Advanced Gas Dynamics

Dr. Rinku Mukherjee
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture – 10
Example Problems in Oblique Shocks

So, continuing from the problem that we were doing. So, we said that this uniform supersonic stream with Mach number 3 right. So, even comes to the corner the deflects the stream by an angle theta equal to 20 degrees right and we calculated the shock wave angle which is beta then the parameter speed pressure temperature Mach number and stagnant conditions behind the shock wave right. So, these are the conditions that we started with right.

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So, these are the conditions and the corner will deflecting it by 20 degrees. So, this was what we did and we calculated the parameters. Now let us do something over here, keeping all these you know parameters the this is the that changing M 1 keeping P 1 same T 1 same to is change theta to 30 degrees right. So, basically what we talking about is that we have an oblique shock resulting right this is a free stream right and then this is something we had to calculate this beta we have something that we have to calculate.

So, this stream line is getting deflected this way and this was getting deflected by 20 degrees. So, this is the theta. So, now, we saying and this is happening because of a corner like that right, this is a physical corner like this right and this corner is resulting in causing this shock wave right and deflecting it by 20 degrees. Now we saying that let us increase this theta by 230 degrees and then keeping these parameters the same let us calculate the pressure and the Mach number right, the pressure and the Mach number on this side.

So, let us see we will do in that case.

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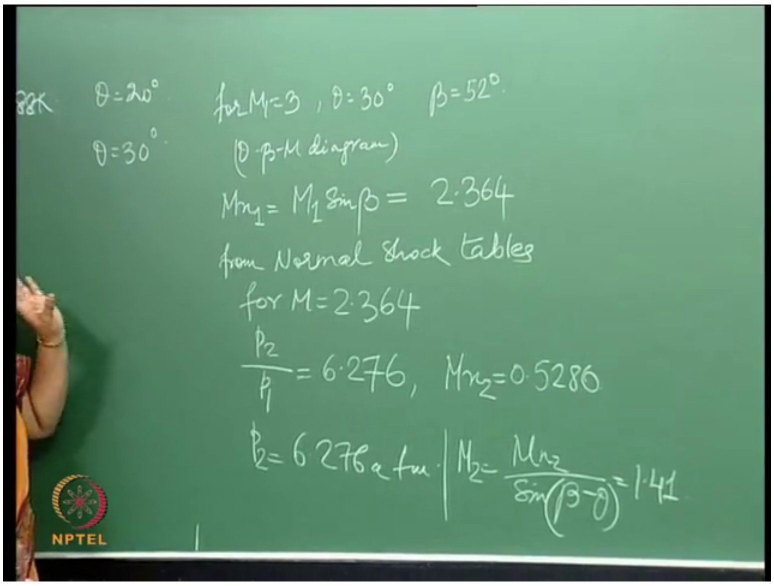

So, for this here again, so for M 1 equal to 3 and theta is equal to 30 degrees. So, from the theta beta M relationship from the theta beta M diagram we get so for this we get a beta this is from the, this is from the theta beta M diagram. So, from theta beta M diagram we get beta is 52 degrees. So, therefore, again right. So, the normal comprehend then comes out to be right.

So, then again using the normal shock tables right, from normal shock table, again what we get in for normal shock tables for this a Mach number right. So, for this Mach number we get and Mn 2 as right. So, from here we calculate P 2 and M 2. So, P 2 therefore, we can calculate as this into P 1, P 1 as now, which is right and M 2 again right. So, that comes out to be 1.41.

Now, what will be interesting is that we were kind of compared this 3 cases in compared the 3 cases and we will do one more case from here and then see the difference then you literally understand what I am trying to get at over here. So, this is what we you know needed. So, this is we calculated that. So, again what we did was keeping the input conditions the same we increase the theta right and these are the values that we get. Now what we basically you going to study now is the what exactly is a doing to the shock it right and the sense there is a shock become a stronger or weaker or what exactly is happening how do we find that out all right. So, let us do another case where we change the we increase the Mach number we increase the Mach number and we keep that same, we keep that same and keep later also same to 20 degrees right, so compared to the first case. So, we just increase the Mach number and keep everything or same.

Second case was we kept everything same just increase the theta in this case we keep everything same and increase the Mach number and for this case let us see what is happening. And the same we still and for this case to we will find out the pressures and Mach numbers behind the shock ok.

So, here to we go about the same way again right.

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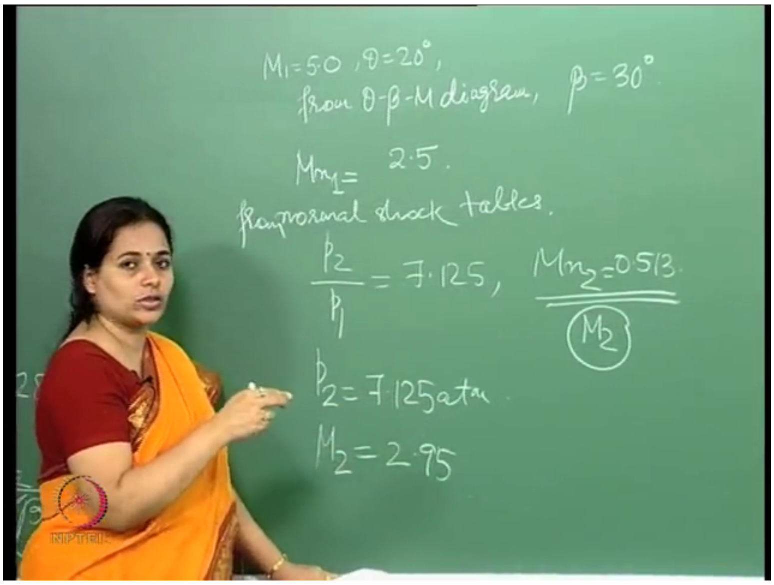

So, for Mach number Mach 5 right Mach 5 theta is equal to 20 degrees then from the theta beta the M diagram right we get 30 degrees. So, therefore, again we calculate the normal component, we calculate the normal component to be 2.5 right. Then corresponding to this Mach number we use the normal shock tables right. So, from the normal shock tables basically from the normal shock tables what we get is right and Mn

2 to be 0.513. So, therefore, from here we can find out P 2 to be right and M 2 to be 2.95. So, pretty much the you know if I were to sort of summarize this. So, what we are given is this is an oblique shock, the oblique shock study that we studying the properties across and oblique shock. So, we are given the Mach number we are given a corresponding deflection angle right, so first things we do recognizing that the flow properties across a shock wave are given by the normal component of the input Mach number right.

So, therefore, from, so this is not the Mach number which it will depend on it will depend on the normal component of this right. So, therefore, first things we do is go to the theta beta M diagram right this is again available in any, in the standard book and then we calculate the beta which is a wave angle right this is the angular which the shock wave is in client or the horizontal. So, then we get the angle and then we calculate based on that we calculate the normal component.

Now, this normal once we get this normal component of the Mach number we just go the normal shock tables which we have been using so for and we calculate all these ratios. So, there are other whole set of parameters right of you using just the pressures on the Mach number so these are all available. And what you should not is that this, this is the M 2 in the tables right. So, for oblique shock though in here it is the normal component of the M 2 that is not exactly the M 2 so we need to calculate the M 2 from there, which we do over here ok.

So, likely use the normal component of the Mach number to calculate these properties. The Mach number that we get here behind the shock is also the normal component and from there we calculate the Mach number. So, now, we will do is we will sort of list out the property change is for the 3 cases that we just did. So, let us let us do it here and see what differences we can make.

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So, say this is M 1 this is M 1, this is Mn 1. So, let us list out what we got for the 3 cases. Now the first case was Mach number was 3, so we got these values theta was 20 degrees beta came up to be. So, this was the first case for Mach number 3 theta 20 and P 1 to be 1. So, this was the first case.

Now the second case is the second case is again the Mach number is the same, sorry this is 2.364, this is 1.41, the theta we increase the theta, beta is 52 degrees, P was 1 and is the 6.276 and the third one was the Mach number is 5 then the normal component is 2.5 2.95 actually, no 2.5memory and M 2 is 2.95, theta is same beta comes out to be 30 degrees ok. This is 7.125. So, say let us let us just say that now 6.3 7.1, I think that is it.

So, this is what now let us look at these this chart little intently. Now just consider case 1 and case 2 now when we go we will keep the Mach number same all we change is the increase in the theta increase in the deflection angle from 20 to 30 degrees what happens and the pressure input the incoming pressure is also same right. Now what happens is what we see over here is that there is a large increase in the beta right. There is a large increase in the beta and this corresponds to the large increase in the pressure. So, there is a large compression right and the Mach number actually decreases from 2.03 to 1.41. So, this is, what we can say is that if we increase the theta if we increase the theta there they there is a pressure increase it calls us the stronger shock with increase in the deflection angle there is a stronger shock with the corresponding increase in the wave angle right.

So, if I want to write that the and if I want to kind of you know say write the term, let us see.

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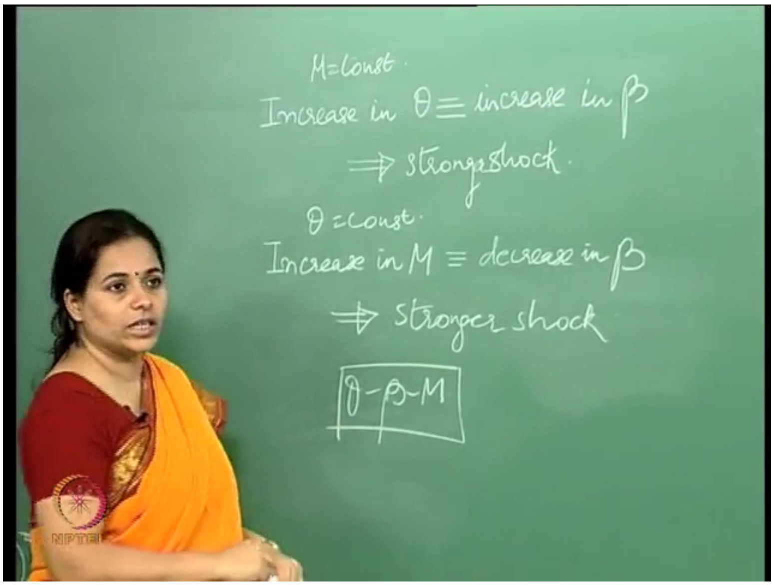

So, increase in theta right increase in theta results in a corresponding increase in beta right and causes the very large compression because it goes you can see this is what defines actually the pressure on the pressure behind the shock is what we will define whether it is stronger shock or a weaker shock right.

So, when in this case what see over here is that we have a large increase in the pressure right. So, this causes a larger compression and this basically results in a strong shock right, it is also in the strong you know strong shock or let us say stronger shock. So, basically if from this, this condition I increase for this condition I increase the theta I will get a stronger shock compare to what we have here that is why we need.

Then say between 1 and 3 this is do we compare and let us compare this and this right. Now, we keep the theta constant we keep the theta constant and we increase the Mach number what we see is that there is a increase in the Mach number behind the shock wave. So, there is an increase in the Mach number from 2.03, 2 point (Refer Time: 16:48), 2.95 there is actually a decrease and the beta now. Smaller beta actually results in a smaller pressure, but here although this is small and this is kind of common set it with the increase in M 1 and the increase in M 2 right. So, which finally results in a large pressure which its results in a larger pressure behind there shock which means we have and even larger you will stronger shock because this change in pressure is smaller than this change in pressure.

So, for this particular case if I keep the theta constant, but increase the Mach number I actually get stronger shock even compare to this right. So, what we can say over here is that increase in Mach number right. So, here, so for this case actually I think we should write that the Mach number is not changing for given Mach number and in this case the theta is not changing right. So, with the increase and Mach number you know causes a decrease in the beta right it causes a right, but this 2 causes a stronger shock I think I should write stronger. So, that is what we can see from here in this particular case right.

And also between these 2 cases you can see that there is an increase in the there is an increase in the Mach number right, there is however, decrease in the theta right we still and there is a corresponding decrease also in the beta therefore, right, but we still get a stronger shock because the pressure still increases. So, there is a larger compression in this particular case. So, basically what we try to do over here it its try to get an idea has to how changing the you know when we using the theta beta M relationship right this is the relationship which is basically governing the nature of the shock is it the stronger shock, is it the weaker shock or what it is right. So, that was we try to show over here there in how that these dependent each other right.

So, this like a small example to show how a change is change the Mach number keep in the theta constant or change the theta keep in the Mach number constant and how the nature of the shock changes. So, basically you know the nature of the shock the strength of the shock is defined by this pressure behind the shock right. So, that you can see therefore, both these cases there pressure is increasing. So, you actually get a stronger shock if you do these all right.

I think one of the more important things also when we study shocks is the interactional shocks. Interactional shocks and let us can understand what the means and let us kind of is do a problem and see if you can yeah get out what you trying to do for over here. So, so let us look at this particular problem say. So, we have a horizontal supersonic flow right at a Mach number of 2.8 and with the static pressure 1 atmospheres and temperature of 288 Kelvin. So, this passes over compression corner with the deflection angel of 16 degrees.

So, the same thing like we did in this case there is slide we will add a little more complexity to this. So, essentially what we have is a compression corner like that right and this is 60 degrees right and we have a flow coming in right this Mach number is 2.8 and the pressure is 1 atmospheres and the temperature, so a this passes over a. Now what happens over here is that there is a, so basically an oblique shock results. So, we have an oblique shock all or let us you know just for this. So, basically this is my this is my Mach number.

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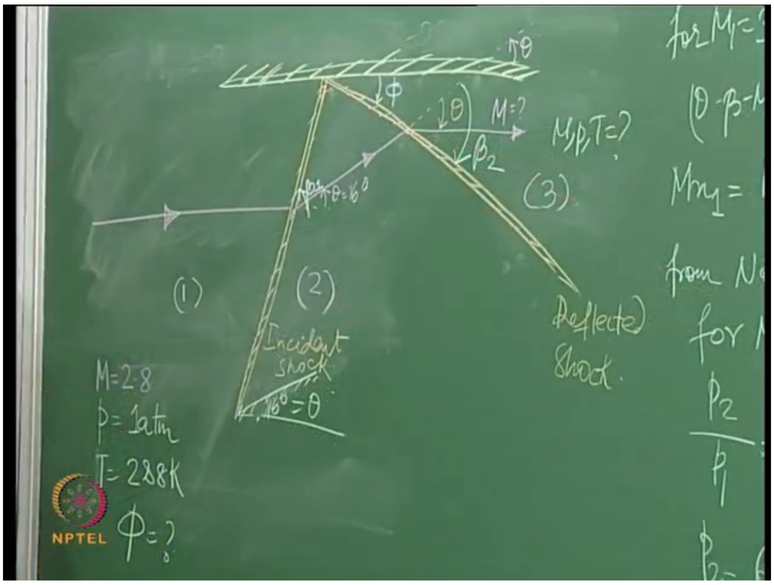

So, we have that and that reflect deflects like that. So, we have that. So, this is essentially by 16 degrees now the oblique shock generated at the corner propagates into the flow. So, this is the oblique shock right, this is generated over here, now it propagates sort of into the flow and what happens here is that we actually have, we have a horizontal wall over here now this is a shock this is a shock with propagates like that and it reaches it heats this wall ok now let me sorted of do this at work a little better ok.

So, say this is my 16 degrees this is the 16 degrees and, so this is it. Now, so this is something there if a familiar with so far you know this is this is something that we did just now right we calculated all the properties etcetera. So, we have a impinging you know flow it heats and oblique shock and there shock wave is happening because there is a shock corner over here, a deflection and that therefore, this stream line then gets deflected because it encounters the shock wave here. What we doing here is adding one more complexity is that this shock wave is now propagating right it developed over here, it is moving in here and it is heating this horizontal wall.

The question is what will happen, the question is what will happen if this shock heats the wall over here and then what happens to this deflection this deflect is stream like, does a deflect further more or you know just remains like that what happens, now that is what we need to study over here. Some these are the properties which are given. So, now, let us just sort of study this little bit. So, first question is what will happen if this is so. Now first things first is that basically if you look at this stream line right. This is this is incoming stream line. So, if you have. So, let us sort of call this region as say 1, now when it moves into say this region to let us call this is 2.

Now this stream liner comes over here right now the when it hits the wall now the boundary condition at the wall will require that this becomes parallel this becomes parallel to this to the horizontal wall that right now it is say at an angle let us call this angle is theta right now. So, right now this is theta right the angle that this deflected stream line is making with the horizontal is theta right, but if this stream line want to hit here hit this wall the boundary condition says the it has to go parallel to the wall right.

So, if it has to do that then this stream line has to deflect again right deflects again and how will that deflect? It has to deflect downwards right. So, it has to reflect that way write here this way. So, let us call that theta. So, in this case, so therefore, this is the theta which is a deflection right, this is the theta deflection. In case here it has to deflect like that and then it becomes this right and how is this possible how is this going to be possible, the boundary condition definitely imposes that is just going to be like this you know, how is this going to be possible this is going to be possible if this shock actually reflects from there. So, we actually have another shock over here, so that this right. So, that this streamline when it encounters this shock, this shock right which is basically reflected from this horizontal wall right then it is gets deflected by theta, it deflected self by theta and it becomes horizontal right it becomes horizontal. So, then therefore, so this shock right this is the incident shock. So, this is called the incident shock and this is called the deflected shock.

And in here what we are asked basically. So, therefore, if you do this, this is the theta and if you look at this so therefore, this is say beta right and, what we are asked to find out here what we are asked basically find out is the angle that this is making. So, what we need to find out is this right. So, now, this phi is basically the angle which the reflected shock is making with the horizontal right and this theta it has basically from our definition right. So, we have this normal shock this is the incoming streamline. So, we draw according to that and based on that stream line this is the deflection and this is the angular of the shock right.

So, in here, in here for example, this is a yes. So, in here this is my theta and this is my beta right for the incident shock, this is my beta. So, therefore, let us just call this is 1 and this will causes 2 and we can call say essentially yeah we can say call this is section 3. So, what we are asked to find over here is that we have a given condition, so these conditions are here right before the incident shock. So, these are the conditions. So, under deflection angle is also given to be 16 degrees right. So, this theta is equal to 16 degrees right.

What we need the find out here is the angle phi, we need to find out this angle phi and the Mach number pressure and temperature behind it and the Mach number pressure. So, basically you can say this Mach number, or we calling this is M 2 let us see what shall we call this, we can call this M 3 or whatever it is. So, basically this M and the pressure and temperature right behind the reflected shock. So, this is what we had. So, how do we go about here?

Now, the only think what you can see over here is basically that, so for what we have done is if you just sort of erase this part of the picture we are familiar with how we are going to tackle if we have an oblique shock right. So, then we had the input Mach number we have a theta, so we can calculate the beta and then we use the normal shock table that is something we have just now done. The other complexity here is now that we have explain what is going on forget about the wall what we have is once this gets deflected it is again heating another shockwave. So, in this case this is the reflected shockwave never mind where it comes from, numerically what it comes down to is once we calculated these values for the incident shock like this it again hits another one.

And then again we just need to calculate the properties across this. So, that is all it comes down to in terms of numeric. So, let us see how we will use just we just need to be little familiar or just careful as to these angles and definitions in so on and so forth right. So, let us see what we were, sort of how will go about calculating the properties in this case all right.

Before we do that let me just emphasis one more time. So, now, this is the compression corner right. So, this is, this the angle of this is given which is 16 degrees right. So, what you saying is the if you have in common streamline like that that get deflected by 16 right. Now, this theta and this theta are same yes because now when this is heating the horizontal wall as 16 degrees the boundary condition forces it to become parallel to itself which is this, so therefore, this becomes a theta. The only difference I hope you can see is that in here it is deflected this way in here it is deflected this way. So, basically their senses are different.

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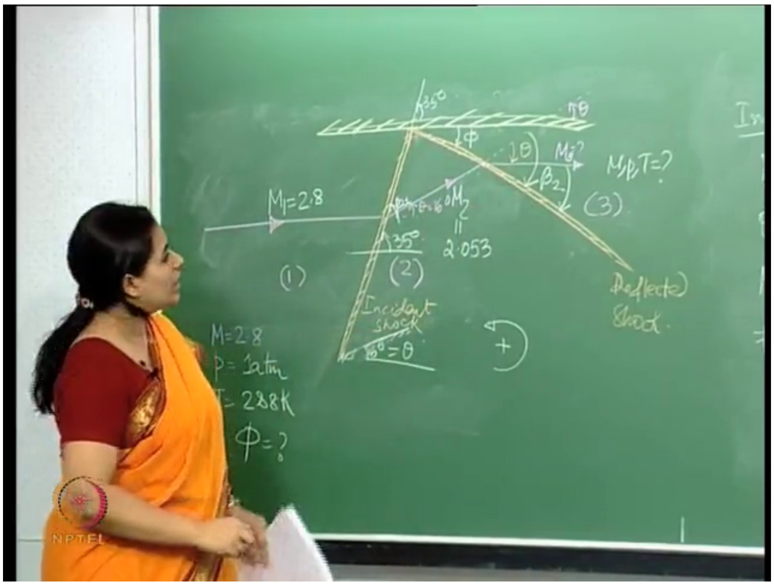

If you consider this to be positive then these angles can be negative, but the theta here though is the same the value of that is same having said that. So, let us again. So, for let us let us now let us look at this part for the incidence shock now for the incident shock, now, we have an M 1 is 2.8 right this is 2.8, we have a theta 1 to be 16 degrees right then like we have done before we use the theta beta M diagram right we use that and what we get is the. So, there we go. One little calculation of we are able to calculate the inclination of the incident shock with the horizontal right. So, now, we know that this is 35 degrees ok.

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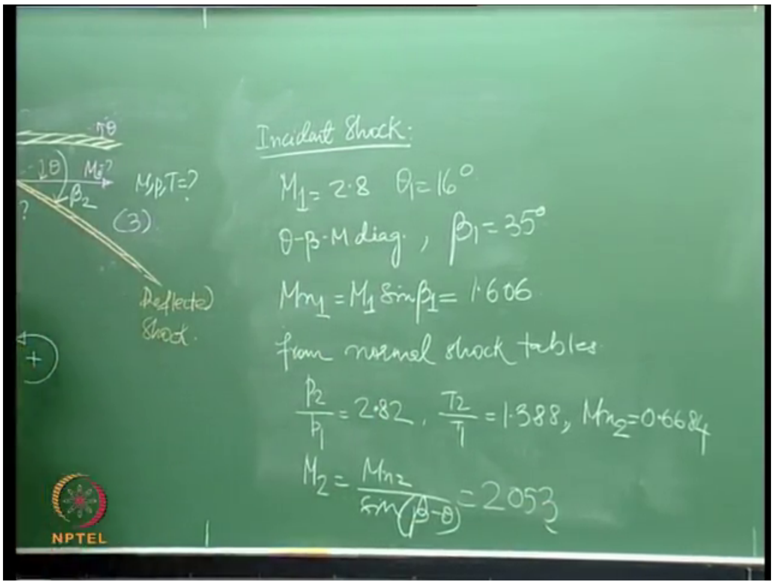

What now we need to calculate. So, our main aim basically is to calculate these properties. So, what we do now, what you thing we should do now? Now what we should get here basically if I had to like for this particular you know this particular shock the incident shock we needed this Mach number right, I mean at the inclination. Now in this case what we what do we need to calculate these properties? We also need the incoming Mach number this is at incoming Mach is this say let me call this as M 1 this is 2.8 and this is M 2, this is the M 2 we need so there finally, we can calculate say that M 3 that region as 3 right.

Sort of this M 2 how do we calculate in case of the incident shock wave? Now what we will get from the tables is the normal component of this M 2 right, since we know beta since we know theta then we can calculate M 2 right. So, that that is the reason the first things we need to calculate is this M 2. So, if you do that, using this, what we will calculate is the normal component of the incoming Mach number for the incident shock and that is I will sort of write this, this is something will be in doing right, it is this ok.

Now, corresponding to this Mach number corresponding to this Mn 1 equal to this from the normal shock tables, now from the normal shock tables for this Mn 1 what we get is and, again what we get is the normal component of M 2 right. So, then from here we can calculate M 2 right now we can calculate M 2 right that which is equal to 2.053. So, basically this Mach number here.

So, therefore, this Mach number out here. So, this Mach number is equal to right. So, it comes from 2.8 and reduces to 2.053. So, this is my Mach number here all I need to do now is using this Mach number and this reflected shock calculate this properties. This is the geometry is slightly different from this let us see what we will do what this values exactly mean to us. So, in here, so therefore, did we calculate by that 2 right? So, therefore, we have this right.

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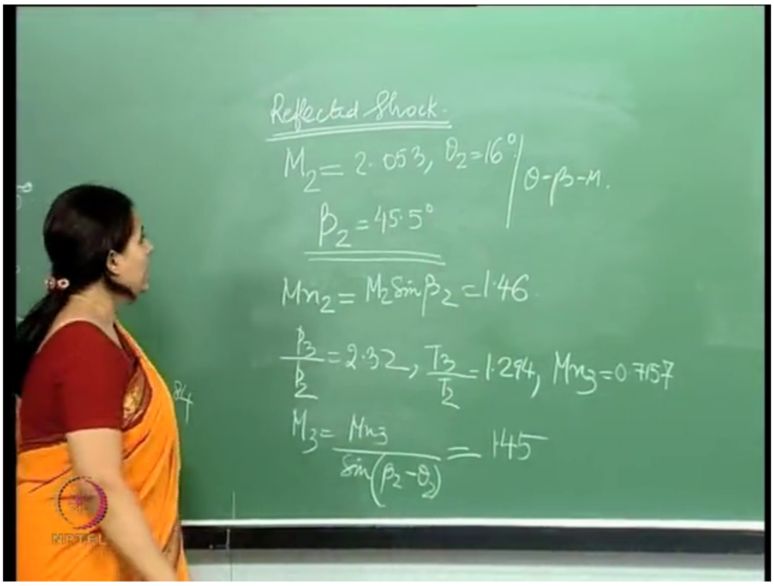

Now, let us do it for the reflected shock. So, we have say, this is between regions 2 and 3 right. So, in the for the incident shock we said 1 and 2 right. So, 1 and 2 we calculated M 2 in this case we will go from 2 to 3 that from I am denoting it, so we know M 2. Just note that in this case the impinging Mach number is the M 2. So, M 2 in here is 2.053, beta 2 actually yeah theta 2 is 16 degrees. Now if I do this, so what do we have over here? So, in this case we have this Mach number, this Mach number which is getting deflected by theta right and that theta is 16 degrees therefore, we can now calculate the value of basically beta 3 right. So, for this and this we use the, from the theta beta M diagram, beta 3 we get as at 45.5 degrees. So, this is the actually I write this as the beta 2, no problems let us write that has beta 2 let us write that has beta 2. So, now, this is basically the wave angle for the reflected shock right and this we get from the theta beta M diagram.

So, we know the incoming Mach number the theta the deflection and let us 16 degrees we get a corresponding beta 2 right, which is 45.5 the only thing is in this case you can see that that the definition of beta 2 is this way. So, this is the angle which it is making with beta 2 what we have to finally, find out is the angle of this shock with the horizontal. I hope you can see what we will exactly doing there right. So, we will find that out. So, this beta to essentially is that from the incident stream line the angle it makes. So, beta 2 is basically the angle between the incident streamline and the reflected shock right, this is the definition so this is the beta 2 that we get ok.

So, now we will basically to see this same way then the corresponding normal component of the incident Mach mark number will be again right. So, this is the 1.46 right and then corresponding to this corresponding to this what from the normal shock tables what we can find out is the P 3 by P 2 which is 2.32, T 3 by T 2 is 1.294 and M n 3 right M n 3 is this right. Having got this now let us see how we will calculate the properties write behind the reflected shock all right. So having got this, this is what we get. So, let us first calculate the M 3 actually now let us calculate the M 3. So, M 3 is equal to M n 3 by sin beta 2 minus yeah theta 2. So, this theta 2 is actually it just the same. So, here what we get is M 3 is basically 1.45. So, now, we will let us calculate the properties. So, what we need to get is P 3 right.

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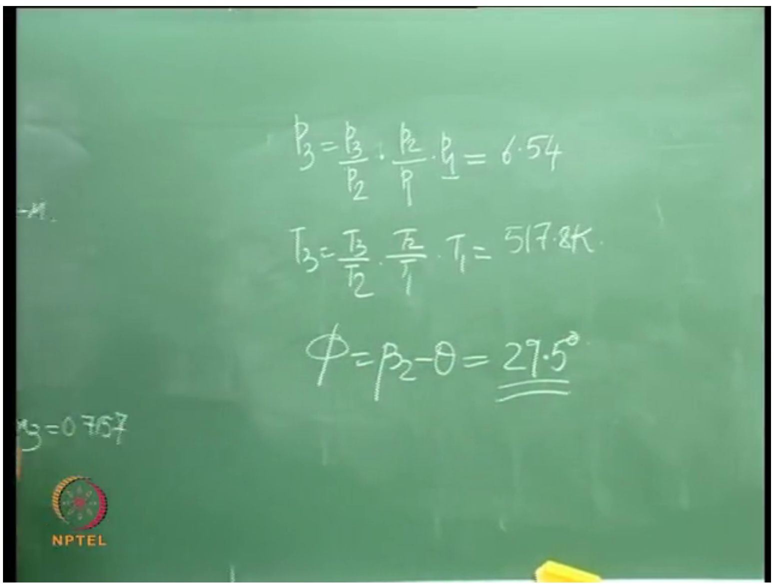

So, P 3 we can now just basically we know play around with ratios that we got. So, we got a P 3 by P 2, we earlier what a P 2 by P 1 into P 1 right and this comes up to be P 3 is this 0.54 you get this. So, P 3 by P 2 is over we get over here this is the ratios of got, I need I also got P 2 by P 1 now if it was required to find out pressures and temperature in this region that also we could have done. We could have done that from here is well because we know P 1 and T 1, but it was not require what we needed to just now as a properties and this region. So, for that we will just play around with these you know ratios that we calculated and which is get that.

So, similarly T 3 will be T 2, T 2 by T 1 it is T 1 and that comes up to be. So, this is what we get. So, essentially what will be unable to do is calculate, calculate the properties in this region right. Now let us see if you can find out what phi is let us see if you can find out the what is phi. So, what exactly is how do we calculate? This is beta, so basically this is the angle the phi is the angle with the horizontal. So, if you look at this just this picture over here this is the beta 2 right. So, what I need is this angle, so what I need is this angle right this angle is what I need this is by phi. So, this is beta 2 mind this is beta 2 minus theta is in that obvious from in this diagram of over here. So, this is the beta 2 and what I need is this, this part of the angle is available. So, essentially therefore, I can write that phi is equal to beta 2 minus theta right and that comes up to be that around 30 degrees.

So, what you see over here is that therefore, in here this is the reflected shock. So, the reflected shock actually what is beta 1 beta 1 is 35 degrees. So, the beta 1 is actually say if you look at this it is making 35 degrees. So, it at an angle of 35 degrees the incident shock hits the horizontal wall right and from there it again deflects downwards it gets reflects it gets reflected from here and in the opposite direction right in the opposite direction it makes around 30 degrees right. So, therefore, if I have a and you know a streamline if you have a flow coming in here it gets deflected by the incident shock as well as the shock will interact with the reflected shock over here and get deflected further right.

So, let me just sort of write down the properties here just to kind of draw a little bit of conclusion a conclusion and let us see what exactly is the happening over here. So, this is the going little congested. So, let me just write it this way all right. So, this is a. So, what we have if these are the properties. So, these are the properties right and T is 288 Kelvin. So, this is what we get much. So, let us erase all of that.

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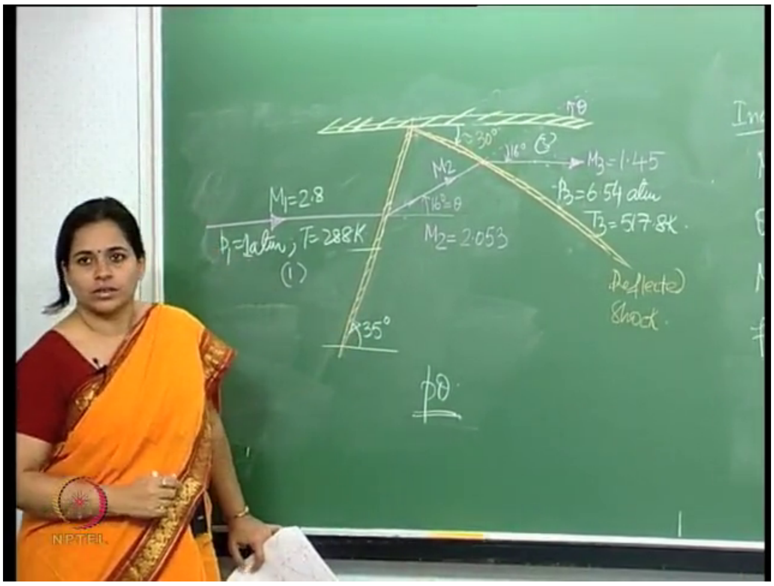

So, here this shock makes an angle 35 degrees and this deflection this deflection is 16 degrees ok. So, therefore, now this is M 1 and if I were to call this as M 2 right, now this M 2 is equal to M 2 is equal to right. Now get a calculated P 2 and T 2 over here actually. So, now we could an actually done that, just because P 2 is 2.82 atmospheres because P 1 is 1, T 2 is 1.38 into this we have got the ratios over here. So, and then it hits the wall over here and this is what it is. So, therefore, now this, then this becomes M 3 right and that M 3 is equal to what to 1.45. So, 1 point goes down 2.8 to 1.45 right. So, then this gets deflected by 16 degrees and the angle that the reflected shock is making with the horizontal is around 30 degrees.

And the pressure here the pressure here P 3 is, I did not write that 6.54 atm. So, this is region 3 right. So, P 3 here is 6.54 atm and temperature is 517.8 Kelvin. So, what essentially we can see over here in the previous example for example, we were having a Mach number of say 3 and we reached a pressure we just passed at across an oblique shock and we got a pressure around somewhere around 6.5 etcetera. Just go and look at that example just the previous example. But here we actually have a smaller Mach number with have a lesser Mach number we kind of get the similar kind of a pressure change, similar kind of a compression with the completely different sort of shock setup.

So, we have an incident shock and a reflected shock and it goes through these 3 regions it deflects twice and this is the kind of pressure changes and temperature changes that we looking at.

So, I think it is a good idea to going to look at the previous example and compared that with the pressure changes and temperature is in this example right. So, that should be all what we would kind of try to do this is the P phi the diagrams and kind of do one more problem on this to kind of get a feel for how shocks will interact and so on.

Thank you that should be all.