Advanced Gas Dynamics

Dr. Rinku Mukherjee
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture – 16
A review of wave propagation

So, what we have been doing so far for with in last lecture was; we started the shock tube, right? And what we have to do now is to study the expansion fan which is traveling to the left of the diaphragm. Once you break the diaphragm there is a there is an expansion fan which travels to the left of the shock tube. And that is something we going to study now.

Now, before we come to that what we will try to understand is the very basic question, as to how do these disturbances travel and what do I mean by that right. In the sense that; for example, the reason I am talking and you are able to hear being at a certain distance from me, is to also say that I am creating a disturbance, right; in the surrounding around me and that disturbance is traveling from here to where you are sitting right.

So, to give you very small examples. For example, on a pond for example, the still water and you throw a pebble. And you see all these ripples being created right. So, although you throw the stone only at a particular point, right. Someone who is standing slightly away from the point it is still able to see some changes in around him right. So, therefore, this disturbance travels. So, what I what I am trying to say here is that the disturbance actually travels.

Now, in our objective here you know in doing these topics in in doing the shock tube for example, is are we break the diaphragm, right. We break the diaphragm. Once you break the diaphragm the effect of that, right. Is felt along the shock tube. It is not just confined to the place where we break the diaphragm. So, that is why we need kind of need to know how does disturbance, right. Which is a normal shock to the right-hand side and an expansion fan to the left of it are traveling. How they traveling, and how are we going to mathematically represent them.

So, that something that we will do today. So, what will start out doing is try to look at the best way we can look at how waves travel. What do I mean by wave right. So, you can like it I gave the example that say we have a still pond, right. And then you throw a pebble and you see ripples.

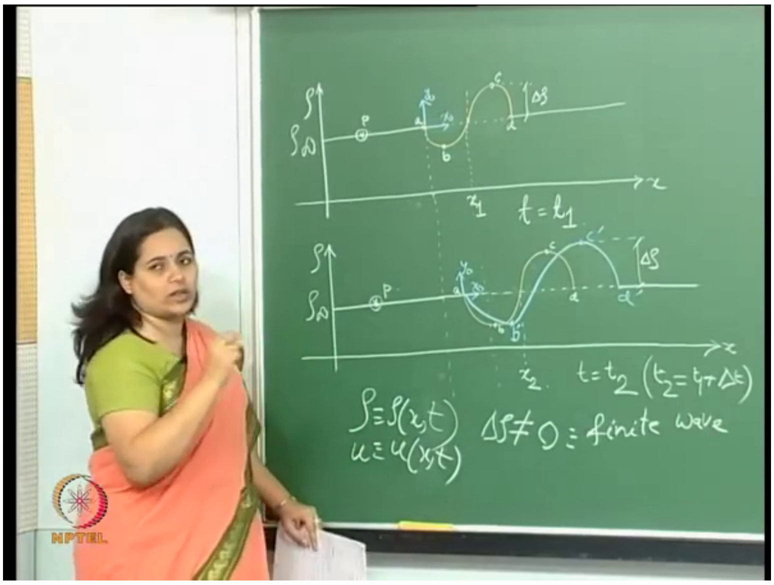
So, you can think of you know, this sort of this sheet of paper, right. You can think of this sheet of paper, say as the still water and the pond, right. You can think of this as that. And then when you say throw the pebble or the stone there, right. Then what happens is that you there was a disturbance created and how is that created. So, then you see this. So, there you have still water like that, and then you see you know a ripple being created. Then a disturbance is created. Now what is this disturbance? Right, this disturbance this disturbance could be in velocity, it could be in density, in pressure, temperature etcetera, right. Depending on what kind of flow you are looking at right.

So, in the same manner, you could also have you know, that you know there is a flow which is moving, right. There is a flow which is moving like this. So, I can just think of that. It is moving uniformly you know along you know moving like that. And suddenly; so, as say as it is moving like this, I give it an impulse and that goes, impulse and that goes. And the question is what happens to this say disturbance, right. What happens to this disturbance over time and also in space.

So, let us go ahead and try to look at this mathematically, and see what we can gather, what we can understand from there. So, will write our equations and see if we can study the movement of waves. Or say basically the propagation of disturbances in the form of waves. So, let us go and do that. Now like I said when I say a disturbance the disturbance could be a change in various properties. You know, density, temperature, pressure, velocity, etcetera.

So, let us just take for example, density right.

(Refer Slide Time: 05:19)



Now, let us say like I; so, let us say this is the x direction, right. And say this is a density. This is a density. And so, at some at some point basically, at some say this is at say at some time at some time we have, right. Now let us just say that this is ambient density or the free stream density. This is undisturbed. So, we have a certain density which is this. And now what I do is suddenly as say some instant of time some x. So, say at this point say x right. So, at this point x I have, I give this ambient there is a certain disturbance what is that? So, that is like a small disturbance.

So, if I do that. So, let us say it is something like this. This is something like this, and what I am to here is there for now. So, what basically I am telling you here is that in so far this long for this distance of x, we have a free stream value of the density. And then we have a disturbance which looks something like this right. So, we have that and this disturbances would in say here. So, and again we have an ambient. So, what we will try to look at is; how this looks like, how this looks like as we go further down x and also in different instance of type. This is what we are trying to look at when I say the propagation of a disturbance.

So, what we want to look here is the this disturbance how this looks like, if I go further down in the x direction, right. If I keep going here, how will this look like in a war or if I go further down in the x direction, what will this disturbance do now? There is nothing new, but the already the disturbance which is caused. What will that disturbance do? And also, if when I look at it at another instant of time, you know forward from time t 1 what will this look like. That is what we are trying to study here. So, let us do this. And let us just you know, what we will do is call this here. So, let us say this is x 1 I am also going to do something else here. I am going to draw a local axis system. Let us say, let us call it x naught 1. I am going to draw this local access system at the origin of this disturbance. And also, we will mark different points different points on this wave.

Let us say this point is a, right. We have another point which is say, b another point which is c and this is d. And this is an, so this is basically an amplitude of the disturbance. So, that this is what we have that time t is equal to t 1. So, let us see what happens to this as we move further down x. So, let us do that.

So, again I am basically try and reproduce the same picture here. So, we have rho infinity. So, what happens to this thing here. So now, what does this look like as I go further down x? The way, so this this we could just sort of draw at this way I am going to just reproduce this. So, which is ok.

So, what happens here is that, what I try to do here is that this is exactly the same as the as this one. This is exactly the same as this one except that this is you can see that now it has moved. So, therefore, this is my this is you can say the wave fixed axis say system, right. It is exactly the same and it moves from here to here. So, again so these are the say this is a. So, this is b, this is d, and again this is and now this is x 2. So, what we can see here is that now again. So, as I move along x right. So, I see this happening. So, I just I just reproduce exactly the same thing, except that it is slightly further down away from this point. From where it originally you know originally happened in the flow, right. This the correct picture now just think of this. So, essentially with respect to this if you consider that b c and d, right. And a these are say particles within the within the wave, right. If I consider these as particles within the wave, then you can see that these are the respective positions of pc and with respect to the that access system right.

Now, that does not change as I did it over here, as I move it along x, but now let us just say that this is a picture we are looking at time is equal to t 2, right. Which is basically let us just say. So, basically, we are advancing in time right. So, in that case, in this particular case if I consider this as unsteady that this also this picture also changes. So, in this progress as of now this it is a function of x right.

Now, let us also add to this that this is unsteady, right. That this picture also changes with respect to time. So, when I go from this time to this time, this picture also changes right. So, if that happens, then how will this picture look like? Right so, let us do this you can probably guess. Now the thing is that, as I just said that when moving from here to here. So, this particles b c and d they are their respect they are exactly in the same respective position with respect to this coordinate system as we move from here to here. But if we consider them as unsteady, if I consider the wave is unsteady, then that will not be. So, which would mean then, that let us say ago say you draw it with this a different color.

So, in that case my wave will become something like this. Say something like that. So, this way. So, then in that case this is my say b or say b dash, right. Then this is my c dash and this is my d dash. So, again right. So, all I have here right. So, you see the difference you see the difference between the blue, and the orange here right. So, this essentially means that as I; so now, for this blue curve. So, blue curve it is no more it is no more just x, right. It is no more just x, right is also unsteady. So, which means that as I go from t is equal t 1 to t is equal to t 2 as I advanced in time, my wave moves along x as well as changes over time.

Now, let us see now the next step of course, you know before we do that of course, is that now this. So, say this, if I consider this now every time we say this, right. Where every time this is not 0, or it is non-infinitely small, then what we have is a finite wave right. So, so this is it. So, therefore, all the properties what I have drawn here or x try to explain here is basically density, now you could basically have the same stuff for pressure temperature velocity. Meaning that I could and so on and so forth.

Now so, therefore, now I can I can just say that now if I look a 2 particles, now one particle is say this this particle at c. This particle c dash or you know the particle c actually goes from that point c to c dash here. So, and let us take another particle somewhere over here. And let us call that as particle as p. So, we have an or another particle which is we call that sp now the density of ps rho infinity of course, p density of p s of course, rho infinity now if I were to look at this. Now this particle say is of course, you can you can see that on the wave it has a certain velocity, right. And that is added to the ambient velocity.

So, let us let us just say that the local velocity, locally this velocity is we can call that as a w all right. And so, when I have mass motion right. So, that local velocity is added to the ambient velocity. So, that becomes the total velocity of the particle over here. So now, also before I finally, move on to the governing equations. If you can just remember or try to just recollect, that what how exactly do waves travel they travel due to molecular collisions, I I hope you can remember that ok.

So, the point is let us go back to the table here. So, for example, say you know if I consider say, I am considering these as fluid particles. I am considering these as fluid particles. So, at some instant of time this is how the particles are look there is a fluid flow happening of course.

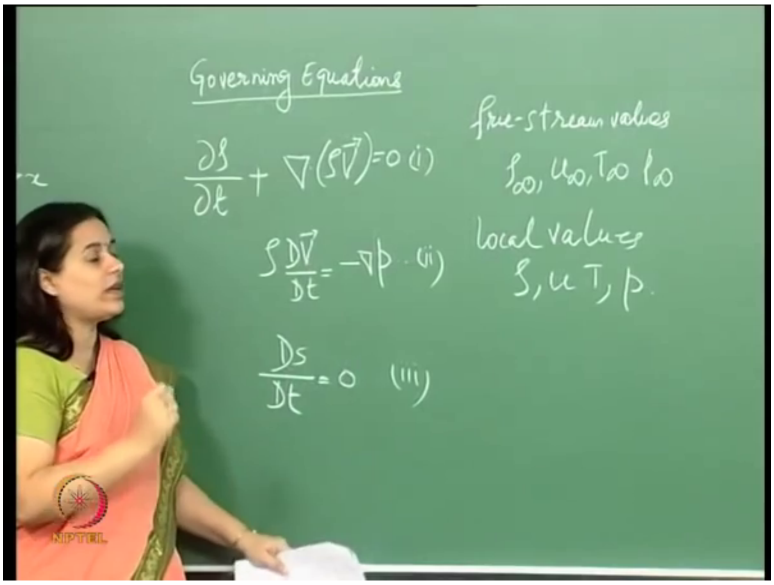
(Refer Slide Time: 19:49)

So, at some instant of time say t 1, these are certain fluid particles and what I do is that I you know give it a disturbance, I apply a certain disturbance to it.

So now I will apply the disturbance to this particle. So, this white particle here. What do you think will happen? Say I apply a disturbance you saw what happened I apply the disturbance only to the white particle, but in that particle, one didn’t hit this one, that went and hit this one. It did nothing to here to these it did nothing to these 3 particles. Let us do that again. So, what I will do is give it to here. So, in in this time it hit the particle propagated the disturbance from this to this to this. So, when I hit this particle this hits the next one, and the next one, and so on and so forth.

So, let us just say I will give this particular slightly larger disturbance. So now, it goes on hits this one or say for that matter let us do it even more. So, if I do that. So now, you see. So, that is what you see here is that when I give a disturbance here. So, it hits the next particle and then it is the next and so on and so forth. So, this is so basically a disturbance will travel due to molecular collisions right. So now, having done that, I think now it is time to go and develop a mathematical theory for all of this and we will start again from the governing equations, right.

(Refer Slide Time: 21:40)

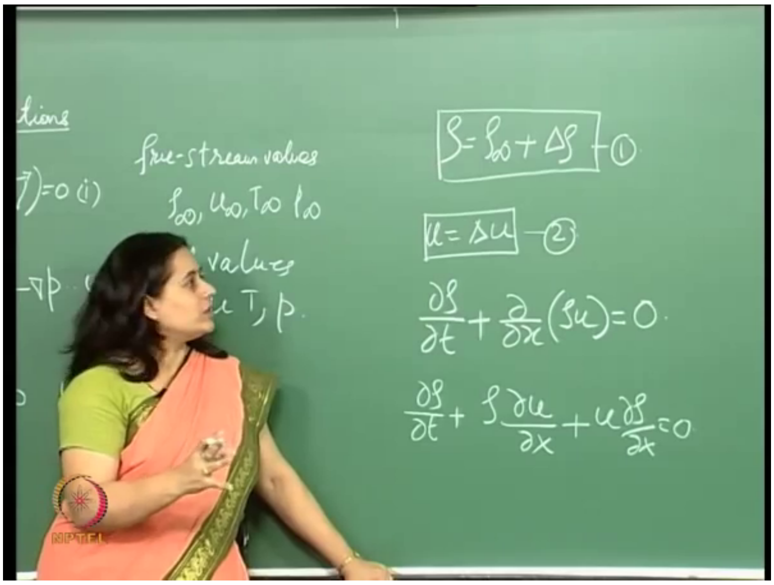


So, let us do that. So, let us write out the governing equations as we know them right. So, the governing equations are, so the governing equations. So, this is as you know it. So, this is continuity, this is momentum and this is conservation laws right. So, these are your governing equations.

Now so, what I am going to this. So, what we will try and do, you see how we are going to apply these equations to wave travel. In the sense that we are giving density, and pressure and velocity how do I incorporate the fact that I have a certain velocity say a density here, and then in this part I have a total you know density which is equal to the ambient plus the delta rho. So, that is how I take into account the fact that I have a disturbance now.

So, let us go and see whether we should be able to do that here. So, let us see like you know we just said. So, let the free stream values. So, we have so free stream values, let us denote by subscripts and so on and so forth. So, let us denote them by subscripts infinity, and local values would be, right. Local values would be this. And so, if I had to do that. So, therefore, what I would write. So, let us let me write this here in that particular case.

(Refer Slide Time: 24:27)

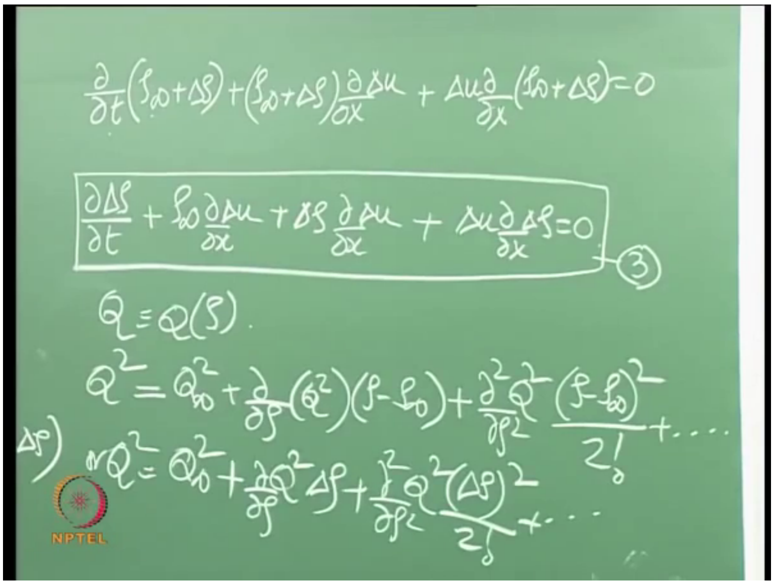


So, therefore, my local say density would be equal to the ambient plus the disturbance, right. This would be it and let us call that equation 1, right. And for the velocity let us say that the initial velocity initial flow field is undisturbed if that is so, then right. So, these are the 2 expressions that we can we can pretty much write that ok.

So now let us do something. Let us go ahead and take the continuity equation here, given in in this one and write it in x direction. And let us write it in x direction. See if I do that what I get is, right we can write this. So, that again what we will do is we will expand this. So, if I do that right. So, then then I can write it like this. So now, what we will do here now is that we will use these 2 expressions which we got in 1 and 2 into this meaning that this density I can replace by what is given in this relationship here in 1 as the rho infinity, has the ambient density, plus the disturbance or perturbation, right. And the velocity of course, is the ambient is say undisturbed. So, then we have u is equal to delta u right.

If I do that then what do I get? Right, if I do that. So, let us in that case what we will get is that, say let us write it here.

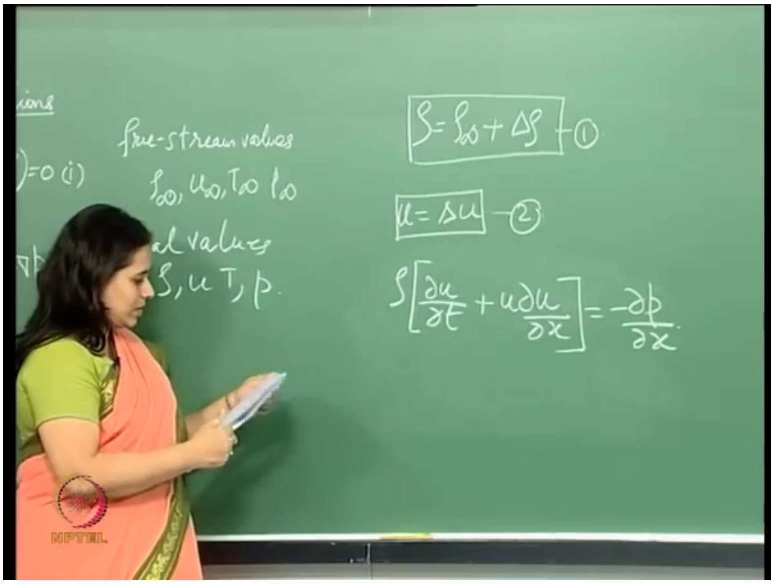
(Refer Slide Time: 26:57)



So, what I will get rho, right. This, now if I do this then, you know we will expand this and finally, what we get here is this. So, what we you can see here, that del del t of rho infinity rho infinity is a constant, right. Rho infinity is a constant. So, then that does not come in here. So, when I or basically what we saying is that the ambient; obviously, remains constant over time. That does not change. So, therefore, this does not come into this equation, and also what else yeah. So, again here this is also change of the ambient density with respect to the x coordinate. That also does not happen.

So, we basically get this equation. So, let us says. So, what we get here is this equation, right. And let us say let us call this as. So, this is something that we get from the continuity equation right. So, again let us come back here, let us come back here, and yeah. So, let us look at the momentum equation, and momentum equation and write this, now let us write this in the x direction, right. If I do that what do I get let us again go ahead and say write it over here. So, if I write this so continuity, so momentum equation in the x direction, if I write in the x direction, what I get is.

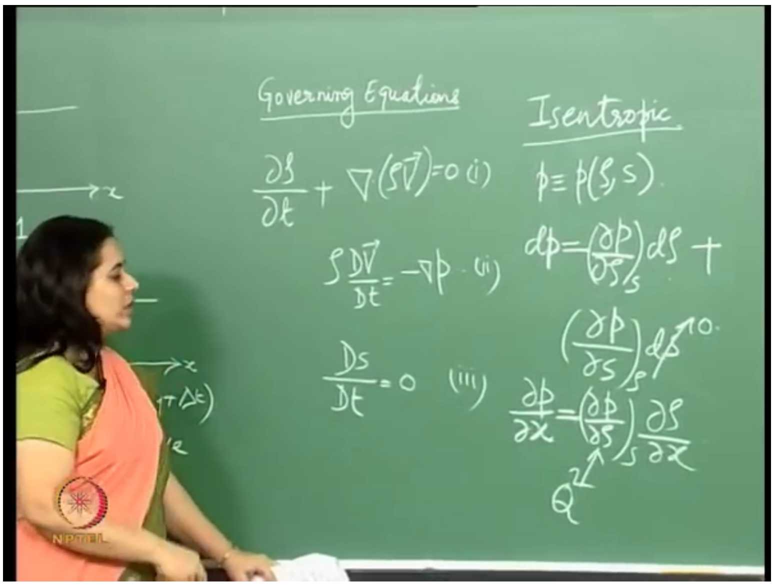
(Refer Slide Time: 30:03)



So, we get that, right. And from the third equation of course.

So, we sort of get that, and from the third equation again, now what we get is Ds Dt is equal to 0. So, which is saying that there is no entropy change over time. What does that mean? That it means that we considering this as isentropic right. So now, when I say that. So, from this third yeah equation here. We saying it is isentropic which means, that one thermodynamic variable can be represented as a function of 2 other. That comes the definition. Which means that; let us just do this here ok.

(Refer Slide Time: 31:21)

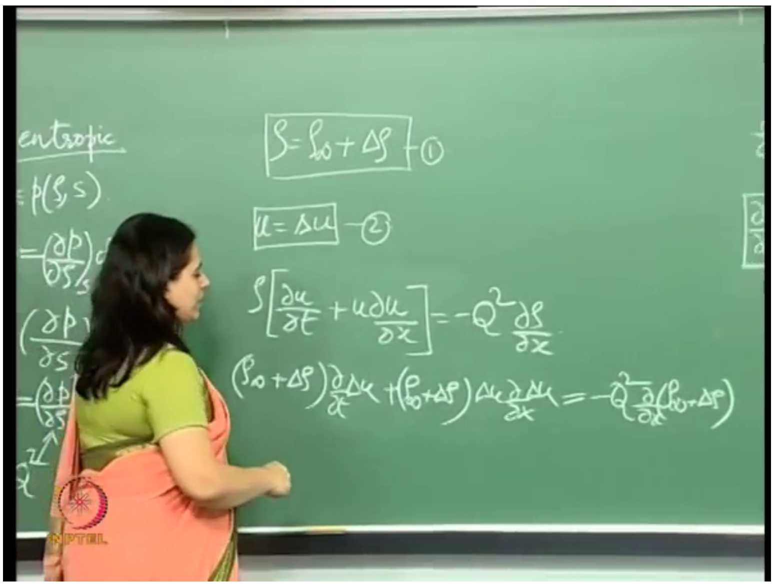


So, let us do this here. So, what I am saying is that we have, we have isentropic, right. If it is isentropic, then I can say, right pressure, right as a function of say density and entropy, right I can write it like that. So, in this particular case in this case let me write dp where write plus write than, just I am differentiating this right. So, in this particular case now we in if it is isentropic therefore, this should go to 0 right. So, if this goes to 0 what we are left out with is this that dp is equal to del p del rho had constant entropy into d rho. So, if this is a thing.

So, let us write the change of the pressure change of this pressure in the x direction that is that is to say that right. So, del p del x I can write as. So, del p del rho at constant entropy into del rho del x. So, this is something that I get from here. So, I find out the change in pressure in the x direction, right. Now at this point let us just call let us just say that this term here. So, let this term here, let us call that as called that as q square it is just a term will come back and find out if; that means, anything let us just call that as q square.

Now, if I do that, if I do this. So, therefore, del p del x. So, del p del x can be basically said it is it is equal to q square into del rho del x. In that case this equation which we wrote out is the momentum conservation equation in the x direction. Then in this one I can write as.

(Refer Slide Time: 34:14)



So, I can basically write del p del x replace it by what we just found out by q square del rho del x, right. I can write this as q square del rho del x right.

So, if this is it. So now, again what we have here is density and velocity. And let us go ahead and again incorporate 1 and 2 into this equation. So, well let us let us let us do that. Let us do that and let us see. So, it is a slightly longer it equation I will write it anyways. If I do that what I get is; and this is equal to this right. So, this is something that we get. So, what we can see here is that we have now basically this term q square right. So, I have something in q squared.

Now, let us see now this is basically now and equation that we will you know; obviously, look to solve. So, let us see if we can you know write this term q square in any other way. So, let us see if I can do that. So, what I will do is that you know we when we let us go back and see. So, we basically we defined q square as a derivative of pressure with respect to the density there are constant entropy. So, we can also say that basically we are looking at the change in pressure corresponding to a change in density right.

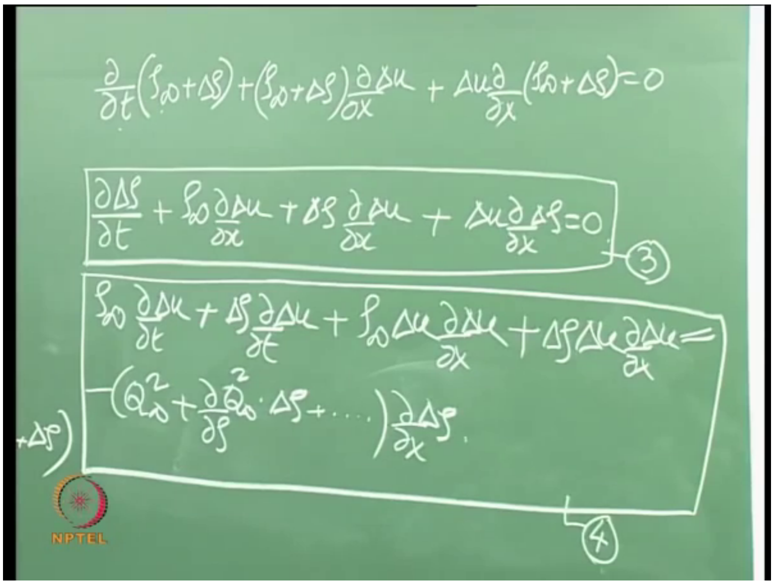
So, del p del rho. So, in that case let us say that is if we start. So, this q square is basically representing that. So, then let us say that we are starting from ambient conditions, right. Pressure and density ambient conditions. And to that we are adding changes. So, therefore, how do I find out that particular change. So, let us use a Taylor sees mathematically Taylor series. So, if I do that.

So now, what we have seen here? Yeah, so, basically q here q is essentially let us see, rho density is ah rho yeah. So, this now this term here q squared this is a function of. So, basically this is a this is a function of the density and entropy, right. It is a function of density and entropy, but that entropy here is constant. So, we can just say that this is basically tracking the change of the pressure, right. Then general pressure in that case what we will just. So, let us say that we will write this as now q say. So, if I do this.

So now I am going to write this in write this the Taylor expansion of this q square, right. See if I do that. So, let us say q infinity squared, right. So, this is the expansion. So, then I can again; so, again write this as or let us write this as delta rho. So, what this becomes? And so on and so forth. So, then this becomes right. So, q square is something that I can write like this. So, therefore, now if I were to combine, you know if I were to combine these equations. So, if I were to combine this 2 and 3, right. For the. So, essentially, we combine that and we got this this in the in the x direction, right. And what we have done here is found out an expression for this q square in terms of the perturbation, right. If I write this out what is this look like ok.

So, I am going to write that down, right. Underneath the equation 3 here.

(Refer Slide Time: 40:45)



So, if I write this what I get is; so, you can see that basically I am going to expand this, and incorporate q square which I just derived. If I do that what I get is so, that is equal to, right. This thing so, this equation. So, to just to kind of summarize; what we have done here is we took the governing equations. We took the governing equations, incorporated the existence of incorporate existence of a disturbance, how? By saying that it comes as a disturbance is basically causing a perturbation to the variables, right. Thermodynamic variables. So, which we did like this. Then we wrote out these equations in these say x direction. So, we wrote out this equations in the x direction, right. And we were able to come up with the equations 3 and 4 right.

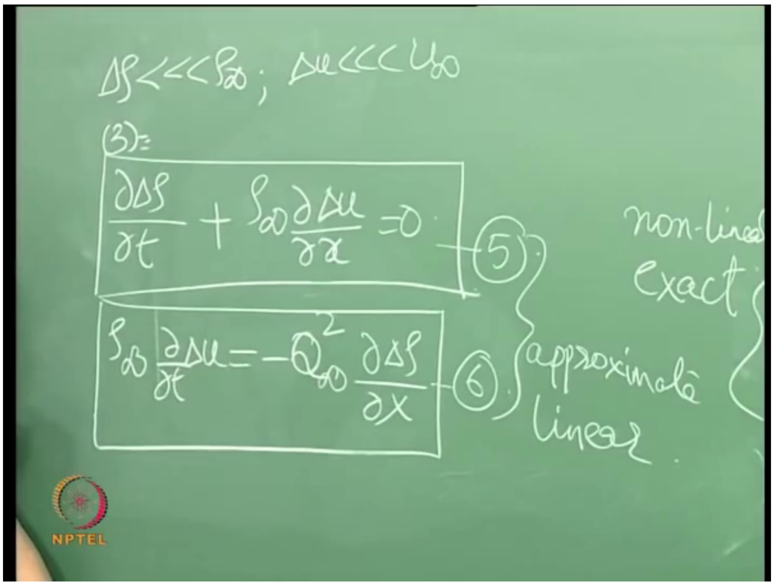
So, this is the governing equations. Like the equation 3 came from the continuity equation, an equation 4 came from the momentum equation and the energy equation. So, when we got these 2 equations. So, this is basically the governing equations taking into account that we have a disturbance, and that is being shown by the perturbations in the in the thermodynamic properties. So, you can see that our task here is to solve for this.

Once we solve for this, then we should be able to get a figure get a hang of the way we were trying to study the disturbance.

So, of course, now you can see that this is a numerically pretty exhaustive right. So, the first thing we will do, right. Is see if we can get something very simple from here. And the first thing that we can see from here, if I take something simpler is that we will consider the wave or the disturbance to be very weak. Like I just showed you as the part when I was hitting the particles never hit slowly then disturbance travels slowly right.

So, let us just say it is a very weak wave, what is that mean in terms of the perturbations here? So, if I do that, let us just say let us do that here now. So, in this particular case. So, so let us just say that right.

(Refer Slide Time: 45:01)



So, this is essentially now we say the disturbance is very small. So, the perturbations are very small compared to the ambient. So, it is a very weak disturbance very weak wave. So, so in that case what does it signify mathematically what happens to these mathematical equations that we just derived from here right.

So, let us look at equation 3, now what we get from equation 3 is something like this, right. If I make this assumption that delta rho and delta u are very small compared to the ambient, you can see here that I am multiplying basically (Refer Time: 46:19) deliver very small amount with a derivative of a very small amount. So, these 2 essentially become very small in a quantity because very small. So, I can basically I will I will take it out of my equation. So, I can therefore, write equation 3. So, equation 3 therefore, reduces to therefore, this is from equation 3. So, this is 3 actually this reduces to and let us call this as 5.

So, similarly we will do the same thing with equation 4. I will do the same thing with equation 4 like, and what we get from here. So, what we get from here is, right. Now we I think we are in a very good space to look at the 2 equations. Now what we can now this equations equation 3 and equation 4, these are exact equations is exactly the governing equation we put in our requirements. So, these are 2 exact equations and hopefully you can see that these are non-linear as well right. So, these 2 equations 3 and 4 right. So, 3 and 4 are accession essentially non-linear and exact right.

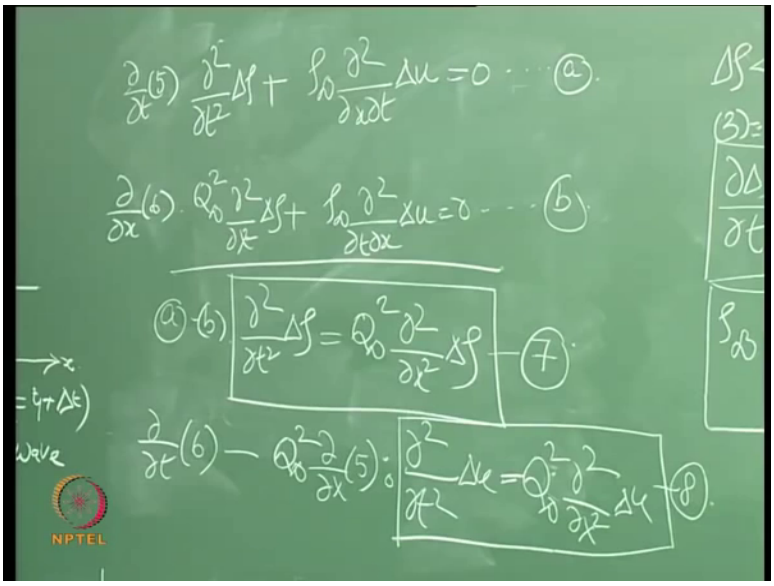
Now, come here and what we do is we assume that the disturbances are very small. So, let us say we make a small perturbation assumption, right. Which means is a very weak wave, what is the weakest form of travis the sound wave right. So, therefore, let us just say a sound wave is something that we are trying to look at here. So, in that case the equations therefore, the exact non-linear equations reduce to equations 5 and 6. And you can see the mathematical property here that these are linear right. So, these are linear, but also approximate ok.

So now these 2 equations on the other hand 5 and 6. On the other hand are approximate, they are also called as a caustic waves, right. I hope that is kind of intuitive because as we as I as we are looking at using these equations to be for very small disturbance. So, when for very weak waves, right. Which could be for sound ray. So, in that particular case the changes would be given like something like this.

So, we come apart this. So, therefore, the way we get equations 5 and 6 is using a small perturbation theory along with along with the linearized theory. So now, what we will do here is a; is there something else now I need 5 more minutes. So, he said 3 50 yeah just 5 more minutes I need. So now, what we will do here is to some manipulations with these equations that we have got, right. With this equations 5 and 6 etcetera, and see if we can get some more information. Because now the point is that once we have. So now, this is basically our governing equations.

Now, what we have to do is find a way to solve these that is our next step. So, what we will do here is in order to solve this. So now, let us in order to do that. So, what will do is first is let us do this.

(Refer Slide Time: 51:00)

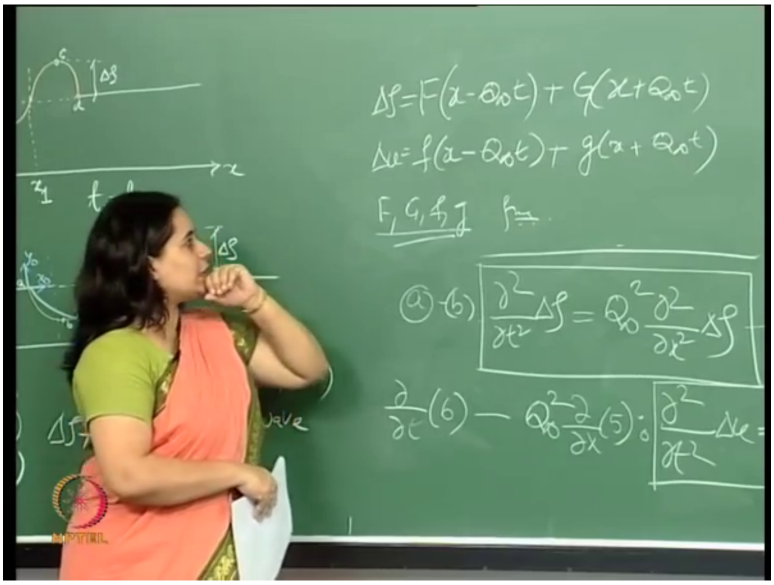


So, let us do a del del t of equation 5, right. What we get here is, right. And similarly let is 2 del del x of equation 6, right. If I do that what I get is, right. Now once I do that then I subtract equation a minus b and what I get from there is this right. So, this let us call this as equation 7.

Now, again. So, again similarly what we will do is del del x. So, I am not going to write that again if you do it yourself. So, we can write del del x, right. Del del x of equation 5, right. And let us let us just do this. So, what will do is del del t of equation 6 minus del del x of equation 5, right. This is what I do this is what I do and what I get is this, 7 and 8 right.

So, what you can see. So, therefore, what you see over here is basically I have 2 equations. I have 2 equations in delta rho and the other end delta u. So, once I am able to solve these I should be able to figure out what my perturbations are right. So now, the beauty of these 2 the way we have these equations, equation 7 and 8; that we have readymade solution. Because this is; I hope by this time this equation should be familiar to you right. So, this is a standard equation, right. And the therefore I can write out the for example, yeah.

(Refer Slide Time: 54:22)



So, this I can write as, right. And similarly, where f g f g are all arbitrary functions, there arbitrary functions right.

So, what we have done if I were to look at this. So now, what we will do is solve for see what this solution holds for us, how this will look. So, this we will do in the next lecture. Now though with the place where I will stop today; is that what we did is that we looked at a picture like this. We looked at a picture like this, and we said that when this is not 0, or if this is not like really really small, then what we have is a finite way. We find a found out the equations how the governing equations would look for a picture like this when I take into account these perturbations of, right. For the disturbance to propagate, and what we came out with are the equations 3 and 4.

Now, to solve this and to decrease the complexity of to solve is what we did was we said hey notice this is complicated this is too long. So, we went and assumed a we made write a small perturbation assumption and what we came up is are these 2 equations, right. Which are approximate, but linear, right. And we played around with those and we found out further an expression for delta rho and delta u which is what we set out to do and at the beginning, right.

Now the question is that the way we got delta rho and delta u, and this is after we made if we used a small perturbation theory and the linear linearized theory now. So, therefore, this now what we need to look at is that how this will change the picture or will it is this, a is this still valid for or something like this, because we have now considered that this perturbation is very small. That is something we look at in the next class.

Thanks.