point. So, this is a vortex sheet, and you know velocity will be deduced by, it will be induced by these vortices on to this, you know at different locations. So, at a particular point say at here .

So, this is the normal component of the velocity induced. This is a normal component of the velocity induced and let us calls that as. So, the reason I show that is that, because this normal component is with respect to, is on S, yes on this curve. So, on this curve; so we have the vorticity distributed, the vortex distribution on this curve S; strength of which it is gamma. The curve itself you can write as say you know zee is, you know X is a function of X right, and the normal component of the velocity induced is this. Now we have to satisfy when we have distribution like this, we need to satisfy a condition. Now our objective here, our objective here is to find a certain a suitable distribution of vorticity; such that this camber line is a stream line.

So, our objective is that the camber line, camber line is a stream line. So, we have to find a suitable distribution of this vorticity; such that this camber line here is a stream line. And of course, this gamma S should also satisfy the Kutta condition right, which is the necessary boundary condition, which means. Why is Kutta condition important, because its nature way of ensuring that the flow leaves the trailing edge smoothly? So, that is a basically a Kutta condition.

So, we have a distribution like this, which looks like this. Now these are the condition which needs to be imposed right. Then the camber line we have to find a suitable gamma, that the camber line is a stream line, and also the Kutta condition right, that gamma of the trailing edge is zero. So, this is the air foil; now when I started out by saying thin air foil theory. Now what happens if I consider this as thin air foil? Now let me. Now draw this again. Let me draw this again, it is basically the same air foil that I am going to replicate here, hopefully I will be.

So, I am more or less right. So, if I do that, and again I do this, this, and I am again going to draw this. So, this is my access system, so in this particular case. So, again this is my S right. This is my surface right, and this is the chord length.

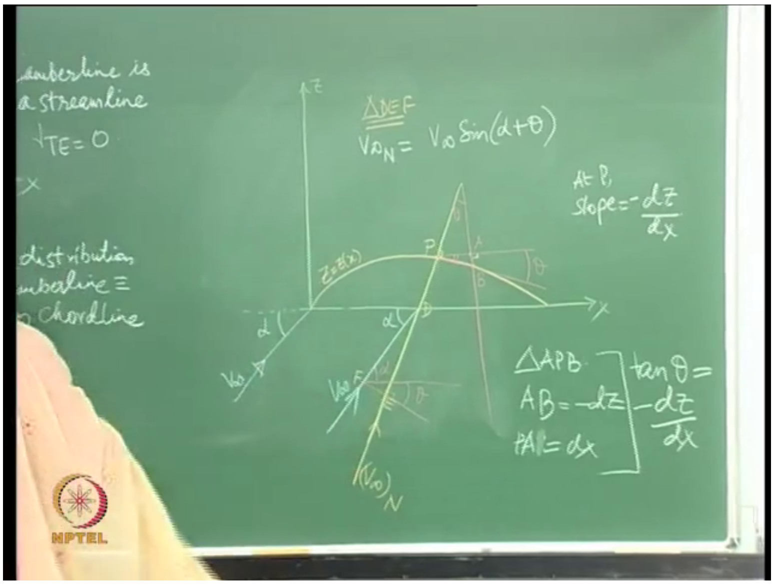
So, similarly this is the chord length. Now what happens if it is a thin air foil? So, when we say thin air foil. So, what we mean to say that, we are ignoring the thickness right or that is to say that this is an air foil of 0 thickness; if it is a thin air foil, then what will happen, is that my vorticity distribution is here. Now you can consider this basically as. Well not consider, this is the chord length, this is the chord line, you can consider this is the chord line. Now if you see the difference. So, in here, now this is basically for a thin air foil

Now this is for a thin air foil, where what we do out here, is that I consider the distribution on the chord length. Now in here we consider on the chord length right and. So, as you can see here right, the normal. So, the normal to this still remains this. This is the normal, and this is. And here of course, the normal is this. This is the normal component. So, vortex distribution what you are basically saying, that the vortex distribution on the camber line, is just the same as vortex distribution on the chord line.

So, for a thin air foil what you are saying is, that vortex distribution on the camber line, is the same on cord line. So, what we are saying is that this is my camber line. Now the vorticity distribution is on the chord line, unlike in the previous case. Now the two boundary conditions still have to be satisfied, which is that the kutta condition has to be 0, and now the gamma S, now this still remains the same, in here although the gamma here. So, gamma here; so we can basically say here. So, we need to define the strength of the vorticity in this particular case, where still the camber line is this stream line. The stream line is not the chord line. The stream line is still the, where the stream line is still the camber line. Therefore, 0 normal boundary condition has still to be satisfied over the stream line. Although, the vorticity distribution here, is on the chord line; so that is basically the essence of the thin air foil theory right

So, when I say that, let us go and do some bit of math and see if we can get some equations. So, in order to do that now, again if I were to look at this; so say I will just draw the camber line here. Let me draw just draw the camber line here.

(Refer Slide Time: 41:31)



And, I will draw the camber line. I am drawing just the camber line here. So, and basically, we have. This is making alpha. This is the free stream. This is the free stream and, or the camber line, the camber line, as we have from before. Now let us consider a point P over here. Let us consider a point P, and drop a perpendicular at this point. So, at this point we are going to drop a perpendicular. What do I mean by that? So, what I am going to do, is drop a perpendicular, this makes. This is perpendicular at this particular point P. So, if I do that. So, now, I have this line over here.

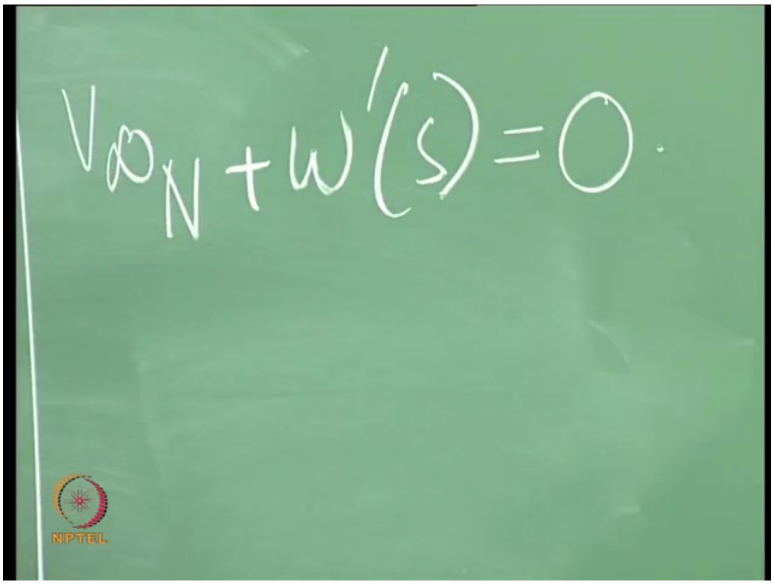
So, now this slope over here at this point, the slope, the slope of this line, if you look at this, the slope of this line. So, this is essentially. So, if you look at this. This is the slope. This is the slope, isn't it? So, at the point P, the slope is equal to d c by d x. So, if I say this is theta; that means, that this is theta as well, this is theta as well. So, then, let me consider say the point, this as A, and say this point say which is B. So, if I consider this triangle P A B. So, if I look at that; so in the say triangle from there because this is theta. I am just considering the slope out here. If I consider that and this is right angle over here. So, A B, if you look at this A B is equal to that, and A P, or rather P A to be technically correct, is d x. So therefore here having said that tan theta is equal to minus d c by d x

So the slope out here is basically minus d z by d x, actual I should you know write it this way again now if you look at this over here. Now let us include this. Now this, I am drawing this line parallel over here. So, this is my angle here. So, this is my free stream, this is my free stream, and if I look at this. So, then this is my alpha, this is my alpha and if I drop a perpendicular over here, this is my theta, this is my theta.

So, now basically what I am trying to look at here, is calculate the normal component, the normal component of the velocity in this direction. So, this is the direction that I need. So, this is the V infinity normal component in this direction. So, basically if you look at this, if you look at this particular triangle. So, let me say in this particular triangle. So, say this is, this is say D E and F; if you look there, so in the triangle D E F. What you would see, is that, is right, because this angle, angle D F E is alpha plus theta. So, all we need is the cos is this way, and the sin is this way; so V infinity sin of alpha plus theta. So, this is it.

So, this is a normal component, which is coming, because the normal component which is induced, due to the incoming flow. Now our job here, is basically in order to satisfy the zero normal boundary condition here, is that we need to find out what is the distribution in such a way, that you know we satisfy the zero normal boundary condition. I think we will start from here in next class, and finish up the problem. We will finish this in our first theory, and we will do that. So, let me at least just say for here basically, is that now that we have done over here, and we have found out that the normal component of velocity is V N. Therefore, total velocity induced at say the point P is this.

(Refer Slide Time: 49:19)



And that total normal component of velocity is 0. So, what we will do is, we will start from here, and complete these thin air foil theory and then take it from there and finish up this problem. We will stop here.

Thank you.