Advanced Gas Dynamics

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Lecture - 27
Quasi 2D Flow - I

So, let us continue from last lecture, shall we? So, let us begin with the Taylor-Maccoll equation which we derived in the last lecture. And then we will go head and see how we can solve for it. We will do a problem.

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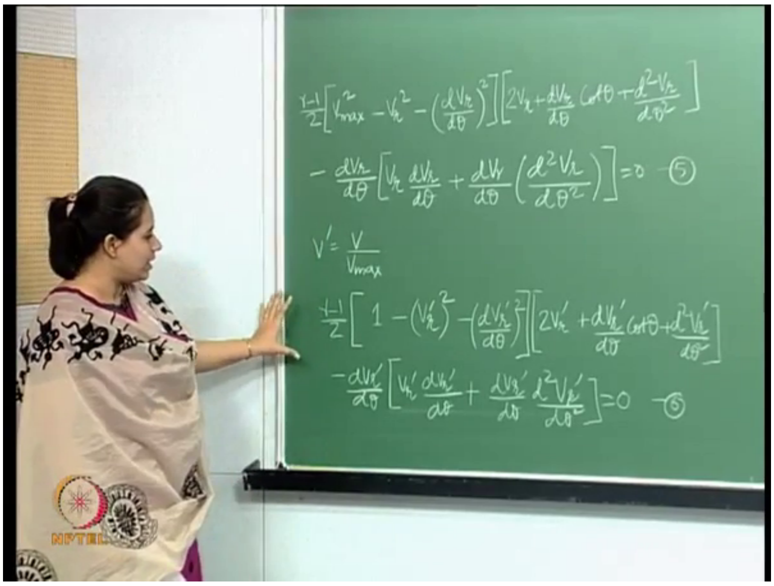

So, let me write this down from you know, just to remind us as basically. So, this was Taylor maccol equation which we derived last time. Now when we start to try and solve this; let us do this, let us define this non-dimensional parameters. So, let us just define here. This is just a non-dimensional parameters. So, let us define here. And when we do that let us see what we get.

So, I am just going to set of write this down. So, what we get here is this. Let me rewrite this. So, what we get as equation is this. And this is the first bracket. So, basically, I you know, I divide throughout by V max square, right. If I do that, what I get is; and V r dash is nothing but V r by V max. So, then this is my first bracket; similarly, the second one, that and finally the last bracket.

So, this was actually they are equation 5 like we did last time, and let us call this as equation 6 a. So, basically what you know we have this Taylor maccol equation; which cannot be solved in the closed in a closed form. So, we have to revert to or you know use numerical methods to do that. So, in (Refer Time: 04:57) of that what we doing is we introducing this non-dimensional parameter V dash which is this. And accordingly, I have been able to transform equation 5, which is the Taylor maccol equation to this.

Now, let us also look at what we did last time. So, we gone a introduced V max.

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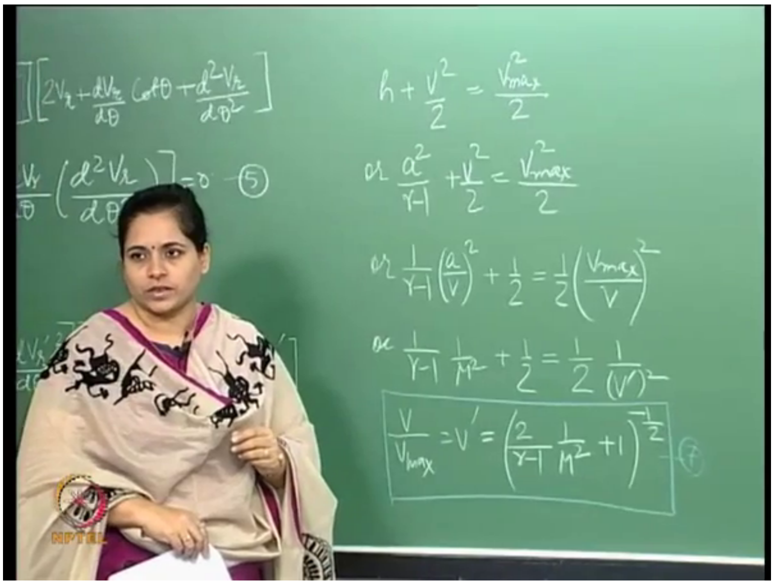

So, enthalpy constant right; so we can write this, this write; if I can write this. So, here let us say what I will do here. So, this also I can introduce this right. So, what let us do something here? Let us what we will do over here is; let us divide by V square throughout. Let us divide by V square throughout, and what we get here is that this, right. So, I divided this equation by V square throughout I divided this. So, you can see what we what we tend to get over here ok.

So, therefore, what we get over here is; so, this is Mach number, right. Is that fine? So, this is Mach number. Well, we can write this as 1 by V dash square also (Refer Time: 07:13) we can write this as this as square. Now from here, from here basically see if I want to write this therefore, I will write in terms of V dash. Therefore, V by V max, right. V by V max which is equal to V dash is therefore, equal to right. So, this is what we get. So, this is a relationship which is interesting. So, let us call this as a 7. So, what we basically have done develop you know got some relationship of this V dash parameter which we have just introduced with the Mach number. So, given the Mach number we in if you see from this relationship. So, if the Mach number is given we can calculate V dash and vice versa ok.

So, let us see whether we should be able to use you know this relationship in our solution to the Taylor maccol equation k. So, let us begin by looking at a problem. So, let us say. So, so we have say 15 dv half angle cone at 0-degree angle of attack in a free stream with Mach number of 2. So, we need to find out the shock wave angle, the pressure temperature, density in Mach number immediately behind the shock wave and on the cone surface. So, essentially what you know the problem is that, we have say a cone like that.

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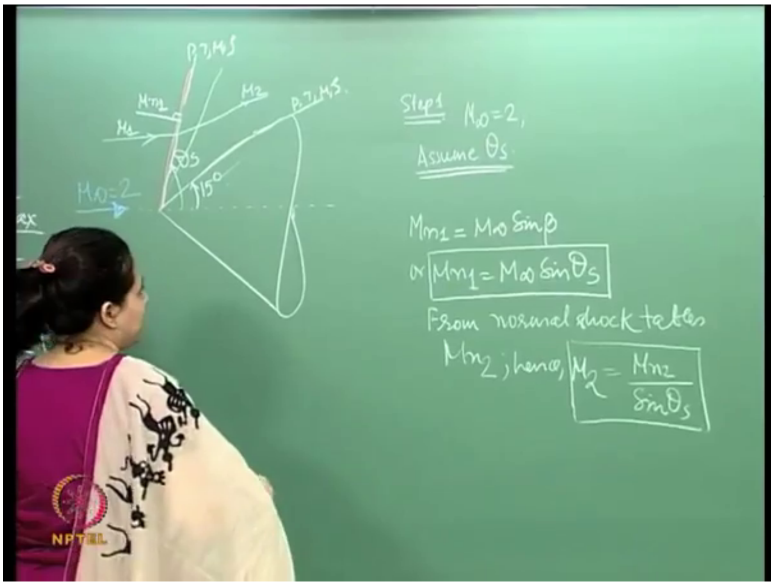

We have a cone like that. So, we have a free this ok.

So, I have a cone the a line of symmetry and this this cone is inclined at 0 degrees to this free stream which is a marked number is 2. And the cone half angle this cone half angle is given to be 15 degrees right. So, what we need to find out is the shock wave angle. So, basically when we have a shock wave over here. So, when we have a shock wave angle over here, what we need to find out is the shock wave angle. And the properties of the of the flow immediately we have a shock wave and all the cone surface.

So, essentially what we are talking about is that if I take several say radial lines, right. Like we discussed yesterday; so if I take say radial lines say just behind the shock wave then we will have several ones in the middle and finally, one which is on which is on the body of the shock wave. So, what we need to find out is the properties. So, pressure, temperature, Mach number density etcetera, right you know behind the shock wave, and then move from there and as closer we get to the you know to the cone and here also on the surface. So, then we also need to find out the properties on the surface. Like we discussed yesterday that these properties are going to change in this direction, in the theta direction; however, on a single radial line these properties are going to remain constant.

So, a problem here is basically the problem to do here is that we have a 15-degree half angle cone, which is inclined to 0 degrees to this free stream Mach number Mach 2 flow. We have a shock wave. So, first job is to find out the shock wave angle, the theta s and then the properties just behind the shock wave, and on the surface of the cone. So, this is our problem. So, how do we go about doing this? What I am going to do is list step by step; as to how we gone a go about this, alright.

So, let us let us begin. So, let us say let us we gone a do this. So, we have a Mach a to flow. And the way we are going to stop solve this problem is like in an inverse fashion, you will see what I mean by that. So, step one is this is given. So, let us assume. So, here let us assume theta s, step one. Just assume theta s. Once we do that; so, theta s is you know theta s is the shock wave angel which is essentially the beta which we have learnt you know earlier for the wedges, right which was the theta beta m relationship.

So, now if we do if we do this basically now, what we will do here is calculate we will calculate, Mn 1 which is the; so, in order to use. So, we basically we have an oblique shock here right. So, we gone a use the tables you know, the tables to calculate the properties. So, in order to do that, I need to know the normal component of the velocity, normal component of the free stream Mn 1 and if you remember right. So, Mn 1 right; so, this was the expression which we use last time. So, Mn 1; so the normal component. So, I have this. So, then I will this is inclined. So, I am gone a find out the normal component of this. So, this is my Mn 1 right. So, Mn 1 and in this, so this is the sign beta which in this case if so in this particular case of course, it is right.

So, in here so now, M n infinity the free stream Mach number is given as 2 and theta s is something that we have assumed. So, therefore, I get a corresponding Mn 1 for this I mean, then we go ahead and look up the charts and get the corresponding M n 2 right. So now, here the next thing to do is from the charts from normal shock tables rather from the normal shock tables we get M n 2, right. And hence we get M 2 which is correct.

So, basically if I if I have the free stream coming here and you know there is a certain you know deflection like that. So, this is my M 1 and this is my M 2. So, this is what we have done so far for an oblique shock. So, you know we will do pretty much that. So, from here what we get is the marked number behind the shock wave. Now let us just for a for a moment step back, and look at the difference between you know in a 3D picture like this and as per 2D thing which we have done earlier. You must do that let us just look at this.

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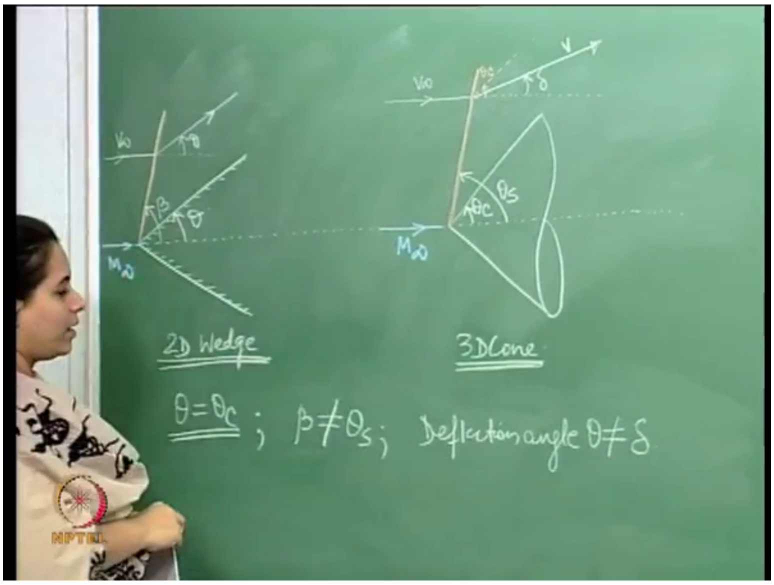

So, let me go and write that up here do that here. So, this is the picture; which we had earlier, right. This was our 2D picture. So, we have this, right. And then say we had a shock wave out here, and then we say had a shock wave like that this way right. So, if we do and say this is our free stream. So, this is our free stream. So, here this was the theta, right. This was the theta, this was the beta right. And So now, say if I have; so, say I have this, I have the free steam coming here, right. Then it would deflect. So, it would deflect and this would make an angle of theta right.

So, this is what we have done so far. This is the 2D picture right. So, this is 2D wedge picture. Now let us look at the 3D point of view. So, what do we get here, now 3D here this. And again, we have a shock wave. This is the shock wave. So, then and we have the freestream coming in here. And so now, in here this is my half cone angle, this is my half cone angle. And this is the shock wave angle you see the difference this is the analogous to the beta over here, shock wave angle.

So now let us do this here. So, therefore so, say I have the free stream coming here. And let us draw a line which is you know draw which is basically parallel to the surface of the cone. So, which is say this or which is say this, and this line essentially theta c now; however, now this free stream which comes in over here. It does not necessarily have to deflect by theta c. So, therefore, it is say it deflects like that, and this is say some and this it makes an angle say here delta.

So, you see the difference between the 2 wedge and the 3 cone. So, what happens here is the free stream comes in here deflects by an angle theta; which is essentially the half wedge angle here. On the other hand, the V infinity here comes in it does not necessarily have to deflect by theta c it deflects by some angle delta which is not equal to theta c. So, therefore, what we set of see over here. So, they both have the same cone they both have the same angles right. So, theta here which is the cone angle here is equal to theta c. They have the same cone angle they which is equal to the same half cone angle there in terms of the similarities. The shock wave angle of course, is not the same. The shock the if you have a 2D wedge the shock wave angle is going to be different than if you have a 3D cone.

So, therefore, this beta and this theta as are not same. And also of course, the deflection angle, right the deflection angle. So, or rather if I may write it like this. Deflection angles are also different. So, essentially this would be. So, this is to basically to explain this; so now, going back. So, what we have done here is; so, we assume theta s and what we have been able to we will basically use a normal shock tables, and found out the Mach number behind the shock wave. Let us move to step 2.

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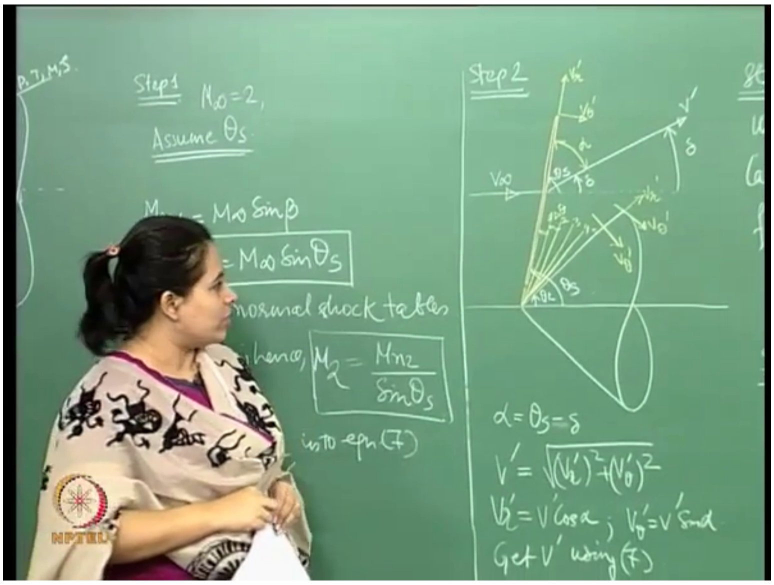

So, once we do this, let us move to step 2 if I do this so now here. So, let me sort of draw a picture here. So now, here so, this is a and then we have of course, the shock wave, or let us say we have the shock wave. I am short of exaggerating this a little bit. So, we have that. So now so, therefore, this is my cone angle, and this is the shock wave angle. Now say we have a freestream which comes in we have a free stream that comes in, right. And that is deflected, let us call that hence and this angle of course, is delta, right. This angle is delta and this angle is theta s, right. This angel is theta s.

Now, you also know which we did just know. So, basically, we have a velocity component in the varial direction, and one in the perpendicular direction which is this now. So, therefore, we could have several radial directions from here to here, isn't it? So now, let us say this angle. So, therefore, this angle here. So, this angle here, we can call this as alpha, right if I have this as alpha. So now, if you just look at this picture itself; if you look at this particular picture. So this alpha is nothing but the, right. If you look at this alpha is basically theta s minus delta; so this alpha s theta s minus delta, if you look at this picture. This is the alpha, right.

Now, again V dash of course, is equal to this, right. And again of course, this V r dash is V dash cos alpha and V theta dash s V dash sin alpha. So, if you look at this picture over here. So, V dash V dash is essentially V r dash square and V theta dash square. And V r dash V r dash is V dash V dash cos alpha and V theta dash is V dash sin alpha. If you look at this picture itself. So, if you have this now if you have this. So, therefore, a now we will use this equation a 7. Now what we will use is equation 7. Now V dash is something that V need now M 2 m 2 is something that we have found out, right. M 2 is something that we have found out over here. So, if we put M 2 here we will get if the corresponding V dash, right. This is what we said.

So, this is an equation which can use. So, M 2 is something that we have found out using the normal chart normal shock tables. Therefore, if I if I use equation 7 now and input M 2 here I will get the corresponding V dash if I get V dash; then of course, I can calculate the V r dash and V theta dash. So, basically what we what you will do here is that get V dash using 7. So, using 7 or you can sort of you know do it here also if you want, we can do it here also if you want.

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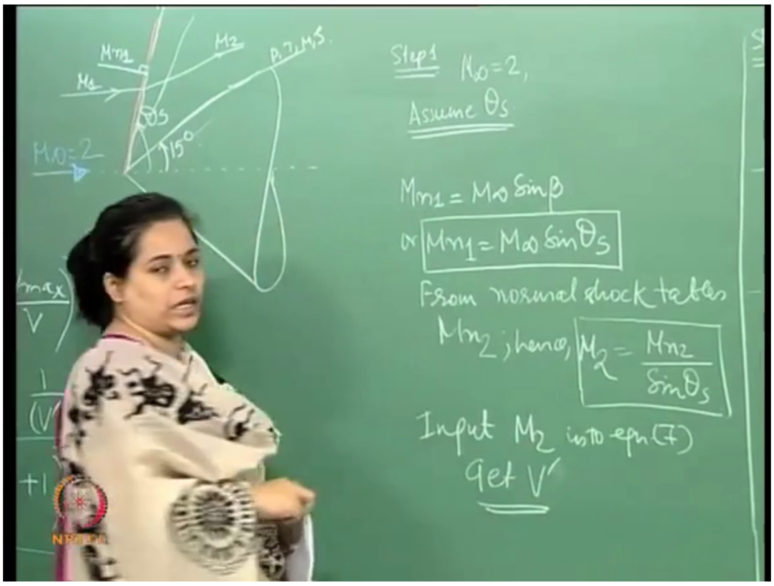

So, if we input this M 2 into equation 7 and get V dash, so the there we are so, once get this once we get this.

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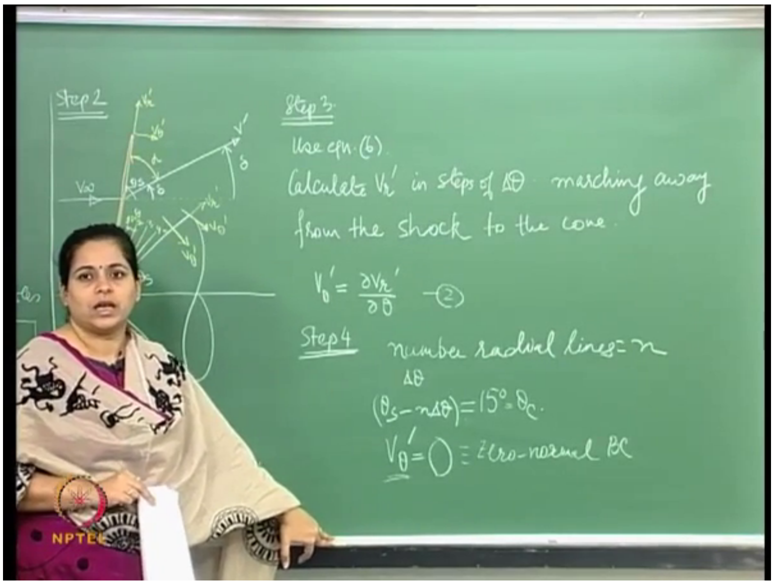

So, then the next step would be; So, step 3 would be, so this is something that we had also written last time, right. Now equation 6: so I am able to write this again basically the Taylor maccol theory Taylor maccol equation when we wrote that using the non-dimensional parameter V dash. So now, we gone use if you see that equation. So, that equation basically is has just one unknown which is V r dash.

So, what we gone a do is now what we gone a do here is in in in step 3 is that we going to use that equation, and calculate V r dash. We will calculate V r dash, and in steps of theta in steps of theta marching away from the cone. So, what we gone a do is here let us say use equation 6, using equation 6 in steps of delta theta marching away from the shock to the cone.

So, let me explain what basically we gone a we are going to try and do over here. So, essentially; so this is the shock wave. So, this V r dash is gone a change, right this V r dash. So, what we gone a do is; when we have the Taylor maccol equation right. So, we gone a take that, and we gone a take several radial lines here. We gone a take say several radial lines between this theta s and theta c. This is theta s and this is theta c, and we are going to move from here. We gone a start from here, let us say let us look at this picture over here. So, what we going to do is we gone a take, so on and so forth. So, what we gone a do is say or now this angles are say these are all delta theta into this.

So, what we will do is we will start from here, we will start from here, calculate the V r dash. And the again come on to this next this come on to this next radial line calculate the V r dash there go to the next radial line calculate the V r dash here (Refer Time: 33:30) can go to the next one calculate the radial line then V r dash there and so on and so forth. So, we will start from here, we will start near the shock march way from the shock, and towards the cone. So, calculate this V r dash in steps of delta theta. Basically, which means that we will calculate the V r in in at various radial lines between the shock and in into the surface of the cone.

So, when we do that. So, we will use we will have to solve the Taylor maccol equation. Now that equation as you see is the complicated one. So, let we will come back to that. So, what we will do is basically use a 4th or a Runge-Kutta method when we do that. So, having we done that. So, once we find out. So, once we find out V r dash once we find out V r dash. So, what we gone a do is calculate V theta, right. If you remember V theta this was right. So, for each so, at each radial line when you get a V r dash you will also get a V theta dash at each radial line you will also get a V theta V theta dash alright.

Now so now, keep going this. Now what was our problem? Now our problem was that we need to find out the properties just behind the shock wave and on the surface of it, on the surface of it. So now, when we start from here; os we going at various steps and we you know keep going further and further away from the shock wave, and at some point, we should be reaching the surface of the cone right. So, like I said this was like a inverse problem. So, we are actually beginning from here and coming back to the cone.

So now say at a particular theta here. So, we are moving, moving away from it. So, say theta s you know minus delta theta minus 2 delta theta minus 3 delta theta so on and so forth. So, let us say, let us say this is step 4, this is set 4. So now, say at we take say n number of say. So, say number of. So, say that number of radial lines between here is say n say we have n radial lines, and you know, the angle step is delta theta. So, essentially n into delta theta is equal to alpha is equal to theta s minus theta c. That is what it is, alright.

So, if I do that now; therefore when I have; therefore theta s minus n delta theta. So, when I get theta s minus n delta theta I should have reached, right. I should have reached the surface of cone, isn’t it? Because what we have done here is that we just dividing this space which is the angle alpha; which is theta s minus theta c, right. And we just divided that that space into n using n radial lines each spaced out at delta theta. So, therefore, one I now start back from here. So, when I start from here once I have covered this. At once I have covered this alpha which is theta s minus n into delta theta, I should have reached the surface of the cone right.

So, I should have reached this surface of the cone, now at the surface of the cone. So, therefore, now at every step when I do this at every step at every radial line I get a V r dash and I also get a corresponding V theta dash. We get these 2 velocities. Now once we come here, once we come to the surface of the cone. So, when we do this, when we have reached this when we are doing this theta at this radial line; which is in a basically surface of the cone then we get a V r dash of course. Now what should be this V theta dash be? Come to think of it if you look at this. This is V r dash this is V theta dash.

So, the radial line now; so essentially the radial line now is here, isn't it? Radial line is here. So, this is your V r dash, and this is the corresponding perpendicular velocity component. In other words, does this make any sense? So, this is a solid cone surface, and I have a component of velocity which is normal to it what should it be 0. So, we have to satisfy the 0-normal boundary condition here. So, as a result of this which would V theta dash should be 0 this V theta dash should be 0.

So, this is essentially the 0-normal boundary condition that we are implementing at this particular point. So however obviously So now, in here so now, this think here this theta s minus n delta s. So, this here of course, this should also be equal to 15 degrees in our particular case, because now this is theta c, so theta s, right. We have done theta s minus n delta theta minus n delta theta that is now equal to theta c, isn't it? This is theta c. So, in our particular case this is given as 15 degrees.

So, at 15 degrees of course, V theta dash this is should be 0. This is what one should achieve, but considering that this is a numerical procedure; obviously, this is not going to be so. So, therefore, there is going to be a certain amount of error when we do this. So, when we reach 15 degree essentially, we will have to implement this 0-normal boundary condition because that will then tell the numerical solution that I have reached the surface of the once alright. So, of course, there will be a the error. So, then are the reason is; now I should have reached 15 degrees here, now reason now what will happen is that

I will get this 0-normal boundary condition. Now why will the error creep in? What is the reason for this error? The reason for the error is that I have assumed theta s, right. I have assumed theta s for the given Mach number I had assumed a theta s. So, what I had to do is to reassume this redo this in a better fashion. So, that I get close to this 15 degrees as well as this.

So, once a so; this theta as minus n delta theta corresponding to V theta dash equal to 0 so this to be implemented. So, I have to continue to iterate over this. So, once I once I continue to do that, so as soon as so that I get close to this I am never going to get you know completely 0. It is a numerical procedure remember. So, there is going to be some errors, but we are going to get we are going to try and get as close to this as possible right. So, that is what we gone a do? So, we therefore, you know continue to iterate will continue to iterate till we get you know this if and once we get close enough we with in a say a 10 to the power of minus 3 tolerance. So, then that is the shock wave angle.

So, then that becomes a shock wave angle which was also part of the problem first thing was we wanted to know is what is the a shock wave angle. So, then we would have correctly assumed correctly estimated the shock wave angle. So, you see the difference here unlike what we have done before where we just look at the charts that is not what we doing here. So, we are going therefore, we are going to do assume and start and then we iterate over this criteria, and then once this gets satisfied with in a certain you know allowable tolerance. Then we get our shock wave angle theta s.

So, after that the stuff is about getting the properties behind the shock wave. So, once you come over here if you remember. So, I am going to stick to say ya well you know I get a couple of diagrams here; now thermodynamic properties now, if you remember.

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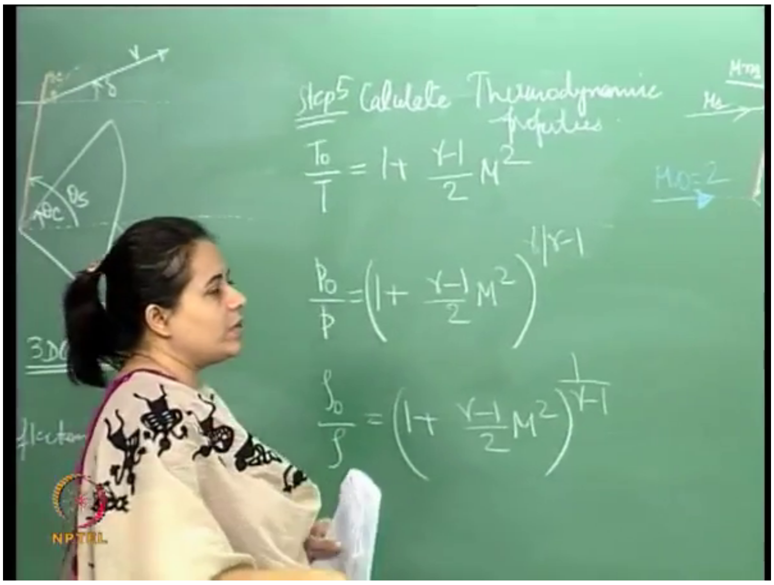

So, these were I am gone a just write this out. So now, the point was to get to calculate the properties just behind the shock wave and on the surface of the cone. Now the way is stands here you can actually calculate the properties in the entire region in the entire region here you do not have to restrict yourself to just here and here. That is what is asked in the problem, but we can actually calculate it anywhere in within this space.

Now, if you look at this here. So, this is the temperature pressure and density, this Mach number now. This Mach number corresponds to V dash. So, that V dash is at every radial location. So, like we did over here, right. If you (Refer Time: 45:32) look over here for at very radial line we gone a have a V dash. So, we gone a have a V dash over here. So, V dash 1 V dash 2 V dash 3 4 and so on and so forth. So, this Mach number actually corresponds to that V dash. So, when we go in step. So, when are basically going from step from here to this particular radial line. So, then we can actually calculate the V dash here, because we basically solve the Taylor maccol equation for V r dash. And then we calculate the V theta dash using this; hence we can get V dash.

So, once we get this V dash. So, then we can input that in terms of the Mach number here, and we can get the (Refer Time: 46:22) properties. So, once we come we keep doing this, we keep doing this over the entire region here behind the shock, and once we come over here. So, basically looking at the radial line which is at theta c which is the half cone angle, so corresponding to that here V theta dash of course, should be 0 because it is the 0-normal boundary condition, we implement that. So, we have a V r dash is essentially V dash.

So, then that is the Mach number which we should be implementing over here. There is the Mach number we gone a implement over here, and use these parameters use these equations rather to calculate the properties to calculate the properties behind the shock wave. So, that basically is the entire procedure; which is I think pretty simple as such. So, that the first step is basically we said will assume theta s given Mach number. So, we get M 2, M 2 correspondingly we put M 2 into the equation when we get you know this V dash. Once we get V dash then we come here.

So, we calculate V r dash and V theta dash using Taylor maccol equation, and this and we march away here. So, at very at the every at various radial locations between the shock wave and the surface, we get the V r dash and V theta dash and we calculate also the thermodynamics properties using these expressions and gone be coming you know we implement this 0-normal boundary condition when we come to the surface it is pretty much it right.

The main the main the complicated part or the majority of the work out here, in the problem is essentially here, isn't it? Essential here in step 3 where we need to calculate V r dash using the equation 6.

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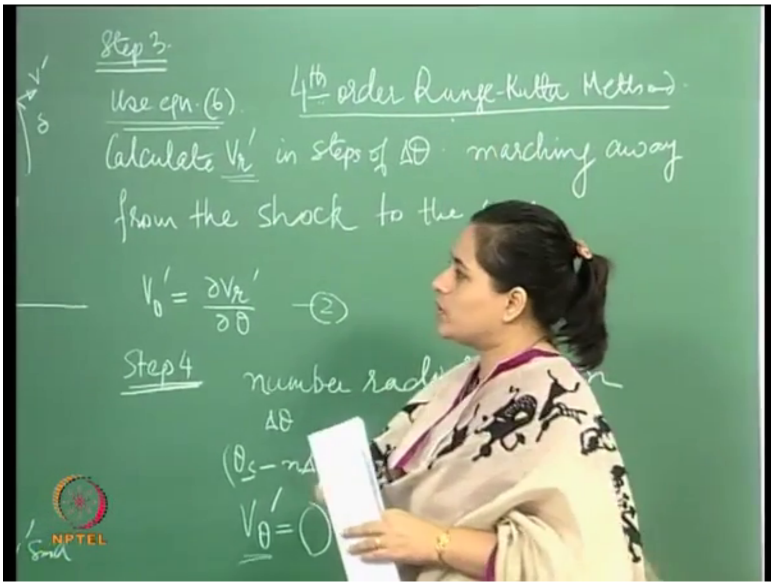

This is the Taylor maccol equation where we gone a basically use a 4th order, right. What we gone a do is basically we going to use a 4th order a Runge-Kutta method to for here. So, what I will do is in the next lecture we will basically go over that. We will go over this particular a step where we use the Runge-Kutta method to solve the Taylor maccol equation for V r dash. We will continue with that and hopefully we should be able to complete the solution since there. So, this is an basically the you know these steps step 4, or you know. So, I guess you can write this as step 5 and calculate thermodynamic properties that are it.

So, these are basically 5 steps and we are done. So, what we gone a do in the next lecture is basically go over in detail about the 4th order Runge-Kutta method and how will you going to use that to solve the Taylor maccol equation, right. That should be all.

Thanks.