Advanced Gas Dynamics
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Lecture - 25
Supersonic Flow over a Wavy wall

So continuing with the wavy wall problem, we came up with an expression for the surface pressure coefficient right.

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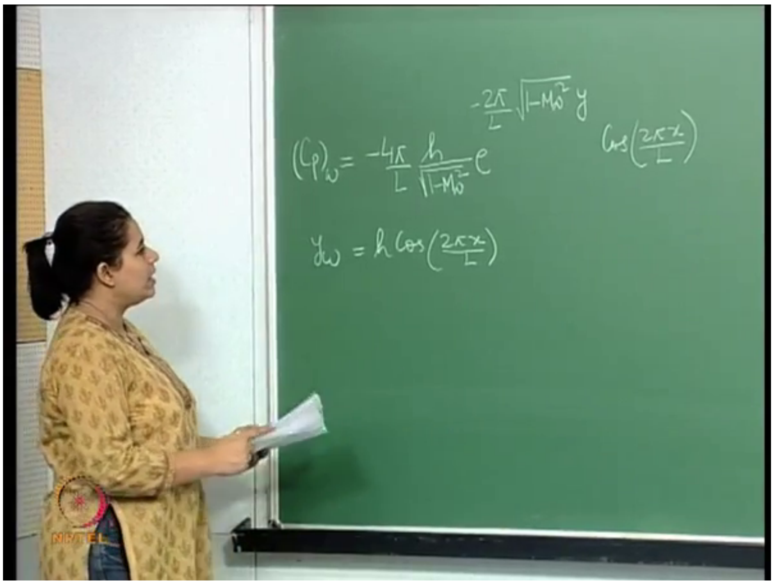

So, wall pressure coefficient it look like this right. So, this was the expression and then also we looks are how this really is acting on the surface, and we saw that you know that the equal let us look at the equation of the surface, the surface equation the control equation rather was given as; the wall control equation was given as.

So, you can see that both these quantities that basically are functions r cosine functions right; however, this is 180 degrees phase like because of this negative sign as a result of which we saw that there was no horizontal component of this pressure, as a result of which there was no drag which was commonly known theorem respire paradox. So, now, let us just look at this a little more closely and see, what exactly this you know means or if any other information we can draw from this equation out here.

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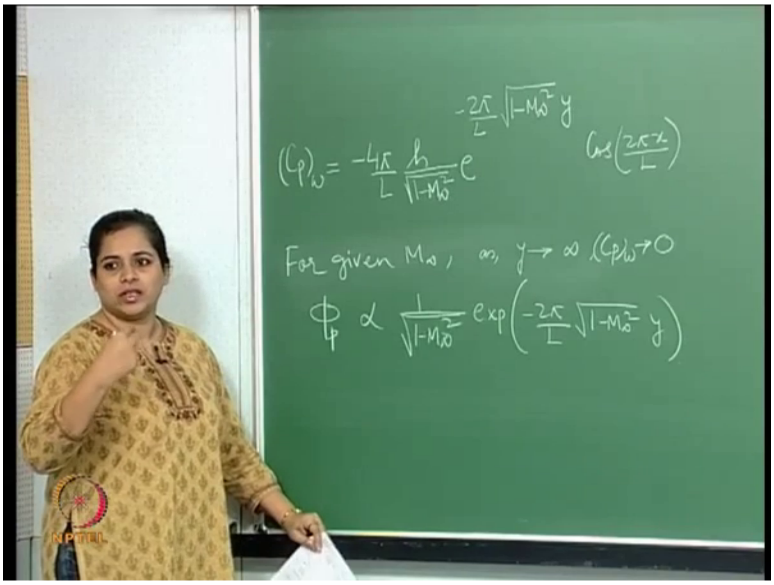

Now let us see say for certain value of M infinity right. So, say as y say is very very large. So we basically, look at a distance which is very very far away from the wall itself. So, why is very very large then what happens to you know C p right. So, if y is very very large then essentially right tends two 0, now what this essentially means that as we basically move farther away from the wall and the disturbances die out right.

So, which kind of make sense right the more we get away the slowly as so basically, we can say that as y increases the pressure coefficient decreases and which in turn would also mean that the velocity the perturbation. Now let us look at the say the perturbation velocity potential right now for example, if you look at this now the perturbation potential now that that is proportional to essentially and exponential. So, if you look at this as well. So, when y increases basically this tends to 0 which means that is

So, if you if you look at this. So, the which means are the perturbation you know dies out or the perturbation velocity is die out, which is essentially means that the disturbances die out as we move farther and farther away from the wall that is what inference which you get from here. Another is that in here we are really looking at. So, let us get rid of this ok.

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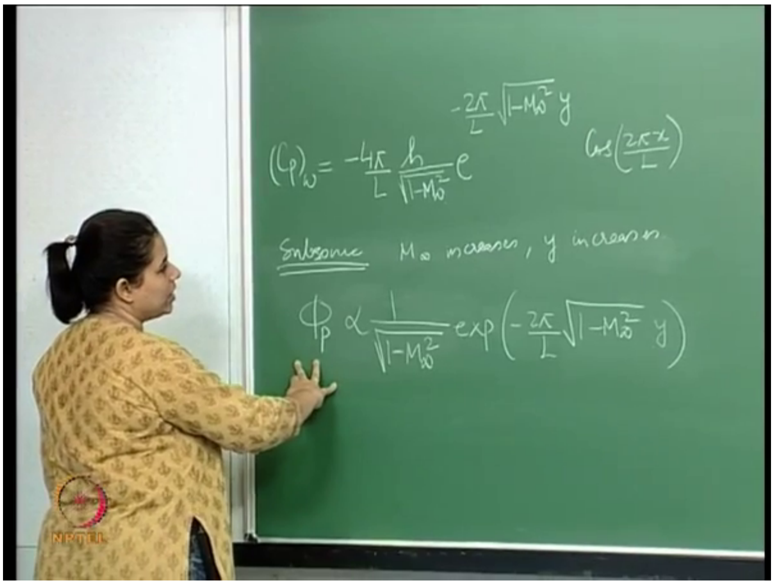

So, now here we are look looking at subsonic right. So, we are looking at analysis that you have done. So, far is for subsonic subsonic flows right.

Therefore, in here now look at this, now if M infinity increases within this subsonic flow, if M infinity if the Mach number increases then what happens right. If Mach number increases look at this term, look at the pressure coefficient here if Mach number increases and this decreases. So, this basically increases this increases now let us say look at also you know look at this and now at the same time let us also say you know phi p that is also this, this is also proportional to this right now see this if Mach number increases so ok.

So, this term basically decreases. So, this term increases; now this has to correspondingly increase this has to correspond this term this term actually increases. So, this term has to correspondingly decrease to keep the proportionality correct. So, similarly you have the same thing here the C p is also, you know similarly corresponding you know it is a function of it is also proportional to the pretty much the same thing right; therefore, if Mach number is increasing; so if this has to decrease.

So, now in here therefore, if Mach number is increasing right, so this term decreases. So, this has to correspondingly increase right. So, if Mach number increases essentially. So, y also increases which means what; what is that physically mean let us look at this. So, we have. So, essentially what we saying is if you look at the perturbation potential here.

So, if the Mach number increases the y also increases. So, this is the relation of the perturbation potential with y, and again this is also you know hence the relationship of the pressure coefficient as well.

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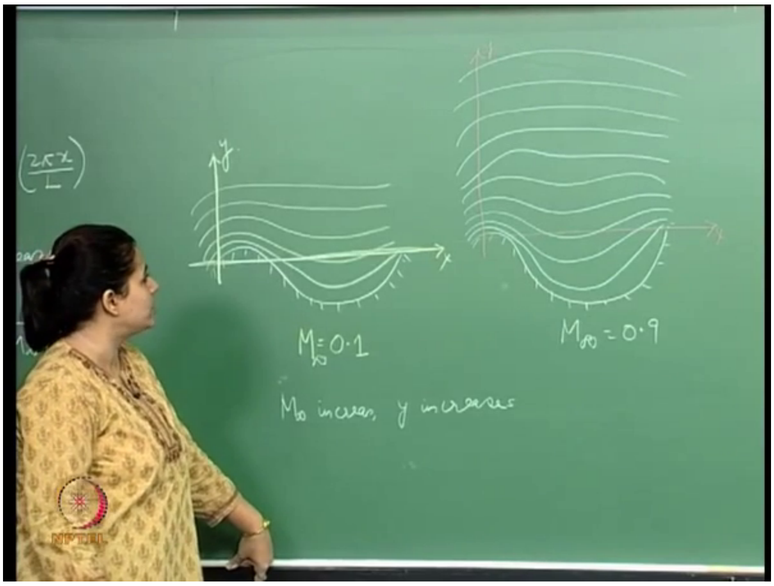

Now, if you look at this, now say this is our wavy wall right. So, this is our wavy wall right. Now say this is at M Mach say this is at M flow is happening here say at a Mach of say 0.1 right say this is happening at 0.1. And another case is that we have say what essentially and this here. So, this says 0.9 this is say 0.9. Now what we said is that if Mach increases y has to increase.

So, Mach is increased from 0.1 to 0.9, now the y has to increase what is that mean what; that means, is that in this particular case you know you have the disturbance. So, say it moves like that moves like that, moves like that, moves like that and so on so forth. So, it goes up to a certain and after that it dies down. So, therefore, you have a certain y distance over here now in here; however, let us see.

Now, we saying that as Mach increases y also increases, what this means is that right and therefore,. So, essentially what we saying is that as for subsonic flows right as the Mach number increases; as the Mach number increases. So, the disturbance also travels to a farther distance right; disturbance also moves you can see here it moves a farther distance in y away from the wall. So, that is what that means.

So, pretty much having done this now let us go and look at the same problem right for a supersonic case. Is in the same linearize theories and see what we can infer from that and what set of the equation we will get right and how do we go about solving those and then we will sort of sum it up alright.

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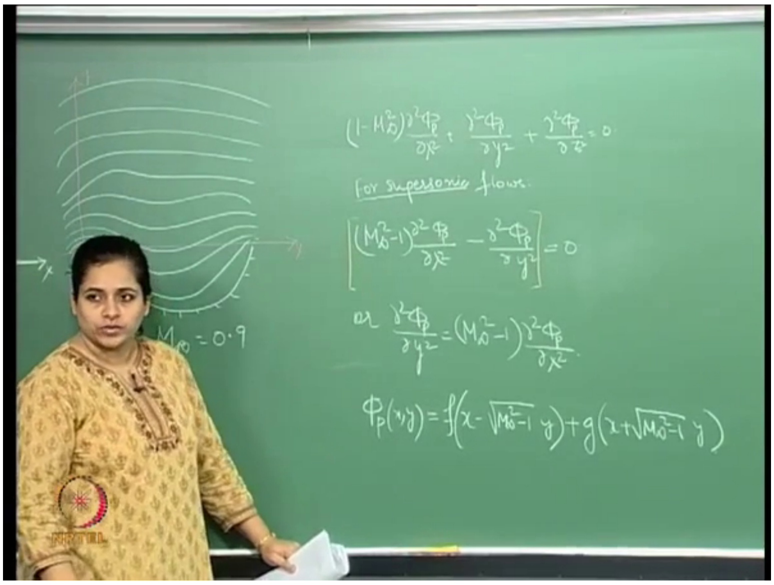

So, here we go. So, we will start with the (Refer Time: 10:41) velocity potential equation. So, the (Refer Time: 10:44) velocity potential equation was this is what we have done right. So, this is what we have we had originally done for in the subsonic case ok.

So, now let us do this for a supersonic case. So, where we will write this equation as say let us write it like this way right. Now if you write it this way then let us look at 2 D picture of this. So, which means that we basically going to look at this right. So, this is our equation this is the 2 D picture. So, if we look at this. So, or we can write this as right. So, if you have this now if you I am hoping that you recognise in this equation is typical wave equation and mathematically this has. So, set for a solution to this right.

So, therefore, we can write this solution to this in this form right. So, we will take two arbitrary functions and say if phi p here which is the function of x and y is therefore, equal to say right. So, this is our typical solution for an equation like this right. So, now, we used method of separation of variables for the subsonic case and this case however, we you know this is in this equation the wave equation. So, we have a solution which will look like this.

Now, let us sort of again go ahead and find an expression for this. Now to do that let us say let us again let us say you know we will set t is equal to 0 in here.

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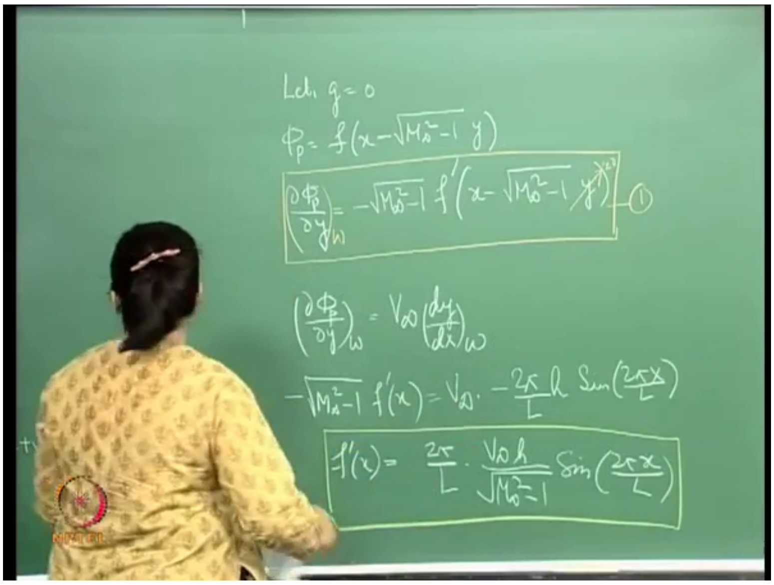

Let us say g is equal to 0. So, to find a solution for that, let us say we will say g is equal to 0. So, that makes my right. So, this is my velocity potential equation now. So, therefore, p del y is let us call this as one, ok

So, this is my del del the del phi del y we dash essentially at the wall. So, now, this is. So, this is not at the wall this is at the wall. So, essentially this will go to 0 you considering small perturbation theory. So, this is del phi del y now as we learnt from our last exercise. So, now, del phi del y right this at the wall is essentially the velocity. So, this is what we have derived yesterday as well, but if you just look at this expression, the loss of the perturbation velocity component at the wall is essentially the free stream component in the direction of the wall ok.

So, this is the slope of the wall. So, this is a what we have done earlier and we have an expression for this d y by d x at the wall, which is what we get by differentiating the equation that we had for the wall which was y w right. So, therefore, let us just do that here. So, like we said now at the wall we will set now at the wall this y considering small perturbation theory right. So, if we do that.

So, therefore, let us equate these two here. So, what we then get is this right is equal to and the slope was given as. So, this is what we get. So, here therefore, we can get our f dash x. So, let us get our f dash x right which is equal to. So, this is my f dash x slope here this is what I get now. So, let us. So, if say we will you know integrate this let us integrate this.

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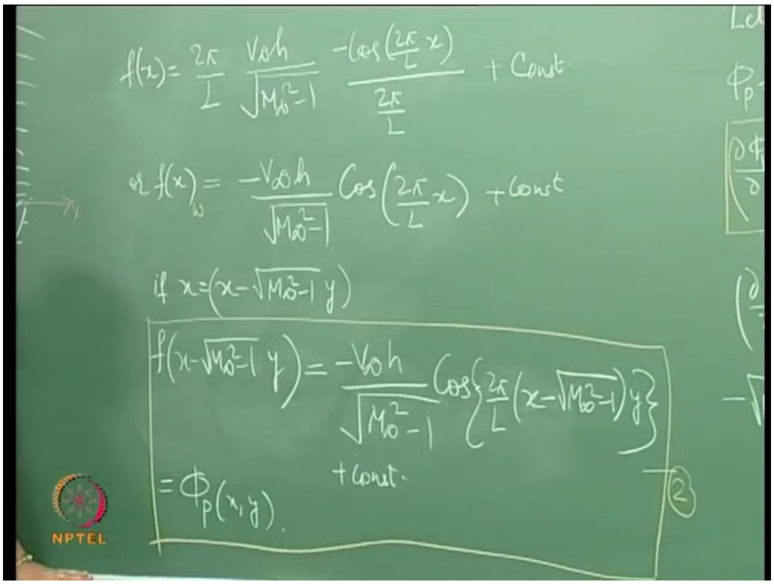

So let us see what we get. So, f dash x is this now if I integrate that right if I integrate it. So, what I get is f x, f x is essentially. So, if I integrate this plus a constant. So, therefore, this or essentially f x is equal to right. Now this f x is a essentially at the wall is not it, we derived all of this essentially at the wall. So, f x of the wall looks like this.

Now, therefore, now if instead of now we know that instead of any arbitrary position in the flow field right. So, that argument here was x minus this. So, instead of x be in just x. So, let us say for any arbitrary x here. So, if I have this. So, therefore, then this equation for any position in the flow field will look like x is x this and essentially what this is equal to our perturbation potential ok.

So, therefore, this is what it looks like. So, let us box this and let say let us call this as say 2; now that we have done this. So, basically we found a relationship found an expression for the velocity potential here. And of course, you also if you will remember that if the we took the; if we take 0, if we take g to be equal to 0, then what this results in is essentially right running characteristics right and then if we take a f to be 0 then what we get is left running characteristics.

So, we if you know done that earlier. So, now, having got an expression for this velocity if the perturbation velocity potential, what we need now is the coefficient of pressure right surface pressure coefficient. So, let see we will two how we will go about that ok.

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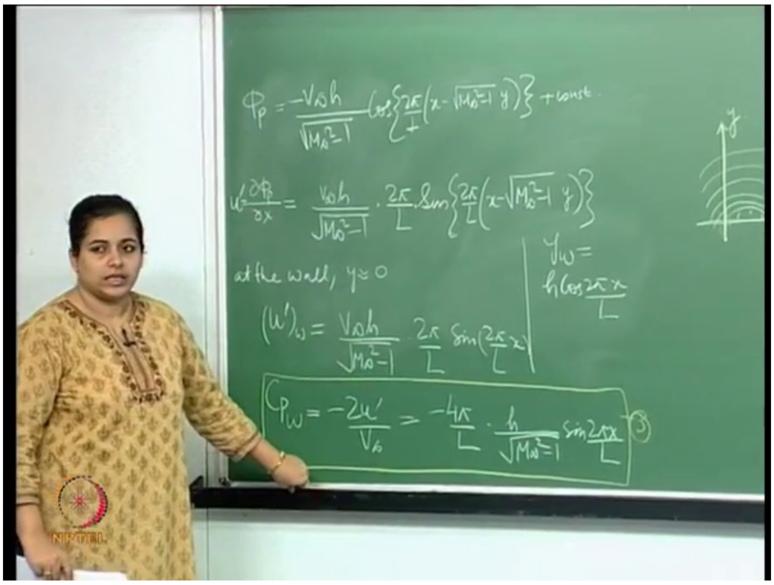

Now, what we learnt from last time is that the C p right C p at the wall is equal to this C p of the wall depended on the u component of the perturbation velocity right.

So, if I have to do that let us say we will try to find an expression for this using the perturbation potential with which expression for the perturbation potential, which we have just derived right. So, if I would do that. So, let us say let me write that again here. So, phi p essentially now is equal to. So, let us see this right. So, this is essentially u dash is not it? This is essentially our u dash. So, del phi p del x now this is u dash what we do get here. So, what we get is right.

So, if am just taking derivatives of phi p with x. So, what we get out here is this 2 pi by l will comes here and the cos the derivative of cos is sin for this is what we get right. So, this our u dash right. So, we took derivative. So, we get negative sign here because of this sin right. So therefore, what we get over here yes. So, let us say. So, these signs essentially cancel out. So, this is what we get right this is all what we get; now again now at the wall taking small perturbation theory.

So, again y will tend to 0. So, therefore, at the wall what we get is. So, this is what we get at the wall. So, therefore, our C p at the wall coefficient surface pressure coefficient right which is the C p at the wall therefore, becomes it is minus 2 u dash by V infinity right this our expression for this. So, what we get from here is this. So, in here therefore, that becomes right into right.

So, this is our expression for the wall pressure coefficient and this is it. Now let us try and understand this little more. So, like we did for the a subsonic case let us try and understand this little more what does this means. Now as you can see that the wall let us just point out a few things the wall equation. The wall equation was if you remember the wall equation was and the pressure coefficient equation here is this, the pressure coefficient equation here is this ok.

So, let us see what these mean and if you can get little better understanding of this in here. Now as we have done before and as we just said that taking either f as 0 or g is 0 we basically get left running or right running characteristics right.

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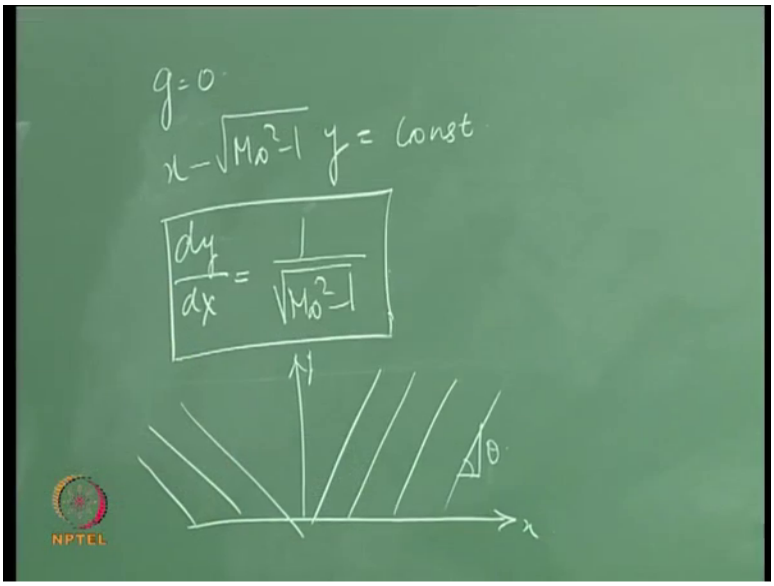

So, now, if we do that, now say if we take g is 0. So, therefore a line of constant if we take g is 0. So, lines of constant phi right.

So, lines of phi constant phi essentially say they correspond to right. So, they correspond to this because phi p is then is function of is f of this right which we just done. So, therefore, in this case I can write the slope of each line, divided by t axis this. So, essentially what we saying is that. So, essentially we are going to get this lines which have a slope of. So, d y by d x is essentially this you know given by this.

So, let us say this is theta. So, these are the right running characteristics we will get and if put f is equal to 0 we will get the left running characteristics this is something that we done earlier on; now having done this now let say then the Mach angle ok.

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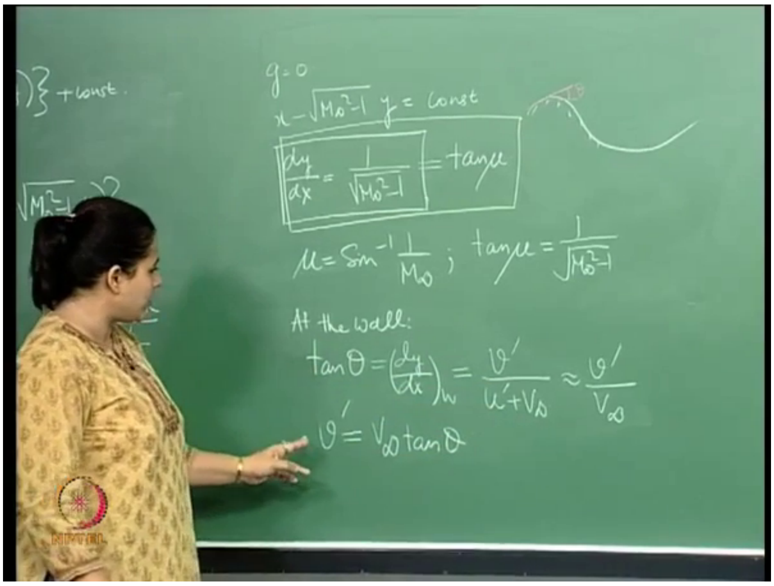

So, now this mu then is equal to this right now and here. So, now, I am. So, if I do a little bit of math out here then I can write this as ok.

So, now, therefore, what we can say is that this is also d y by d x is also equal to this is also equal to 10 mu. So, say let us keep the entire thing looks the entire thing right. So, d y by d x is given by this and this is also the slope is also equal to the tan to the tangent of the Mach angle. Now see at the wall. So, theta is tan theta so this d y by d x at the wall right. So, this is tan theta is the physical slope right.

So, tan theta at d y by d x is right this is what we have done before right and taking small perturbation considerations. So, which would mean that this is very very small compared to v infinity, what we get here is this right.

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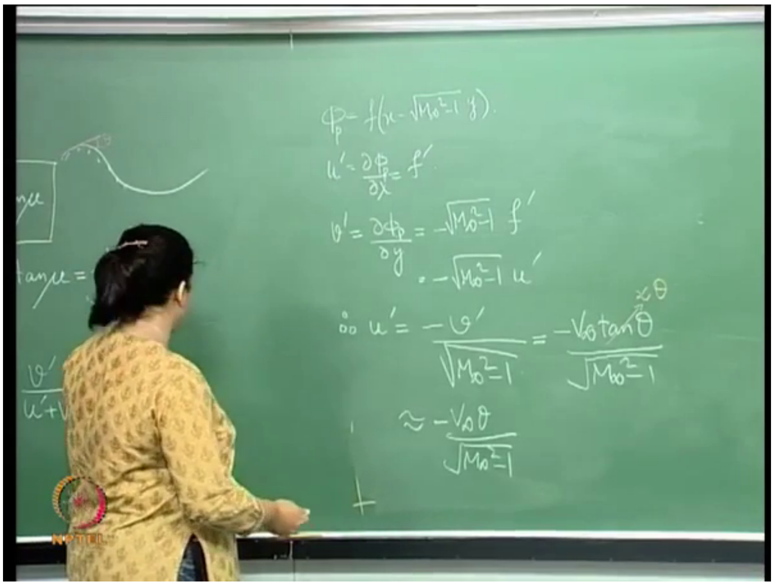

Let say what we have out here is that phi now is equal to we have taken g to be 0 right. So, f of into this; so then this is the perturbation process. So, u dash is which is equal to f dash right. Now then v dash is equal to and this is equal to.

So, this, but f dash is also is equal to u dash. So, therefore, I can also write this as this right. So, therefore, I can write that u dash is equal to; now what we saw from here? What we saw from here is that we can write at the wall tan theta is equal to this v dash is here. So, let us say from here let us say v dash we can write as v infinity tan theta. Now please try and understand this here that this theta is the physical slope right.

So, if you have when we have the slope here. So, when we have this actually wall over here right. So, then this is the actual slope physical slope of the wall that we are talking about. So, that is the slope right and we have also said this is the d y by d x at the wall and we equated that to the to this to this which essentially the angle which is made by the velocity components right and this and since the velocity vector is tangent to the wall itself we are able to connect the physical slope to the direction of the velocity and. So, therefore, I can write that this is what we basically trying to do here is that this is the perturbed velocity component in the y direction and we are connecting that to the physical slope at the wall right; which is this.

So, now, we have written here that the u component of the perturbed velocity is equal to this and we therefore, we can also write this v dash right, we can also write that as this because this is this can be written in terms of the physical slope at of the wall which is this right and considering that this is again small perturbation. So, therefore, we can say that this is theta is really small. So, then what we can write here is this right.

So, what therefore, we can write is this is equal to. So, we can write this because for small perturbation. So, this I will take as very small and therefore, I can write it as theta right. If I do that then what happens to my coefficient of pressure at the wall. So, therefore, what happens to my coefficient of pressure than is this?

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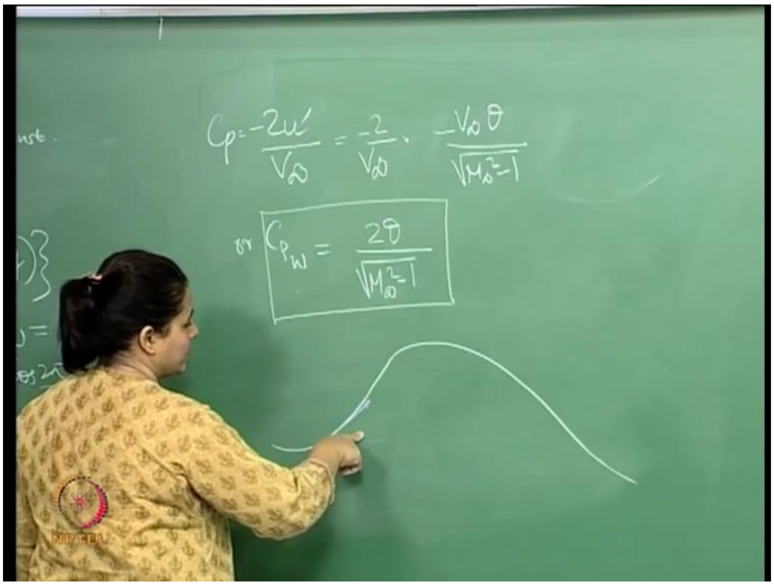

Now, coefficient of pressure is right this is my coefficient of pressure and you can now I have an expression for u dash; however, in terms of the physical slope at the wall right.

So, therefore, I can write this as an u dash we will in incorporate this right. So, what we get C p at the wall. So, now, this is what we get at the wall, now this is very interesting because what we see from here is that if let we will have let us now take a look at this. Now say if this theta is positive right if this theta is positive we get a positive C p w and if this think theta is negative then we get negative here which means what which means that. So, let us draw a wall here. So, say I have a positive like that. So, I have a positive and then I have a negative.

So, which means that as long as this is positive right. So, if this theta is positive I have a positive C p. So, let us do that right a let me draw the wall right ok.

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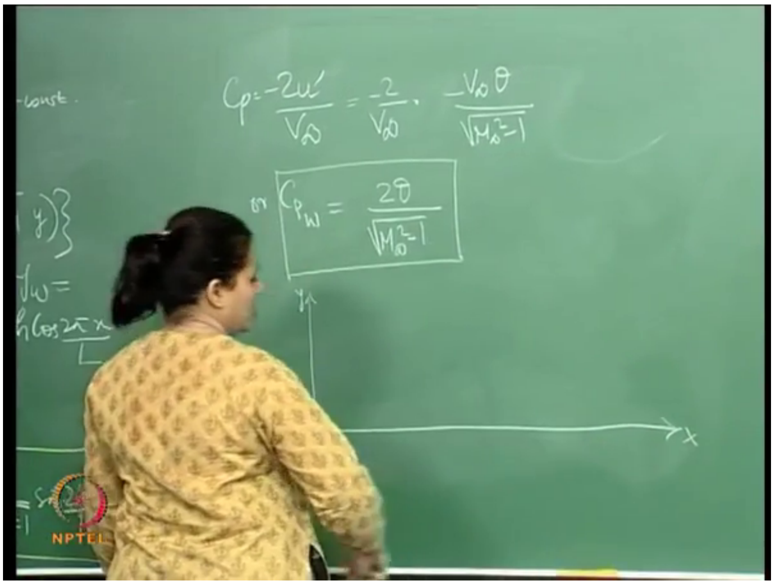

Let me draw the wall right. So, or say let this take a little bit of space here and do this.

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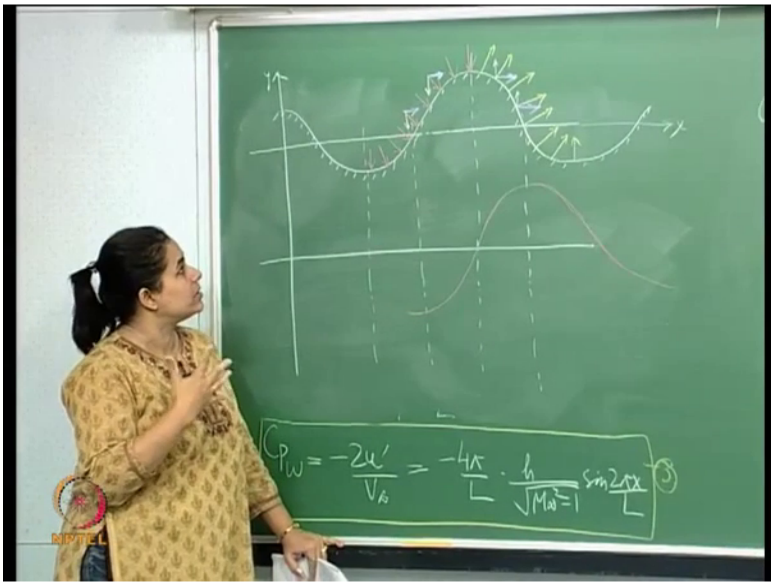

So, let us do this over here. So, if I look at this. So, say this may this is the wall this is my original this is my physical wall ok.

So, if I look at that, now when I have say let us look at this part out here and so on so forth. Now this is the region right this is the region where theta is positive. So, I have a positive C p. So, I have a positive C p and then I have what and then if you look at this theta is negative C p is also negative. So, if that is true then this becomes negative here. So, then C p I am just using the conventional that ok.

So, this what this means now I think by now you should be able to kind of figure out what this actually means from here from these arrows right. So, these are basically the showing the direction of the pressure. Now before we go back there let us look at this here. So, this is the other C p expression that we had got right. So, this is what this says this varies a sin and this varies a cos. So, essentially what we have from here right. So, is that. So, if I wanted to do this. So, let us say how this how should this look like.

So, if I want to do this what this would like is that. So, that should be what we get in from here and I hopefully should able to correlate between the two. So, now, let us go back to what we doing earlier, now if you look at this over here. So, now, let us take say this and this velocity vector sorry the pressure vector if look at this, I get this and if I look at this and I get this ok.

And let us take say another one. So, let takes this let takes this. So, let us take this and let us take this what do we see from here? What we see from here that unlike in the subsonic case right when this was symmetric when the C p was symmetric. So, we had a C p distribution which was symmetric as a result of which we did not have any drag right unlike here in the supersonic case you can see that the drag the horizontal pressure component is not cancelled each other.

So, we actually have a drag in this particular case. So, the drag does not blow away or there is does not shown no drag. So, this is the prime difference between the subsonic case and the supersonic case. So, we basically have an asymmetric distribution of pressure as you can see as a result of which. So, if we if you just look at say this half lambda over here. So, you have a positive pressure and negative pressure and. So, as a result of which we do have a finite drag. So, what we will do now is got of you know let sort of take a minute and summarise the whole thing ok.

So, do that let us let us say and also before we do that where was our phi p ok.

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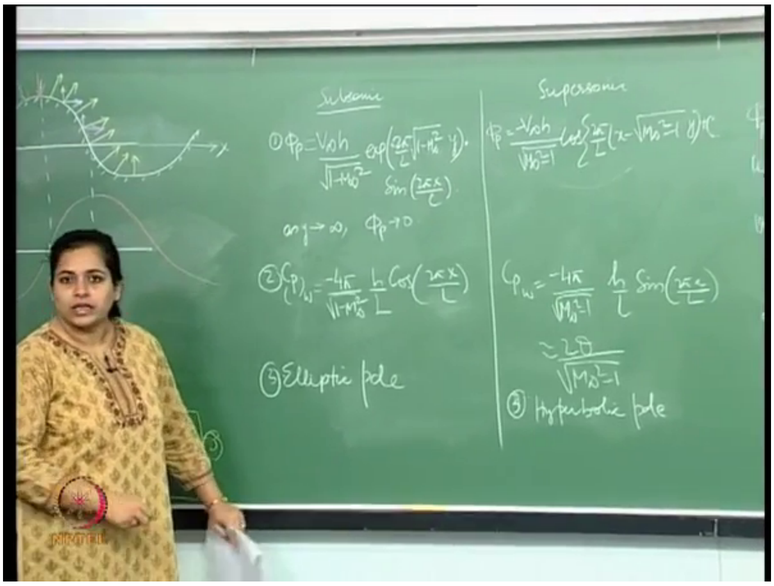

So, we got an expression for phi p. So, let us see let us look at that say let us say some of the things. So, this was our subsonic and this is our supersonic right. Now, we got an expression here for phi p right which was some of this sort. So, which was which look like this exponential into sin and for supersonic case, supersonic case we got phi p which looks like this ok.

So, first of the very first of we can see over here there is no exponential dependents of the perturbation potential on y x or y there on y actually and for phi p there is in for the subsonic case, we do have an exponential dependents of the perturbation potential right and here as we had seen that as y tends to infinity as we go farther away from wall, this disappears right the this disappears. Now if you look at here if you look here.

So, this does not sort of this does not die out in the supersonic case; however, for we cannot say the same thing it is not like if you move away farther from the wall the disturbance dies out it does not. So, essentially like we said we basically the in here. So, we have the disturbance which is propagating to infinity right on straight lines with the slope of 1 by this, which is basically in the right running or left running characteristics ok

So, in here C p at the wall was this right and here of course. So, here on the other hand C p at the wall this look like right and this was also say this was also equal to this. So, the this is how the C p look like and we saw that this cause no drag where as this because of its symmetry where as this did cause a drag, and finally, what we have here is an elliptic p d e.

So, which we solve using the method of a separation of variable that is what we did, here on the other hand right we have had an hyperbolic p d e right. So, for which is basically the which is the wave equation and that is how we know we also had solutions for that and we came up with which eventually resulted in solutions, which you know which were valid on right running and left running characteristics. So, that should be all.

Thank you.