Advanced Gas Dynamics

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Lecture – 11

Pressure- Deflection relationship of Shocks

So the problem that we did last time, which was reflection of shock wave right from a vertical wall.

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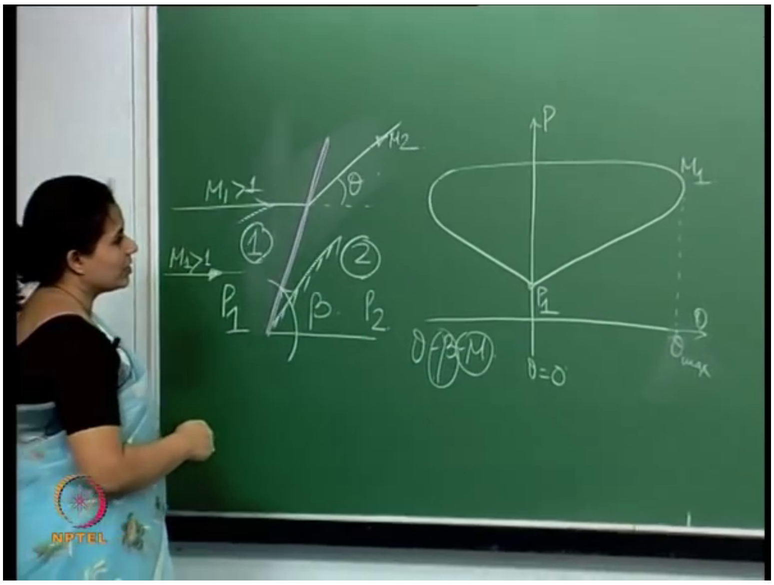

So, basically what we have is a compression collab or so with this and we have a flowing coming in right supersonic flow coming in and it encounters this sort of a corner which results in turning the flow right and this is due to a shock wave.

So, let me sort of draw this a little right. So, we have inclined or an oblique shock wave. So, therefore, when we say these encounters; this is flow coming in, then this gets deflected right and say this angle is theta and the shock wave angle is beta. Now let us try this here that now we know we have talked about the theta beta M relationship. Now this is a diagram which you can which is a standard in standard books they should this diagram is available basically it is plotted for various combinations of theta beta and M right.

So, let us say for a given shock wave angle right and for a given mach number, we change the theta it goes through a range of thetas, then we will look at the pressure change right with the change in the deflection angle. Now this is for a given mach number and a given shock wave angle. So, for this particular shock say right and this particular mach number, if we change this theta can be changed for various range now in that case how does the pressure differ?

So, in such a case for example, now say in here this is say region 1 and this is region 2. So, the static pressure here is P 1 and the static pressure behind this is P 2. So, when you see basically here say this is the point where theta is equal to 0. So, this point is P 1 right P 1 and if we plot a pressure diagram, it will look something like this. So, this is basically for M 1 right and as you can see that there is a maximum value of this a maximum value of this theta right and maximum value of this theta and as we have talked about before if the deflection angle is more than theta max right, then we will not have an oblique shock, then we will yes. So, this; what we have in terms of the P theta diagram or pressure deflection diagram. So, now, the one region here is denoted by say this point P, now let us do a particular thing over here.

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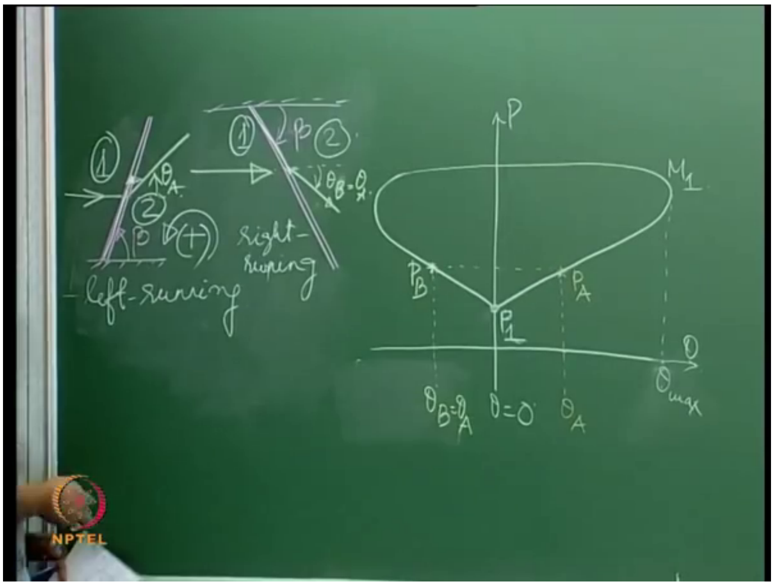

Now, let us do 2 things; let us remember the; now, say I have a shock wave like this right. So, this is a shock wave say it happens from somewhere over here right. So, if I and the flow is from left to right over this. So, flow is this way right. So, say I move; I move this way stand here on the wave on the shock wave and look downstream and I look downstream over here then this shock wave is to my left right. So, therefore, this is called; it is called a left running shock wave right.

So, essentially what I am doing is this shock is generating from here and propagating this way right and if I say I move along with the shock stand here on the shock and I look downstream right. So, then this shock will be to my left right and therefore, this is called a left running shock.

On the other hand, like we did last time right say we have right and we have a shock wave like that right. So, in this case, this angle is say beta and this angle is say also is just say beta, but you can see that the senses are different it is just in a different direction now in here too. Now say I the flow the direction of the flow is the same. So, here again this is the direction of the flow. So, this is downstream, now I travel like that along with the shock right. So, I the shock is generated here. So, I start travelling with it and stand here on the shock and look downstream right then the shock is to my; so, I look downstream like that then the shock is to my right. So, therefore, this is called a right running shock this is called a right running shock. Now if for this particular case, then right now this is say a deflection angle theta.

Now, for example, say this here say for this particular case that I am showing you. So, for example, so, say it comes here and it gets deflected right it gets deflected by say theta A here it gets deflected by theta A and this comes here right and this also gets deflected by theta A, it is just that it also gets deflected by say let us call it theta B.

B which is also equal to theta A; no, in magnitude both are same it is just that this is deflected in a clockwise fashion. So, if you consider this as positive and this is negative. So, if I were to come here. So, therefore, say I am going from. So, let us call this, here this is say section one and this is section 2 for the left running wave and this is section one and this is section 2 for the right running wave, for the left running wave.

So, P 1 is this condition which is also the condition for the right running wave. Now for the left running wave when we come to section 2 here the deflection angle is theta A. So, basically what I do is come here. So, this is the point say right. So, this is theta a. So, this the corresponding pressure right. Now similarly we come on the negative way to theta b. So, if we come if we do that. So, it is the same distance over here. So, say this is right this is theta b which is equal to theta A. So, therefore, this region here is P b this is how we represent these 2 points on this P theta diagram. Now let us do a thing over here.

Now, this becomes interesting now if we consider the diagram consider the problem that we did last time let me just sort of kind of remind ourselves what we did.

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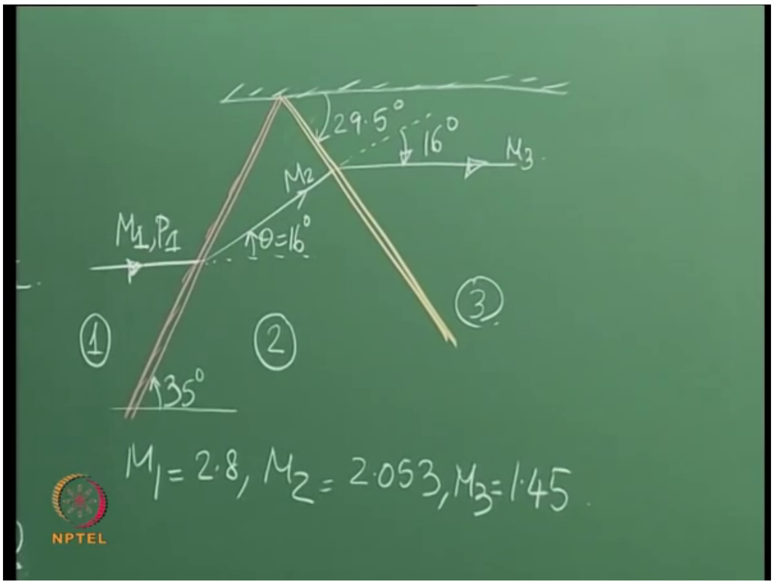

So, basically we had a shock wave right which was travelling right and this angle, I will have to look that up. So, what was the wait a while. So, this was 35 degrees, this was 35 degrees and then it hits a wall over there right it hits a wall over there and what we get is the reflected shock which we found out to be finally, this reflected shock was I think 29.5; yes. So, that was 29.5.

Now if so; let us call now this region as 1, 2 and 3 and what we are going to look here is basically we will try and look at the pressures P theta diagram. So, if we want to do that let us look at the deflection angles. So, this was the incident flow. So, this was M 1 and P 1 right, then it got deflected and this deflection was 16 degrees, if I am not correct yes. So, this was say theta is equal to 16 degrees and here again this deflection was 16 degrees. So, this is region and this say M 2 and this is M 3. So, now, we found out now we know that M 1; M 1 is 2.8, M 2 is 2.053 and M 3 is 1.45 now these are things that we know.

Let us see if we can plot the how these points will look this 1, 2 and 3 on the P theta diagram how we going to do that. So, so let me go to the other corner ok.

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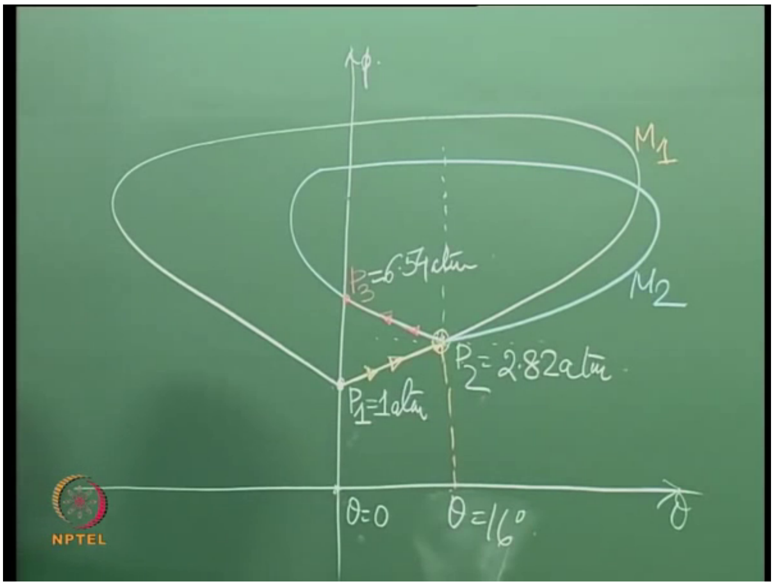

So, let us just start doing this theta and this is P now. So, now, for one if we come back here M 1 is here in one in one region. So, this is the incident wave. So, let us for M 1 we will get this curve. So, this has this has to be you know symmetric and stuff I am not doing this outline right. So, this is basically P 1 and this here theta is equal to 0. So, this is P 1 now, then let us look at what happens when we go to 16 degrees now, then at 16 degrees now. So, this gets deflected by 16 degrees right. Now let us go to 16 degrees here. So, this here this point here. So, this here theta is equal to 16 degrees that is the 16 degrees. Now you see at that particular point. So, I basically deflect this by 16 degrees and reach this point. So, this will be my P 2. So, what I do here; what I do here is that I move. So, if you look at this point this is my theta 16. So, I move from here to here.

So, I move here to this point which is P 2 and corresponds to a deflection of 16 degrees now the next thing is that the next shock wave the next shock wave is this one. So, whatever I did here was for the; this is for M 1. Now this is for M 1 and the pressure of pressure of the flow static pressure in front of the shock is P 1 and behind the shock is P 2 right for this corresponding for this M 1 right and the deflection is 16 degrees. So, that is what is that is the kind of information we are getting from this kind of a plot. Now we come to the reflective shock. Now this is the reflective shock; now what happens here is that the incident shock itself is at an angle 16 degrees right if you look at this; this is M 2.

So, unlike here where the incident shock the theta was 0 the deflection here was 0. So, therefore, we got P 1 over here. So, the corresponding deflection is 0 and in the second region the deflection is 16 degrees. So, corresponding to 16 we got P 2. Now for this region; however, the incident way there are 2 things now why is that the incident mach number here is M 2 which is different from M 1 and it is also inclined at an angle 16 degrees.

Therefore what happens here is that on this diagram the origin of my next P theta diagram moves here right moves here because the it is inclined at an angle of 16 degrees already. So, this is for M 2. So, if I have to do this over here. So, say let us do this. So, therefore, the origin here shifts over here for the next one. So, therefore, what we get here is this and. So, this is for M 2 which is already inclined at 16 degrees. Now how do we look for the point P 3? Now what we look for P 3 is that for M 2 right for M 2 which is inclined at 16 degrees this moves 16 degrees the deflection is 16 degrees in the negative sense. So, therefore, here on this diagram what we do is for in on the M 2 plot. So, we move in the opposite direction exactly for 16 degrees. So, this distance is 16 this is also 16. So, we just move in the opposite sense right and this becomes P 3 right. So, this becomes P 3.

So, as you can see here again that corresponding deflection here is 0 right which is what is being shown here because once it deflects by 16 degrees it is parallel to the upper wall. So, therefore, so, this is how we represent basically the 2 this was the incident shock and this is the reflected shock. Now let us do something one more example like this. So, here let me just sort of write down now this pressure P 1; P 1 was one atmospheres. Now this P 1 is one atmospheres; this is the theta; this theta is 16 degrees, then P 2; P 2 is 2.82 atmospheres and P 3 is 6.54; I think right yes P 3 is 6.54 atmospheres.

So, if you look at this diagram you should be able to tell the pressure in the 3 regions here and the corresponding deflections that the incident wave is undergoing. So, this is. So, what we did regarding the P theta diagrams and this is the problem which we did last time. Now let us look at something else over here something a little more interesting. So, say we have this say we have. So, we had a compression corner and we had an incident shock wave right and it was hit in the other wall we had an incident shock wave and a reflective shock and so on.

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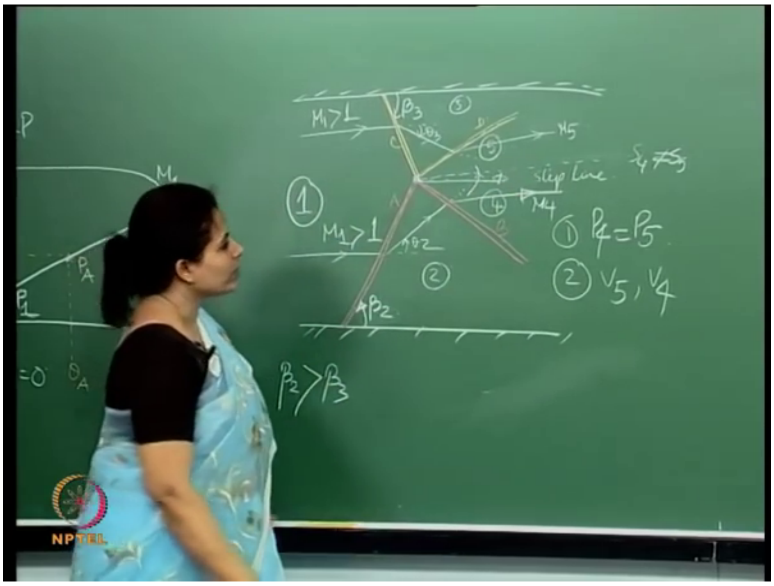

Now, let us have 2 walls like that comes in area. So, let us have say ok.

So, let us say we have 2 walls over here. So, then. So, say we have a shock wave which emanating from here and this is a shock wave which is emanating from here. So, let us say that is what am I going to call this. So, let us call this say this is region one this is region one and this is say theta 2. So, or say shock wave angle is this. So, and we have another shock wave which is generated from this side from the other wall and this angle is say beta 3 and let us just say that beta 2 is greater than beta 3. Now if that happens; now what we have in this scenario is that we have 2 walls we have 2 incident shocks and they seem to be hitting for each other. So, what happens ok.

So, basically. So, we go here and they do meet at a common point. So, this is a common point now, then this continues as a refracted wave this refracts from here and similarly this continues as a refracted wave. So, when that happens now there is a certain. Now let us sort of discuss this a little bit. So, let us call this region as a 2 and this region as 3, this region as 4 and this region as 5.

Now, what we can see here is that. So, if we have a. So, if we have an incidence like that and if we have an incidence like this now these 2 deflection angles are going to be different; obviously, right. So, then these 2 will deflect in a certain it will deflect in a certain different way and then it hits here. So, let us say this is theta 2 and similarly this will deflect and this region is 3. So, this is theta 3. So, that is theta 3 and then it comes here and then it deflects again; now let us talk about this a little bit here.

Now, as you can see that the shock wave angles are different now we did a problem last time and we showed that if the mach numbers are same then by changing the by changing the shock wave angle beta we were able to change the nature of the shock which means that the shock becomes stronger or weaker right keeping the mach number same. So, now, when you keep the mach number same your changing beta as a result of which the deflection angle is changing right now because of the 2 different deflection angles. So, therefore, these 2 are different sort of shocks right and therefore, the entropy change across the shock structure say this one which I am showing in pink here. So, say which is say A, B right is very different from the shock structure C and D.

So, when I have an incident over here which interacts with the; which first gets deflected by the incident shock A and again gets deflected by the refracted shock B. So, now the entropy change between say region 1 and 4 is different from the entropy change between regions 1 and 5. So, therefore, basically what we are saying is that S 4; S 4 is not equal to S 5. So, therefore, now we have basically the discontinuous entropy change ok.

Now, if you look at the structure over here if you lo0ok at the structure over here. So, now, this is a place right. So, this is a place across which there is a discontinuous entropy change and this is called a slip line this is called a slip line now there are some there are some physical reasoning behind this; however, or say downward conditions. Now the where this is these are horizontal these are straight walls. So, this line has to be this line will be a straight line. So, it is inclined it is fairly inclined at some angle say uh phi here not know what and this and to be a straight line instead of a curved line the; to be imposed is P 4 is equal to P 5; P 4 is equal to P 5 and also as you can see the velocity is here. So, basically if it comes like that. So, then it gets say deflected like that right. So, say this is M 5. So, similarly if this goes here and it is this is. So, this is M 4. So, therefore, now this is P 4 is equal to P 5. Now the velocities V 5 and V 4 are different in magnitudes are different in magnitudes, but they are in the same direction.

Now, what we shall do is now for this sort of a shock structure let us go ahead and look at the diagram and see if we if we can get you know anything interesting from here what we will do is. So, clearly what you can see here is that when this the M 1 is using is going across the shock structure C and D. So, this deflection is going to be different here the M 5 deflection is going to be different than M 4. So, they are going to follow the same the slip line. So, if we do this. So, let us go here and try and plot.

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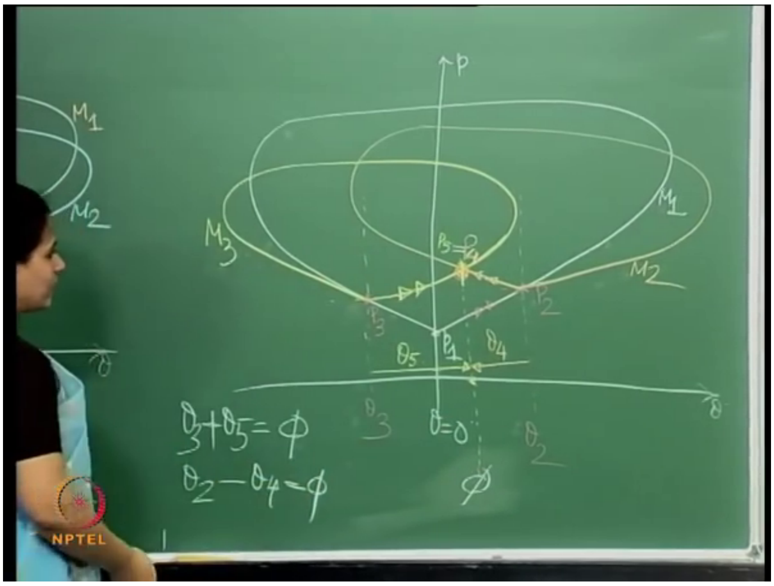

Say the P theta diagram. So, like we said this is for the here. So, as we said before this is for region 1 and take theta 0 here and this is P 1. Now for the first case. So, this is say if this if for M 1 this is for M 1 and if you go from say region say 1 to 2 which is beyond the shock structure a right this is incident shock A, then it deflects this theta 2 it deflects through theta 2 and then it get and then it encounters the reflected shock B which is incident. So, that is inclined the M 2 here already is inclined at M 2 here is already inclined at say theta 2. So, anyway let us look at the point P 2. So, that is for M 1.

So, if we go on M 1. So, this is M 1 right and if I go along M 1 if I travel along M 1 to theta 2, then what I get is P 2; P 2. Now for the refracted wave; now refracted shock wave which is already therefore, inclined e at the point theta 2 origins has not shifted here. So, for that, basically this is the origin or the refracted wave. So, if I were to do this over here. So, this is already inclined at the point theta theta P 2 and now for the other case for the other case is that it moves here and it this is the incident wave now this is for

M 1 again. So, let us go back to the M 1 curve and deflect it by the theta 3 which means. So, this is the M 1 curve and what this is this is for M 2. Now here is the M 1 curve on that what we need to do is go negatively to theta 2 or if you remember theta 3 is less than theta 2 and that is how I have drawn this here. So, what you get here is this P 3 and the refracted the origin of the refracted wave of the shock structure d is moved here right. So, then what we get is this is the new shock structure.

So, what we get is this. So, now, let us sort of go over this; what we are looking at is that this is this is one shock structure right. So, basically we are looking at 4 things here. So, we have 2 incident shock waves right now for both these incident shock waves we have the same mach number right. So, therefore, this is region one. So, for region one; we have mach number one and here. So, we have this incident shock wave which; then get refracted as b right. So, therefore, we have this shock wave for which the incident mach is M 2 and we have this shock wave for which the incident mach is M 3 right. So, this is this is M 3 over here. So, which is what I have drawn; If you see the white plot here that is for mach number one. So, that is the incident mach number and then the second this M 2 is the refracted is the refracted shock wave for the for the for shock a actually and that is shifted and that that is basically incident the incident wave is now at theta 2 deflection. So, therefore, the origin over here and the other refracted wave is at angle minus theta 3. So, therefore, the origin is shifted over here ok.

Now, let us look at one thing now you can see here what is interesting is that that these 2 curves interact right these 2 curves interact here, but now for say this curve shock wave this is a right for shock wave b right this is the refracted shock wave B. So, we go along the curve of M 2, we go along the curve for M 2. So, this is for M 2 and like we have drawn here this is for M 3. Now we go along the curve M 2 right at some angle. So, we go along here, right. So, we go along here and reach now the point is that we said we said here that because of this physical condition here P 4 is equal to P 5. So, what we are saying is when mach one here goes past the shock structure A and B it. So, the pressure right behind the shock structure is P 4 and when this flow goes along the shock structure c and d then the pressure here is P 5 these 2 pressures are same.

So, for the second shock wave here b right for b here we are going here right. So, we have this pressure here. So, at some point we will have P 4 we do not know the say the angle here the deflection. So, now, let us say this is say P 1 2 and is that 3. So, 1, 2, 3. So, this is 4; this is 4. So, similarly if we go along mach 3 curve right mach 3 curve and we move like that then at some point we also get. So, if we move along this way right at some point we get P 5, but like we said like we said over here that P 4 is equal to P 5. So, there is one point here where these 2 curves interact. So, this is the point that we reach. So, this is the point we reach. So, that P 4 is equal to P 5. So, this is the region. So, you see this.

So this is the point now what we are saying is that from theta 2 therefore, we moved this much. So, we moved this much and this and this say the amount is say theta 4 and from theta 3, we moved this much here which is theta 5. So, therefore, what we can say here is that and let us call this angle let us call this angle as say phi. So, just from this diagram what you can say is theta 3 right plus theta 5 is equal to phi right and theta 2 minus theta 4 is equal to phi. So, if you just look at this plot over here. So, theta 3 plus theta 5 gives you phi and theta 2 minus theta 4 gives you phi.

So, this is what this set of diagram here. So, this is a P theta diagram for a shock structure of this nature now so having talked about oblique shocks and normal shocks in some detail. Let us now look at 2 things which I think is the right time to look at. So, now, I have talked about detached bow shocks right detached curve shocks behind.

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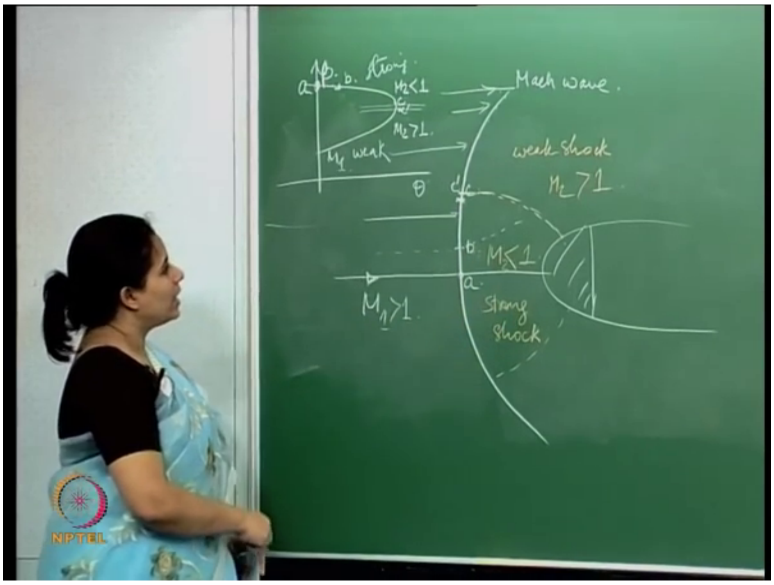

So, if we have say a blunt body right if we have say a blunt body like this right. So, we have these detached shocks like that. So, now, if we were to look at; so, let us just let us do something over here. So, let us say this is the. So, let us say this is the incident mach number here. So, this is the incident supersonic flow; now, what we are going to look at what we are basically going to look at the theta beta M diagram, right.

So, we have the theta beta M diagram. So, if you have theta and if you have beta, then you have plots you know like this right. So, this is what we have been talking about so far right. So, this for a particular mach number. So, let us say here we have this mach number this. So, what we are basically looking at here is that for a given mach number here you know for a given shock wave angle what sort of deflections is going to is the flow going to suffer at various parts of this that is what we are going to look at over here.

Now, let us say let us call this point at say a right. So, say this point is a now if you look here the flow is. So, the deflection is pretty normal right. So, therefore, it corresponds here. So, if I look here. So, for this point a, it corresponds to say this point. So, this is say point a right now if I move slightly further up right if I move slightly further up. So, say this is say b right. So, say this point which is b right. So, there is a little more deflection right instead of being straight normal, which means that I increase the theta a little bit. So, I come over here.

So, then this is beta now in here what you will basically see is that in this particular diagram over here right. So, here this is for M 1 and the further you come down this is for M 2 less than 1 and this is M 2 greater than 1. So, basically in this part of the curve what you have is that you have a supersonic flow right where it goes past the shock wave it becomes sub sonic. So, it is basically region of strong shock right and here what you have in this region is that you go from supersonic flow and the mach number behind the shock is also a supersonic this is the region of weak shock. So, let us just say this is a weak shock and this is the strong shock. So, say we move at the region b.

So, therefore, we are still in the strong shock region. So, we keep moving. So, we keep moving say at the point c at the point c now which is just about here. So, we still are in the strong shock region if we move from here just slightly a little bit over here say c dash. So, if I move to c dash over here I an in the weak shock region. So, there this very thin sort of region over there a little thing region over here where I transition from a strong shock to a weak shock region and after that it becomes weaker after that it keeps becoming weaker and beyond a point, it we get what we call as a mach wave I will talk about that just in a little bit.

So, now let us look at the let us come back and look at this region. So, therefore, if. So, in this region therefore, if I were to mark out this region if I were to mark out this region, something like this. So, basically from this region here this region here is a strong shock region right like will explain this. So, mach number is here say M 2 is sub sonic. So, this is a strong shock region and. So, beyond this you have a weak shock region this is a weak shock region. So, that M 2 is greater than 1. So, what you see is for a bow shock region you basically have a combination of supersonic and sub sonic flows. Now as the shock becomes weaker finally, we get what is a mach wave. So, what exactly am I talking about when I say mach wave.

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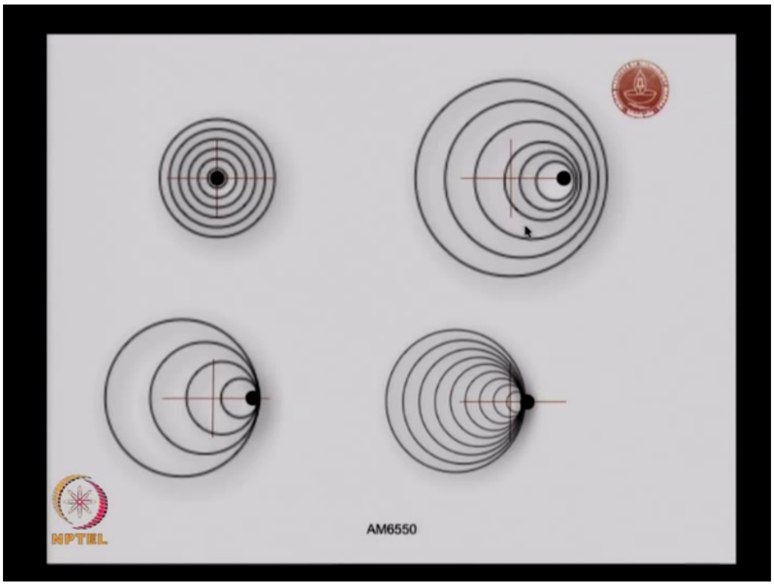

Now, let us sort of go back and look at this plot which we had looked at before. So, if you remember this in one of lectures we did this ok. I now say we do this; now if you see this is the; so, basically what we have is a stationary source of sound and which is emanating sound waves; this is the one for this here the source of sound is moving right and you can see that this is moving sub sonically in here it moves sub-sonically and if I come here hat you see here is that this is a sonic movement and now let us look at this. So, this is moving super sonically is it not. So, what we are going to be interested in are these 2 cases.

These 2 cases. So, these subsonic case and this is the supersonic case. So, let us just sort of look at this one I say this one is moving sub sonically what does it mean in terms of say numbers. So, for example, basically we have we have you know this is source of sound like a car horn say right.

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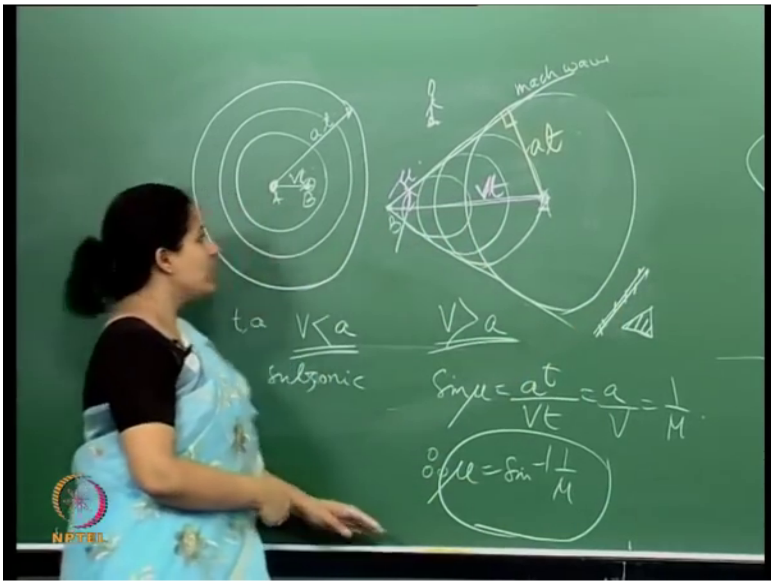

And say at its emanating you know sound waves. So, say at after I say time t where the speed of sound is a right. So, there is a sound wave right this is the sound wave which has travelled say for time t. So, this is at point a and this is say a t right it is continuously emanating. So, within this time t. So, we have other source of sound etcetera, but this source has actually moved over here right. So, this source has moved over here point b. So, and say it is moving with a velocity V t. So, this distance is V t. So, what you can clearly see from here is V is less than a. So, therefore, this is subsonic. So, this is subsonic now just now just let us look at what will happen if it is supersonic.

Now, so, if you look at this plot over here right if you look at this diagram here what you see is that there is cone of disturbance which is travelling right, there is a cone of this disturbance which is travelling and so, there is a large. So, this is the disturbance right this is a disturbance and if I would do this; this is again travelling at in this a; the source of sound has actually moved from a to b right. So, what you see is a cone right. Now if I draw a tangent. So, basically a tangential surface. So, if I am going to look from outside. So, all I look at is a surface right which is engulfing all this disturbances which is travelling at me. So, basically what this has is several other shock several other disturbances like this. So, this is what I am looking at now in this same time t the source of sound has travelled from point A to B. So, clearly V is greater than a. So, this is supersonic.

This is supersonic now just think about this if I were to stand here right if I were to stand here what I am going to see is that there i8s this surface region which is moving towards me at some sort of speed and this is engulfing a region of disturbance now this and this is moving at an angle say this is an angle. So, let us call this angle as mu now if you look at this triangle over here; let me make this a little more clear let is make this a little more say this. So, this is a t and we have this which is V t and this angle here is mu. So, if I do the simple math here is that sin mu is equal to a t by V t which a by V which is 1 by M right therefore, mu is equal to sin inverse 1 by m.

So, like I said if I were to stand here and look at this right all I would see is that there is this region which is moving towards me and it is engulfing a region of disturbances right. So, this is basically a mach wave this is basically a mach wave now consider this if instead of this car horn or small beeper or very small source of sound instead of that if we have say a cone if we have a cone which is blasting through the atmosphere at supersonic speeds then instead. So, what we will have here instead of this you know a weak sort of a wave travelling engulfing disturbances what we will have in this case is very strong shock wave is a very strong shock wave right. So, therefore, when we say here that we have a shock wave.

So, just think about it in this case we have a shock wave and as this shock wave becomes weaker it becomes a mach wave. So, basically mach wave is a limiting case of a shock it is a very very light shock wave. So, the more this becomes strong we engulf into that disturbance basically becomes a mach wave basically becomes a shock wave that should be all.

Thank you.