Advanced Gas Dynamics

Dr. Rinku Mukherjee
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture – 22
Application of The Method of Characteristics: Flow through a diverging channel

So, let us sort of continue and everyone into the design convergent divergent nozzle right for a given exit mach number right and we said was that it is a minimum length nozzle that is when the flow comes into the throat area which is the sonic area, then it expands right and this expansion is not over length right, it could be over length right as it is in like a wind tunnel span then we decrease that lengths to a point so that we have a centered expansion wave right and that is really a minimum length you need. So, that in the flow diverges and then finally, we get parallel a uniform panel flow to the walls of the nozzle. So, that is where we were ok.

So, let us sort of we will go ahead and do that and what we said was that.

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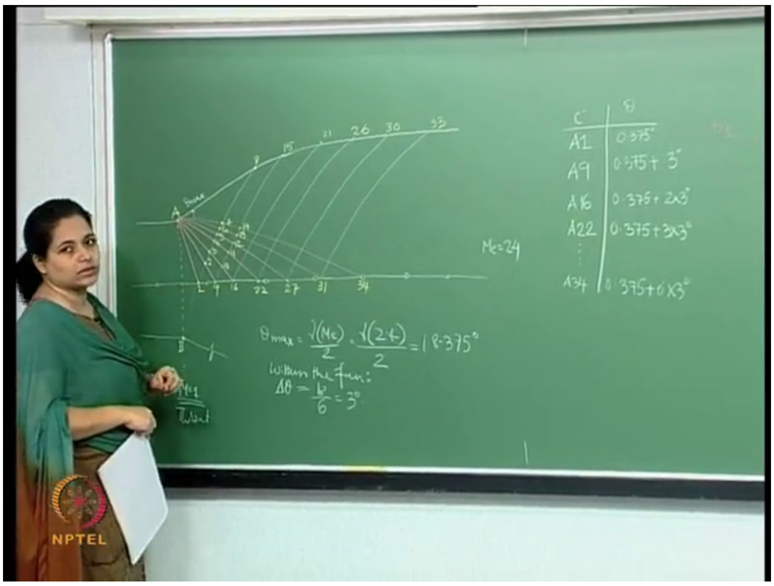

So, basically what we can a do is that we have to design and nozzle like that right and of course, we said that we also going to take the line of symmetry; this is the line of symmetry. So, as long as we just calculate in one part you can get it for the other right ok.

So, basically this is going to be repeated. So, at this particular point of time; so, this is the sonic region this is the throat, right and the what we know here is the exit mach number. So, the exit mach number is given as 2.4 right, this is the problem.

So, therefore, what we calculated is the maximum angle here right this was the maximum angle here and this was theta max right and we were able to calculate that as right; so, which is the polymer function with respect to the exit mach number half of that. So, that is your theta max. So, that is all the information we have what we do not have right now is this contour and this is something that we need to design we need to say where you know various points of this surface is going to be look at. So, that is what is our problem is. So, the what we said was therefore, and essentially we have centered expansion wave say we are going to call this say let us give it give some name, we are going to call this say A, let us call this point say A all right.

And so, let us basically we have an expansion fan going from here. So, now, so, what we basically have is 1, 2, 3, 4, 5, 6, 7, 8; 1, 2, 3, 4, 5, 6, yeah I think that should work. So, what I have essentially drawn over here is an expansion fan now what is happening is this thing is good and we will have a similar point down here right. So, because this is I have just taken the symmetric part of it. So, I will have one more over here. So, similarly I have say at this particular point B out here right if I have a. So, let us just cut that there right.

So, at this point B we will have a similar expansion fan. So, when we have that. So, therefore, these are going to interact and then what are you going to get is this and so on and so forth, right. So, therefore, what we are going to do is just ignore this as you know bottom part of it and just consider this. So, essentially we will have one more like that.

So, that is what we need to have. So, we are going to pretty much have all this and this is also generating from here, we are not going to look at this. So, this is how the expansion fan is going to look like and if you ask. So, now, let us do the next thing to do out here; now once you draw this little fan expansion fan, what you essentially see over here is this is actually a grid right, we will do is a special kind of grid because what we have is these are left running characteristics the or red ones of pick ones and the blue ones essentially right running characteristics and we know from analysis the last couple of lectures that there are certain relations compatibility relations which are valid on these characteristics right .

So, now let us know the next thing to do here is to say number these points where these are intersecting. So, if I will do that. So, I am going to call this as say 1, this is 2, this is 3, 4, 5 6, 7, 8, right. Now just remember at this point of time that we have not actually; we do not know where this point 8 is at this point of time we have to find out where this is 8 and then again we come back and here.

So, say 9, 10, 11, 12, 13, 14, 15 and so on and so forth instead of going on each characteristic and labeling it. So, we come back in here. So, 16, 17, 18, 19, 20. So, we got say 21, we come back in here 22, 23, 24, 25; saying 26 come back in here 27 to that is it ok.

So, essentially. So, what we; what you can see from here if you now just sort of take a look back here what we have to do here is to design this nozzle which means I need to find locations of various points on the surface and I have basically 8 fifth 1, 2, 3, 4, 5, 6, when I consider an expansion fan with as many mach waves out here which is say 1, 2, 3, 4, 5, 6, 7; ok.

Including that if I have like if I am considering a wave front, which I am dividing into say 8 divisions I have 7 mach waves going in here then I have essentially 1, 2, 3, 4, 5, 6, points on this surface. So, I should be able to do look at these points with respect to this and hence, once when I connect them I will have a surface. So, clearly; obviously, if you can have more number of points here more number of points within this here etcetera, etcetera, then it gives up you know a better estimation of the nozzle.

So, for that all you need to do is take more and more number of markings as you can you can you can see clearly see that from here right. So, hence computational you know tasks will be more efficient if you write a computer program and then if you do this you know we can do this better all right now. So, now, so, therefore, I have taken this just to kind of run you through the actual procedure through which we are going to do this, right.

So, let us go step by step. So, what we have found out over here is our theta max right. So, our theta max is therefore, runner mach function and this is in 18.375; I think we did this; now the thing is how we look at these mach waves is really up to us this is are up this is really up to you I took I have taken like 8 here; 1, 2, 3, 4; yeah I have taken like 7 mach waves out here. So, you can take 100; you can take 200, you know you will give a better estimation of that. So, at this point. So, this is really up to us how we orient this is really up to us.

So, what we said was last time. So, this is my theta max. So, now, let this angle A 1. So, let us say; let me just develop a table.

So, let us say; this is the say left running characteristics right and we have the theta corresponding theta right. So, for A 1. So, for A 1, let us say, we will take as 0.375. So, what is 0.375. So, what we are left out with is essentially 18 degrees which we will divide amongst this which is 1, 2, 3, 4, 5, 6. So, 3 degrees each right.

So, therefore, what we will do is this is same within the within the fan delta theta within the fan. So, let us write it this way. So, within the fan delta theta is 18 by 6 degrees . So, therefore, for the characteristic say A 9; A 9 theta is going to be delta theta which is 3 degrees. So, again for A 16. So, we considering A 16 here. So, then we get there is A 16, it is 0.375, then similarly for say A 22 it is and so on and so forth. So, again for say 27 you have you know 4 times, 5 times, 6 times, right.

So, essentially; so, you know the. So, let us say fairly A 34 ; A 34 before this is. So, this is 3, 4, 5, 6; right. So, and you can see this gives 18.375. So, this is 18.375. So, now, that we have; so, basically we have calculated the thetas for each of these characteristics. So, what essentially we can say therefore, is that this 0.375. Now let us look at these each characteristic separately now if you look at this characteristic. So, this has a theta 0.375 which means this is equal to say 2 1 this point one is on this characteristic. So, this is corresponding to the theta to the theta of theta 1.

Similarly for this characteristic theta is you know there is 33.375 A 9.

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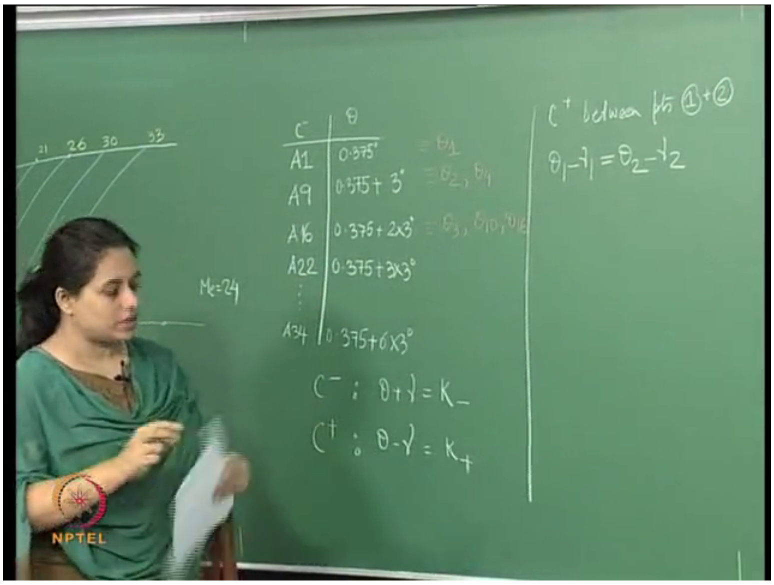

So, therefore, theta 2 and theta 9 are the same also this corresponds to and so on and so forth. So, similarly 16. So, you have A and so, theta 3. So, theta 10 theta 16 right. So, then here you have right. So, similarly for here you have theta 4 theta 11 theta 17 and theta 22 and so on and so forth; ok.

So, having done that now let us see how we can use what we are going to basically do now is going to apply our compatibility equations and what we saw from last time, right is that for C minus C minus characteristics for C minus characteristics, we have theta plus mu which is equal to constant and for C plus characteristics means theta minus mu which is constant right. So, therefore, ok.

So, now what we will do over here is that let us say look at this characteristic let us let us look at this characteristic the right running characteristic at one . So, the characteristic which is say what we are going to look at now is this characteristic. So, 1, 2, 3, 4, 5, 6, 7 and 8. So, let us look at this right running characteristic; now if we if we do that we need the right running characteristic. So, theta minus mu is a constant right theta minus mu is a constant. So, basically now on this right running characteristic we will say consider points one and 2 and apply the compatibility equation between these 2 points.

Now, let us see; so, what we are going to do here is the right running characteristics between points one and 2 right yeah between points 1 and 2. So, then what we get here is right. So, this is my compatibility equation because on the C plus characteristic theta minus mu is a constant right now look at this angle the this angle is like 0.375 which is like nearly 0. So, now, what we are saying is that the flow which is coming in here right flow which is entering here is sonic right. So, therefore, the polymers function for that is 0; is it not.

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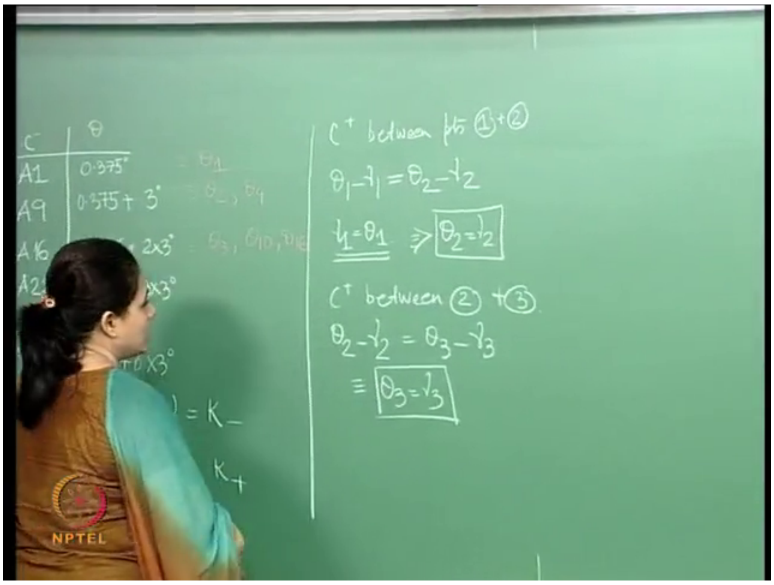

So, therefore, mu 1. So, mu 1 out here is equal to theta 1. So, which means that this is 0 which means that theta 2 is equal to mu 2 theta 2 is equal to mu 2 on this characteristic line on this characteristic line. So, theta 2 is equal to mu 2 theta 1 is equal to mu 1; now let us do that at say b; now let us consider say points 2 and 3. So, if I consider again C plus right between 2 and 3. So, what we get is theta 2 minus mu 2 is equal to theta 3 minus mu 3. Now we have just find out found out a theta 2 is equal to mu 2. So, therefore, again this means that theta 3 is equal to mu 3 and so on and so forth.

So, what we see is that in this particular characteristic level 1 8. So, this characteristic line at each point theta is equal to the polymer function corresponding to the polymer function right now having now if we do that; now let us now come here to the point 8 on the same characteristic line. So, now, we are at this particular point 8 which is the point which we actually need to design. So, if we come there.

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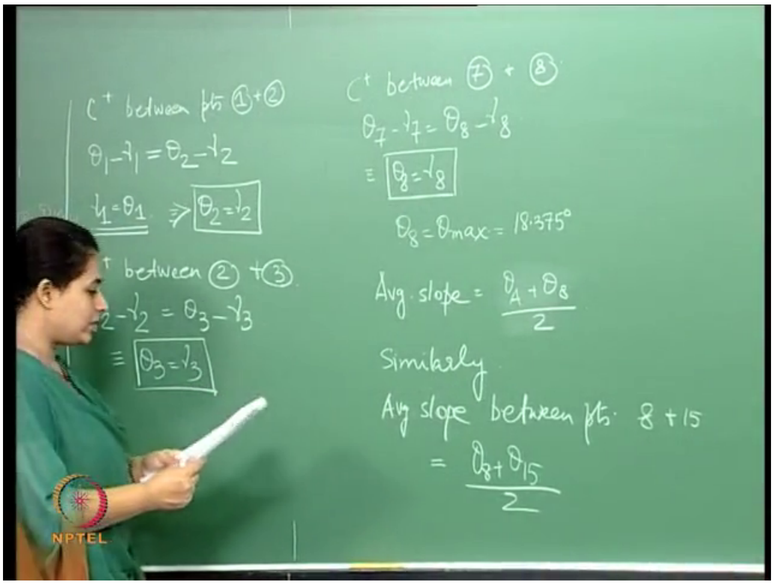

So, we will say now let us lets go here. So, again C plus between points 7 and 8. So, what we have is right. So, yeah; so, as you know we found out that theta 8 is equal to mu 8 theta 8 is equal to mu 8 right; now the point is how do we how do we look at this now theta 8 and mu 8 both are unknown. Now let us say here that finally, when we do a computer program and do this we are going to have many more such characteristics we should be able to do that. So, therefore, this point will actually probably get closer to this ok.

In this particular problem I have sort of exaggerated that just too sort of walk you through the procedure of doing this. So, therefore, a let us say that the point A is connected by a straight line to this point 8 it is connected by a straight line A and 8. So, that. So, essentially what I am saying is. So, say we have a point A over here ok.

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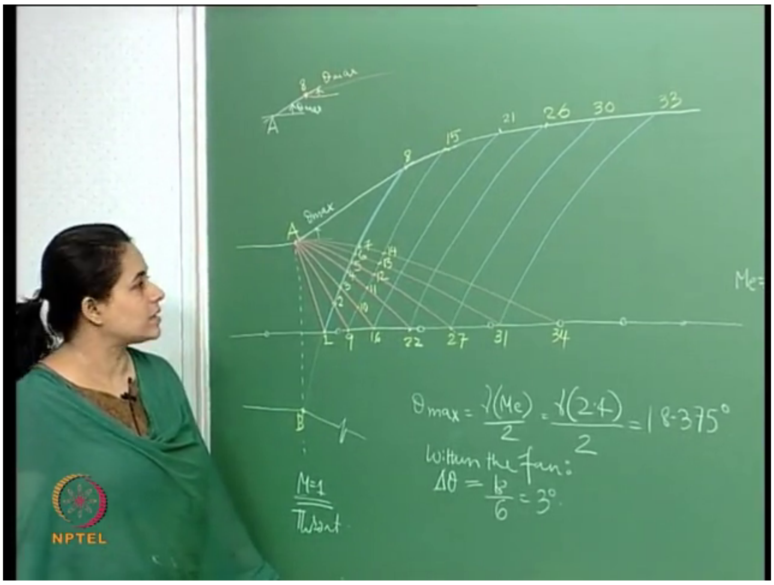

And if I draw a tangent to the curve at A; it makes theta max and I am saying that the I am going to draw a curve; I am going to draw a curve through this point a right that is my job out here.

So, what I am saying is that I will take a point here which is say in this case point 8 which lies on the same tangent which goes through goes through a which is we are going to connect A and this point 8 by a straight line which means what which means that this 2 is also theta max we are going to connect it by a straight line right. So, therefore; so, let us say that theta max right. So, the which is which is 18 point and 3 3 7 5; right. So, therefore, now if I if I sort of do that.

So, now what we will need to is to look at this point 8 right we will we will need to look at this point eight. So, what we will do is we will just sort of. So, theta max out here is again theta 8 is again theta max right theta 8. So, we have the same straight line going through here going through here. So, this is our theta max. So, the average slope between A and 8. So, the average slope this is now just geometry right. So, average slope is theta 8 or say theta A; right. So, which is which is what I am going to calculate from here right. So, we connect essentially. So, therefore, we connect this by a straight line to this particular point right.

So, now, if we do that, essentially what we do is we; you know continue to do this we continue to do this and again when. So, once we have done this out here. So, now, we have been able to look at this point eight. So, then we do the same thing coming back to this characteristics out here we come we come here and do the same thing over here and then we are going to look at point say 15.

Now, again how do we the point is how do we connect 15 and 8 again, what we will do is we say connect that by a straight line. So, say I connect that by a straight line and then the average slope is going to be theta 8 plus there are 15 by 2. So, say between the surface between. So, similarly you know between the points 8 and 15, right if we do 8 and 15 what we get is average slope here is and that is it. So, this is this is pretty much it.

So, hence we will be able to find out the point look at this point 15 similarly we go ahead and look at points 21, 26, 30 and 33 and so on and so forth. So, well, but that is all actually, there is to this you know this this sort of an exercise. So, this is actually the procedure. So, as such it is simple, but in order to do this in a more efficient manner; it is always advisable it we have to write a computer program for this. So, that we can you know get more denser grid and unless and until we get these points like really close to each other. So, then we have you connecting these by straight lines. So, you can understand you know.

So, we will get this points. So, we will get more and more points. So, that they are closer to each other. So, if you guide them by a straight line you know you know it should give us less error get the disco for error is very less. So, well that is it. So, we started out with a problem where we had just the; we said we have a design A; a minimum length nozzle which has an exit mach number of 2.4.

So, we use just this mathematical; you know method where we said its going to be a centered expansion wave. So, we got a grid something like this now which is a combination of left running and right running waves and from that information we were able to calculate what would be theta max here and then we got several points on the surface which we connected by is like connecting the dots right you connect that by straight lines and we basically what we are armed with is that on this grid we are solving the governing equations by means of the compatibility equations right. So, on the right running characteristics and left running characteristics we use the corresponding compatibility equations and hence we are able to look at points on the surface. So, yeah. So, that really is that is all there is to this problem.

Now, well there is there are essentially several applications of the method of characteristics. So, this was one of them. Now let us just see one more application and then then we will close. Now for example, so, say we have the problem here is that we essentially have a channel we have the channel is essentially it has the walls are straight. So, we have a channel like this right and its going to provide give us radial flow it is going to give us radial flow.

So, we have say a supersonic flow coming in we have given mach number then the incoming mach number is available to us. So, the flow comes in and then it diverges and we get like a radial flow out. So, we just need to calculate the properties. So, we have flow coming in with which is essentially diverging. So, if you do that. So, essentially what we have here.

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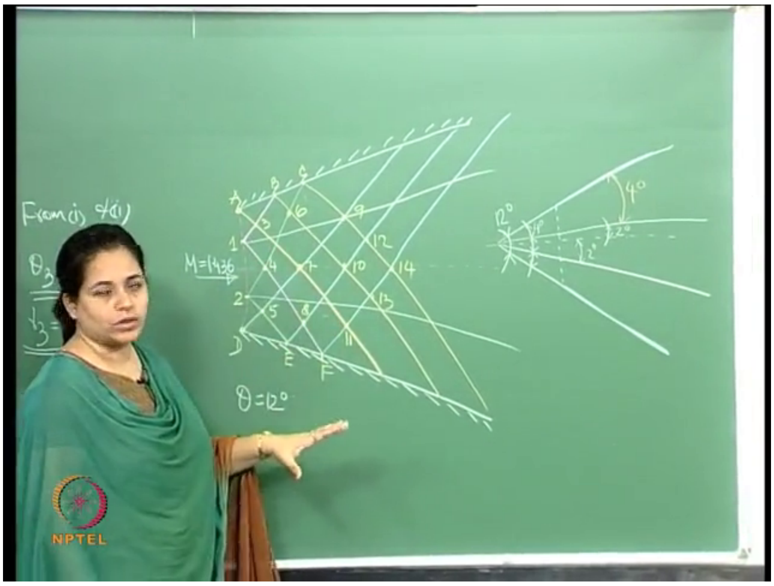

In this particular case is that we have in our channel like that we really need to be symmetric over here. So, I have a channel here like this we have a channel here like this and the and this is diverging by 12 degrees, it is diverging by 12 degrees and input mach number mach number coming in is 1 point say 4 3 6. So, how do we go about this? So, what we will do is if we.

So, now this is again this computational space is mine is it not; this is for the user defined. So, you can; obviously, when you write a computer program you can you know do more with this. So, let us just say right now what we will do is. So, let us define the.

So, we will have say A; we will have a center line somewhere right. So, let us say we will take let us say we will take we will consider this these 2 points. So, let us say I am going to call this as a I am going to take a point out here which is one and I am going to take a point out here as 2. So, we will basically take this now what we will do is that we what is happening we see well the flow is expanding is it not. So, it is an expansion fan. So, we will draw this expansion fan and unlike the minimum length nozzle. Now it can expand over lengths this is just a charm. So, we do not have to like it is not a centered it not necessarily a center expansion with.

So, if we have this now let us say now at every at you know at each point, we will have you know an expansion of the. So, we will have say you know the flow going in directions like this. So, say at this particular point. So, let us say we will have something like that and so, let us say we will take say intervals like this B and C you know some lengths from that and then and so on so forth, then we have ad over here I am trying to be as symmetric as possible, but sometimes my approach is pretty bad.

So, then yeah; so, then we will go like that see there I go this is supposed to be like really symmetric which is which is what I am trying to do here that is how I am going to do it in a computer program. So, then correspondingly. So, if I go I am going to take this point right here and say I have E and then run another one say through this point like that and then we have another one say F right and then I am going to run that one through this and so on and so forth. So, we have this o.

Now, let us look at this. So, we will have 2 more from there; now if I am and do this. So, essentially what is happening is now if I look at this if I look at this. So, essentially all right that goes through there and we have goes through F. So, again and you have one which goes right up to F. So, then you get do that and then similarly this is like what you confusing. So, then we have say this goes through yeah 2 goes through E. So, we have these series of; to goes through and goes through to C.

So, I have been as still not you know I wish I was little more my artwork was a little better. So, if this could run a little parallel to these things I would have it a little better, but anyways I am just sort of you know trying to be as you know symmetric as possible. So, essentially what we start getting is. So, we know we will get lines which are sort of parallel. So, what we are trying to do here is that the flow will come in and then it will diverge in a radial flow.

Now, let us look at some things you know we again before we do that; let us go ahead and number these like we have done before. So, we have one two. So, let us call this as 3 4 and 5 hopefully this will make a little more sense 6, 7 and 8 and you got say 9, 10, 11, right and 12, 13 , 14 and so on and so forth, right.

So, we have a sort of a non simple region out here. So, several right running and left running characteristics. Now if we look at this how are we going to go ahead and solve for solve for this. So, now, let us first get the geometry right. So, we said that the entire the total the total divergence is 12 degrees it is this total is 12 degrees. So, if I were to do this geometry right essentially what I have is say. So, say I am going to look at it like this right.

So, what I am looking at is essentially this is my channel out here right what I am looking at is like this. So, this is the part of the channel this I am drawing just to you know show you the geometry part of this. So, if I do this right. So, then this total angle is 12 degrees; right. Now I have taken basically 2 points out here. So, then I will have one like that. So, that now each of this right. So, each of these is 4 is 4 degrees right. So, therefore, now this is 2 degrees and you can see my artwork is not; it is not symmetric please bear with me.

So, essentially therefore; so, this is the streamline this is a streamline which is kind of going to you can see that sort of become curved it is not going to B. So, it is going to be a straight line. So, bear with me. So, I am going to sort of just run this through. So, I will have a streamline. So, essentially this point is going to be here somewhere. So, 1, 3, 6 and 9 and then again we will have this you can see we will just draw line like that. So, 2, 5, 8 and 13 and that is going to be another stream line. So, those are going to be on there.

So, now, this stream line essentially a is making 2 degrees and this is making 2 degrees the senses opposite right. So, what we have over here. Now let us do the numeric; let us do the numerics again, we are basically going to use the same right running and left running characteristics. So, if we do that now.

Let us say look at this point 3; let us look at this point 3. So, maybe I will sort of do that here.

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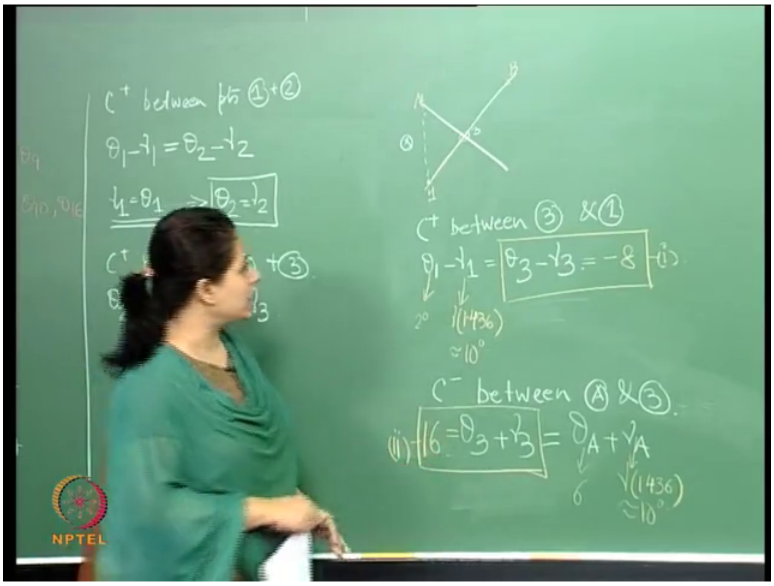

So, this is the point 3. So, at the point 3 this is my point 3. So, here is. So, we have here we have A and here we have one and here we have B and so, that is it. So, if we if we do this. So, let us look at the let us look at the right running characteristic right running characteristic between 3 and one between 3 and 1. So, what we are looking at is the C plus characteristic between points 3 and 1.

So, therefore, which means that if I look at that. So, therefore, theta 1 minus mu 1 is equal to say 2; 3 minus mu 3. So, now you of course, know that this is at the a and one these are at the; these are at the is the entrance right is the entrance the mach number is known here mach number is known here and we also know the theta here which we know because of the geometry which we know because of the geometry. So, therefore, theta 1 and mu 1 or known.

So, theta 1 essentially this right corresponds to a mach number of 1.436 which is around you know 10 degrees and theta 1 is the 2 degrees as we just saw from this this is the geometry definition right it is given that the total divergence of this is 12 the channel is diverging, but 12 degrees this is given to us and since we have taken basically divided this radial space into 3 parts. So, we found out that this is going to be 2 right. So, therefore, theta one is 2 which means that theta 3 minus gamma 3 is equal to minus 8, right.

So, therefore, this let us say this is let us just say this is this is one relationship that we get. So, theta 3 minus mu 3 is equal to minus 8, all right. So, then this is it now next thing what we will do is take a left running characteristic between A and 3. So, let us take left running characteristic between a and 3 right. So, again and this is my compatibility equation right. So, now, if I have this now what is theta A and what is mu a now mu a is again this right. So, now, mu a mu a out here is again mu 1.436 is it not.

So, this is again because this is also at the inlet. So, this is also equal to polymer function corresponding to 1; 1.436 and this is again around 10 degrees and theta A; now what we saw from here theta a is this my wall out here.

So, as you can see from here this this here is; so, therefore, theta a is 6 degrees. So, 6 degrees. So, which means that theta 3 plus mu 3 is equal to 16 right. So, let us call this as. So, what you basically have here what you basically have here is these 2 equations and you can just eliminate theta 3 and mu 3 from these 2 equations simultaneously.

So, what you get from here. So, what if you do that what you get is what you get is. So, if you eliminate theta 3.

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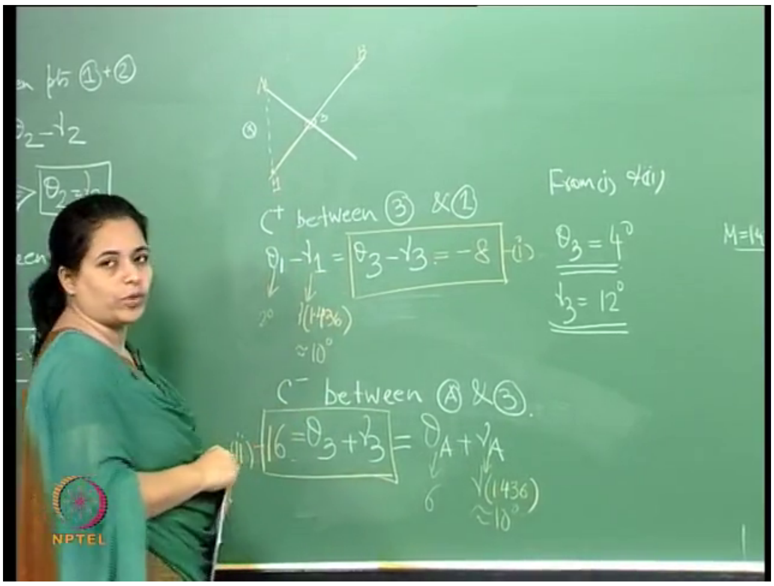

So, basically from theta 3 is equal to theta 3 is equal to 4 degrees and mu 3 is equal to 12. So, therefore, you we are basically able to find. So, now, we can find out the properties now having known the polymers function. So, we can correspondingly find out the rest of the properties there.

So, therefore, theta 3. So, we take this particular point and we get 4 and theta 3 is 4 and the theta 3 is 4 and mu 3 is 12 right. So, yes right. So, therefore, the point 3 is done now once we do point 3 now let us let us look at say point 4; point 4 is here. So, on the center line it is on the center line, but let us go ahead and solve for properties at point 4. So, similarly basically we will be solving for other points; I mean this is the procedure to go about throughout the entire you know you know we basically divided this thing into a grid which is found by the characteristics ok.

So, let us do that let us go and see; say of point 4 right.

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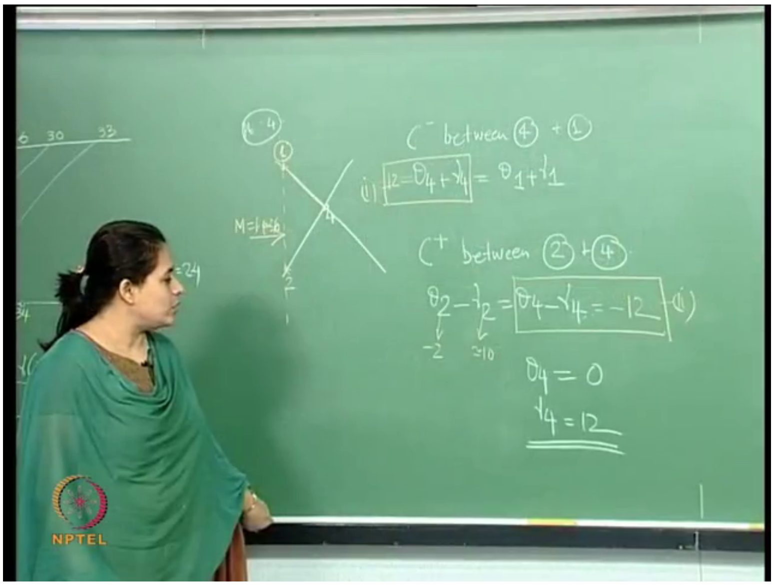

So, let us see what we get; now at say point 4. So, this is point 4 now let us see it. So, this is point 4. So, what do we have here? So, we have now set of characteristic passing through that. So, then we have a point 2 over here it passes through 2 and it passes through 1 and this goes through to 6 and C that is fine. So, the here this is what we have 4 and 1.

So, now let us look at this. So, this is a left running characteristic. So, now, let us look at the left running characteristic between 4 and one if I do that. So, this is C minus between 4 and one. So, what we have is theta 4 plus mu 4 is theta one plus mu one right now theta one is 2 and mu one is 10 at which we found from before. So, therefore, now. So, therefore. So, this is equal to 12 this is equal to 12. So, let us again call this as say one and again what we will do is we will take 2 and 4 right 2 and 4. So, it is C plus; so, now, we are going to look at the right running characteristic C plus between 4 and 2 basically right if we do that then.

Now, again theta 2 here. So, now, theta 2 now mu 2 is something that we know again which is around 10 degrees right because again this is at the inlet right. So, this is all at the. So, inlet. So, mu one mu 2 is all the same. So, this is the inlet. So, now theta 2 now let us come here now theta 2 we are looking at this stream line which is the stream line. So, you can see that this angle is just minus 2 it is it is on the other side of the centerline just the sense is difference the same as the stream line going through 1. So, therefore, this is minus 2.

So, then this is minus 2. So, what this becomes is essentially this becomes this is equal to minus 12 that is all. So, then we get this and again we will now solve for theta 4 and mu 4 from here if we do that if we do that what do we get which is correct which should be theta 4 is 0 right because it is on the centerline theta 4 is 0 and mu 4 is 12.

So, then now we basically go around you know doing this for each and every point and then and then we can basically find out the properties. So, this is another essentially application of this method of characteristics divide the space into a certain grid, this is user defined of course, and then, use the compatibility equations and find out the properties. So, that should be all.

Thank you.