Advanced Gas Dynamics

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Lecture - 29
Similarity Rules and Transformed Coordinate System

So, we have been doing linearized flow right and the primary reason for doing linearized flow is what, we have exact equations which is very large, and then we said no we are going to linearize this use small perturbation right and then we are going to use linearized equations. So, essentially what do you think is the reason for that.

The main reason to do that is for simplicity right, for simplicity, it is just too difficult to solve those equations and even if we do how do we validate them right that is the mid concern. So, if we can do with something that you know at least we are sure off we kind of better off. So, let us see something a little more interesting in terms of linearized flow, we this is well you could say those classes about similarity rules.

So, we will see what is I am talking about. We will do a transformation in space and space meaning you know in domain rather x y to another to transform it to another domain of space and then see that what linearization actually does simplify or does it have any more physical meaning or what happens. So, a linearization of course, when we say linearize we basically saying we are you know bringing in some simplicity in the way we solve our numerical equations governing equation solutions to them. So, and the physical analogy to that is that we basically small if you assume small perturbation then that results in linearizing you know the equations the numerical part becomes simplified.

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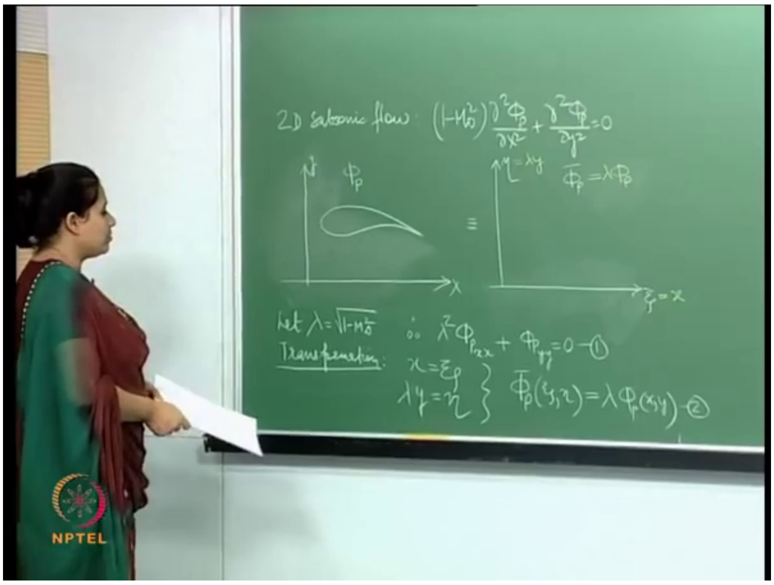

So, like we have done earlier right. So, we had for 2D a subsonic flow right. So, if you remember right, this was our governing equation. So, this now this phi p is the perturbation potential and you can see that this is in our usual x y or Cartesian coordinate system. Now, so if we have this in our. So, let us do this here. So, essentially what I am saying is that. So, say we have right say we have an air foil like that in this x y you know coordinate system then the governing equation for a 2D subsonic flow will be so, so phi p and perturbation potential and if I know the perturbation potential I should be able to find out the velocities and so on and so forth enhance the properties.

Now let us use say what if instead of this x y in a coordinate system what if I change the coordinate system a little bit, now let us I just assume this. So, let us say we will you know change this coordinate system to something like this. So, it is a orthogonal system, so similarly, I am just if I change the coordinate system to say something like the two questions now what exactly is the transformation what is the connection between the relation between x and z, and what is the connection between y and eta right, and what happens to this shape how does this air foil look in this coordinate system how does this look in this coordinate system and another question is what happens to this equation, this is the governing equation in the x y coordinate system and how does that change over in this coordinate system right. There are some other questions will try to answer you know.

So, having said that let us use, now let here whether just denote we are going to just denote you know the lambda square to be equal to this. So, therefore, our governing equation basically becomes this is I am just writing the governing equation in the x y coordinate system in this form right. Now if I do that now, therefore, like I said the relation between x and z and y and eta. So, let us use this transformation. So, let us use this transformation where we say that x is the same as z path.

So, essentially what you are saying is that z is the same as x, but eta is equal to lambda in to y where lambda is this. So, this is a transformation that we are trying to use here. So, if I do that right if the y axis is somewhere connected to the free stream Mach number in this fashion and z z axis remains as it is then 2 questions one is what happens to the air foil shape in the x y access how does this transform in to the space number 1 and number 2 is how what happens to the governing equation how does this look in this transformed space.

Let us see. So, since we have this here right. So, therefore, what we will say now we writing the governing equation in terms of a perturbation potential. So, therefore, let us call say this equation as 1, right. If I do that, now using this transformation of course, if I use this transformation then I am going to denote this as a bar right it is called that as 2. So, basically what we have here is that this is our the perturbation potential and the way I denote that here is pi bar and what I note is that this is equal to write or let us say I will write it this way. So, this is basically the transformation. So, now, let us do some Mach with this.

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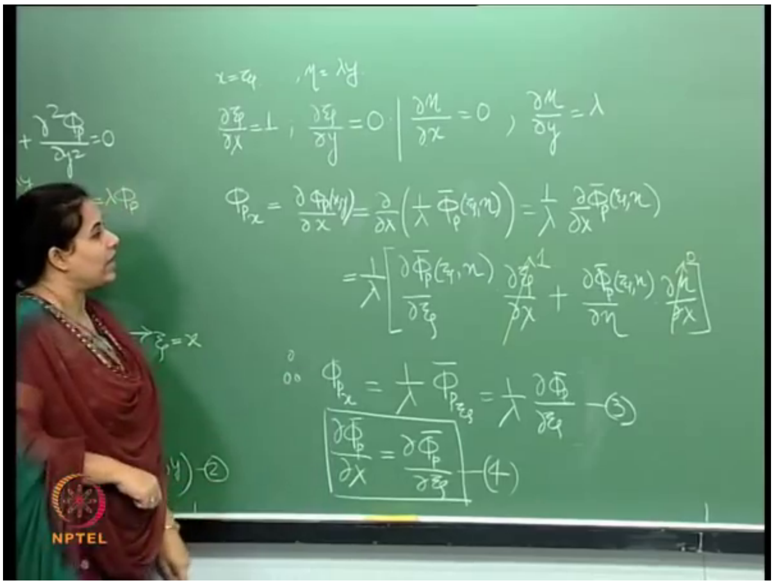

So, now, if you look at this, we said that x is this and eta is equal to lambda y this is our transformation. So, if that is true then what we get is, what is this del z del x is 1 right if you look at this if you do the math from here then is of course, 0 del z del y is of course, 0. So, what I note here is the relationship or change of z if I change x or y. Similarly let us do the same for eta for eta, del eta del x is 0 and del eta del y is equal to lambda right. So, this is basically the change in the coordinates transform coordinates with respect to the Cartesian coordinates or the original coordinate system that we are using right. If I do that, then now phi p x, now phi p x, or this is now this is what this is basically right. So, then this I can write if I write this in terms of the potential perturbation potential in the transformed coordinates right. So, this is this, is not it, this is what this is all about.

Now, if I were to write this more clearly you can see that this is this now this is in the phi p is essentially in the x y coordinates and this is in the zeta eta coordinates and I need to take the del del x of this. So, basically what you are saying is this we can write as del del x of phi p see to this. So, the way we will write this is like this. So, we are basically going to take a change of this perturbation potential in the transform coordinates with respect to x. So, that would mean that we need to also check we need to take that change with respect to the change in coordinate z and eta with respect to change in x right. So, therefore, what I can write here is. So, I take this out here. So, this would mean del zeta in to del theta del x right plus the change in del eta right, makes sense.

So, what we are writing here is with writing phi p x, which is essentially del phi p x with respect in the x coordinate. So, then a I write that in terms of the perturbation potential in the transformed coordinates here, which is essentially this now. So, I say transformed coordinates with respect to change in x right, which means that if you change x we need to change take the change in phi p where with the change in z as well as the eta coordinates. Now as we have just seen now if you look at it del z del x del z del x this is one right this is one and del eta del x del eta del x is 0, so then this is 0 right. So, what we essentially get over here. So, therefore, you can say phi p x is equal to 1 by lambda right. So, you can write it like this phi p zeta or this is nothing, but del z is not it. So, therefore, this as we call this as equation 3. So, phi p x is nothing but phi p z by lambda this is the transformation.

So, del phi p this just this term out here, just this term del phi p bar del x and that is what we expanded in this bracket here turns out to be del phi p bar del z there is no really no difference. So, therefore, let us write that down as well. So, we can write that down that del phi p bar del x is equal to del phi p bar del z and let us call that as 4. So, we basically get 2 information here. So, what we get is a connection between the del phi p del x and del phi p bar del z and also del phi p bar del x is also equal to del phi p bar del z.

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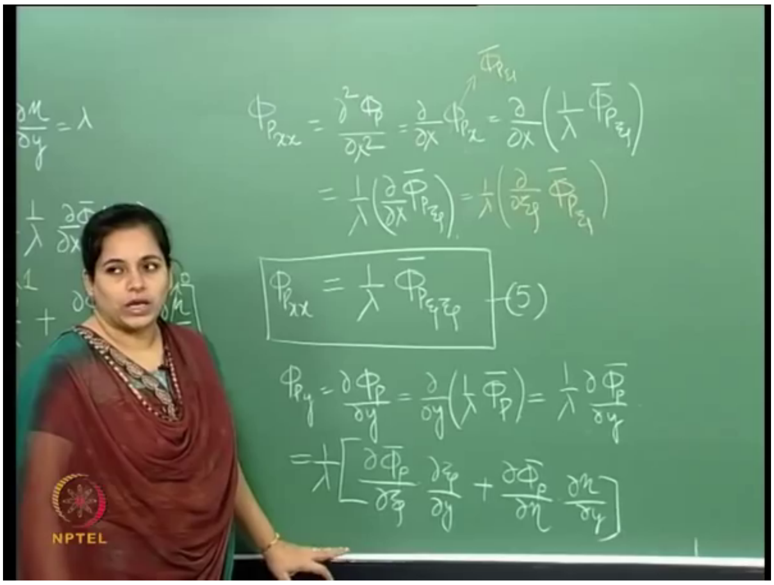

So, essentially having done that, so let us using this, phi p x x which is nothing, but right. So, if this is it. So, what this is nothing but del del x of phi p x right and as we have seen from here if I were to write this in you know the phi p bar then the way I would write this is. So, this is what we get here right. So, then, this what we get here is this and we get del del x of phi p z phi p z now as we have seen over here you can see this phi p del del del x of phi p is equal to del del phi p bar of del z. So, then again we can also write this as. So, therefore, we can write this as one by phi right. So, and let us call this as say five hopefully you got this one right. So, what we are doing here is taking del 2 del x 2 right. So, we write this, phi p x I write in terms of the z here and what we have seen here that is this is phi p x is equal to phi p bar z. So, this phi p x this phi p x you can also write as phi p bar z.

So, therefore, if you do that, essentially what we have here now is that del del x of this which is again the same as del del z of, so therefore, this actually you can write this as one more the bracket this also you can write at del del z of phi p z. So, you can write that. So, therefore, what we get is this relationship for the second order derivative. So, we have this here. So, having done that obviously, now we will do the same thing for a phi p y and you know phi p y y. So, without sort of, let us go ahead and do that. So, now, if I do that right, phi p y which is nothing, but right, del y, this I can write as del phi p. So, this is del del y of 1 by lambda bar phi p is not it, bar phi p. So, again if I expand this, if I expand this also essentially what I am getting here is one by lambda del phi p bar del y right now let me go ahead and expand this. So, what we get here is that this is what we get now del phi p del z del z del y then plus del eta del eta del y. So, basically we are tracking the change of the perturbation potential in a transform coordinates with respect to the change of both coordinates with respect to y and again as we have seen before del z del eta.

So, if you look here del sorry del z del y which is 0. So, del z del y this is 0 del eta del y if u del at a del y is equal to lambda right this is equal to lambda. So, what we get here is that what we get here, phi p y is equal to you know this lambda cancels out, so what we get here is phi p bar eta, we can call this as 6.

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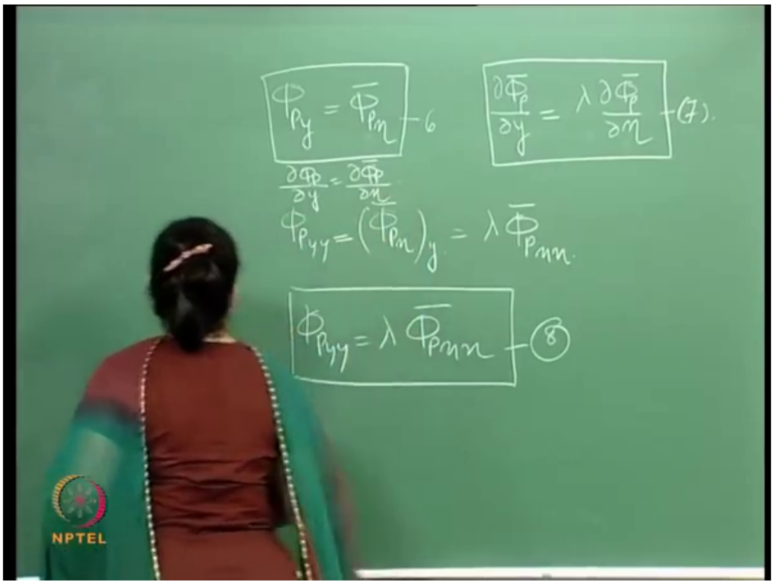

This and also if you look at this here del phi p del y this is just determine the bracket. So, also what we see is that del phi p bar del y. So, this del phi p bar del y is equal to lambda in to del phi p bar is equal to lambda del phi p bar del eta and let us call this as 7. So, this is essentially the different the relationship between the perturbation potential in the x y coordinate and in the transform coordinates, and this is the transform perturbation potential transfer coordinates with and the differentiation with respect to the y axis and the eta axis

So, having had done that, similarly what we need to get now phi p y y. So, phi p y y is what, phi p y y, so I can write this that phi p y is equal to phi p eta right. So, this is. So, basically this is. So, I can write this is phi p eta y, so this is nothing, but ok, this, so phi p eta y. So, this is again equal to right if you look at this over here. So, this term is nothing, but phi p eta this phi p eta and, so I introduce this here. So, therefore, what we get is, having got all these terms right. So, the reason we are doing this hopefully you can see because what we are looking to do here is how does this transform in to the transform coordinates or equation. So, we basically need to get a term equivalent to del 2 phi p del x 2.

So, we need to get a term here which is del 2 phi p bar del z 2 del 2 phi p bar del eta 2 right. So, that is why we went about doing all these things. So, let us see how does the our equation transform, how does our equation transform. So, now, let us see, so we have say let us say erase this. So, this is our equation.

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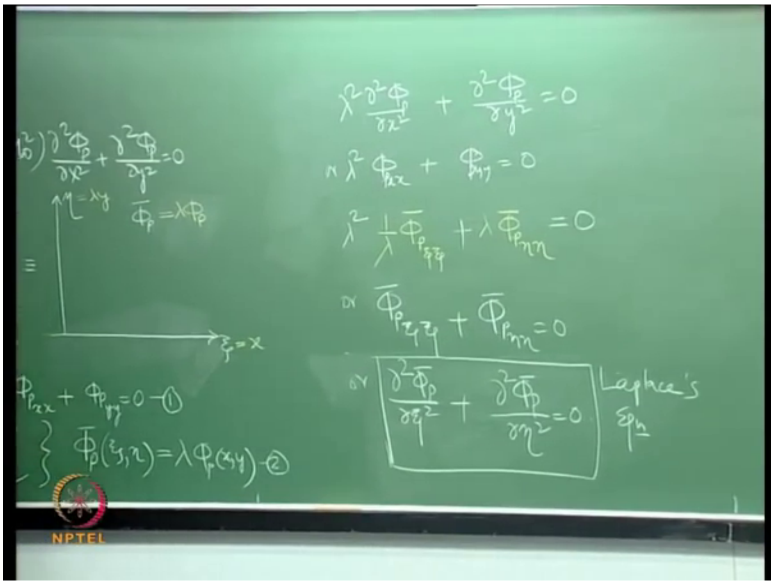

So, now here, what we can here. So, let us we can also write this as lambda square as we have assumed. So, then if I write this how does del 2 phi del x 2 transform in the transform coordinates. So, if you look at this, this is your del 2 phi del x 2. So, this is this all or we can write this as around the square phi x x plus phi y y 0. So, if you look at this, this is the perturbation potential of course, now this phi p x x is nothing, but one by lambda phi p z phi p bar z z right. So, therefore, this I can write, this I can write as 1 by lambda phi p bar C C right, this and what happens to the next to the next one phi p y y. So, let us look here. So, phi p y y is lambda in to phi p eta eta. So, again this is equal to lambda in to phi p bar eta eta right. So, this is our x y coordinate system and this is our transformed coordinate system right. So, when we have this. So, now, if I write this out now let us see what we get. So, if I do so, basically this becomes lambda, so the lambda cancels out. So, what we have from here do you recognize this let me write this out another form.

Now, do you recognize this? Well if you are saying Laplace equation will you are right. So, this is therefore, our transformed equation this is therefore, our transformed equation and this is nothing but the Laplace's equation right. So, therefore, let us say, this is our equation here, and the governing equation here is lambda square phi p x x plus phi p y y is 0 right.

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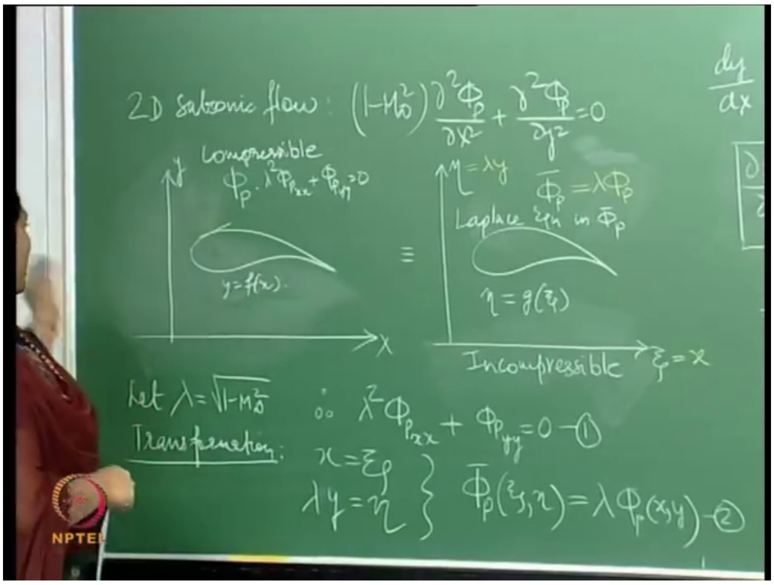

And what we get over here is that here, so in here our governing equation basically becomes phi p bar or let us just say Laplace equation in phi p bar, Laplace equation in phi p bar. Now that is very very interesting that is very very interesting why because this is a 2D subsonic flow, 2D subsonic flow when you see the Laplace equation what happens to the Mach number is not there right anymore. So, Laplace equation is basically the governing equation for 2D, in this particular case 2D incompressible flow. So, therefore, if I transform my space in the way that I have done here my 2D subsonic flow governing equation actually gets transformed numerically to the Laplace equation which is the 2D governing equation for incompressible flow my life gets easier right, could have it better.

So, having you know done that having got that out of the way. So, the next question to answer is, but what happens to my air foil now does that does that become a square or a cylinder or what does what happens to that, you know what happens what kind of shape is it because the equation has transformed itself to the Laplace equation which is great. So, this phi p, so this is essentially this is compressible flow here. So, this is a compressible flow right. So, this space is however, it is right here. So, when I transform it here it becomes incompressible all right now what we need to look at is how does this shape change. So, let us look at that. So, in order to look at that let us go ahead and look at a few things. So, if you remember what we did in terms of the boundary conditions right.

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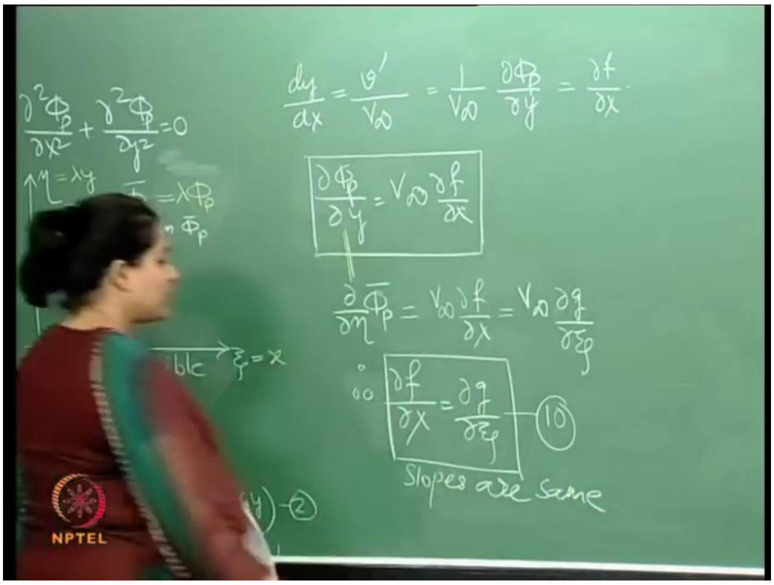

So, now dy this is in the x y coordinate system that Cartesian coordinate system. So, this is the physical slope of the body right this is the physical slope of the body here right this is dy by dx here and this is this was equal to right. If you remember, these were the perturbation velocity in the y component and this reserves assuming small perturbation, which is what we are doing here linearizing the flow. So, what we have is this at we had this. So, in this case how does now this transform here, how does this transform here. So, therefore, in this case yeah and this also one could write of course, this one could write as if I write this in terms of the perturbation potential then of course, I can write it this way all right. Now if the like it was for the problem that we had that the contour was given to us let this function let that function be this, let that function be f x which gives the contour of the body here. So, I could write this I could write this therefore, as equal to.

So, dy by dx would be the df d x. So, therefore, I can write let us do that, so del phi p del y is let us give the function to be say y is equal to f x let us give the function to be here the when I draw the body out here let the function be like this y is equal to f x and here. So, let that be eta is equal to. So, we do not know how the body will look like here we still do not know. So, let that let us write that as eta is equal to this one. So, this is basically the definition. So, from there, when I write dy by dx is essentially df by dx this is what we get in the Cartesian coordinate system. So, now, we will do the similar stuff in the transform coordinates.

So, in order to do that, essentially what you are looking at is right. So, this is the. So, the corresponding, del phi p bar del eta right, del phi p bar del eta is equal to, if you look at this particular which is just now, del or this is basically del phi p del y is equal to del phi p bar del eta. So, therefore, this I can actually you know this is actually equal to from our transformation del phi p del y is actually equal to del phi p bar del eta.

So, therefore, therefore, in this particular case is this is also equal to del this thing, but again eta is equal to g. So, del phi p del eta, which is basically d eta d is e. So, this also again is equal to right again this is what you get. So, therefore, from here what we get his del del f del x is equal to del g del zeta. So, what does that mean that this del f del x del f del x which is the slope the slope in the x y coordinate is also the same as the slope in the zeta eta coordinates in the transform coordinates. So, what we are basically saying here the slopes are

The slopes are same slopes are same which means what the slope of the body in the x y coordinate system is exactly the same as that in the transformed coordinates what does this tell us that the shape of the body does not change which means that here too I have exactly the same shape as we had in the x y coordinate system is not that nice. So, what we have done here is we have taken the coordinate system this is the original coordinate system on which we have a body which looks like this an air foil and it is a compressible flow as a result of which has governing equation is this right.

So, what we do is we take the same body which looks exactly the same, but transform these coordinates in to eta and zeta in such a way that zeta is x and eta is lambda y and the perturbation potential in the x y coordinate is phi p here it is phi p bar this is the transformed this is a relationship with the phi p of the x y coordinate system and what we see is that we get the same body, we get the same shape, but the equation transform the governing equation transforms to the Laplace equation in phi p bar which is nothing, but the governing equation for incompressible flow.

So, this like really makes our life a lot more easier then you have done even with linearization whichever we had. So, if you do that now let us also you know just take a second and you know write out the expression for the coefficient of pressure in that particular in this, in this case how will C p change from here to here.

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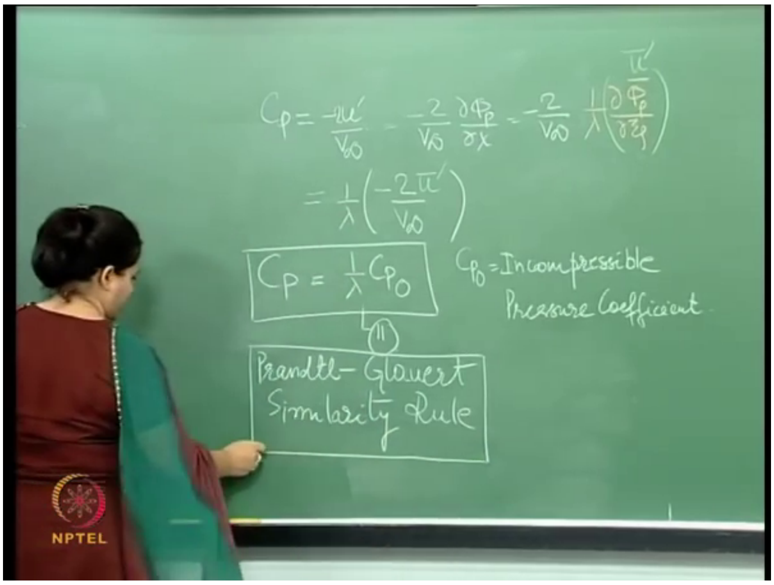

Now, C p if you remember right, C p was this. So, if C p is this what do we get this is nothing, but del phi p del x right this is in the x y coordinate system and what is the connection of del phi p del x with how does that transform to the zeta eta coordinate system. So, what happen? See this is equal to right. Now, when if you transform this what you get is right. So, this, then I am going to write this as. So, if I am going to write this. So, let me write that as, if you look at this if you look at this, so let us del phi p del z. So, this term I can write as basically the u component of the perturbation potential or this is basically the yeah the x v z component of the perturbation potential this thing. So, this is essentially nothing I can write this as bar. So, this is the u velocity perturbed velocity in the z direction in the transformed coordinates right.

So, then I can write this as u bar and let us just call that as dash just to denote that as the perturbation potential perturbation velocity right. So, this is what we have. So, what we can see here the C p which is minus 2 u dash by phi p it has the same form also out here it has the same form here in the transform coordinates as well where this is nothing, but the perturbation velocity component in the z direction which is the analogous of the x direction right. So, therefore, what and now of that we have found out now that we have been able to establish that this z eta coordinate system basically it is an incompressible flow.

So, then I will write this in a special notation I am going to call this as basically. So, the C p naught is nothing is basically the incompressible pressure coefficient right. So, therefore, for any you know in this in the case that we are considering for any 2D subsonic flow right. So, this is a compressible pressure coefficient is basically given as 1 by lambda. So, now, lambda is basically the transformation used in to the incompressible pressure coefficient. So, C p naught is basically incompressible pressure coefficient right and this let us call this as and, so what we basically see that we have the same profile we have the same body is not it, we have the same body and what we have basically done now is that we have related the C p in this compressible flow field to the to the C p in an incompressible flow field over the same profile, meaning say we have this particular body in one case we consider a compressible flow over it. So, we get a coefficient of pressure which is C p.

Now consider another case where we have the same flow field we have the same body and we have an incompressible flow over it and the corresponding pressure coefficient is C p naught right and what we have just now established is this relationship which is a relationship between the coefficient of pressure in the compressible flow field to the coefficient of pressure in the incompressible flow field over the same body over the same profile and this is basically credited to Prandtl and Glauert and this is, so Prandtl and Glauert. So, essentially Prandtl and Glauert similarity rule. So, this is the. So, basically this connects the C p in the compressible and incompressible flow fields over the same body.

So, briefly before we sort of you know closed for today let us also I will just write down coefficient of lift. So, taking from you know take a cue from this similarity rule, taking a cue from this similarity rule.

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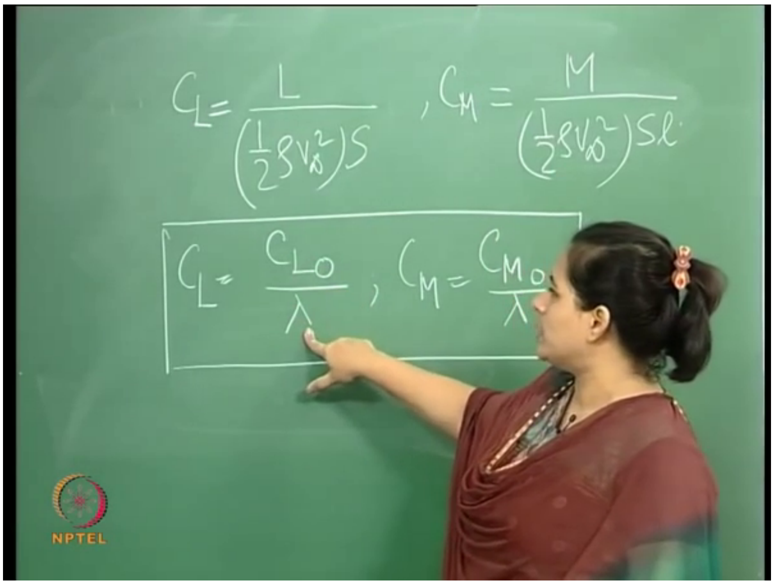

So, coefficient of lift is lift by dynamic pressure in to an area and similarly C M is nothing, but this again in to yeah because I have M there, in this case also right. So, C L is if I were to write this what is our lambda yeah, so lambda. And, basically what we do here this these are the two incompressible C L and C M right and this is the Prandtl Glauert correction toward you can add this correction toward and use it as the compressible C L and C M over the same profile.

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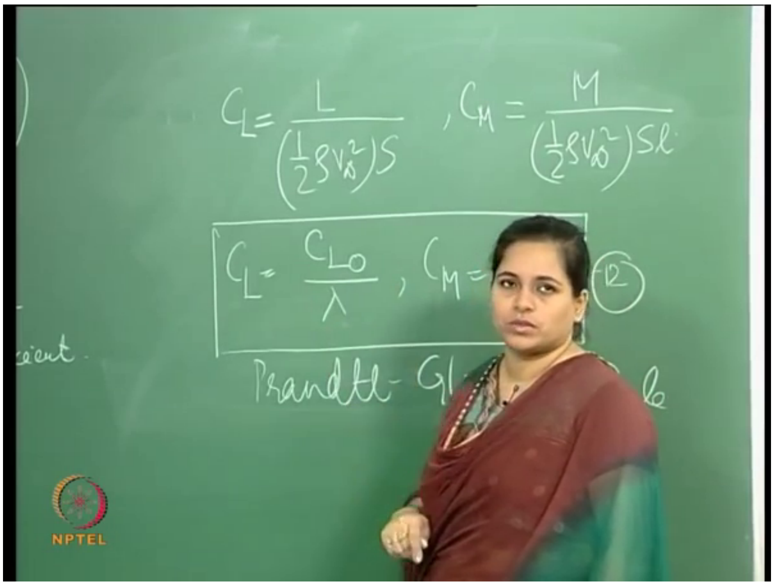

So, this you can say this is the yeah you know rules, so that manner. So, basically they we can use the incompressible C L and C M add a correction to it and use it as the compressible C L and C m. So, I think will stop here and continue with this next class.

Thank you.