Advanced Gas Dynamics
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Lecture – 19
The Shock Tube: Propagating Expansion Fan

So, continuing from last class. So, basically we were trying to figure out the expansion fan in a shock tube right and we saw we kind of began studying the method of characteristic, so that we can study the change in properties behind and in front of the expansion wave as well as the property changes within the expansion fan.

So, we stopped kind of to the last lecture, where we just at the point where we were about to find out relationships for the property changes within the expansion fan and behind and in front of it. So, let us go ahead and continue from there and see where we go.

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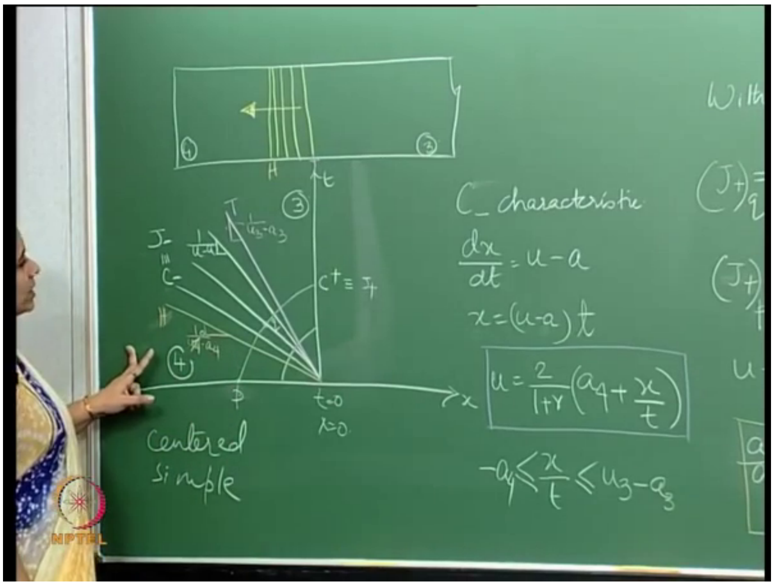

So, just to remind you this is what we had right. So, we had this was our shock tube right just drawing this half of it.

So, then this was region 4 and this was region 3 and we had a; you we have an expansion fan all right moving into this quiescent flow quiescent region right, we had that and oh the this whole these a way is basically originating at a single point. So, we found out that if I were to draw an xt curve.

So, if I were to draw an xt curve like that then I could actually represent these waves on this curve this being the head of it which is this right and the other being the tail of it which is this and essentially these are; so, this was our these were characteristic lines, these are lines in which the governing equations are have their solutions. The solutions to the governing equations lie along these lines and hence characteristic equations.

Now, we also talked about and we said that the slope of this was basically u minus a right yeah sorry. So, u by a and this was. So, therefore, this the head is you can see is moving into region 4 and this is region 3 and so, therefore, the slope of this becomes that and this becomes slope of that that becomes. Now in this region is 0 is it not because this is quiescent.

So, therefore, this is minus 1 by a 4. So, this is where we had stopped. So, this is essentially the as you can see the wave is basically centered, this is a t is equal to 0 centered and we also showed that these. So, these are the C minus characteristics and we also drew see these could be possible C plus characteristics and essentially what we see over here is we also talked about whether these characteristic lines would be a straight lines or could be curved right.

Now, in general case, they do not have to be straight lines, for this particular case, we prove that yes these are straight lines. So, hence we said that in a shock tube essentially what we have is a centered simple wave simple because a straight line centered because is centered and we also found out that we also said that on C minus the C minus corresponds to J minus these were the Riemann invariants and on C plus corresponds to J plus.

So, we and we then must showed that is along the C plus for such a such a case the J plus is well you know J plus is constant on any C plus a characteristic, but then we took to C plus characteristics and we showed that essentially within the expansion fan essentially within the expansion fan J plus is a constant, right.

So, therefore, let us start from there.

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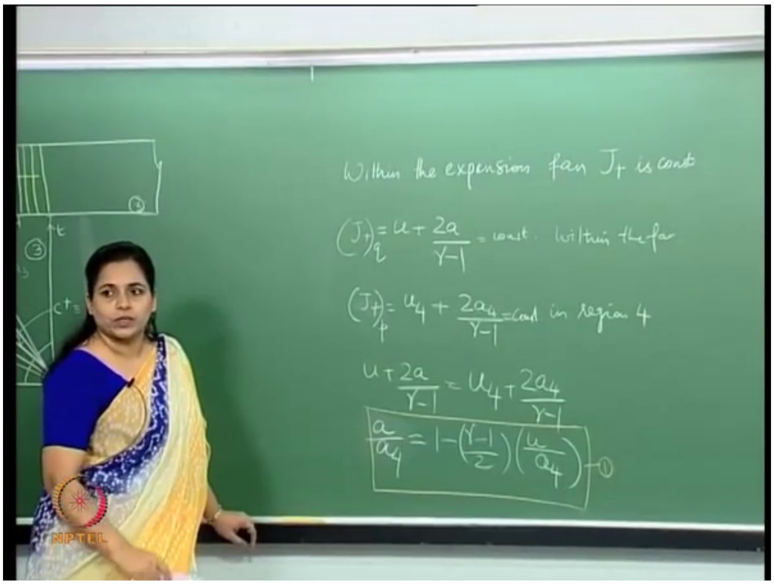

So, say what were basically saying is that right. So, within the expansion fan J plus is constant. So, which means that I can take any point out here for any see any positive characteristic and I will have the same J plus and so, let us do thing; let us apply this for a point within the expansion fan and a point in say region 4. So, now, J plus is essentially right. So, J plus within the expansion fan; so, is equal to u plus 2 a by gamma minus 1 right this is a constant and the J plus if I apply this if this I apply to region 4 then what I get is that this is say within the fan right and J plus is u 4 plus 2 a 4 by gamma minus 1 this is this is in region 4 in region 4.

So, therefore, basically if I; what I am saying is let us say consider a region a here I say let us consider a point say p let us consider a point say q. So, say q. So, this is J plus q and J plus p right; now they both lie on the same characteristic. So, therefore, from here what I can find out is that hum. So, they lie on same characteristic. So, J plus is going to be constant. So, therefore, I can say right. So, now, from this what we can get is if I work with this what I get is this from this expression what I will get is this.

So, we get this sort of an expression let us say call the cells one now basically what we do here is find out properties right find out the relationship properties within the expansion fan.

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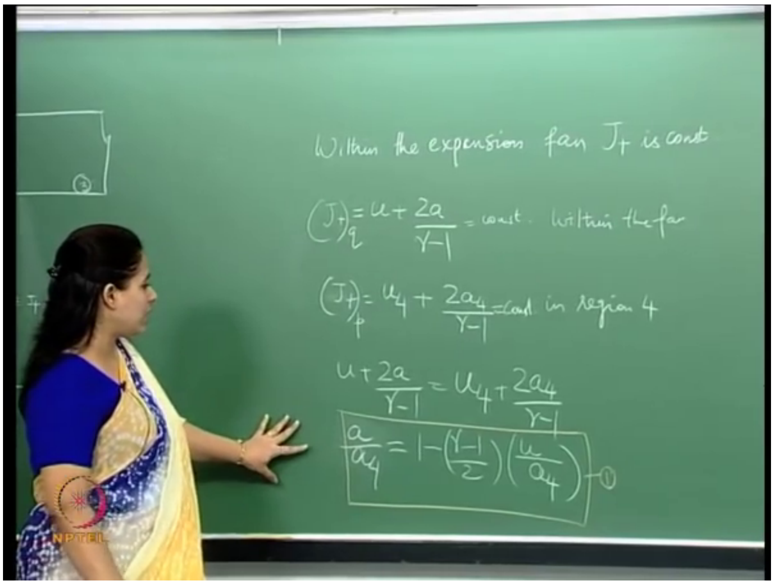

Now, what we know is that right and what the relationship that we have is a by a 4 right. So, therefore, I can now the therefore, write. So, from this basically what I can write is t by t 4. So, you could sort of do that yourself.

So, this is t by t 4 is 1 minus gamma minus 1 by 2. So, what is interesting here is that this is the any local velocity within the fan this is the in local velocity within the fan and this is the speed of sound into the of in the region into which the fan is advancing and this is where the flow is stagnant or quiescent.

So, similarly now since this is an isentropic; you know it is an isentropic process. So, again we will have this. So, now, you can see from here. So, therefore, we can write p by p 4. So, therefore, p by p 4, we will get these 2 relationships here now. So, p by p 4 therefore, becomes I am just rewriting that this is for completion. So, this is the relationship of the pressure this is relationship of the pressure and let us also write the gamma.

So, and sorry rho. So, rho by rho those densities is essentially equal to this t by t 4 to the power one by gamma minus 1. So, which is I thought this should be 2 by gamma minus 1 to the power gamma and rho by rho 4 should be equal to 1 by gamma minus 1 this should be I think this should be this I should be I think this should be this.

So, I will have to cross check that if I am getting that wrong or not. So, I will just cross check that you can you can check it also it should be given in any standard book. So, just check this I think this should be 2 by gamma minus 1, but I see in my notes I have written one by gamma minus 1. So, you just have cross check that. So, essentially what we have done now. So, therefore, what we have done now is got an expression for the properties within the expansion fan right.

So, we have a; this is the looked temperature within the expansion fan this is the pressure within the expansion fan this is the density within the expansion fan and these all are you know with respect to the temperature pressure and the density in the quiescent region. So, having done that now let us see now this is a velocity u which is within the expansion fan.

Now, let us see now we know that within in a C say C minus characteristics. So, we have a C minus characteristic here where J minus is constant and the slope; obviously, the is dx dt is it not. So, u minus a is it not. So, dx on a C minus characteristic; now dx dt is u minus a which means that essentially right if I do this. Now let us look at this here this equation here and therefore, from here I am going to write x using equation 1.

So, when I can basically get rid of this a. So, if I do that what I get is this, now what is interesting to this is a interesting result. So, what we see what we see is that the local velocity is given in terms of the speed of sound in the in the region in which into which it is moving which is the quiescent region and x by a t; now what you can see from here and also you should instead of cross check yourself that 4 right.

So, essentially what you are saying is that the x by t here right. So, that is u 3 minus a 3 which is here right and it is minus a 4 which is in this region. So, all we are saying is that the; you know basically the inverse of slope which is here inverse a slope is essentially the u 3 minus a 3 in the region 3 and here it is u 4 minus a 4, but this is quiescent. So, it becomes minus a 4. So, this is a limit this is a limit of the x by t.

So, what this is essentially giving us an idea is the range of the expansion fan in the sense that within a given time more or less what would be the distance to which this disturbance will travel. So, this is what we get from here now. So, before we you know sort of plot this before we look at this and see how they look now let us go and find out how we going to calculate the relations in this region and this region because in the especially in this region because now we have done the other part of it where we had the contact surface moving and let us draw that. So, we had this was the shock part of it.

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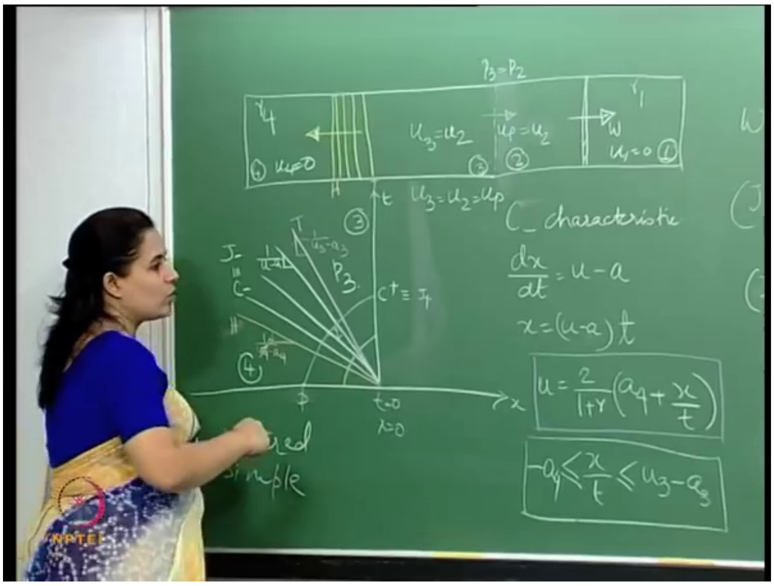

So, we had this was the contact surface and this was we said moving with up, right and we had a shock wave and that was moving yet into another quiescent region. So, this was region 2 and we said this was moving with yeah. So, this is equal to u 2 actually is it not and this was moving with cell velocity w into a quiescent region.

So, now the thing is we have adequately calculated relations in region 2 and one. So, we kind of no relations in region 4 and we are still left to find out relations in region 3 and we have now found out expressions for the properties within the expansion fan. So, to do that now here u 3, now once this once the diaphragm is broken this is the original location of the diaphragm is you know it sends out a shock wave which induces motion in the fluid behind it. So, in this case therefore, now this is a contact surface now as we said it is like a slip surface. So, therefore, the pressure across it is the same. So, we have said the p 3 is equal to p 2. So, would mean that u 3 is also equal to u 2; right.

So, having said that. So, so essentially what we get from here is that u 3 is equal to u 2 is equal to up. So, if I do that let us say some now and as we have said that it does not need to be the same fluid it could be 2 different gases right. So, let us say we are going to say called the gas in we or the region one and for say corresponding gamma is gamma 1 and gamma 4. So, let us just say for generic case that the; you know the ratio of specific heats is 1 and 2. So, we have 2 different gases.

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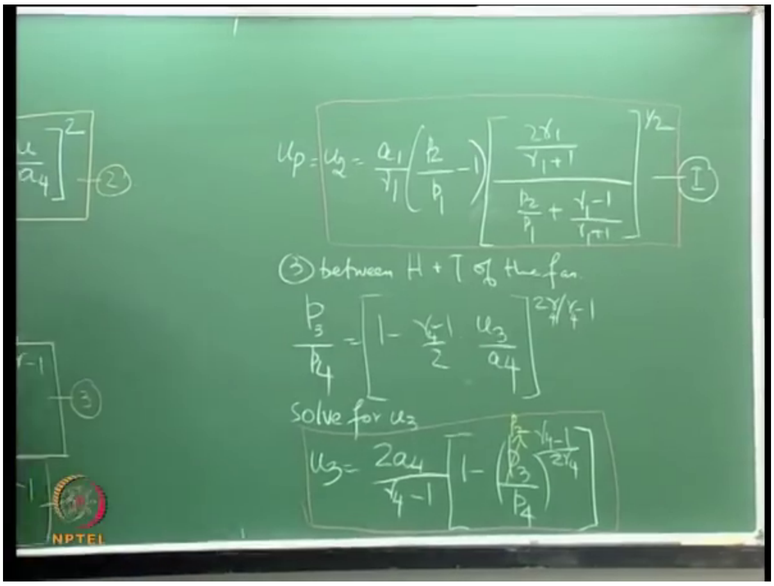

Now, this is something now up. So, therefore, so, this is now this is what we did earlier with the shock wave this is what we did earlier with the shock wave and we you can see that this is dependent on the pressure ratios in the regions 1 and 2, right. So, we had a value we had an expression for the velocity which is u 2 up right in terms of the pressure ratios in the regions 1 and 2 and in this case we are basically considering the gas in region 1.

So, therefore, this is what we have this is what we have from the shock analysis which we have done before; now what we will do is who will use this equation the pressure equation here the you can see that this is pressure equation here and we will apply to you know anywhere between the head and tail of the expansion wave. So, this is the pressure anywhere in a within the regional anywhere within the expansion fan.

So, if I look at this fan and I apply this between the head and the tail of it. So, head is this region which is p 4 and tail is essentially this region. So, which is p 4 sorry p 3; this region. So, what I am going to do is apply this equation 3. So, equation 3 between head and tail of the fan right if I do that. So, essentially what will I get; what I get is this yeah.

So, let us write the gammas out. So, we will write it this way gamma 4; gamma 4; gamma 4. So, we get this now what we will do is we will solve for u 3 from here we solve for u 3 from above if we do that what we get is this. So, one. So, I just solved for u 3 from this expression and I get this right, but of course, we know that p 3 is equal to p 2 right because this was the contact surface we said that there.

So, therefore, here p 3 is of course, equal to p 2 as well. So, essentially this becomes the relationship. So, u 3 is to you know u 3 we get in term the velocity in region 3; we have now got in terms of velocity of sound in the in region 4 the pressure in region 2 and region 4. So, we have got u 3 here let us look at this now.

So, essentially this is my u 3 and we have u 2 from here and we have u 2 from here. So, all we will do is now equate u 2 and u 3 because these are the same if I do that then what I get is this. So, say coming back to this here. Now if I do that if I equate that the slightly longest kind of thing.

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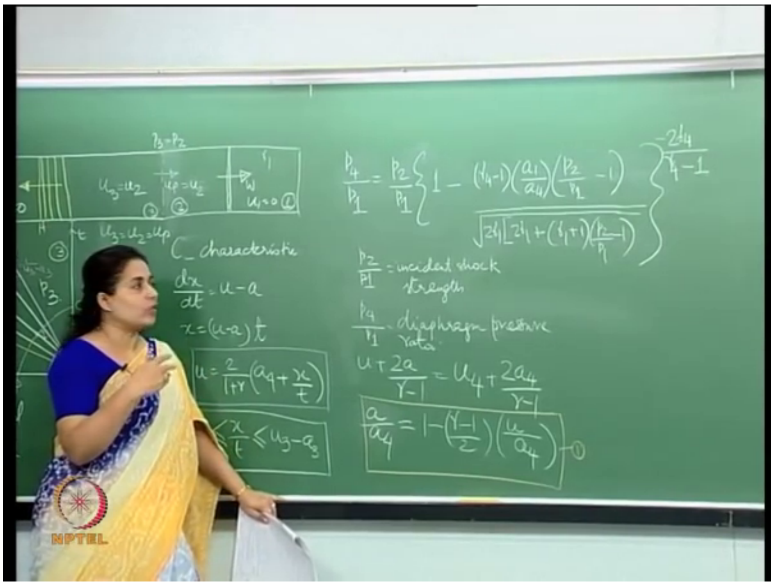

So, I will just sort of write it out a little patient. So, so p 4 by p 1; a 1 by a 4 and all of that is raised to ok.

So, if all are done here is essentially equated u 2 and u 3 and this is what I get. So, you can see that this relationship when I equate these velocities this relationship is all in a given in terms of the pressure ratios. So, now, p 2 by p 1 is the incident shock strength is it not. So, for a shock tube unlike in the shocks which we have done before in such a case the strength of the shock is driven by the pressure ratios across the across it.

So, which is what we see over here right. So, p 2 by p 1; p 2 by p 1 is basically the incident shock strength right and what is p 4; p 1; this is essentially the p 4 by p 1 is essentially the pressure ratio across the in across the diaphragm. So, this is say diaphragm pressure ratio. Now the point here is if you look at say if you look at this particular expression over here.

So, now for a given diaphragm pressure for a given diaphragm pressure and for given specific ratio of specific heats of you know 2 different gases in the 2 sections which is 4 and 1; you can actually calculate the strength of the incident shock wave directly from this expression directly from this expression. So, therefore, let us sort of look at you know I will just illustrate point by point as to sort of what how will we go about this.

So, now, we are in a position where we can essentially solve for properties in the entire shock tube let us see that now that we have found we have we have an expression say like this what is the first thing we do. So, I will leave these expressions over here. So, let us say hum say let us say over here.

So, first things first of course; so, first things first is that for a given diaphragm pressure for a given diaphragm pressure. So, let us call this equation something say for let us call this 5.

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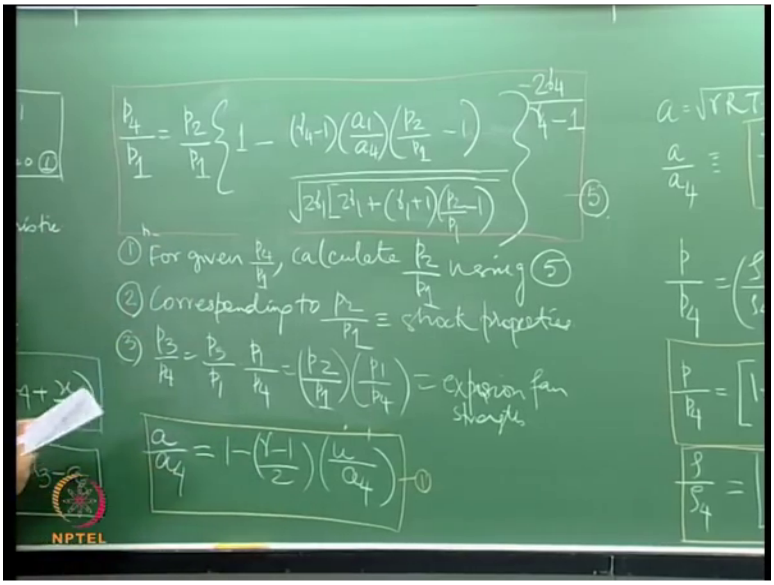

So far given diaphragm pressure ratio which is p 4 by p 1; so, you evaluate or calculate insulin shock strength right you can see from here directly.

So, for a given pressure ratio diaphragm pressure ratio you can calculate the incident shock strength p 2 by p 1 using five this particular equation and then as we have already done for shock waves once you if for an unsteady shock structure like this for unsteady structure like this the properties basically depend on the pressure ratio unlike the steady cases which we have done before where they would dominated by the incident mach number.

So, therefore, once we have calculated the shock strength we can totally calculate the properties across it right. So, yes; so, using the shock; so, for the corresponding to. So, p 2 by p 1; we calculate all shock properties. So, we can calculate all the shock properties now once I do that now what I need is essentially p 3 by p 4. So, p 3 by p 4 or the strength of the expansion fan. So, I mean the see that is easy to do. So, so essentially p 4. So, I will just write that as you can see p 3 by p 1; p 1 by p 4, but then this is p 2. So, which is p 2 by p 1; so, now p 3 by p 4 if you see this is the region. So, p 3 by p 4 is given by this now p 2 by p 1 is known at this point of time and this was given.

So, then this is your expansion shock strength sorry; sorry, expansion fan strength right. So, this is your fan strength. So, now, what we will calculate is the properties right; now within the expansion fan now that we know p 3 by p 4 right if we know p 3 by p 4. So, let us go back to these relations over here. So, if you look at this now p by p 4 here.

Now, once what we can do now is essentially use isentropic use the isentropic relationship which is it is here I have it written over here. So, in when I have this relationship over here. So, but at this point of time what I have found out is p 3 by p 4. So, I need. So, let us now calculate the properties in region 3 and region 4 in region 3 and region 4. So, we are not talking about the inside the fan we just talking in these in the region behind and front of the expansion fan.

So, now this is the isentropic relationship. So, now, if I apply this now here p 3 by p 4 is what I have just calculated. So, p 3 by p 4 is known and therefore, you can calculate the density and temperature ratios. So, that can be done and after that what we need is properties. So, what we need is properties within the fan. So, within the fan we use equations say 3; 4 and what oh; 2, 3 and 4.

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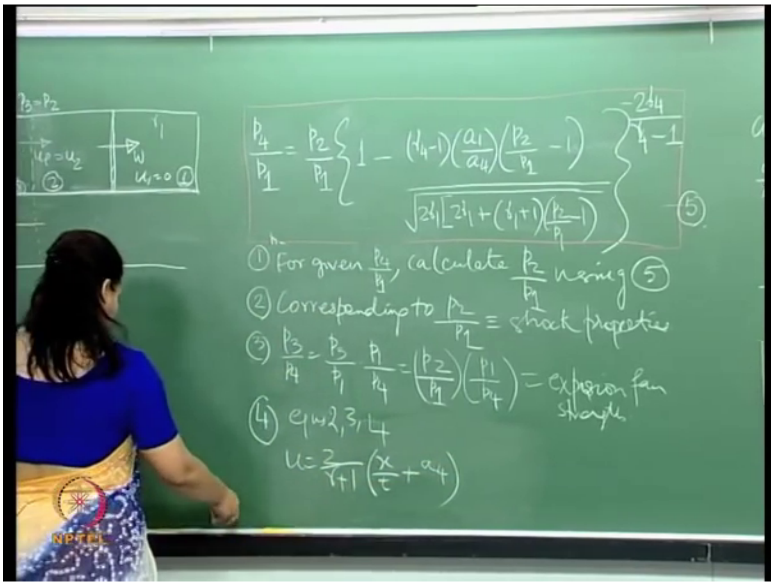

So, within the fan then we should use these equations. So, this is 4 within the fan my temperature is this temperature ratios is this 3 and 4. So, with using those equations I can calculate for the properties within the fan. So, that pretty much gives us solution for the shock tube. So, now let us just sort of go back to the picture that I had drawn right at the beginning you know which you did not have any way of sort of deciding whether it was correct or to agree with it or not agree with it. So, let us sort of revisit that one more time ok.

So, essentially what I am going to try and do is you know plot the pressures temperatures and densities and velocity and see if what I had drawn right at the beginning is something that you agree with. So, say this is a little asymmetric, but that is let us draw it over here. So, so that essentially this is my shock tube; so, essentially this is my shock tube this is my shock tube is it not ok.

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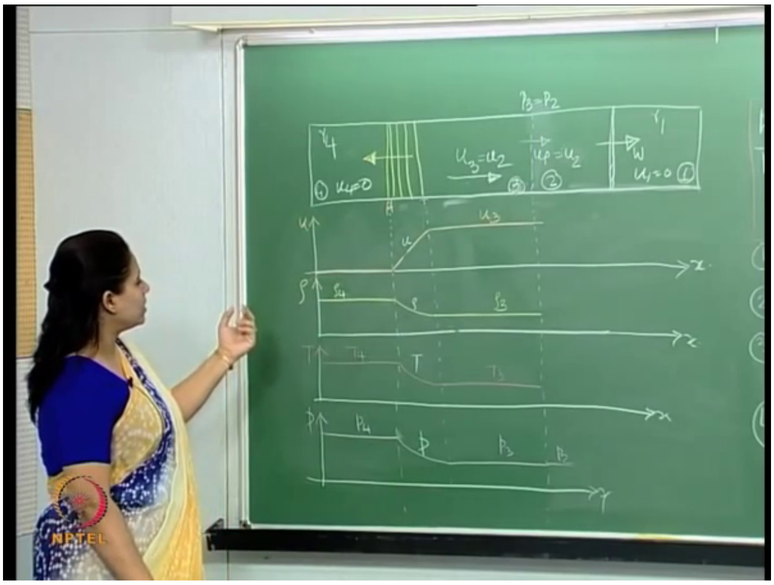

So, now let us just draw this. So, let us say and I am going to concentrate this is something that we have done before right let us just concentrate on this region till here. So, this is in region 3 and region 4 which is what which is what we have been studying the past part to lectures right. So, this is basically regions concentrating in front and behind of the expansion fan. So, if I do this. So, let us mark this out. So, this is the head and this is the tail, ok.

Now, if I do that let us take a different color. So, if this is this. So, let us say; this is let us call the velocity now you can see that the velocity a here is 0 is it not its u 4 is 0. So, therefore, this is 0 this is 0 and then in u 3. So, u 3 is some velocity which is you know given by the expression. So, it is some velocity it is some positive velocity. So, this is essentially u 3 and at this point, it is equal to u 2 over here. So, we are not going beyond that. So, this is u 2 essentially. So, u 3 now what is left is what is happening inside the expansion fan.

So, let us see if we have any relationship for the velocity within the expansion fan. So, if you go and look at it; if you go and look at this we just wrote out the equation some time back. So, what we had was what we had was; so, this is the velocity relationship and this is essentially x. So, over x what is the relationship of the velocity?

So, you can see it is a linear relationship if you can see from that from this particular equation that the velocity here rest everything is constant the velocity and x is a linear relationship right., within this region. So, essentially what you can say what basically we are saying is that we have the; you know a velocity which is u 3 and then that linearly decreases to a 2 to 0, right as the expansion fan moves into the quiescent region. So, that is the velocity. So, then let us now next look at say the density. Now let us then look at say density again this is all x . So, let us say this is say density ok.

Now, rho 4; then there is some density here. So, say this is rho 4 and rho 3 now rho 3 rho 4 by rho 3. So, in here I think we will have to kind of do the numbers for calculate something by numbers and then you will see like we had seen for the shock wave as to where the pressure decreases increases in this case, I can say that the fluid here is moving in here right the fluid moving is moving to its right. So, u 3 the velocity of that is this way and therefore, we did C minus characteristics right.

So, the wave is actually. So, this is the movement here this is here, but the particles within the wave are moving opposite to it hence we had within the waves or within the disturbances moving away from the way. So, therefore, we had C minus characteristics. So, that is the reason why we will have now the density will decrease in here. So, density is say rho 3. So, rho 3 this is rho 3 and if you come here to this expression over here. So, this is the relationship of this is the relationship of the densities and you can replace this by the x relationship with x by t here.

So, what we get is which is a non-linear variation of the density within the wave. So, we can I think write that any point within. So, this is u this is rho. So, again if I similarly if I draw again x and let us say; this is the temperature. So, similarly now you know this is this will kind of follow I guess this is t 4 and then we have t 3 and again this is a non-linear relationship.

So, you can sort of see from there and again finally, the pressure. So, then we have pressure right. So, p 4 we have p 3 of course, this is equal to p 2 that is all we know and again in here we have a non-linear relationship. So, therefore, this is p. So, therefore, what we can see. So, these are essentially if you look at this relationship. So, this relationship say if I call 1; ok.

So, let us say I call this as you know. So, something; so, you know the velocity basically velocity we plotted this using this relationship and these are rho t and p with 2, 3 and 4 using equations 2, 3 and 4 we were able to kind of figure out the nature of the changes within the expansion fan and you know behind and aft of it. So, essentially what we have done so far is that we have been able to calculate the change of properties in the entire shock tube and we have derived expressions using method of characteristics in this particular case.

So, method of characteristics is something that we will do now a little detail and apply it for some more different you know problems and we will see how that works. So, in this case what we just to sort of you know summarize this in this case what we therefore, saw was that this expansion fan is it basically results in a straight line characteristics moving left which was negative essentially because my fluid the disturbance here is moving away or in the opposite direction to the to the you know fluid here does when the shock show when the diaphragm is broken and the shock starts moving to its right in this particular case into the quiescent region it sets the sets this; this fluid which is behind it into motion as well it induces velocity there.

So, this starts moving to the right; however, an expansion fan therefore, travels to the left the disturbance right. So, therefore, the particles within the expansion fan they are moving to the left where the fluid the fluid in totality is moving to the right. So, and you and that is that is something that you can see represented in the properties as the velocity decreases the; is in an isentropic process it is an isentropic process. So, as the velocity decreases velocity decreases linearly within the expansion fan and then the density density temperature and pressure all increase as in a non-linear manner non-linear manner; it is an isentropic process and therefore, and you know reach the constant values in this quiescent region and we found out that the characteristics for these are straight lines which is and this is centered because they originate at the same point and move into the quiescent region.

So, it is a simple wave; now we will see different applications of these characteristics like I said when we you know continue the next couple of lectures and also what we saw here for the shock wave is that this is an unsteady motion right and the way we were; we solve for properties in this particular case is basically change the reference frame to change the reference frame.

So, as to do; so, as to get a picture which we were familiar with the earlier which was the steady case and for that all we did was superimpose these 2 regions in here with a velocity which is exactly the same in magnitude, but opposite in direction to this the velocity of the shock wave and that with that we were able to calculate properties behind and in front of it and what we have seen here of course, is that unlike in the steady case unlike in the steady case where the incoming mach number governs the nature of properties behind and after the shock in here that is governed by the pressure ratios across the shock wave and similarly. So, for a given for a given diaphragm pressure right.

So, initially when we have the diaphragm we have a pressure, we have just this region 4 and 1 and all of this is not happening at the time. So, then we have a pressure say p 4 and p 1. Now if for a given pressure there is a given there is a given shock strength. So, shock strength and using that we can therefore, calculate these; the properties across it. So, all right I think that is all about the shock tube what we shall go further with a some more applications of the method of characteristics.

Thank you.