Advanced Gas Dynamics

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Lecture - 17
Wave propagation: Small Perturbation Theory

So, let us sort of continue from that. So, the wave is discussed that waves are basically produced by a perturbation over ambient conditions.

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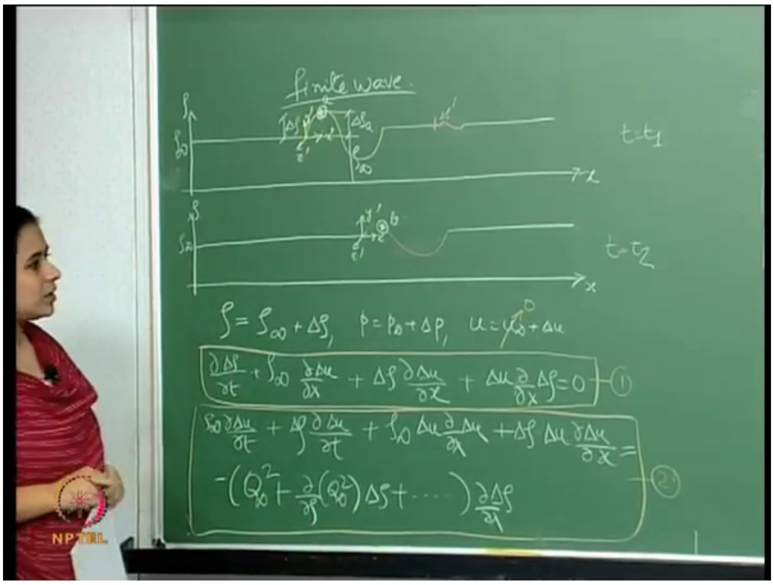

So, we said that if you have for example, say if I were a plot say serve this, and say this is say density right. So, density and say at some at some time say t 1. This is at some time; this is a picture that you are seeing at say some time t is equal to t 1. So, if I have a an ambient conditions of say gamma infinity, then at some point which one impulse I have a certain disturbance which is in the, which is introduced into this ambient conditions. And then so, we said delta gamma and we can have the same picture for pressures, velocities, etcetera.

Now, then what we said also was that if I look at the same plot, I would say at another time instant which is at an advance time. So, then again what happens is; that is this disturbance also travels again this is gamma infinite this distance also travels, but now it looks different as well. So, say, right. It looks different as well. So again, therefore, if I possibly took a reference frame say here, at the start of this. Say if I took a reference frame at the point where the disturbance has started. So, I do not take the same thing over here, right. Then if I have, if I say I take a particle; if I take a particle call that as particle a. That particle is at a different location at time t is equal to t 2.

So, what we are basically trying to say here; that particles within this disturbance move differently. So, they also the each particle is moving differently within this disturbance, all right. Now mathematically what we said that is the governing equations the continuity conservation of momentum and the energy equation.

So, if I you know if I introduce the perturbations into that. If I say that is the density term is the total density is actually the ambient density plus an induced plus and induced perturbation right. So, that gives me the total density, because of what I or the local velocity the local density velocity here is basically due to this perturbation, right. But the total velocity density is basically when you add this local velocity to the ambient conditions, isn't it? That is what you will see.

So, here this is the delta t isn't it? So, add this for a for example, this is the perturbation, or this is the change in the, this is the is density when I consider is just within this just within this wave, or just within this disturbance. So now, the total density; however, here is when I add the ambient condition to that. So, therefore, I get den total density of came infinity plus this. So, similarly we can have you know the and so on and so forth.

So, in our derivations yesterday in the last lecture what we did is that we said this was initially at rest. So, we said initially this was at rest. So, therefore, the entire change in the velocity is happening because of the disturbance of the wave. So now, what we did was we introduced these. We introduced these perturbations into the governing equations, and what we came up with was; well, it was some slightly a large sort of equation, but I will go ahead and write that.

So, what I came up with was this. So, we came up this from the continuity equation. And the and from the momentum conservation and energy equation we came up with this. So, this is; so, actually essentially have quite an extensive. So, that is enough to confuse us right. So, we have this. Let us call that a same 1. And so, we said finite waves what a finite waves when this perturbation is finite. It is not it is not very small. Or it is not something that you will ignore. So, you do have a finite perturbation. So, you have a finite wave.

So, for these picture out here the governing equations in are basically these. So, this is the continuity, and this is a combination of the momentum and the energy equation. So, this is what we get. So now, we saw if you look at this. So now, we said this seems to be too complicated. So, let us do one thing.

Let us consider this to be very weak wave. Like a sound wave, and what we will do there is we will we will go ahead and say that delta rho all these perturbations etcetera.

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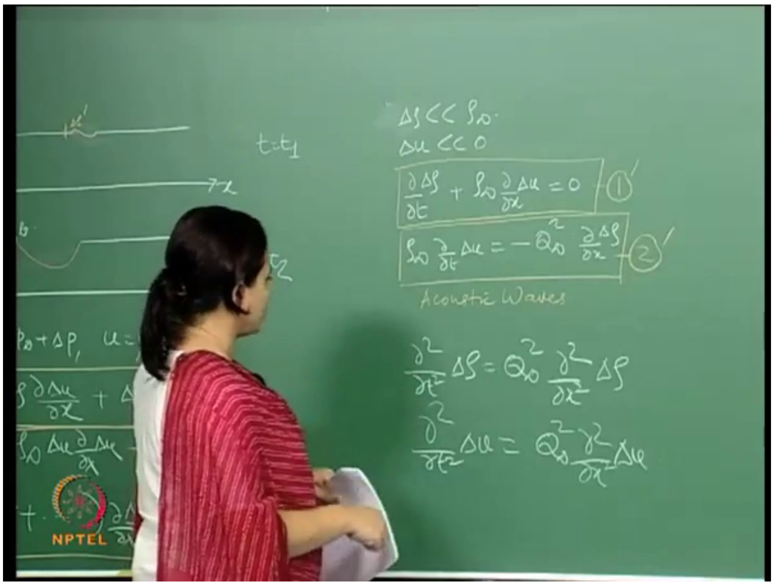

So, for example, is very, very small. So, this is very, very small, then delta u is very, very small.

So, essentially what we are saying is; that if you have a wave like this, where it is the finite disturbance and or perturbation, then you have these are the governing equation. Instead if you have something like this, and you say you know you have say a disturbance, say you have a disturbance like this. So, as you can see, if you if you say if you say look at this part here. So, if you say if you look at this part here ok.

So, in here this is my delta rho say. So, you can. So, let us call this as say dash. So, if you look at this clearly. So, this disturbance or perturbation is very small. So, for this particular case we will just ignore it, and we will say that is they know the perturbation is really very small. Then what do we do? In that case how does that get taken care of in here? So, there is the perturbation it is very small. So, therefore, it is a very weak wave, that is a very weak wave.

So, in that case what do we do? What happened in that case was that; we were able to reduced again these equations or transform these equations in the following way. So, our continuity equation becomes this. This is from the continuity equation. So, what we have here is. So, let us say I am going to call this as one dash, because this is what we get from 2 dash. So, this is so essentially if I assume this, if I assume this then I by continuity equation gets transformed accordingly, and the other equation gets transformed in this fashion.

Now these are therefore, because we said it is a very weak wave is it is like a sound wave. So, therefore, these are acoustic waves. These are acoustic waves, and the important thing here is that you can see here that I had an extensive you know, non-linear equation. And I have been able to convert that to a an linear set of equations when I consider a very weak wave ok.

Now, there is another advantage to this. I think I may have kind of done this last time probably. I think I have done this right. So, essentially what is we then came up with is for a solution of this. I came up with a solution for this. Where I said that is you know now if you make combinations of this and so on and so forth.

So, if I you know combine these 2 equations suitably, then I will get these 2 equations. I will get basically these 2 equations. And so, if I combine these 2 suitably then I get these 2 equations. These 2 equations mathematically are termed as the wave equations. And they have a defined solution to them right. So, say in this particular case. So, therefore, if I have an equation of this form mathematically, say of any say parameter say phi, let us just call that.

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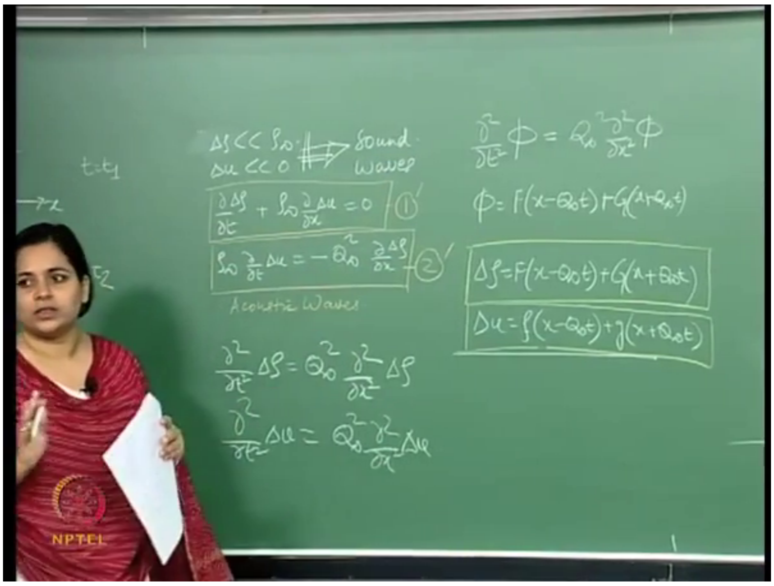

So, that is so if I have say something like this right. So, this is a wave equation, and this has a solution of this nature. So, this has a solution of this nature this ok.

So, therefore, what we get from here also is a solution like this right. So, which is the delta rho is and delta u is again. So, this f all these fs etcetera is are basically arbitrary functions. So, what I essentially have therefore is that numerically now. If I am I have basically solved my governing equations using the perturbation theory, for small for small perturbations, I have been able to linearize these equations. And from there I get a solution for these equations like this ok.

Now let us just see what this how does that change actually this picture. I have a wave here, I have a wave like this. So, in this case just thus this perturbation is slightly is very small. So, for this the governing equations you see the perturbation is this. I was able to assume that is I was able to basically say that this is a weak wave. So, consider these as very small. And so therefore, I get a solution for the perturbation in this form. So, how does that is now based on this solution, how does that change the picture? That we have been dealing with so far.

Now, let us we do that let us do this. So, for that now let us say first things first, we will say that is say G is 0. So, let us consider these 2 solutions over here. Let us say so we will take this; so having taken this.

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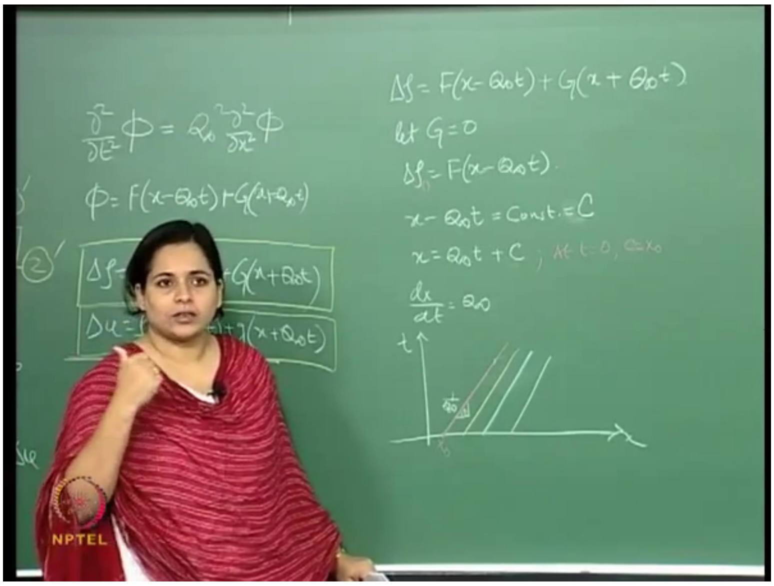

So, essentially what we are saying is the delta rho is. So now let us say so therefore, what we have is; so, if I have that, now let us say this delta rho is also a constant. Now say this is also a constant. So, let us call that as say delta rho naught, let us call it that.

So, in that case if that is so, then let us call that as let us call that as C. So, essentially, so delta rho is a constant. So, in that case this is an arbitrary function. Therefore, this also becomes constant. Let us call that as C. Now what we see from here is that if you look at this this term here, now this relationship here was basically giving us the relationship of the perturbation with respect to space and time. And as we saw in the finite wave, that this actually the way the particle moves within the within the within the wave is different at different times. And it changes at different space also which is one dimensional x time right.

So, essentially this is giving us how the wave is traveling, how a particle within the wave is traveling. So, what I see over here from this relationship, that is the way the particle moves within the within the wave is actually a straight line, isn't it? If you look at this, it actually moves like this. And if I look at this, it moves with a velocity which is Q infinity; so therefore, if I have an xt diagram. So, let us say x and t. So, this is basically have equation of a straight line and for various values of C. For various values of C which corresponds to various values of delta rho naught, isn't it? Which taken that as a constant we will essentially have different sets of lines.

Who? So, if I were to draw that on this xt diagram for example, say dx dt is this. So, I will have a straight line which is like this. And that c is here. So, essentially say let us say for. So, at t is equal to 0. So, let us see over here. So, say at t is equal to 0. So, x c is equal to x naught. So, let us say this is x naught. This is my x naught, and the slope of this line is 1 by Q infinity, isn't it? Because dx by dt is Q infinity. The slope of this line by in this plot is actually dt by dx, if you look at this plot.

So, what I have is a line like this. So, again if I take another value of this C which depending on instead of this being this being say rho naught one for example, So, I will get one more plot. So, I will actually get one more line. So, let us sort of; so, let us say let us say we will we will do it over here. So, say this is my this is my slope right. So, in here this is actually the slope is actually 1 by Q infinity.

So, then what we get over here is a series of line, right. And each of the a here and the slope is the slope is same. The slope is same for all these lines, isn't it? Is just that this value is changing, that depends on this rho the delta row actually ok.

So, what we see is that the solution. The solution that we got from here in for the finite wave is; we see that it basically emerges into several lines, several straight lines, right. Which means, that on every on every line the particles, right. On each wave the all the particles are traveling with the same velocity, isn't it? Dx by dt, so on every line here the velocity is same. Now this is different from what we saw for our actually finite wave right.

So, what we saw over here was; that the particles in the wave all travel differently. On one particular wave the particles are traveling differently, they change with x as well as with time. At all the particles are traveling differently, but if you look at the solution here, which we numerical solution that we got is that all the particles in a particular line are traveling with the same velocity, which is Q (Refer Time: 22:09). So, that is basically the difference between; the finite wave and the linear perturbation theory acoustic waves here ok.

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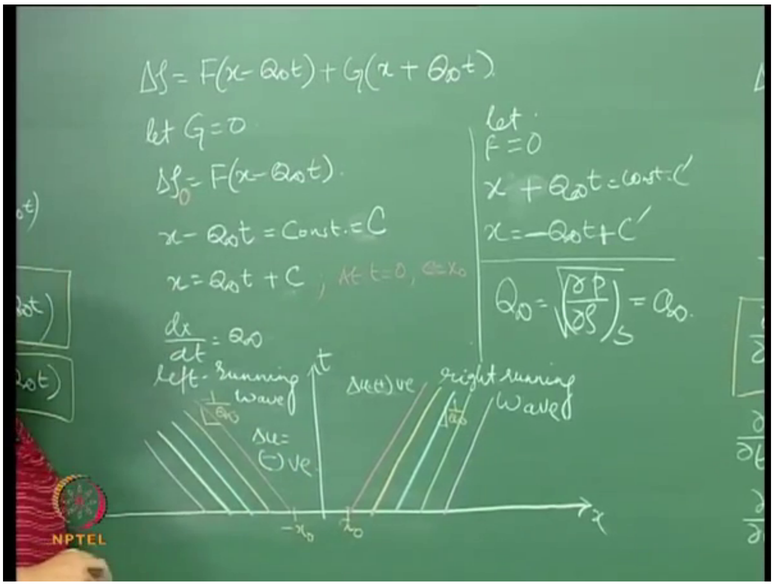

So now this is the picture. So, let us sort of redraw this picture this way. So, let me sort of redraw this that is for a reason. So now so let us this is say x and t. So, I essentially have a series of waves like that. So, they all have the same slope. So, let me just say draw the slope over here which is 1 by Q infinity. And say this is x naught. Now we what we have taken is; G is equal to 0. So now, in this case what we what we call this x. So, we will we will we will come to that later on.

So, we said G is equal to 0. So, instead of G now we can also we will do the other way around we will say that f is equal to 0. So, what we will do is; we will do the same thing and say that f is equal to 0. So, we will give we will basically get the same you know, procedure to do this and then what we will get for the solution for f is equal to 0. So, if you see, so essentially what we will get is that x is equal to this is equal to a constant. So, let us that we say c dash ok.

So, what we get over here therefore, our solution is; so, therefore, what we actually get over here. I am trying to get basically the same picture out here. So, what I get from there is this that this and this. So, let us just say this is negative x naught and the slope of this is nothing but that is all ok.

So, therefore, what we have what we see over here is essentially that this is moving this is moving just the direction is opposite. So, what we if you look from here in the xtt diagram? Therefore, this is actually called a right running wave this is a right running wave which is has which is moving with this Q infinity velocity, and this is a left running wave. This is a left running wave which moves with a negative velocity the negative is basically the direction. So, the solution that we get from the linear perturbation theory is essentially this.

So now so which is Q infinity. So now, let us do something else now we know. So, we know, that is now you if you look at this, that Q infinity the wave we defined it was del p del rho right. So, in this case we now we also define this as velocity of sound, isn't it? Because in this case we said we have assumed that our perturbations are very small. So, which means that that we are really talking about very weak waves, weak disturbances or we are basically saying that we are causing the disturbance by sound waves. And therefore, we can say that this Q infinity is basically speed of sound, because that is how we define this is also the expression for speed of sound.

Therefore, we can say that all these waves are essentially moving with the speed of sound. And the solution gives us this picture. The difference between the finite wave and this is that here particles in a particular wave are all moving in the same velocity unlike in the final wave picture.

So now, so let us let us do some little bit more on this, and then we will kind of move to other questions. So, having done this now let us say the. So, we can do the same thing for. So, we did all of this for say delta rho, isn't it? We got this solution we understood this for say delta rho, but we can do the same thing for the perturbation and the velocity as well.

Now let us see let us sort of just you know check that and see the relationship therefore, between the velocity change and the density change. So, if that gives us any more information now. So, therefore, we will do a little more math here.

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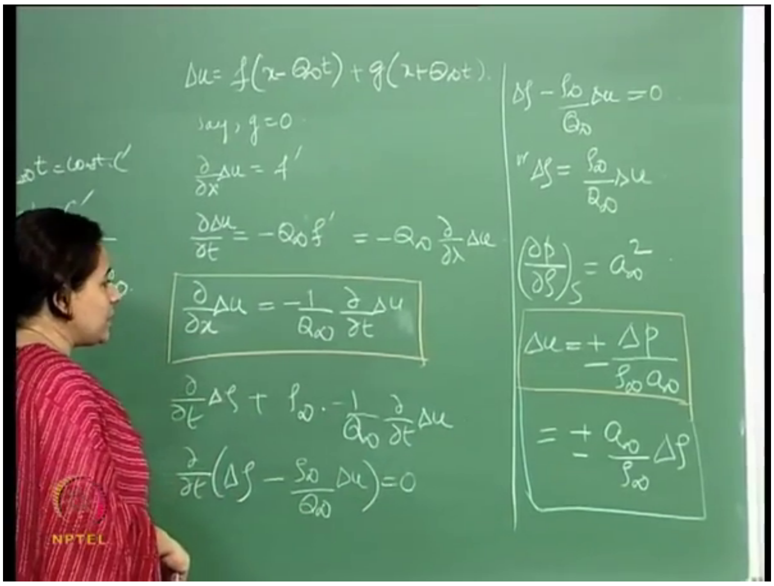

So, we basically said that delta u is f. So, anyway I am I am going to write Q infinity, but this is actually the speed of sound, this right.

So now again let us say that; I will say G is equal to 0, right. If I say that and then, so then we will just say right. So, we will say del del u is that, and also delta t. So, delta t is minus Q infinity f dash; therefore, if I incorporate that here. So, this is equal to minus Q infinity. So, that is essentially what I am sort of looking for here. That, so del so let us sort of write it in this way. So, this is equal to; so, this is the tale of that.

So, this is what we get, right; now we have this linearized continuity equation where is that go. So, we have the linearized continuity equation over here right. So, we have basically we are trying to sees that if there is a perturbation in the velocity, right which is how accurate the wave, what happens to the density what is the connection, how are these 2 related if at all. So, what we did just now is we found an expression for this delta this delta u del x in terms of delta d. So, if I use that into this acoustic equation. So, then what we get is this.

So, what we get? So, this essentially; so I am not rewriting the equation. So, I will just straight away. So, put in for del delta u del x, I will put in this which gives me. So, if I get this then. So, what essentially, I will get from here is that I can write it delta rho minus, right. This is what we get. So, what this means. So, in in a in essence what this means is that if I do this what this means is that delta rho minus rho infinity by Q infinity delta u is equal to 0, or delta rho is equal to rho infinity by Q infinity delta u.

So, this is essentially the relationship. This is essentially the, relationship now again. So, delta p delta rho is equal to a infinity square, right. Now if I then incorporate this into this then what I get is I will leave you to do the detailed or you know simple math now. So, essentially what I am going to do is incorporate this into this.

So, what I get now is delta u is delta p by rho infinity a infinity. So, I get this this. So, let us just we remind as us. So, what we did was; we took we took the delta this solution. We took this solution and did the same thing as we did for this. So, at first, we said let gb equal to 0 which is what we did and then we sort of worked with this and incorporated this term into the continuity equation, the linear perturbation 3 theory continent continuity equation. And what we will combined this expression for the speed of sound and we came up with this relationship.

Now, we can do the same treatise by saying that this goes to 0. We can do the same thing with that and what will up with for say you know, f is equal to 0 what we come up with there is essentially I can write this as. So, in here so basically let me sort of write this totally. So, say delta u is this and again this is equal to; so, what we see over here is actually, for a disturbance or a perturbation in the velocity of delta u the connection of that, what the connection of that with the kind of pressure perturbation and the kind of density perturbation. And so, if there is a if you disturb the flow velocity in this fashion. It will create a disturbance it will create a perturbation in the pressure and density in this fashion. So, therefore, what we can see over here? So now, plus if the if the velocity if delta u is plus. So that means, it is a right running wave. If it is negative it is a left running wave. So, in here delta u is positive this is positive. And in here delta u is negative. So, therefore, delta u is essentially for a right running wave.

So, let us just tried understand what this information that we got here what is this mean to us ok.

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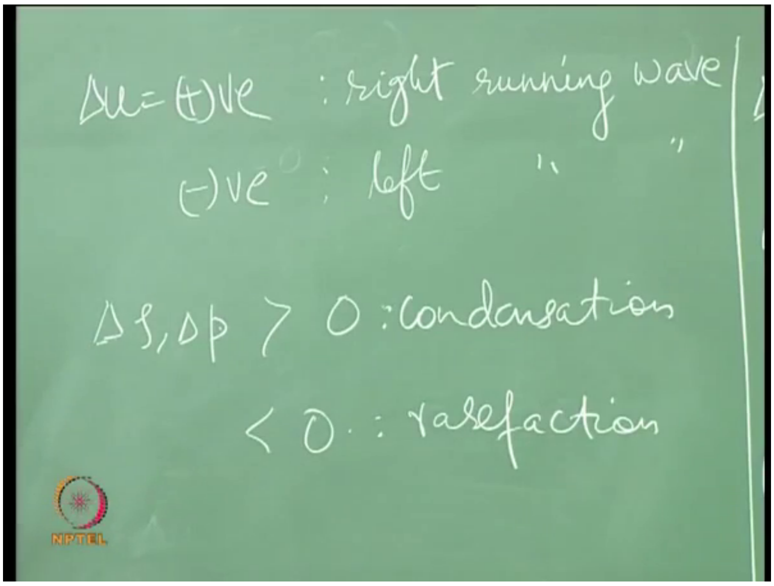

So, essentially what we are saying is; if delta u is positive then it means a right running wave. And if this is negative then it is a left it is a left running it is a left running wave. So, essentially what we are saying is that delta u is the perturbation. So, therefore when it is the right running wave in this case, ok.

Now, if this is positive it essentially means that the perturbation is in the direction is in the direction of the wave. Delta u is actually in the direction of the wave. And if it is negative essentially means if the direction is opposite to the wave. So, in here, so say delta u is positive. So, then I have a positive delta p, and I have a positive delta rho. Now what does that mean in here? So, say delta rho and say; so, delta rho so, it is greater than 0. So, we have a perturbation in the density and pressure which is positive. But this means is that there is a condensation. And if it is negative, if it is negative then it is called rarefaction. Just think about this. Just think about this. So, I have a I have a certain wave right. So, I have a certain wave ok.

So, say this is a fluid which is which is moving, right. Now I perturb this. I perturb this; I apply by a velocity which is positive. By a velocity which is positive. So, the perturbation is such that, this say this is the velocity with which is moving, because of the perturbation suddenly the velocity you know pushes it this way because the plus u is in the direction of the wave, right. If this was moving this way and then I perturb it. So, it moves in this way, what this does is condenses it. The pressure and the density of the fluid now increases from what it was when it was moving like this ok.

Let us do that again. So, you have a fluid moving like this. You perturb it. So, you see it condenses, because the pressure and the density increases. Instead now the fluid is moving this way. You now the fluid is moving this way. And if it is move you apply a delta u v in a direction which is opposite to that of the wave. So, I have a fluid moving this way and I have applied this way, what did I do; so it kind of fans out, right. It spreads out little more; therefore, the density and the pressure actually. So, the perturbation actually decreases. It decreases it is negative from the ambient ok

Therefore, if I have a wave which is say moving this way, and I have a delta u like this. So, what that we end up in rarefying. So, this is your rarefaction. So, this is how a wave actually travels. This is how a wave actually travels and this is what we see from our right running wave and left running wave and so and so forth. So now, basically we have enough information about say weak wave which is our sound wave.

Now, what we have done essentially so far is that, we set out trying to study waves how waves travel and so on and so forth. So, then we said you know we are going to incorporate small perturbations into it. And we were able to do that. And then we assume that the perturbations are very small. So, that the wave is that of very weak sound wave and we have certain information, you know regarding the numerical solution and all that.

Now, then next thing to do; however, is to is to check who is to see that if we can actually go ahead and solve the governing equations for a finite wave. You know, where we do not assume that these are very small. When we assume these to be very small then we said it is found waves and we have gone ahead and done this whole treatise which is interesting. Having said that though what if you do not assume this I will be still able to solve this equations, we will be able to sort of get more information out of it, yes.

So, what now what we will do is not assume this. We will not assume this. And we will deal with this instead of this. So, we will deal definitely with a finite wave. So, this is my finite wave. So, I think we will just sort of begin this and can take it you know take it forward in the next lecture. So, let us see how we can go forward with this. So, we will go back again to our basically our governing equations, right. We will go right back to our governing equations.

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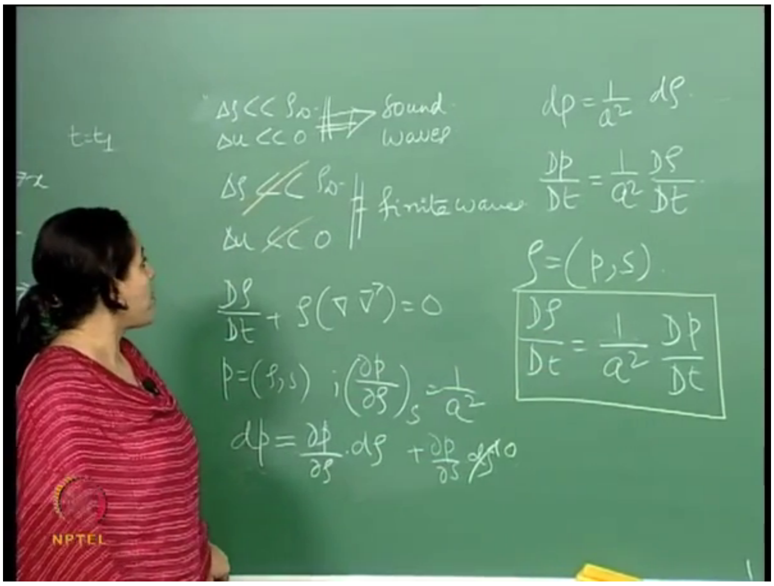

And so, what happens here is that, delta rho is not you know is not, this is not true is not less than rho infinity. And delta u is not less than this. So, what we get now is essentially, finite waves what we get now is finite waves. So now, let us go back to our continuity equation, which says and then we also said like in in the last lecture we will represent a thermodynamic variable. And using this we also got and this is isentropic. So, the change in density is 0. So, therefore, ds dt is 0 ok.

So, what we got from here was. So, we got this. So, therefore; so, what we will write some over here, is that now when I ge get from here actually. What I get from here is that dp here if I look at this. And the other term and the other term which is del p del s into d s now that basically this will go to 0, isn’t it? Because we have because this goes to this is an isentropic process.

So, basically, we are left with this and we know that del p del rho is 1 by a square. So, we can incorporate that here. So, this can be incorporate like that. So, therefore, what we have is this that dp is 1 by a square d rho. This is what we have. So, therefore, I can write it like this. Dp dt is or what I say why d rho dp; does not matter. So, I can I can write it like this as well.

So now if I. So, basically, I can write this in the similar way now. Instead of p k instead of p written in terms this is what we had done earlier. So, we can also, right. Say a density, right. We can also write density in terms of p and s. We can also write the same thing same treatise, then in that case we can also write as say d rho dt is actually in that case it will become a square, right. So, this is a square dp dt. So, we will just they use that because this is what we need, here is the same treatise we can write one thermodynamic variable in terms of the other 2 state variables. In this case the ds isentropic. So, ds is 0. So, therefore, we can write this d rho dt is this.

So now if you look at this equation here, basically I have an expression, this d rho dt I can write in this form, isn't it? So, if I do that. So, what we will get is this. So, we are going to basically introduce that this expression into the continuity equations and see what we get.

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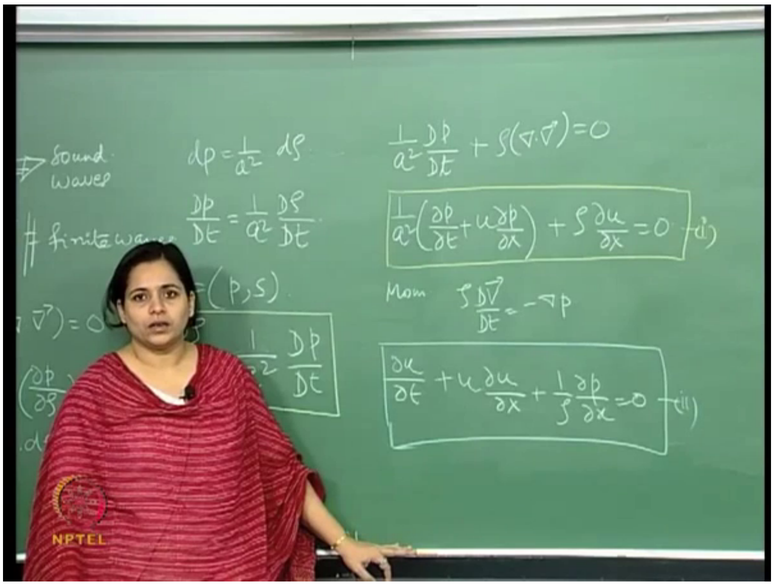

So, essentially what we will get is, right plus this is what we get. And let us write this for a one dimensional one, one dimensional case if I do that what I will get is this, right. So, this is what we get from here, if I write this for a non-dimensional for one dimensional case. Let us call this as 1.

Now, I can again you see I have this momentum equation. Now from the momentum equation what we get is that; this is what we get from the momentum equation and we will write this again in terms of if were for an for a one-dimensional case. So, again what I get here is. So, I will rearrange this. I will write this for a 1 d case. And rearrange this. So, what I get here is essentially this. So, this is what I get from the momentum equation. So, this is essentially it is.

Now, what we will do in the next lecture is take these 2 equations. All I have done here. All I have done here; is just taken the governing equations. And I have written them in a form that is I am going to use and it is going to give me some more information. So, all I will do now is take this and this equation and sort of work with this I want start from here.

So, what we will do is essentially this is actually the step where we introduce the method of characteristics. A method of catalysis is a very strong tool in how we solve problems in especially compressible flows, right. And we will see if we will be able to get some make some headway into getting us a solution for the finite waves. So, we will continue this in the next lecture.

Thank you.