Advanced Gas Dynamics
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Lecture - 3
An introduction to Normal Shocks

So, let us say this is a Normal shock.

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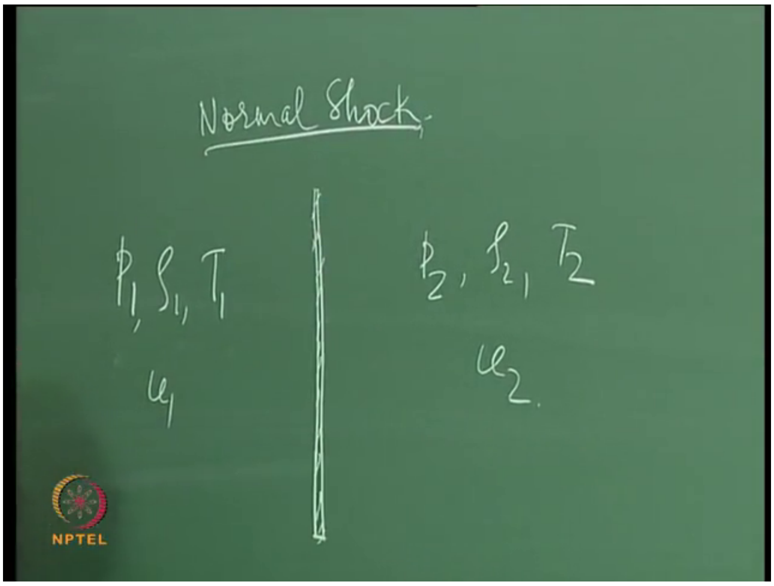

So, this is say a normal shock and I have say. So, what I am doing here is that I am taking p 1 rho 1 t 1 u 1 and so on so forth on the other side right. Now this is a what I am calling as a normal shock. First things first, what exactly is a shock anyways right? What exactly is a shock and what exactly is a normal shock; I am saying a lot of things and what do these you know subscripts denotes you know on each side of the normal shock.

So, to define or to explain or to give you some ideas to what exactly a shock is, just think of an example say a lot of you are travelling on your bikes. So, you are going on your bike and in the middle of the road you see a lot of people standing. Now you are at a speed where you are able to see the people standing in the middle of the road, so then you know ring your bell. People are able to hear you in time they move and you are able to effort them and if they are able to give you way then you go fast. And you are travelling slightly faster than that you know, so people really do not have a chance to move, but then when you ring your bell they are able to see you and you have enough time to kind of divert your way and you know you are able to avert any sort of accident and you move fast them right.

Now think of this: now you are going a such high speed you are riding your bike so fast that you ring your bell, but by the time the people are able to hear you are already there and you are at a speed where you are not able to divert them, and now what happens? You know you fall you hit one of them there is an accident right. That is why they keep constantly telling you, drive at speeds which you are able to control. Now a shock is something like that, ok.

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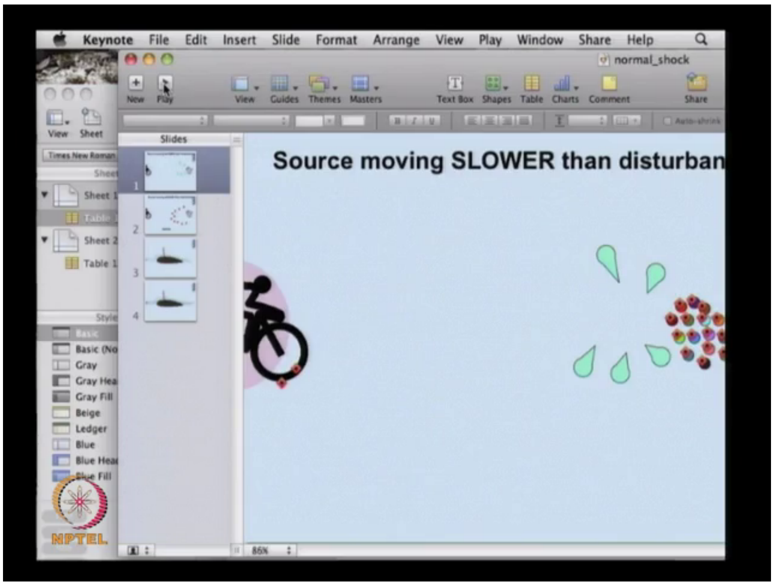

So let us see whether we can make you know some headway and understand this concept of shocks a little bit. So, let us sort of visualize this you know cycling and heating a couple of people a little more and see if that makes any sense to us.

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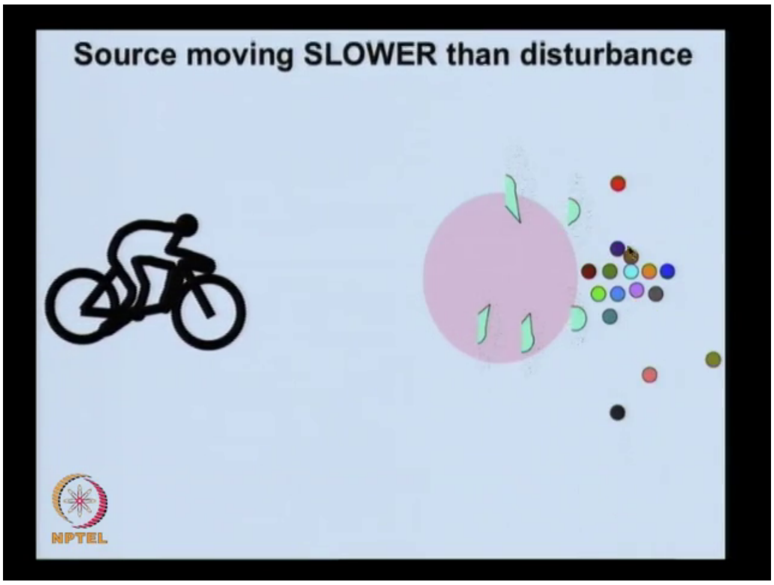

So, let us say you know these are the people standing in a group and then in a at some point somewhere. And then like you said you come into the picture, you are here on the bike let me stop you right there. And, because of you coming into the picture you are also carrying with yourself the disturbance right. So, let us say that disturbance is shown by this you know hallow think sort of thing over here. And this disturbance is obviously going to you know travel in the atmosphere right.

So, these are group of people standing at some distance from you and you come into the picture you are riding on a bike approaching them, and because of you approaching there is also you are also disturbing the air flow around yourself right and let us say that disturbance is travelling also. And the question whether we will have a shock or not is whether this disturbance will travel faster than you or slower than you. Let us see what that means.

So let us say: let us look at this case first. So, essentially the disturbance travels quicker than you and that warns these people right that there is a cyclist behind itself. So, the people get warned right they move apart, they move away giving you way and you are able to sort of you know go fast without hurting anybody, without any anything dramatic happening ok.

You just repeat that one more time. So, we have a group of people standing here. So, just you know some group of people, you come into the picture, you carrying with yourself disturbance the disturbance is travelling faster than you, it goes in four once the people the people therefore, move away realizing that they a cyclist is approaching them right and they give you way and you are able to just move fast right and that prevents any major disaster. So, essentially you can say that now the source of the disturbance which is the cyclist in this particular case is moving slower than the disturbance. Now, let us look at this scenario again.

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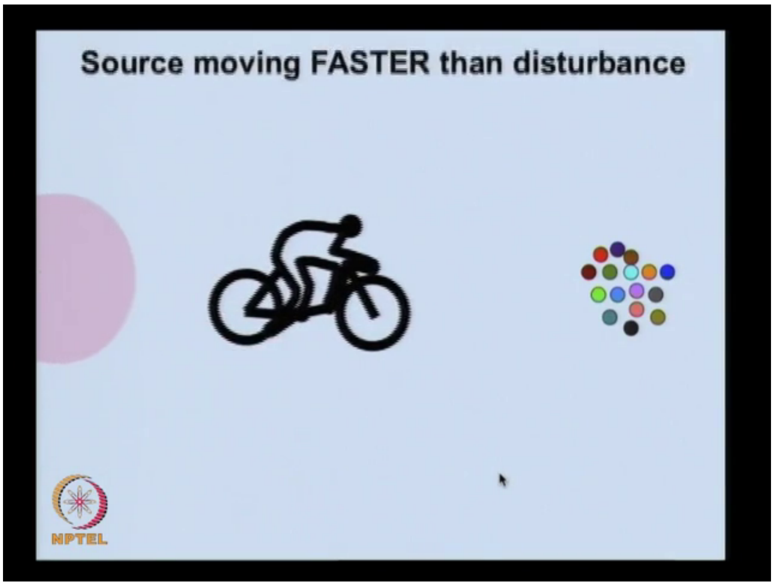

In this case what we will say is that the cyclist is moving quicker than the disturbance itself. Therefore, let us see this first. So, the cyclist who is quicker than the disturbance this is moving behind, it is goes and hits the people right.

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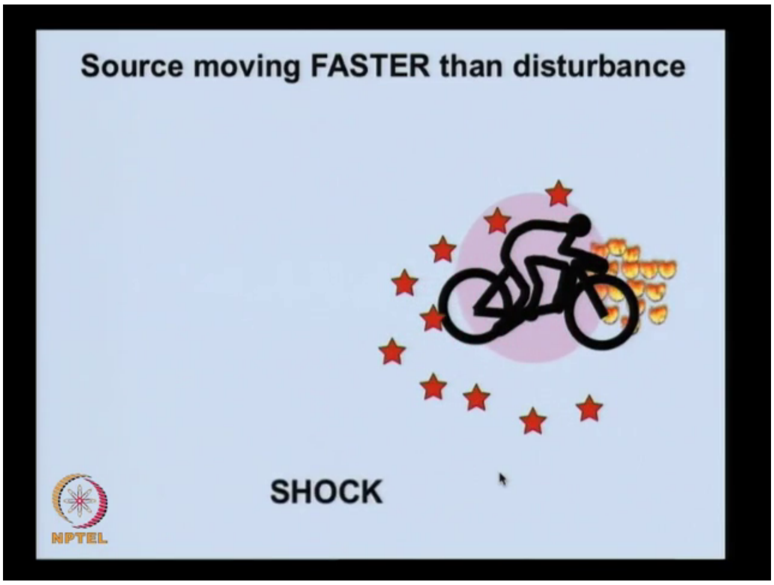

Oops there is an accident alright and well the people do not have enough time to move away and they really. So, what we get in principle out here is a shock. So, let us sort of do that again. So, we have this group of people, we have the cyclist or the source of the disturbance moving quicker than the disturbance itself. So, the people are not forewarned the cyclist is coming right hence they you know not able to move away, you are going and hitting those people and hence what we have is a shock right.

Now, when I say this here; so source moving like I said at the beginning that the disturbance here is going to travel in the air right and when it travels in the air it travels with the speed of sound. So, these two cases essentially mean that the source is either travelling faster than the speed of sound, which is it is travelling a supersonic speeds which is this case when we have a shock. And the previous case when the source was travelling slower than the disturbance is when it is the source is travelling or the cyclist is travelling in this particular case at subsonic speeds right. So, this is essentially you know very sort of a very crude way of trying to understand what a shock wave is. So, where could this actually happen of course, I mean a cyclist you know in principle is not going to be able to travel at supersonic speed.

So, what you need to visualize here is there instead of the cyclist if you had a bullet, if you had a rocket you know moving at supersonic speeds then you know it would be passing through the atmosphere at such high speeds that the area around that would not be able to just you know itself as quickly as this bullet is travelling. And then you know the changes would happen very quickly or in a very short period of time and that would mean that you would have a shock wave.

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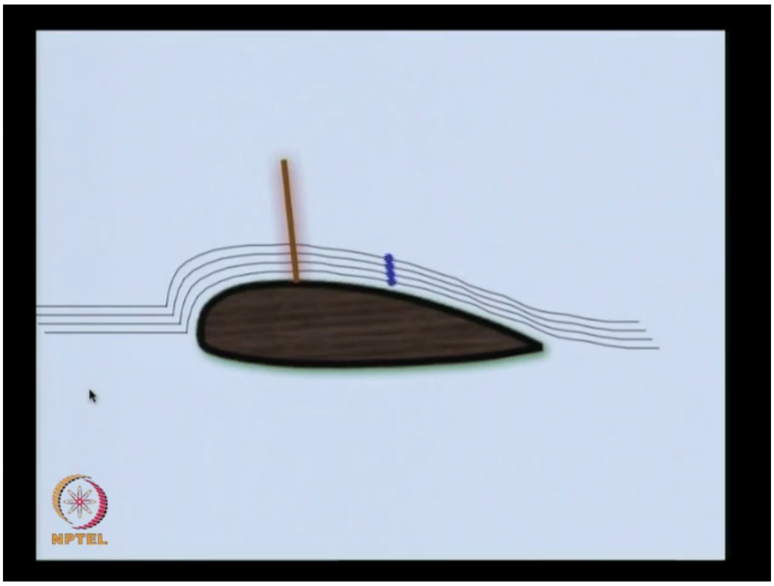

So for example in this picture here say you know you have this (Refer Time: 08:45) out here is important the important is to how the shape of this (Refer Time: 08:51) is. So, you can you can see that well you know this it is a (Refer Time: 08:59) and what I have again try to visualize here is that you know these are you know typical streamlines past the (Refer Time: 09:07). So, just taken like a four say particles of you know air pockets of air which are travelling over these streamlines, I would want you to watch first what is going on well let us let us sort of do that again. So, what I would let us play that again and then we will talk or an hoping you have noticed here.

Now what you see is that this is a free stream you, it is a free stream is come it travels it moves in the particles are moving here. Now over this portion of the airfoil, the particles actually speed up and again just this is the around this location which is the location of maximum chamber. So, here the chamber is increasing and after this point the chamber is decreasing. So, the particles come here over this increasing chamber region, the particles accelerate and just as they cross and then suddenly the at around this point the chamber changes from you know positive to negative and they the particles need to slow down again.

So, they slow down just beyond this point and then they keep moving down. Now at this in this region when the chamber is suddenly changing and the flow itself also has to adjust itself going from faster speed to slower speed that is when what you get is a shock wave. So, let us say that here the flow is you know subsonic it comes here, here in this region it is a subsonic. Now it becomes supersonic and this speeds up and again as it goes fast this region, it becomes sub sonic it slows down. Therefore, in this region right as it goes from supersonic to subsonic right and the changes really subsonic you can see. So, in this region what you have is a shock wave it typically this would be a normal shock wave and this when I say normal, this normal is normal to the surface.

So, now you could look at this picture a little differently as well.

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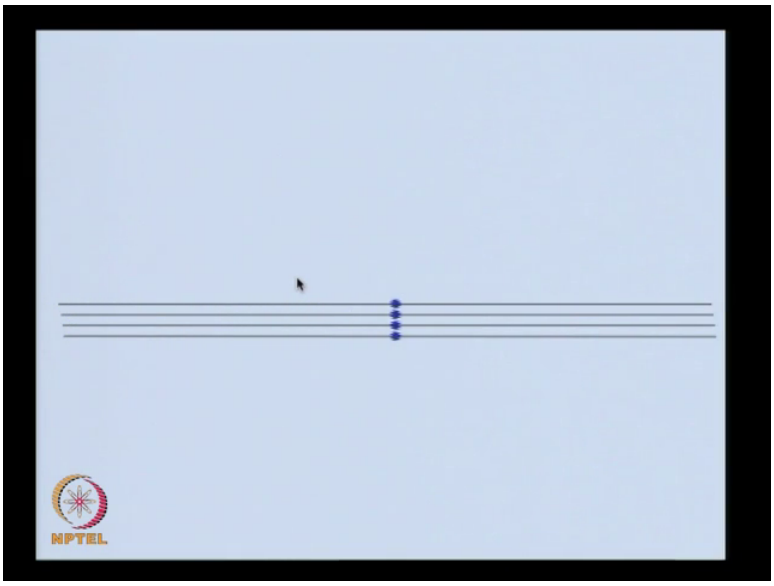

For example I mean you just have a free stream, you just have a free stream again I just you know just trying to visualize this flow particles free stream and then suddenly you have this airfoil coming in.

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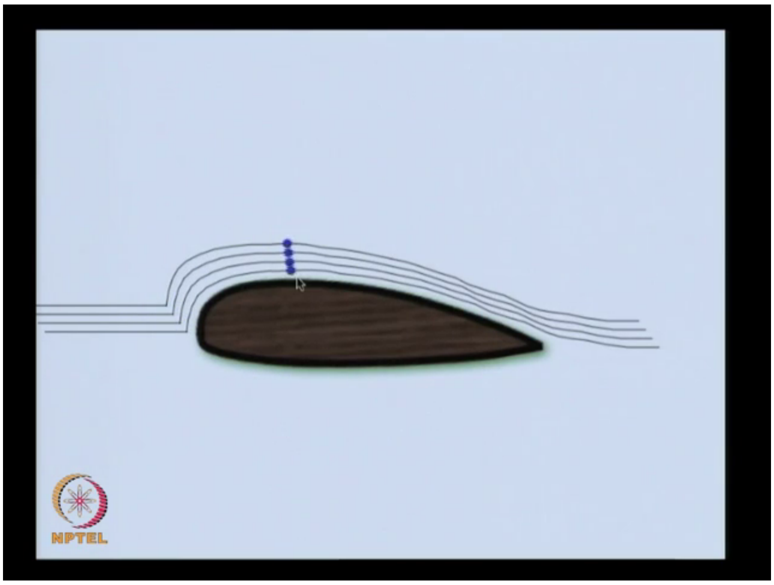

So, the flow the free stream has to adjust itself right move fast the you know the chamber airfoil and the way it does that is that it in a increases over this area, goes through a shock wave and then goes down to subsonic. So, this is essentially again we will just look at this one more time. So, this is essentially basically to give you know a brief idea or conceptualize exactly what exactly do we mean by shock wave. Of course, you will see a lot of (Refer Time: 12:38) and photographs and other process of visualization. So, this something that I thought could be interesting or you know could be to understand at least what shock waves mean or how they develop ok. So, let us start and do it with the.

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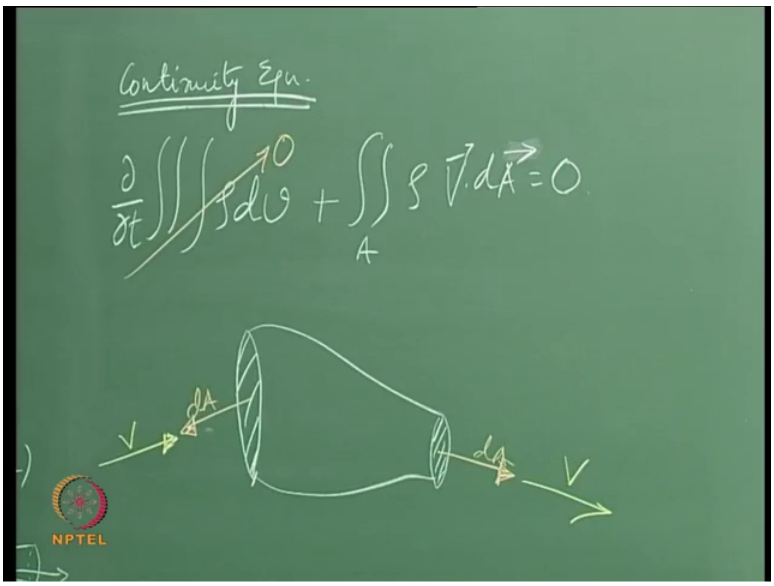

So, I am hoping that you are familiar with this continuity equation right this is the integral form of the continuity equation. So, we will do first things first right we will say this is a steady state case. So, we will say this goes to 0. So, we were going to work with just this part. So, we are left out with this, let us look at that equation.

What is this telling us right we have a density we have a velocity which is the vector right and this d A also. So, let me sort of re write this. So, I am writing this as well as d A right. Now I am writing the d A also with as a vector, I always taught you a was a scalar right well why am I writing that d A with a with a vector sign what does that mean? Let us let us sort of just you know do that a little bit now for example,. So, say this is the nozzle here right.

So, say this is the now for the entrance the area vector always points outwards. So, you have an area vector. So, that points outwards and for these points outwards. So, area vector always point out at the face or the area that always points outwards. So, this is your d A this is your d A. Therefore, in this case if I have a velocity vector which comes that way. This is my velocity and in this case this is the exit right. So, I have the velocity this way. So, this is now the velocity vector and that is what we and this is what we mean by the area vector. So, now, this equation actually is developed remember we use a control volume approach for that.

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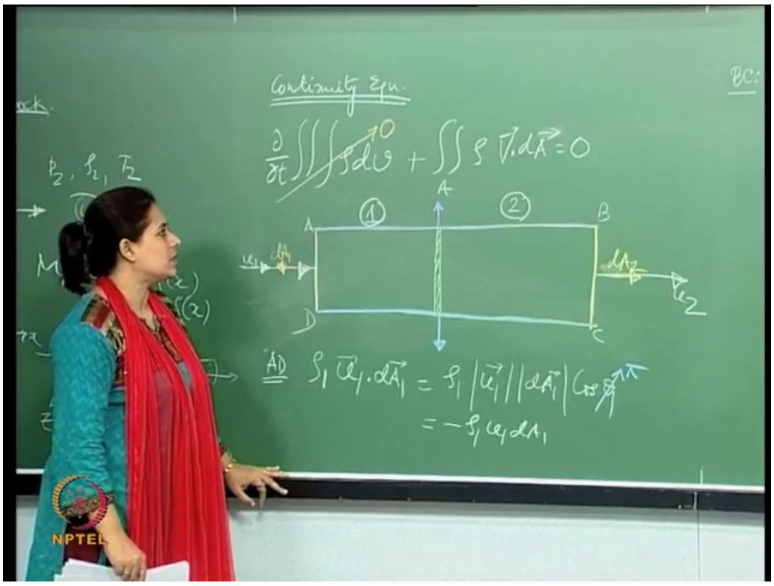

So which means now, say this is my and the question to ask is what is this mean. That we have a box and if the shock wave starts from here and ends here, it is really not like that. So, what we have basically are talking about is that this is a region which is kind of demarcating there is nothing like a line or anything like that and this is not a solid boundary anywhere. So, the way we you know do these things is that there is a certain space in which they certain space which we are demarcating where we have a very sharp change in properties right.

So, the properties as we go far away from that section are not that are not that effected or they are not changing the you know dramatically if this here that they are changing. So, I am. So, there is nothing like you know the shock waves stops right here, it is nothing like that. So, all I am saying is that this is the region where we have a sharp demarcation and the properties are really changing very sharply and there is the thing is how far or how big should be this control volume. So, therefore, this is you have to just draw a control volume which is on your judgment and where basically you would think that is you know with the property changes will be seen right. So, that is how we sort of draw upper control volume which in which my property changes will show up right. So, if I draw a too large volume it is not probably not going to show up any you know show any property changes at all. So, we will take a realistic control volume and look at the property changes in this control volume.

Now in this case, we are talking about the area here. So, what do we go ahead and do over in this case. What we are going to do here is that we take this integral right. So, we are talking about the area right. So, say in in here if you are talking about this area say. So, say we have just unity on the on this side, you just take unity. So, if we do that then we are going to apply. So, we have flow coming in this way right this is my v here. So, v this is my entrance and this is my exit. So, in that case you can see there is this dot over here which is essentially a dot product. So, if I had to show my area vector. So, in this case for the A D phase this is my area vector this is my area vector right for the B C phase this is my area vector, for the A B phase this is the area vector and D C phase this is the area vector. So, if that is the case then how do we write this integral for each of these phases?

Now, for A D case now for A D case how do we write this? So, say we write that as rho. So, we will write that as, let us call say the region before the shock as one right. So, that I can write all the properties in this region as rho 1 u 1 2 1 v 1 at u 1 etcetera and this is the region which is after the shock. So, if I do that. So, then then I write this rho 1. So, I am going to call this u 1 over here, u 1 dot say d A 1 right this is for the A D case. So, what is the angle between this is a dot product. So, this comes up to be rho 1. So, this theta is nothing, but the angle between u 1 and a 1.

So, this like I said just now. So, this is d A 1 right this direction and this is the velocity this is u 1. So, what is the angle between them that is that angle is theta. So, as you can see I can say that this angle is actually pi which means that my integral comes up to. So, this is a negative sign over here the negative sign comes from here. So, all we have is rho 1 u 1 d A 1 and the negative sign. So, that is for the entrance. So, now, what happens for the exit which is the B C part, what happens for the B c part?

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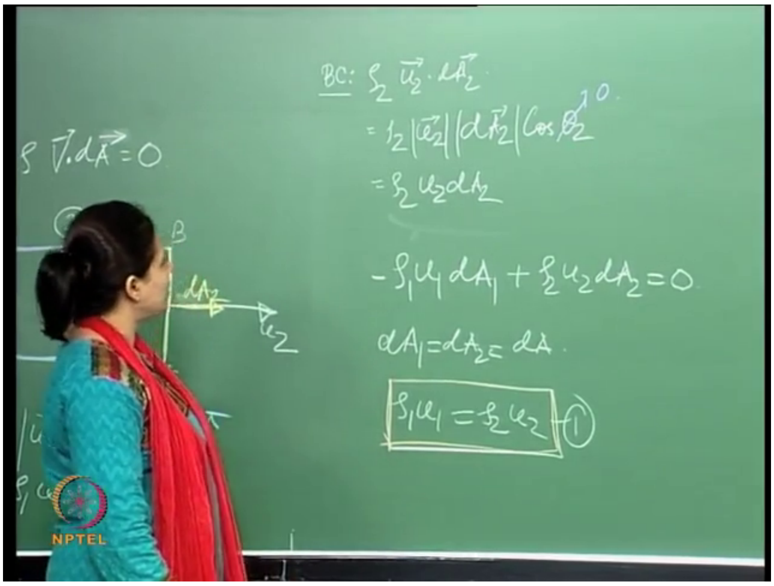

So, again rho 2 u 2 d A 2 this becomes; right. So, this is let us call this as theta 2 and call this as theta 1 actually. So, if I do that. So, in this case what is the theta if you come back to this picture and see. So, this is essentially this is my u 2 actually now, this is my u 2 and this is the velocity right. So, what is the angle between them is 0. Therefore, if this angle is 0 cos 0 is 1, so then this is what you get. So, I think that is simple to see. So, now, next the next question is what happens for A B? For clearly if you do the math you will see the I mean there is no velocity in component in this direction anyways.

So, this integral does not does not amount to anything for sides A B and D C; so all we are left out with the contribution by the entrance and the exit right. So, therefore, what we get is this right. Now for these particular cases you can see that the area is constant right. So, if may say like because this is d A. So, if I write d A 1 right. So, we can take that out. So, all we are left out with is. So, all this is the final expression that you get from the continuity equation. So, essentially what is happening is that the relationship between the density and velocity this is before and after the shock, this is how they are related and this is the relationship which we get from the continuity equation.

The continuity equation in case of the normal shock wave boils down to this. Now we will do the similar thing with the equation of conservation of men linear momentum. So, let us do that. So, what I will do here. So, let us do that conservation of momentum.

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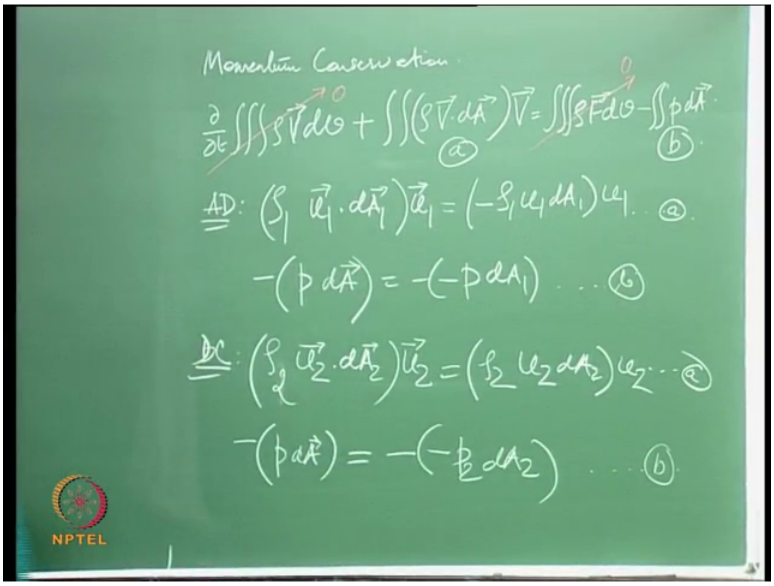

So, this is the conservation of momentum equation. So, I hope you at least you are familiarize with this equation. So, now, when we do this again we will do the same thing as we did for the conservation of mass will consider a steady state case right. So, that is why your term goes, then this term comes with the body forces right like gravity etcetera. So, we will take that out as well we will just say that goes to 0. So, what we are left out with is this expression and this expression and then again we will do the same thing like we did for the previous case and we will apply that and you can see these are the area integral. So, we will apply it over the control volume control surface rather. So, if I do that than just term A D again let us come back and do it for the entrance. So, A D is the entrance and B C is the entrance over here. So, if I do that. So, so what do we get it for A D right. So, this is what we get.

So then if I get this how should I write this. So, basically what we get is this right and. So, this is let us call this term 1. So, this is term one right this is say term 1 term a and term b. So, this is the term a. So, this is a and what happens to the term b which is the pressure right. So, we have a negative term so then p into a right. So, d A right. Now this again here let us think about it what should we have over here now pressure. Now let us come back to the inlet over here inlet and outlet now by definition pressure will always act on the surface right. So, the pressure is always in this direction right. So, the pressure is always this way. So, p 1 will always act the way. So, in B C phase as well pressure will act in this way right. So, that is always the direction of pressure.

So, based on that we will calculate the say pressure vector in terms of the area vector right; so here again see if you agree with this, because for the inlet the pressure is the pressure where the pressure is pointing in the direction which is opposite to that of the area vector. So, this is what we get for the second term right now for the exit which is the d c actually. So, this is basically for the exit. So, for the exit again what we have is rho 1 sorry rho 2 rho 2 right and u 2 now this comes up to be again.

So, in here the angle between u 2 and d A is again 0 that makes it rho 2 u 2 d A 2 into u 2. So, that is the first term right and what happens to the b terms second the second term rather. So, the again it is p d A right. Now in this case see with the direction of the pressure is always opposite to that of the area vector. So, again we get the same thing, did I say that right let us just. So, the total equation therefore, boils down to. So, what we get is this. And, so let us write that.

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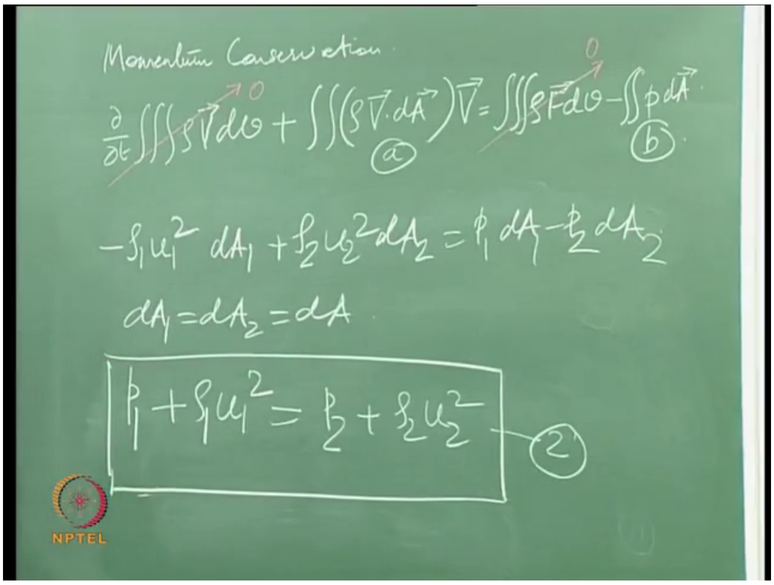

So, d A 1, this is what we get d A 1, d A 2 and as for this particular case right. So, the area into the areas all cancels out. So, we can take that out. So, what we essentially get is this right. So, what we get is if we rearrange a little bit; so p 1, so this is what we get. So, therefore, now from the momentum equation what we are able to get is the relationship between pressure density and the velocities right. So, this is what the momentum equation boils down to ok.

So, we have got a little more work to do after this. So, we need to work with the energy equation right. So, let us come up with that. So, let us see.

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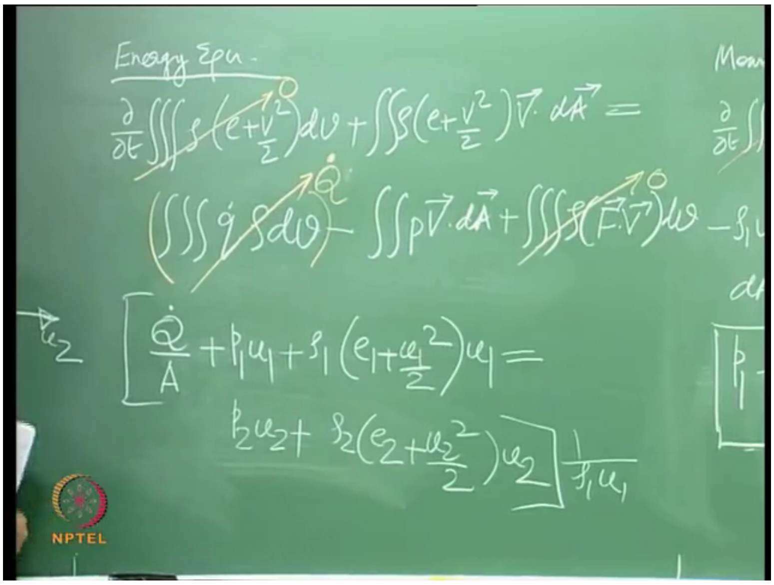

So, now, the and now, we finally, come up to the energy equation right now the energy equation is this now again, I am going to write this in full form and then. So, as we go further the equation seems to be the equation seems to get longer right. So, I think the best thing when you look at equations is that whether scary and then that they do not make any sense is not to get scared actually, because as you see here we will kind of get rid of a few terms right based on assumptions. So, that will take care of the length part.

So, it will become smaller and the next thing is once you understand what is going on here once we understand where it came from I think it becomes a lot easier to understand equations because equations are not just math they actually represent the physics. So, once we at least can understand what exactly each term means its likely better. So, at this point if you have kind of forgotten what each of these terms; mean, I think it is a high chance that you just go and refresh your memories that is all just go over it read a very basic book and find out what exactly each term means and I think to do the derivation once gives you that idea.

After that, if you forget how to write this equation which is I forget it all the time which is fine. So, here 2 again, this is the unsteady part. So, we get rid of that then this here this f is the body forces. So, we will get rid of that. So, now, therefore, we are left with three terms right we are left with three terms. So, this is the; so that is the. So, e here is the internal energy as you can see. So, we will now denote this q dot is the mass flux this is the heat flux per unit mass right. So, we will write this for the total, we will write this we can write this as a total for the total given mass and we will write this as q I am sorry. So, this is what the entire equation boils down to. So, we will write this in terms of total heat flux.

So, once we do that essentially again what we are left out with is to calculate these two integrals at the entrance and at the exit right that is what it comes down to. So, say right that is equal to. So, this equation now if I write each of these terms for the entrance and the exit and then come and rearrange a little bit, but this is what we will get I think it is advisable to do the same and you can go ahead and do it yourself. So, just write out these expressions for the entrance and exit, just do it once and you should be able to get this and this area I write as a you can write that d A if u still want.

So, this is the expression now. So, another thing you can do here is that if I write this if I divide now this equation by rho 1 and u 1. So, if I divide this entire equation by rho 1 and u one. So, what will happen here is this will become p 1 by rho and this becomes rho 1 u 1 and so on and so forth and then the reason why I do that; slowly you will see that we play around with these terms, so that in all these terms start appearing in our equations. So, that is basically the reason why we do that. So, what I am doing is I am diving this entire equation.

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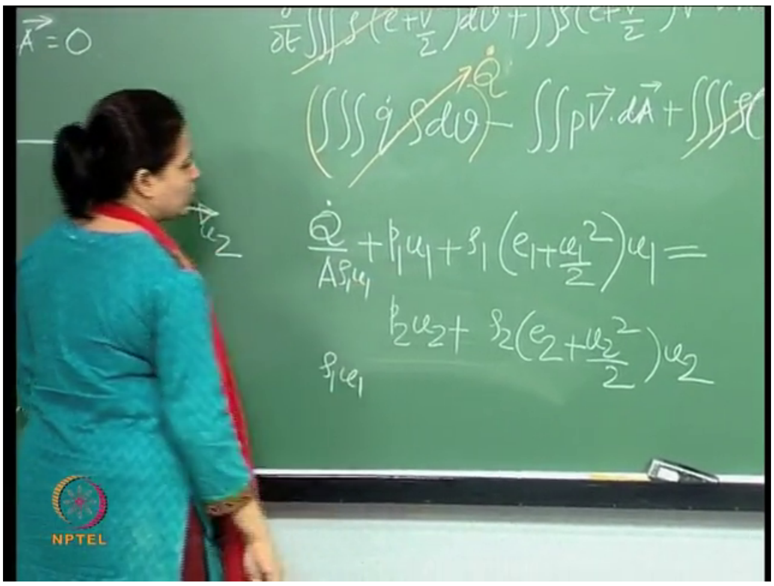

By rho 1 u 1 if I do that what do I get? So, if I do that what do I get?

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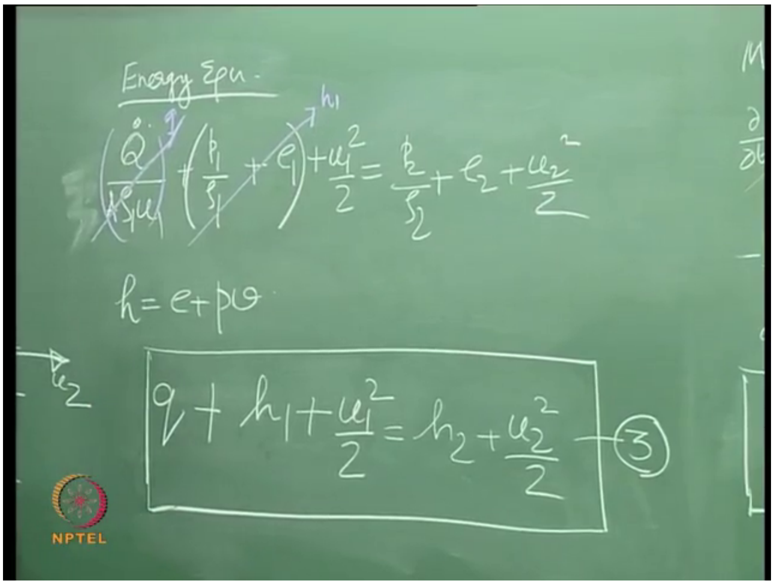

So, what I get is this I get that. So, this is what we get; do you think we can introduce another term over here? When you see these equations do you remember do you see that I can probably introduce another term over here what is the term for enthalpy right. This is the enthalpy term right. So, how do we introduce that over here, do you think we can do that p into v in any case what about this term here, is it possible to get you know p v over there do you see if we can do that right. Now this this is a specific volume right this is the internal energy per unit mass right now. So, therefore, here one by rho is nothing, but v 1 actually 1 by rho here is v 1.

So, therefore, I can actually write this term as the enthalpy term. So, this is nothing, but h 1. So, if that is essentially h 1 you see it you see how I get that right. So, if I do that then then in that case and we will let us call this as say a q. So, if I call that as q, let us let us write it this way. So, this part we will call as q. So, if I do that then this equation boils down to and that is what I get from my equation of conservation of energy right. So, therefore, I have introduced the enthalpy term here right and I have the velocity and the velocity. Therefore, now what you can see from my three equations. So, let me write out the three equations. So, that you have a feel for how the governing equations have boiled down to for the, for a normal shock wave. So, this if I go back here. So, this is the equation of the second one is the equation of momentum.

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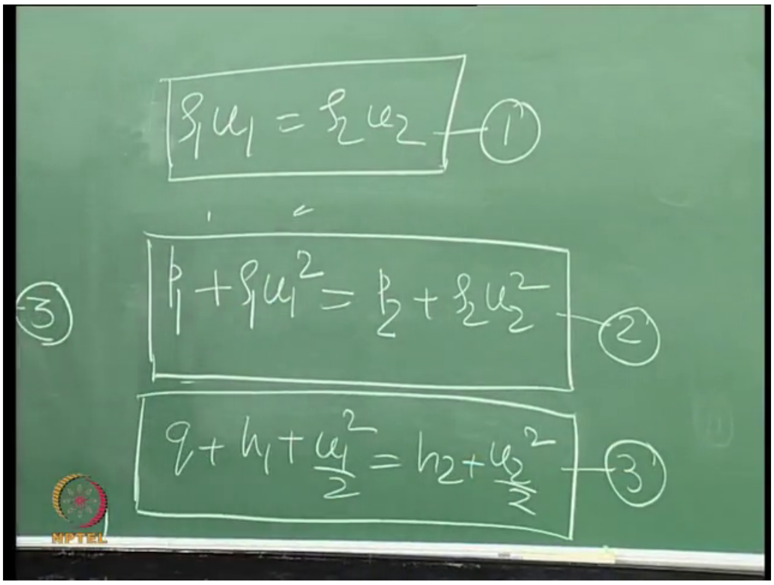

So, this is the equation of continuity this is the continuity equation this is the conservation of momentum and the other equation that we just arrived is this. So, the essentially using the three governing equations what we were able to do is find out the relationship between density velocities pressures enthalpy etcetera. And we are able to get to these three relationships.

So, now using let us using these equations I think in the next couple of lectures what we will do is try to study the we will do some problems and try to study the parameter of changes, that say a given property say the density and the pressure or the pressure and temperature and the velocity in this section is known. Then if it goes through a shock wave then what should be the changes in pressures and temperatures etcetera. Now that question is something which is not a linear question it is not like you know I can just plainly say that if I make a change in the pressure in terms of say 10 percent pressure change, I can just say oh. So, therefore, it will be 10 percent temperature change no. The relationship is given by these governing equations right the governing equations basically defined how the flow should be changing across a shock wave.

So, these basics of the fluid flows; the fluid flow when it is changing its properties across the shock wave we follow this rule let us put it that way. Therefore, we will have these equations to work with which we will now before. So, in the next class what before we do you know couple of problems using this, what we will do is also find out the like I said I think in the previous class is that what is the relationship of the speed of sound. You know we have talked about the Mach number here as well you know going from subsonic to sonic etcetera. So, what is the relationship of Mach number and why are we so concerned about the speed of sound.

So, to speak and what is the relationship and that does this speed of sound change like along with the pressure temperature density etcetera. So, we will find out find that out you know about the relationship of the speed of sound in Mach number, and then we will use these governing equations the way we used this for a normal shock. And we will solve a couple of problems before we go further that should be all.

Thanks.