Advanced Gas Dynamics

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Lecture - 15
The Shock Tube: Propagating Normal Shock and its reflection from end wall

So we shall continue with the Shock Tube little bit more, ok.

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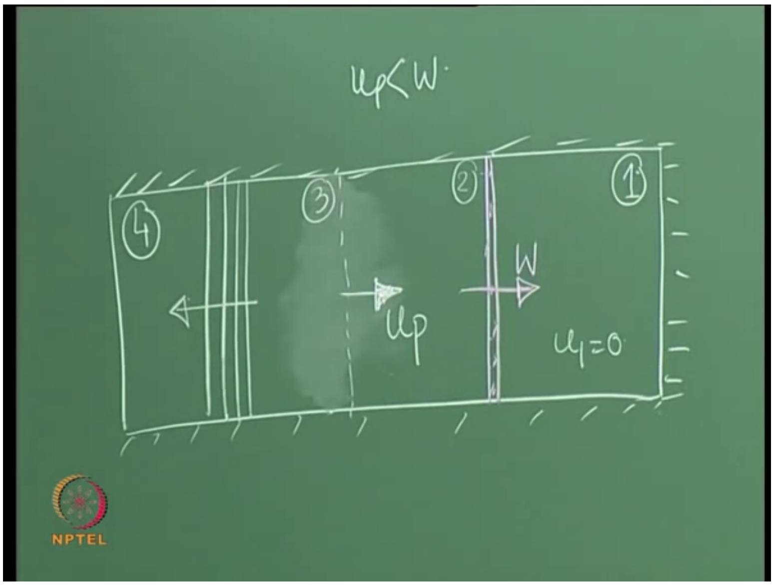

So, like we said that is the last lecture. So, we have this shock tube. So, we had a diaphragm which we break. So, let us do this right back. So, this was our diaphragm, so this is our shock tube, so this was a region 4 and this is a region 1, this is what we said and the corresponding values. So, then we break this diaphragm. We break this diaphragm and what we have in that case is that; we have a contact surface which is similar to a slip surface.

So, what we essentially have when we break the diaphragm is there is a normal shock which starts moving into quiz and flow. And this induces mass motion in this region. So, we are call this is region 2, so we have the contact surface also. So, this causes mass motion. So, let us say the velocity of that is say u and this is obviously. And on the outside we have an expansion fan which moves into region 4. So, this is the basically the shock tube.

Now what we are concentrating is on this normal shock which is propagating. So, let us do that. Therefore, now let us just concentrate on this region here.

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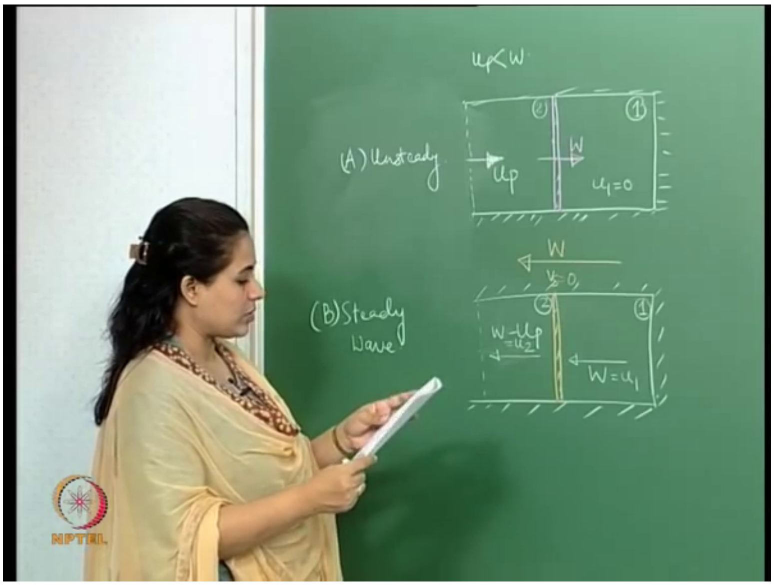

So, we have a shock which we have a shock which is travelling in a trough its right; this was is this is the unsteady shock wave travelling. If we have to convert this to the steady state case that we are most familiar with then all we will do is superimpose this flow with a velocity W in this direction.

So, if we do that then what we get is essentially; now sort of redraw that. So, what we get now is that that this is the shock wave, now the velocity of that is now 0. So, velocity of the shock wave is now 0, so it is basically a standing wave and well then the velocity here it becomes and this becomes W. So, this is my region 1, this is my region 2; so this is u 1 and this is u 2. So, all we have done is taken this and imposed a velocity W in the other direction. So, what that gives us is a velocity W in here and a velocity W minus u p here. And this is my steady wave; this is in a steady state k. So, this is what you know we did it in the last lecture.

So, what we did is we wrote out therefore. So therefore, W 1 is this and u 1 is W and u 2 is W minus up. And using these we wrote out the three governing equations. And we were also able to write the (Refer Time: 05:13) equation which turned out to be exactly the same for both these cases. So, now let us look at some things here.

Now we said that this velocity let this velocity u p is going to be less than W. Now, so the that the question to asked here is that is there basically an idea or can we estimate how much u p will be depending on the speed of this shockwave. Let us see. Now let us write down the moving shockwave. So the, right; so the shockwave is basically moving into this quiescent region. So, the shockwave Mach number we can write like that, and then I can or I can also sort of rewrite this in this form which is without really going into the descriptions or basically this is equal to just little space, ok.

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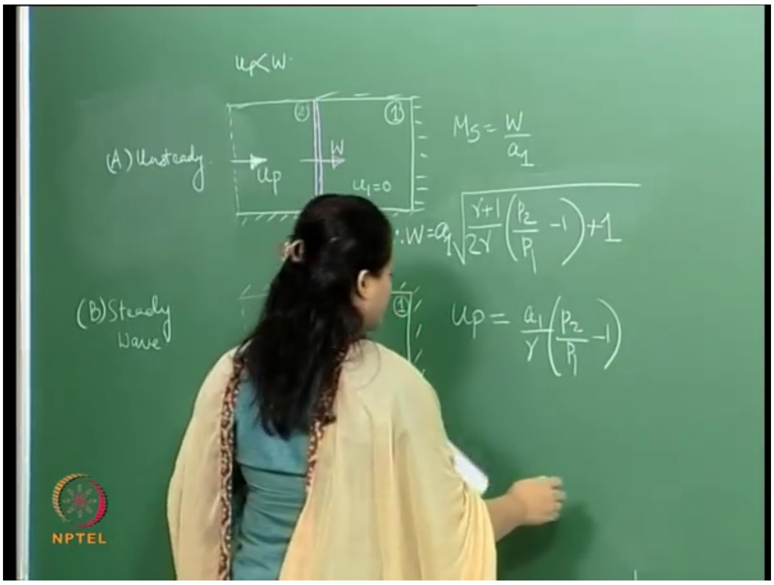

So, what we have is. So, like we said in the last lecture is that basically the shock here that the strength of the shock is governed by the pressure ratios across it, unlike in a steady state case where we deal with basically the incoming Mach number. So, this is the Mach number of the wave and therefore I can also write this. So therefore, W is basically a 1 into this. So, in that case; so let us sort of write that. Therefore, W is equal to a 1 into this, this is it.

Now let us write up. Now, u p can be written like this. So, this is all from the governing equations. So, you can just sort of cross check this if you so want. Let me move this place.

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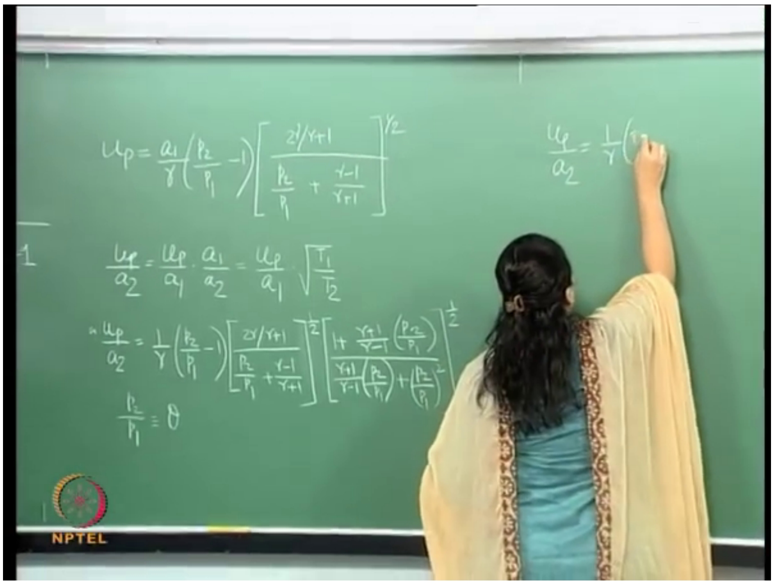

So, u p can be written as. Using three governing equations we can just come up with this, I am not doing the math which by this time hopefully you should be able to do that its preliminary elementary. So, this is more or less; this is the expression for u p and you can see we have written it out essentially in terms of the velocity of sound. In the driven section which is section 1 and the pressure drop of pressure ratios between the two sections.

Now let us see what now. So, basically so this is the region. So, this is the region u p's functioning. So, if I have to write this u p by a 2 we can write this as u p by a 1 to a 1 by a 2. So, that will also be. Now the reason I do that is because then I again I can write all of this just in terms of p 2 by p 1 and (Refer Time: 09:51) which is what we get. And therefore, u p by a 2 comes out to be this. So, this comes out to be this way. Now let us do a little bit of math out here, ok.

So, now just for the ease of working with this sort of a little elaborate expression, let us just denote this I am going to just call it theta. This is just there is no physics in here it is just representing this as another variable, ok. This is just a dummy variable.

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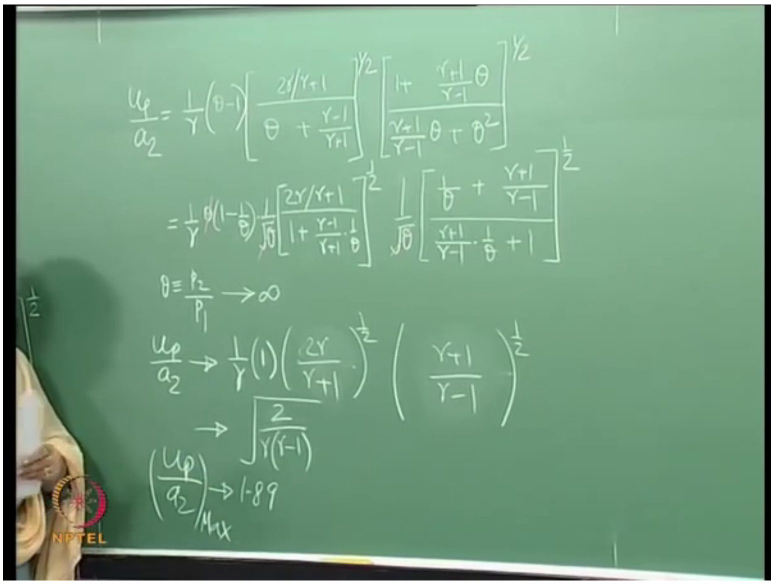

So, if I do that what I get is this. So, that is the second term. And then we have this which is. So, we do that. Now, I am going to do something a little one more step out here; I am going to write this like this I do that. Then again what I do here is that basically I take theta out, so what I get here is 1 by root of theta. So, I get 1 plus.

So, hopefully you can see what I am doing. So, what I to did is, took theta out of this you know bracket here, so what I get is this. Then again from the denominator I take a theta out, so then I get this here and this expression becomes like that. Notice we have a square root, so hence the root of theta here. So, we do similar stuff over here. So, what again I get, I take theta out of from the numerator, so I get this and I take theta square out from the denominator then I get this. So, get 1 by theta, so what we get out here is this. So, this root theta this root theta cancels with this so let us just cancelled that out.

So, now, that we have this expression and let us look at this. So, what we are basically saying is that theta is this. Now let us just say that this is its tends to infinity; it is infinitely large what that means is that we have an infinitely large shock propagating, because that depends on this so larger the pressure ratio between the two regions. So, let us come back in here. So, p 2 minus p 1 is the larger the pressure ratio between these regions you have a larger shock which is propagating.

So, let us just say we will take as you know an infinitely large pressure ratio between these two, which means that this say mathematically this is tending to infinity. If that is so then what happens to this that tends to what. What we get there is this, this becomes 1, that becomes 1, then in here what we get is 2 gamma by gamma plus 1, at bottom we get 1, this is square root, what we get over here is again 0 ,and 1 and what we get is this. So, let us just say we write it a little better here. So, what we get 2 gamma like gamma plus 1 and gamma plus 1 by gamma minus 1.

So, what we get over here is essentially this. So this tends to; what this tends to is basically 1 by gamma into gamma minus 1, is that right let me just sort of cross check that. Sorry, we have the 2, so it is basically 2. So, 2 by that is what it is. So, what we get is this.

So, what we are saying here is that for an infinitely large shock, if you have an infinitely large shock the Mach in the region behind it right tends to this. So, for a given gamma of say 1.4 so therefore this becomes 1.89 which is less than 2 Mach. So, what this tells us that if we have an infinitely large shock which is propagating then the velocity which is induced in the drive section here; the velocity which is induced is it reaches a maximum of around 1.89. Now, which means that the Mach number of the different section, so this is the maximum. So, the maximum value of this is 1.89 which is less than 2 Mach.

So, the basically the Mach number of the induced flow it cannot exceed more than 1.89, alright. So, that is a little bit about, so the incident shockwaves in the shock tube. Let us sort of move a little further and see something else now. So now, that we have this. So, we have this incident shock here and this is the steady case of this, now let us look at another picture. Now what happens is now this shock moves is moving into the tube right. Now you can think of this that suddenly we close the valves in this region or say these hits against this wall and then it reflects back. So, usually in various industrial applications there is a sudden closing of a valve and what that does it creates a reflected shock wave in the other direction. So, if you have say something like that this is travelling into the tube and there is a sudden say closing of the valve out there, then what happens.

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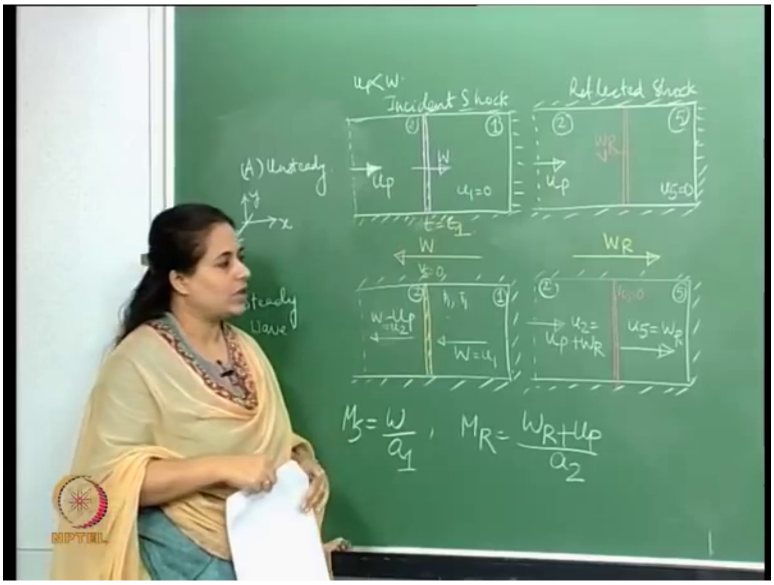

Well what will happen is; I still have this, this is my region 2 and then this shock meets that region and its reflected back and obviously the strength of that is now different it is not the same as W. So, say it starts moving in this direction and let us call this as W R, and let us call this region as 5. Again here also because it comes to us very sudden stop or maybe even if it is facing in the wall is may basically comes to sudden stops the. So, the flow of velocity here is 0, ok.

So, this is essentially the reflected wave. Now again this is essentially the wave picture of the reflected wave. So, we can of course convert this to the steady state case by imposing a velocity which is exactly in this direction. So, this is what we have over here. Now, again this is the unsteady case and this is the. So, let us just say that this is the incident shock; this is the incident shock, this is the reflected shock; this is a reflected shock. So, let us just you know convert this to the steady picture of that let us do that.

So, if I had to do this. So, we have a shockwave out here. So, this is moving this way. So, now, what we will do is impose the whole flow with a velocity which is this in this direction. So, what that would mean is that the velocity of the shock wave; the velocity of the reflected shock wave is 0. That is 0 and then of course in; so this is region say 5 and this is the region 2 in that case what we get is u p plus W R. So, this is equal to say this and u 2 is this and u 5 is equal to W R; u 5 is equal to W R.

So, that is all there is. Now, how do the governing equations look like for the reflected shock waves? So, all we need to do is, therefore input these velocities into those. So therefore, what we have for the governing equations, if you were to go back there.

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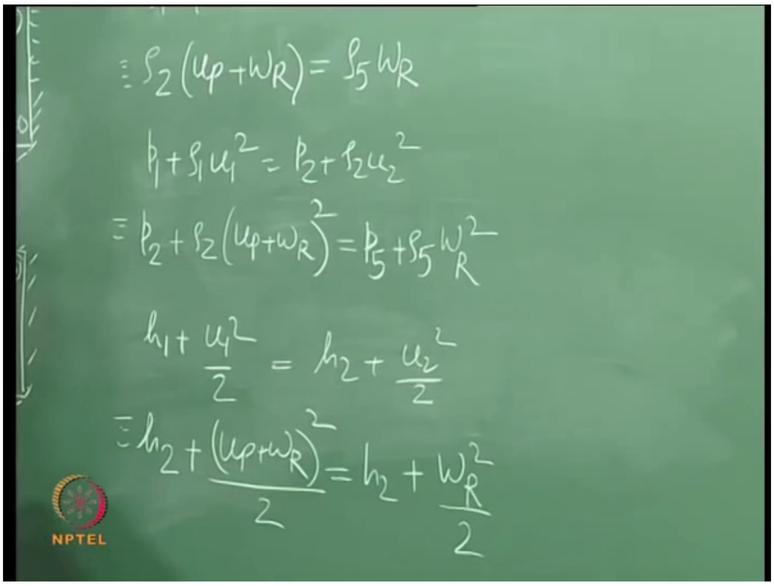

So, what we had is rho 1 u 1 is rho 2 u 2. So, in that case what this rho become is that rho 2 u p is W R is rho 5 W R. So, that is the continuity equation then similarly, so that this is the continuity equation. So, then we have the momentum conservation, so similarly this then becomes p 2 and that is equal to p 5. So, this is the momentum conservation. And finally, we have the energy equation. So, if we have this so then this gets transformed to and that is equal to. So, this is what we get from the governing equations.

Now the question is; we have two more relationships if you remember here, we have two more relationships. For example, for the incident shock we had this and for the reflected shock then we shall have, ok. So, now for the incident shock if you look over here this is for the incident shock. So, the Mach number of the incident shock was W by a 1. So, if you look at this steady state picture here we have a flow coming in the intersecting the shock and moving passed it. So, this is the Mach number that we are concerned about. So, this is W by a 1 in this region.

Now for the reflected shock the steady state picture looks like we move from here into this region. Therefore, this is the shock Mach number that you are concerned with, this region. So, that is W R plus the velocity there is this and a 2. So, this is the Mach number of the reflected shock wave.

But these are the relationships that we know. Now if we use these two relationships we can see that we can get some relation between the reflected shock Mach wave and the incident shock Mach wave. So what that means is that for a given incident shock I will have an estimate of what sort of shock the reflected shock will be; which in turn means, that if I am aware of the pressure ratios between 2 and 1, right between these two the driven and the driver sections then I have an estimate of the nature of the shock which in turn gives me an estimate of the nature of the reflected shock.

So, again without getting into the elaborate derivations let me sort of write this out.

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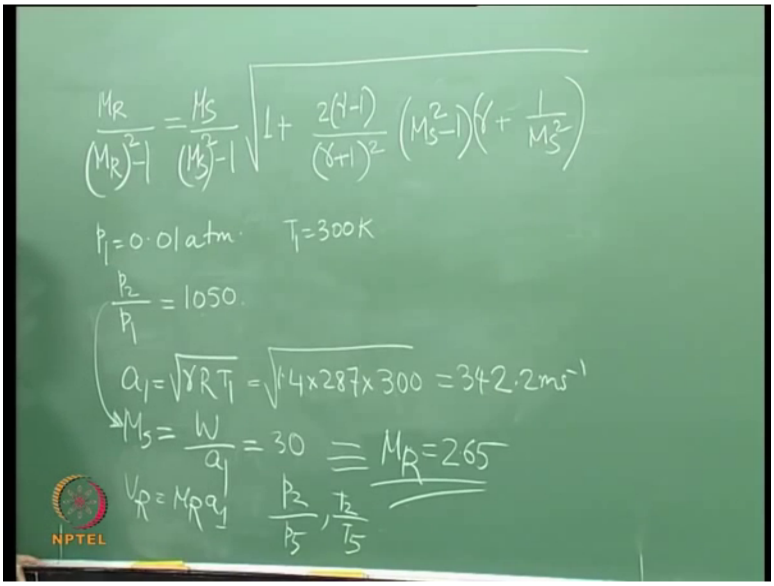

So, then what we get here is this is a kind of an expression that we get, ok. So, this is the relationship between the incident shock Mach wave and the reflected shock Mach wave. So, this is essentially the relationship between the incident shock Mach wave and the reflected shock Mach wave. So, if we know this then we shall have an estimate of this. So, let us sort of do a small problem based on this, and see what exactly we are talking about.

So, the problem says that an incident normal shock reflects from the end of a shock tube wall, if the air in the driven section, right. So, this is the driver section, because the shock is moving into the right. So, as we said we said in the last lecture that the pressure out here before region is much more than 1. Therefore, when we break the diaphragm it moves into the quiescent flow here. So therefore, this is the driver section this is the driven section. So, the air in the driven section of the shock tube is at p 1. So, p 1 is given as (Refer Time: 30:02) and T 1 is 300 Kelvin and the pressure ratio across the incident shock is 1050. So, let us say what is given.

So, the problem here p 1 is given as this and T 1 is given as 300 Kelvin, this is given. And the pressure ratio across the incident shock which is its quite large actually. So, the pressure ratio across the incident shock if the way you choose to locate it if you look at say the study version steady picture of this case, so basically I have a flow which is moving across a standing shock. So, the pressure in this region is given p 1 and T 1 are given right, and the pressure ratio between p 2 and p 1 is given as 1050. So, what we need to find out is calculate the reflected shock velocity with respect to the tube.

So therefore, we have something like this. So, we have an incident shock moving there is a sudden closing of a valve and we have a reflected shock propagating. So, what we need to find out is the reflected shock wave velocity with respect to the tube. Now you have to remember that, because the reference from with respect to the tube and outside it in the laboratory is going to be different and pressure and temperature behind the reflected shock wave. So, pressure and so behind this shock wave which will be in region 2. So, in that case how do we go about this?

So, we will basically look at this picture out here. This is for the incident shock, so we will look at this picture over here. So, in 2 region this is my velocity and this is my velocity. Now what we can find out here is the a 1 which is gamma R T 1. So, then taking gamma to be 1.4, so what we have is 1.4 287 and T 1 is given to be 300 Kelvin. So, this comes out to be 342.2 minutes per second, so I get this. Therefore, what I get from here is the incident shock Mach wave, isn’t it. So, that is W by a 1. So, what I get is W by a 1. Now how I we going to get I have a 1 here how do I get M S because we do not know W. What we know however is p 2 by p 1, because that the problem that we did last class so we knew or we calculated the incident shock Mach wave, in this case what is given is p 2 by p 1.

Now this p 2 by p 1 corresponds to this incident shock wave. Therefore, we just go and look at the normal shock tables corresponding to p 2 by p 1 which is equal to this we find out the incident shock Mach wave then Mach number; which in this case comes out to be 30. How do we therefore calculate? Now that we know this, now that we know this let us use this relationship here which is the relationship between the incident shock Mach wave and Mach number.

We put this value in there with a gamma of 1.4 then we will be able to get a value for M R. So, without going into the details of that calculation what I get from here the reflected wave I get as 2.65. Now, basically this is reflected wave shock wave I get 2.65. Now, basically this if I say look at this picture over here; so now we need to find out the reflected shock wave velocity with respect to the tube. So, with respect to the tube we need to find out the reflected shock wave; which is if you look at this picture here, what I can find out. So, I know a 1; right I know yes I know a 1. Therefore, I can calculate the velocity the reflected shock as a 1.

Now having done that what we need to calculate it are the pressures and temperatures behind the behind the reflected shock. So, this is the reflected shock, so behind the reflected shock is basically in this region and this region is quiescent, because again it has it initially the shock was moving into the quiescent region and again this has brought you a sudden stop and its sort of reflect moves in the opposite direction. So, this p 5 in here, so p 5 and T 5 or it basically equal to p 1 and T 1.

Now corresponding to this Mach number; so corresponding to this Mach number again we go to the tables and find out. So, basically p 2 by p 5 actually and T 2 by T 5 and then we shall be able to calculate the pressure and temperatures, because like I said p 5 and T 5 is equal to p 1 and T 1. That is a small problem basically to see what we mean by this I know how we are going to basically calculate you know these properties. Now, let us some look at something little more interesting over here.

Now let us go back to this picture over here. So, let us look at these two pictures. So, we have an incident shock which propagates and then it reflects from the end of the wall and propagates in this manner. So, what we are going to look at is essentially, what is happening over time.

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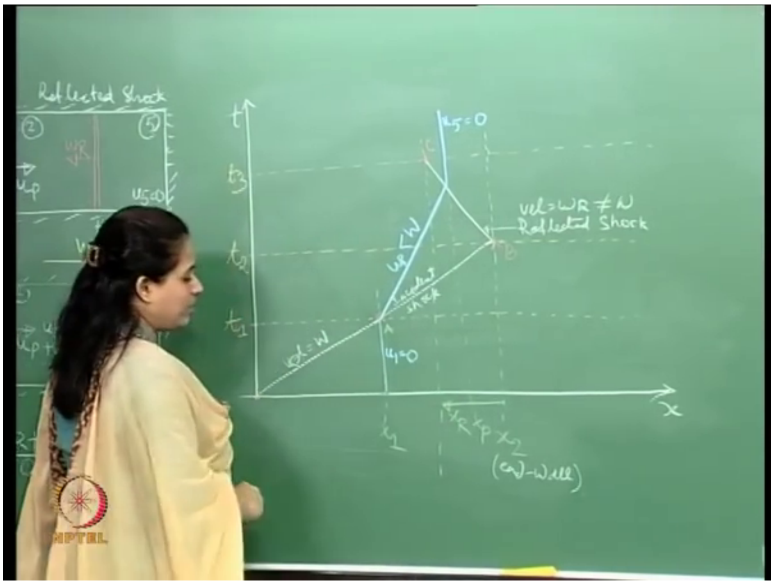

So, this is essentially an unsteady case isn't it? So, I have things travelling in this direction isn't it? So, say I have this is say x direction over a period of time. Now at T is equal to 0 there is nothing. So, basically T we can say that I am here, so essentially say I am at this point. Now, at T is equal to T 1 say the diaphragm is broken and we have a shock wave which starts at some instant of time. So, just about at T is equal to 0 say in a little bit away from that so the shock of the diaphragm is broken and we have a shock wave travelling.

So, say the shock wave travels for around say T 1. So, this is a picture for example. So, this is a picture for example say- T let us just say that this picture that we are seeing is at a time T 1. So, what has happened is that this has travelled a certain distance the shock wave. So say at time T 1, let us just say this is a time T 1; this is time T 1 and at that point it has travelled a certain distance say which is x 1. Therefore, what we see here is that I am at say this location. So, basically therefore, right. So, if I may call this as; we call that at A.

So, the A point that; the point A in that x T diagram is essentially this picture here. At 0 there is there is no shockwave and just you know just at that point we break the diaphragm, the shockwave starts travelling. And this is a picture that you are seeing at say some time instant T 1 when the shock wave is travels a at distance x 1. So, at time T 1 it has travelled a distance x 1. So, this is the x T diagram.

Now after that again what happens is the at say time T is equal to T 2 it hits the end wall. So, basically what happens is that it travels some more distance and hits the end wall. So, the reason we do that is because this is going to continue, isn't it; the same slope is going to continue because the same velocity is going to continue, is going to travel with the same velocity W. So, this is essentially. So, this is the incident shock.

And then at time say T is equal to T 2. So, at time T is equal to T 2 it hits the end wall say which is at a distance after travelling say a distance this much. So, let us call this point as B and then this is the point. Therefore, this is essentially the incident shock. So, this is the incident shock. So, this is the end wall; this is the end wall here; now after this what happens is that there is the reflected shock. So, the picture that we see here so at this point here there is no reflected shock, but the moment it hit this immediately after that there is the reflected shock and this is what we are seeing at say time T is equal to T 3 the picture that we see here is at T 3 when the reflected shock has again travelled with some distance.

So, now, let us say at time T 3. So, this is T 3 and basically now the origin of the reflector shock is going to be here at this point p, but the velocity of the reflective shock is going to be different than W. Therefore, the slope of that is going to be different. So, say therefore, it travels by some distance. So, travels by some distance and say which is here. So, essentially what I am saying is it travels from here in the opposite direction like that. So, let us just say it travels by some distance x R, the origin now being here. So, therefore, what happens is that.

Therefore, I have this reflected shock wave. Therefore, this is essentially a let us call this. So, this is the picture that we are essentially seeing x R and this is the reflected shock. So, this in here this is the; and velocity is W R which is not equal to W, which is not equal to the incident shock. So, this is what we see over here.

So, therefore, this is this is basically the x T diagram and if we have a plot like this you can see pretty much the way the shocks are moving etcetera. Let us also look at this at a certain in a in a different perspective. Now say- in the shock tube at a location x 1 a and we have a particle right which is sitting here and what happens then is its just sitting there; it does not move right it does not move it just sits there. Until this incident shock meets it isn't it. So, it meets it at time T is equal to T 1. So, therefore, I have this say particle which is sitting at this distance x 1, it continues to sit there for a time T 1 there is no velocity there it just sits there. It continues to sit there, so let us I am going to call that as say; its sit there like that.

And after that, so after that what happens is that the shock reaches it and displaces it from its position and impose the velocity u p into the it causes mass motion. So, then it starts moving with a velocity which is u p. Now that qp as we said the slope is less the velocity is less than that of the incident shock. So, then let us just say moves with this velocity, it moves like that; it moves like this. So, this is my u p which is less than the incident shock. Now what you can see out here is that this particle as it is travelling here because it has been displaced from its position due to this incident shock again it meets the reflected shock, which is what you are seeing over here.

So, again it meets this reflected shock right, it means this reflected shock and the shock is reflecting because there is a sudden stop so that the entire fluid here is going to come to a stop. So, the moment the particle comes here intersects the reflected shock it again stops completely. Therefore, the velocity out here again will be this now it has reached the region 5 which is again equal to 0.

So, essentially this is kind of a little graphical way or an interesting way to kind of understand the functioning of the shock tube. So, what essentially this diagram is giving us is a picture of the movement of the various shocks out here with respect to time, because this is an unsteady case. So, we found out relationships between the thermodynamic variables etcetera, but what this is giving us exactly that what is happening at various instants of time along with the in the particular directions in the along the length of the shock tube.

So, again here basically what you can see here is that you have a; that therefore, this could be call for interaction of shockwaves because you have this incident shockwave right that reflects. And what you see over here that this is a fluid particle which was originally at rest it was displaced from its position by the incident shock which from, and it starts travelling with a velocity u p which is less than that of the incident shock. It is travelling and would have reached the end of the tube, but before that it intersects another shock wave which is the reflected shock wave. And what that does it completely stops this particle in a spot, right. And therefore, the particle again continues to being at a stop.

Therefore, the particle basically has been displaced from this position x 1. So, say two the position x p. So, essentially the particle is travelled from x 1 to x p in given this time.

So that should be all.

Thank you.