Advanced Gas Dynamics
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Lecture - 01
Introduction to Gas Dynamics & Review of Basic Thermodynamics

Welcome to the Advanced Gas Dynamics class. So, as far as the name is concerned it seems like we concern only about gases, now this course could have been easily been called as compressible flows.

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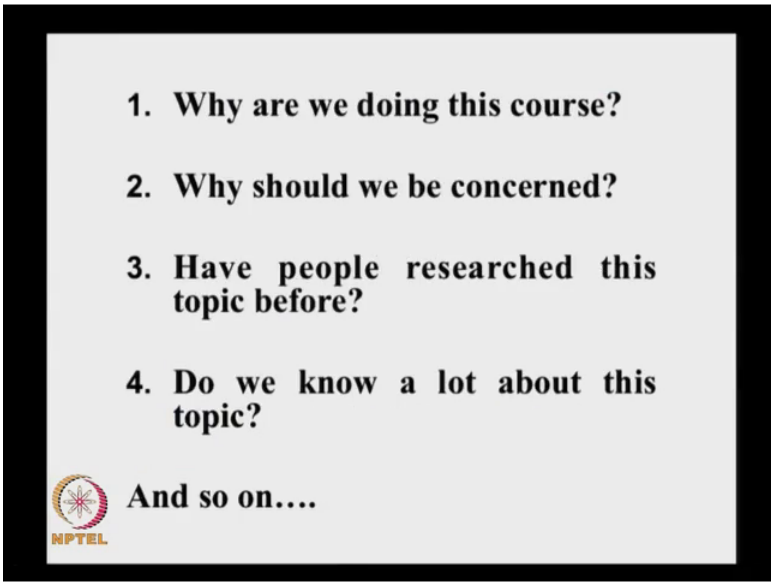

So, some of the questions I always requests students to ask and also instructors to answer is that, why are we doing this course? These are the obvious answers to this. So, should we be concerned about gas dynamics is that important; do we see that around examples this around us all the time. Is this a new topic or people have enough knowledge about this from the previous researchers etcetera. And do we know a lot about it or we still learning about it so on and so forth. So, like I said this course could have been easily called as compressible flows.

There is a compressible flow. So, does that mean that a compressible flows or compressible properties are only a phenomena of gases or others fluids for that matter can be compressible as well. While we will try to answer some of these questions like we said you know the purpose of this course, the importance of this course a little bit about history and what exactly we should be interested in. And of course, if we can find normal ways of you know implement in the knowledge that we acquire as we go long in this course.

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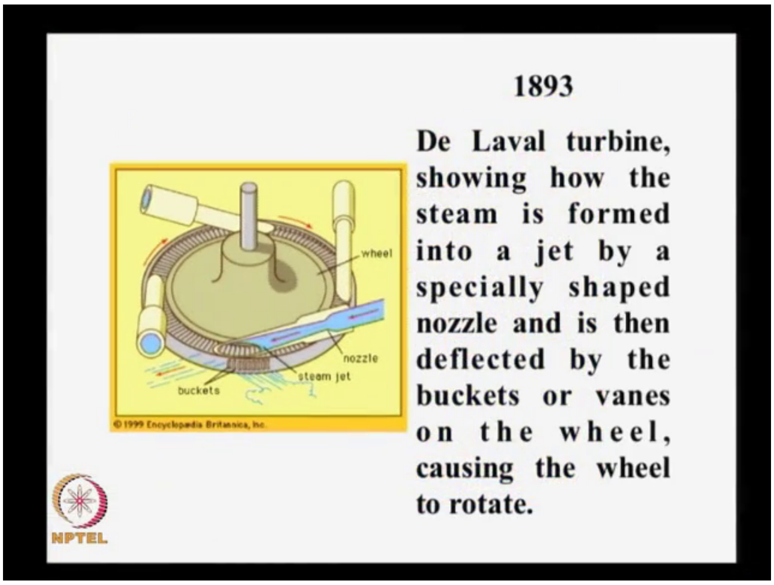

So, what will this by starting is a little bit of history and try to understand how the study became the study of compressible nature of fluids became important and what exactly you know triggered studies or that today we are in a place where actually doing a whole course on gas dynamics.

Now this is an example which you probably familiar with. So, this is a wheel which you can you see, this is a wheel and these small little things out here are blades buckets if you wheel. And essentially the wheel is going to rotate that is you can see. There is no mechanical mechanism to a move this wheel. Instead we have these four things here right, but these things are called as nozzles. Let us call them just ducts. And let us see, let us sort of try and look at what is the special about you know that.

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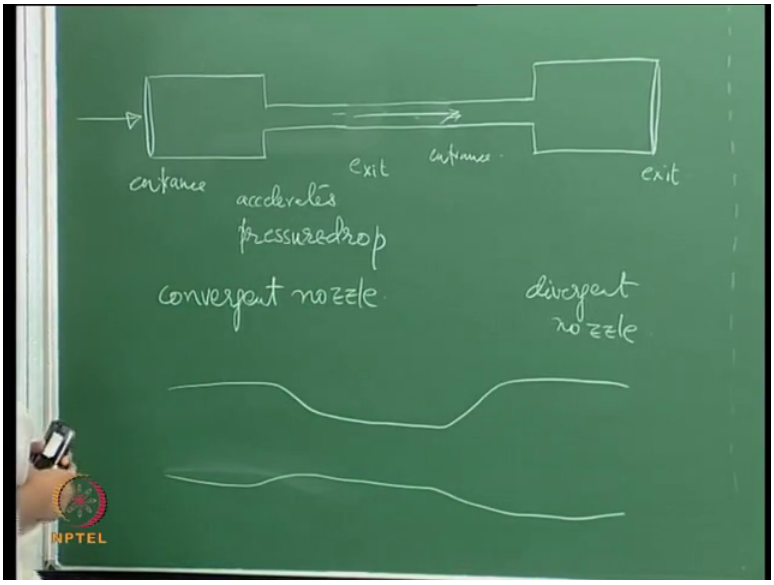

So, essentially what we see is that there is a. So we have a large slightly wide entrance and a slightly narrow. So, we have a large entrance and a narrow exit. So, let us complete the story first.

So, what you see on the wheel is that we have nozzles like this four of his kind. And the blue here is essentially steam being injected into these nozzles; steam is injected into this nozzles it enters through this a wider entrance and comes out through this narrow exit. And it comes out and hits these buckets or blades with such an impact that it is able to rotate this wheel at 30000 rpm. And this engineers way of doing this was first time accomplish by the Swedish engineer called de Laval. And this is also called Lavallel's wheel in 1893. So, this was the first time he was actually use something like this.

So, essentially what is it that this is causing it? So, we have a steam coming in jet a steam coming in and producing a lot of impact. So, essentially what is happening is steam comes in and it is choked; you know that is the technique term to use. So, essentially it comes here it accelerates then the corresponding pressure drop. And when it a hits the buckets it is able to rotate it with as high speeds of 30000 rpm and that was pretty larger that point of time.

So, this is a essentially a convergent nozzle. We can have another type of nozzle as well like that, where the entrance is narrow and the exit is large. So now, this is the entrance and this is the exit, and this is a divergent nozzle; this is a divergent nozzle. Now what we can do is combine these two. So, what we can do is essentially combine these two. So, what you know as probably something like this. So, this is nothing but a convergent divergent nozzle. And this is the modern day convergent divergent nozzle.

So, works on the same principle of the compressible flow which was use by Laval at the first time round. So, this was the first of his kind we back in 1893.

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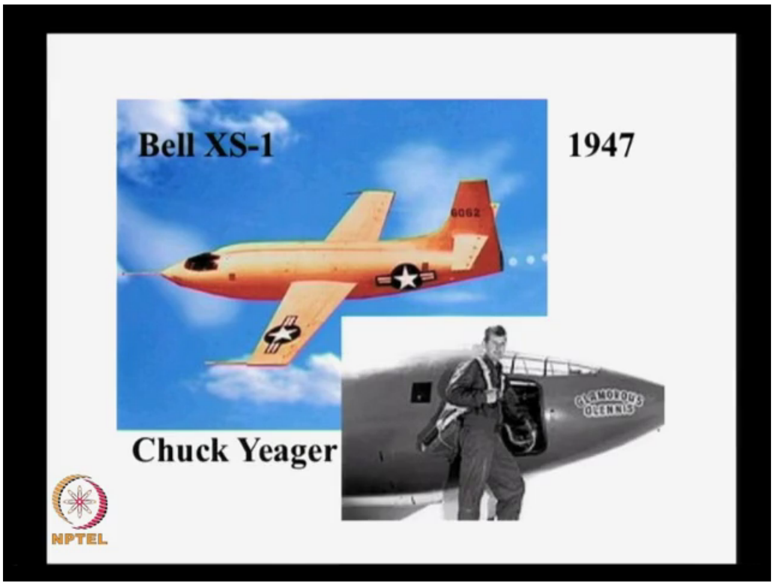

While go of a one of these a examples you know jump over to 1947; if all you were familiar with this a picture out here, so this is the Bell XS-1.

And this was the pilot Chuck Yeager and this was the first time in 1947 that this plane and Yeager was able to break the sound barrier, which means that he was able to fly at a speed which is higher than the speed of sound which is Mach one. So, basically he was able to break the sound barrier. And why is this important? Why am I bringing this up, in terms of a you know Laval's experiment? Well, the reason being that the thrust in the Bell XS-1 the thrust in this plane was provided by 4 convergent divergent nozzles. And that was the one which was able to create speeds or a move this Bell XS-1 at speeds if for which he was able to reach mach.

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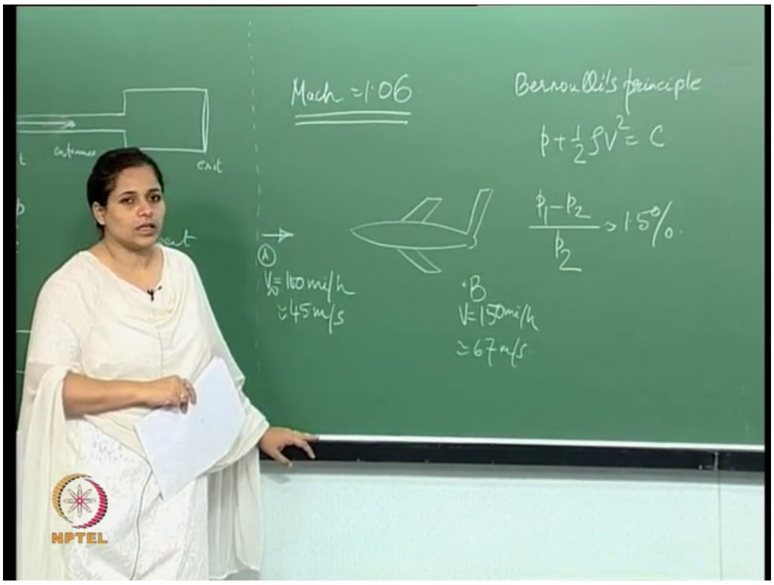

So, essentially for a very simple experiment which shall have all used with a basically a Swedish engineer to a rotate a wheel he used essentially the principles of compressible flow. And we have been able to catapult that and use it for something else sophisticated as supersonic flow.

So similarly, we have been able to see application of compressible flow theories in literature so far, then.

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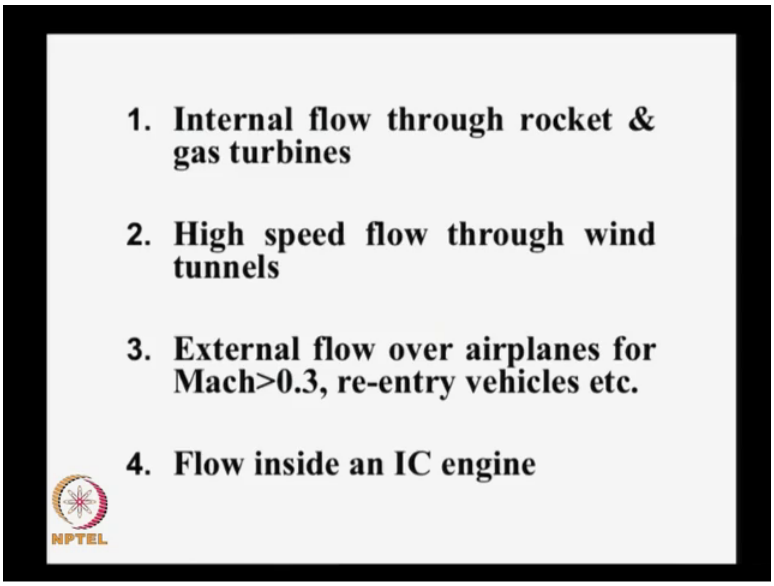

So, some of those applications could be a internal flow through rockets and gas turbines high speed flow through wind tunnels; external flow over airplanes for Mach 0.3 reentry vehicles etcetera flow inside an IC engines. So, what I mean by this is that a all people, so from mechanical background or aerospace background or you know and so on and so forth all you know could all interested in flows like this because compressible flow theories are applicable to a wide range of disciplines, ok.

Now, I said something about external flow over airplanes for Mach 0.3 it seems like a thumb rule that if the flow over planes is a less than 0.3 then the compressible nature of the fluid may be over load. So, will I try to see what that means? So, we try to see that with say an example. For example, let us take an aero plane. And say at this point say A right which is far off from the aero plane and how many have a the speed here, the speed here is 100 miles per hour which is around 45 meter per second, right.

So, we have; so let we call this as a free streams- a free stream upstream of this plane is 100 miles per hour. Now this comes over here and it accelerates over the plane, right it accelerates. And a at some point here say the speed out here is 150 miles per hour which is around 67 meters per second. Now as we saw in the example which I showed you just earlier over here that the flow accelerated, right there was a corresponding pressure drop.

So, here to what we would like to know is that if the flow is moving say from or its been accelerated from 100 miles per hour to 150 miles per hour then what is the corresponding pressure drop. And what we should; I mean how we should we will go about finding this. Now first thing first let us do this. Let us just say that this flow out here is incompressible, it is not compressible in nature. So, if that happens and I might sure of that no; I am not so I am going to just assume it is incompressible and then go and verify that whether mine assumption was correct or not. So, we will just say it is incompressible for the time being. And we use the Bernoulli's principle between A and B.

So, this is the a Bernoulli's principle. So, we will apply that between A and B. So, what we will get. So, I get a melting pressure drop off pressure change of a 1.5 percent. And now we want to see a whether you know I used an incompressible theory over here if I was correcting doing that. If not then what we will do. So, let us move a little forward. Now I keep saying that it is compressible in nature; now how do we quantify that, how do you put math into this. I understand that there is a flow over here, there is a corresponding pressure drop; the question is how much. How do I relate this pressure drop to this speed or do I have to do that, can I do that.

So, I need some sort of quantification right to say how much a gas or a fluid is compressible and only based on that can we say that this is an incompressible fluid or this is a compressible fluid. So, let us try finding out some sort of quantification for this.

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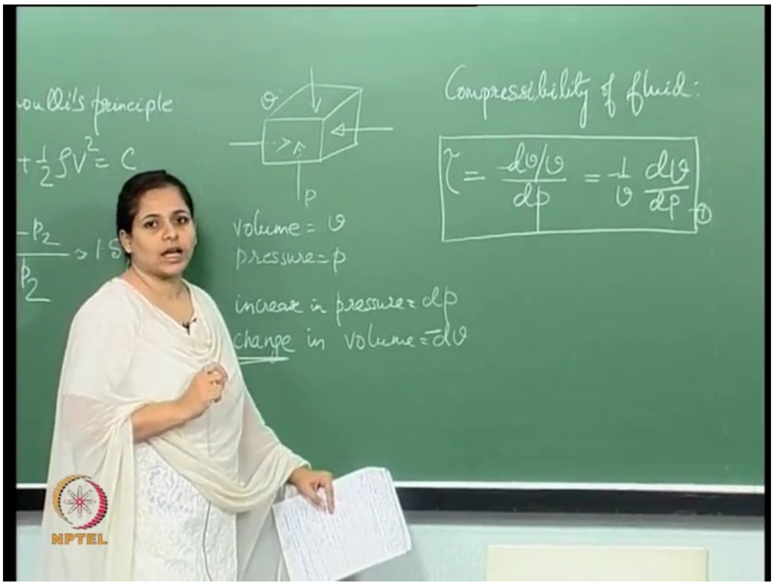

So, a let us say we have a fluid element like this and this is the pressure and this is the volume. So we have; now say there is an increase in the pressure I increase the pressure; so there is an increase which is an dp and there is a corresponding change in the volume. So, I will write this as; so this is a corresponding change in volume. Now if there is a increase in pressure the volume will decrease, that we know from our you know known principles of gases etcetera.

So I will note that, when I write change here actually mean there is a decrease, so we will denote that by n negative sign. So, if I have a positive pressure change there is a negative volume change. So, negative change basically indicates there is a decrease in the volume. Now we shall define what I understand by compressibility. So, let me write this here. So, compressibility is defined; so let we call that as tau then I shall write that as: every time I write an equation I always like to say that in words where I understand what it means, so that I do not have to remember the expression itself. If I know what that means, I can always develop the expression.

So, what this means is that fractional change in volume which is dv by v per unit change in the pressure. So, that is what compressibility means. There is a fractional change in the volume per unit change in pressure and I can write this as: let me call this as I like to name my equations for further reference.

So, that is how I define compressibility. Now the next thing we can see here that if there is a change in the pressure and there is a corresponding change in volume what happens to the temperature. Now if this is change in pressure there is change in volume there should be a corresponding change in temperature as well. Now if that is so what is happening to that over here I were taking care of the temperature or no. Now let us there therefore, defined this compressibility for a few cases where we do that, we say something about the temperature right how do we do that.

Now first thing we will see is that will say is that the temperature is constant right and how do we hold keep the temperature constant; that there is a positive heat transfer that there is an non zero heat transfer. As a result of which the temperature is constant, ok.

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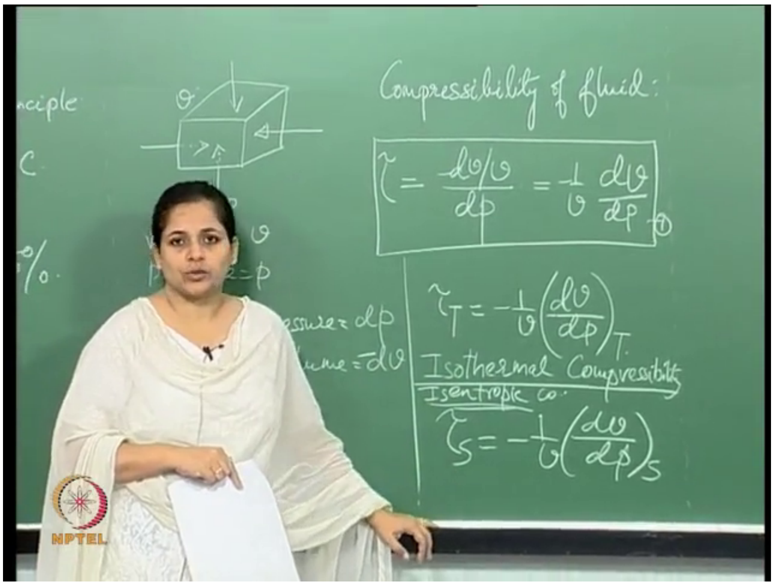

So, then I will write my compressibility in this form. And we can call this nothing issue of guessed by now that this is isothermal compressibility. So, this is isothermal compressibility. So, in this case the temperature is constant. Now a something is to notice here is that when I write these subscripts out here, so I write T over here, this is a denotion and then when I write here I am just bracketing this part of the derivative and I did not include this; is that the right thing to do or wrong thing or I should have include that any ways. Well, the point here is that the compressibility when I am defining right the process; the process under which the volume is changing based on this pressure. Now this process is happening under an isothermal condition.

So, when I increase the pressure there is the change in volume, now that is the process during which the temperature is a constant. And this process I am comparing with what the volume was before that process started. So, this basically the state volume, this is state parameter or a it is a representation of the state. And then we started a process in which say there is an increase in the pressure and a corresponding decrease in volume and that is the process which is happening under isothermal conditions. Therefore, I am going to write it in this fashion.

So, one condition is where I keep the temperature constant, now what if I do not keep the pre temperature constant. There is no a heat transfer happening, there are no other dissipative forms, but there is no heat transfer happening then what happens. Do we have a condition for that or what do we do. Now we will define something else. Now we will define an isentropic compressibility. We will define an isentropic compressibility and I will write that as: now what is this a S that I am writing here.

Now, this is entropy, this is the property called entropy and for an isentropic compressibility, if I am a write it this way so this is isentropic compressibility. So, here for this the entropy is constant right, in the isothermal case it with temperature was constant for an isentropic case the entropy is constant. Now what is this S, now entropy I guess you are familiar with this, but what I will do in the next lecture is do a basic review of thermodynamics because this entropy is a thermodynamic variable. So, let me not deal on that too much, but we will go ahead and do a brief review of thermodynamics and so we will come back to what we mean by isentropic and entropy and so on and so forth.

So now, let me give you some numbers; numbers always help to get feel for you know how compressible or less compressible and so on and so forth.

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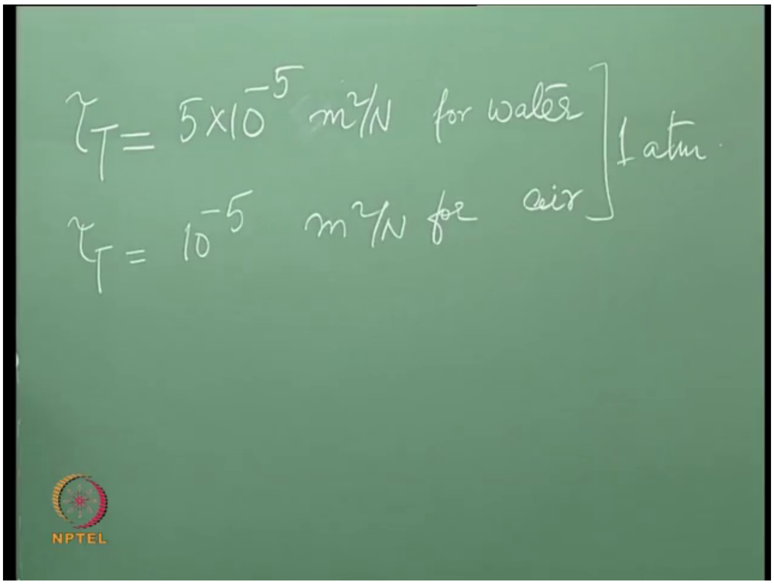

For example: this is the isothermal compressibility for water; this is for water and this is for air and both these are at one atmosphere. So, what do you notice from here, what you see these are the numbers that we have. So, this for water and this is for air.

So, what do you notice; what exactly can you (Refer Time: 23:06) from here. What you can see is that the compressibility of air is much more than that of water, which is essentially what it is that for gases the compressibility is much more than water. So, we kind of beginning to see a few things about the name of this course on the first slide is to why we are doing this course or why do we call it advanced gas dynamics as it were. So, essentially that is a thumb rule gases have more compressibility.

So, let us find out. So, we define pressure, volume, density, temperature, etcetera, etcetera, so what is the relationship between them if we do have changes in them how they are related or if at alright. So, let us do something simple and develop a few expressions.

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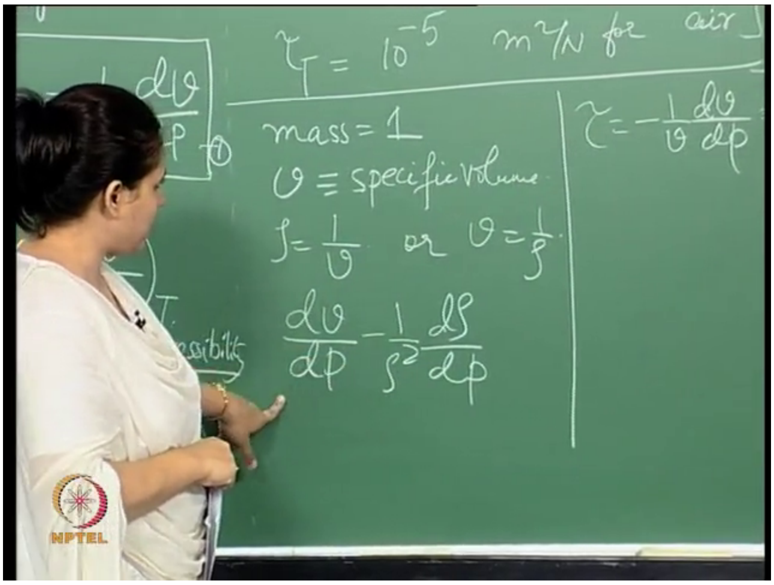

So, say we have a mass of gas; a unit mass of gas, so we define a specific volume. Therefore, we have density right or we can write this as then let us do a little bit of math and do this. So, basically what I am you know this is simple here what we all we are trying do is trying to get a relationship in terms of the compressibility. So, this is the equation one that we wrote about, so this was we need derivative something like this. So, when I have this expression over here, so dv by dp; if you can do this math. So, this is a simple over here, so I will do this. So, this is the derivative.

Therefore, now I will try and write the compressibility. So, now if I write the compressibility how does that look.

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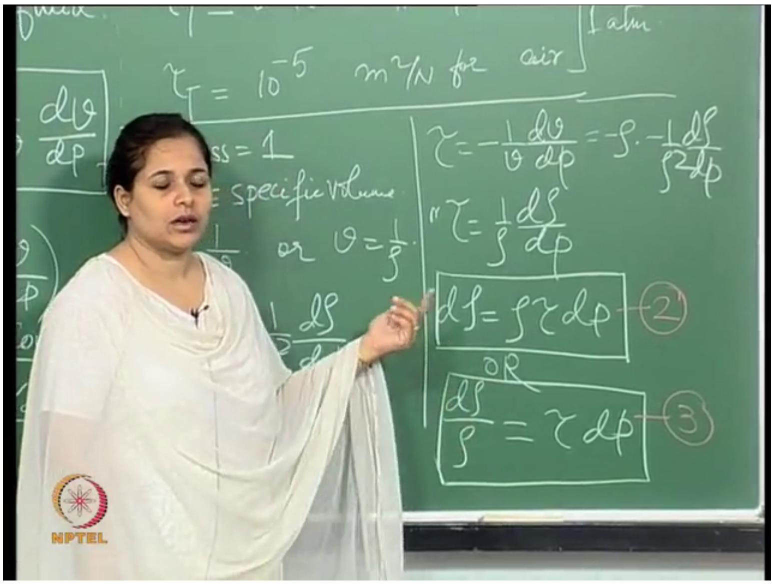

Now the compressibility is nothing but 1 by v dv by dp and we got 1 by v as rho into dv by dp which is over here, so we will put that in there which is minus 1 by rho square d rho dp. So, what we get over here. So, this is what we get. So, what I will do is write this out this way. So, essentially d rho is equal to rho tau dp, we will write that. And, or we can also write this as. So, let us say this is.

So what will say over here is that in here what you can see now; you have just now said that for a given for a gas the compressibility is much higher than that of a liquid. So, therefore, for a given pressure the density changes in a gas is much higher than a liquid, because this a compressibility is much higher for a gas. Therefore, to bring about a for a given pressure you can you can see here this is the fractional change in density. So, for a given pressure for air for example, for given pressure change there is going to be a much larger density change than compare to say water. Essentially because this compressibility factor here is much larger for air than for a liquid.

Having said that now let us go back to your previous problem. Therefore, we have this problem over here right. Here and we been said that the pressure change is 1.5 percent. You can do the rest of the math yourself and relate the densities, relate the densities etcetera if you want; in this case it was a incompressible, so there was we said there is no change in the density. When we consider no change in density what we found is that the pressure change is 1.5 percent.

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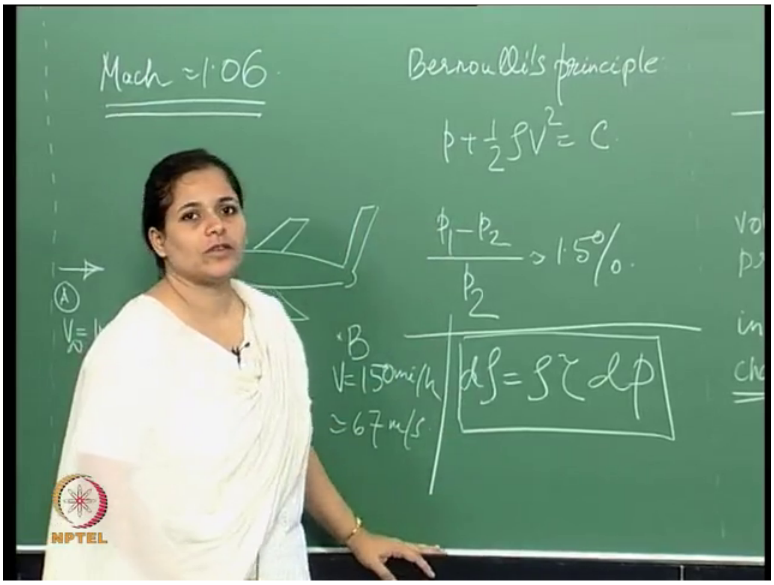

But what I can say here is that, since the corresponding relationship is for a compress; if I were to consider this as compressible than the density change is essentially given by this relationship. Now in here since this pressure change is very small, it is just 1.5 percent. So, I can say that we have infinite small or very small change in density. And hence I can, it is for me to ignore that. And that is so I can say more or less safely say that it is to say that this is an incompressible fluid. Therefore, changes in density here and not very dominant. So therefore, it is to use the incompressible theory over here. Hence I was fine doing this.

Then of course, the new you know the immediate question which we should be asking that; ok so if we had a flow which was going from 100 miles per hour and it reached up to 150 miles per hour and that cause changes in density in which we were able to ignore, but then how far would be we able to do that or for how largest speeds or how smaller speeds can we do that.

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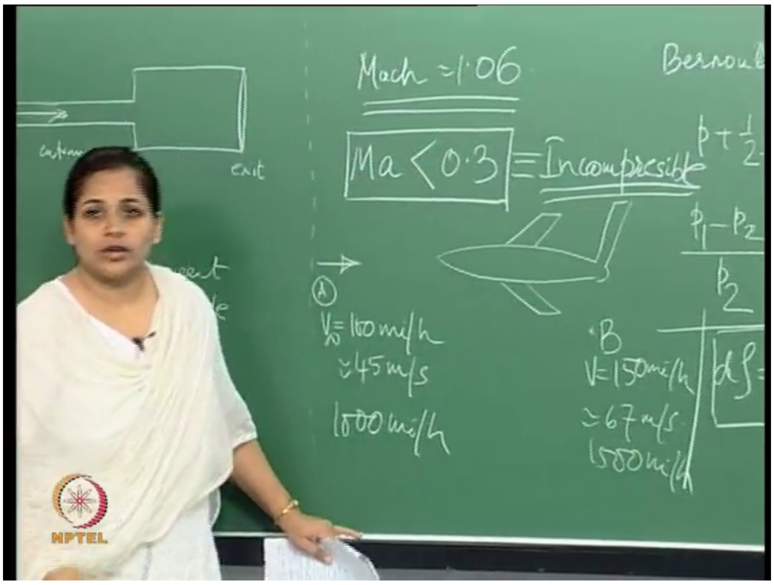

If I had a velocity which is going from say 1000 miles per hour and going up to say 1500 miles per hour, where I still see changes in a pressure which was very small and causing changes in density which I could ignore could that be a possibility. Therefore, I need to relate this now to the speed here and that is why that is where when I first said that for flow over aero planes for Mach numbers less than 3 it is incompressible.

So, now over a period of time with experiment etcetera been able to see that more or less is like thumb rule that; so I mean just write over here that for Mach numbers less than 0.3. For Mach numbers less than 0.3 the changes in density can be ignored. Therefore, as more or less thumb rule for this we can consider the flow to be incompressible. So, to answer the question that whether we know a lot about this topic from you know history and literature etcetera; well, I we know enough to know that these thumb rules. So, people have worked on this over a period of time, so we kind of know these sort of things.

Now the next thing like I said the next thing to do would be to do a brief review of thermodynamics, ok. Now, what is this connection of thermodynamics with this? I said something about entropy when we defined isentropic compressibility, but what exactly why do we need to study the thermodynamics when we are dealing with compressible flows.

Now we saw that there is when we have say higher speed flows or essentially to generalize when we have changes in pressure which is causing changes in density and a these are changes in density which are not small enough to be ignored. Now there should be also changes in temperature. Now when we consider those changes in temperature that is when we need to study little bit of thermodynamics before we proceed. So, I will do a brief review of that.

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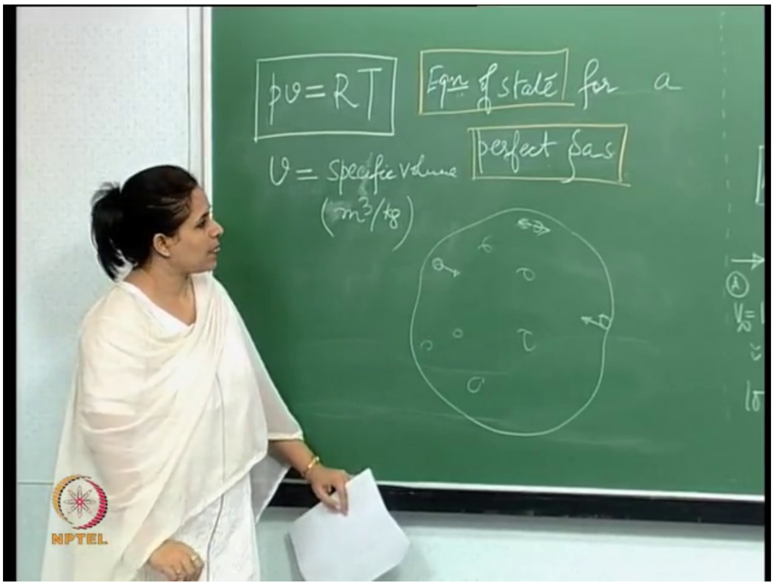

Now, let just say let just you know began with something like this. Let us just began with. Now, this p here is pressure, v here is a specific volume which is volume per unit mass, so I will just write the unit here and R is the specific gas constant and T is the temperature. Now this is something that we are familiar with. So let me write here, what exactly is this. I hope you can recognize this. This is the equation of state. So, this is an equation of a state for a perfect gas, there are two things over here: one is equation of state and the other is perfect gas. What do these things mean? Equation of state perfect gas, what do you mean by a gas which is perfect.

Now, all we have done here is taking the pressure the volume, temperature, etcetera connected them by some sort of a constant etcetera and we are saying that this is the equation of state; some sort of equation. And we are saying this is for a perfect gas. Now what do we understand by this perfect gas. Now let us just define that at first.

Now a fluid essentially is consist of molecules I think you all aware of that and these molecules are very widely spread about for a gases and more closely packed for a liquid. And this particles or molecules are in constant random motion, there are in random motion. So, now if I am takes a volume like that say if I take a volume like that; so there is a constant in flow and out flow of molecules across this boundary. So, the number of molecules out here is constantly changing and we have free flow of molecules on either side which have a constant motion. Now when we have these particles, if by nature there is intermolecular forces we know that right. Now if you have two particles which are like really far off; we have particles which are very far off. Then, they exhibit very small or very weak attractive force. And if they are very close say like that then they have a strong repelling force. So, these are the intermolecular forces.

Therefore, these intermolecular forces kind of define or kind of control for that matter the movement of these particles; I think it is kind of to see from here. Now the thing is what is this perfect gas: the perfect gas is one when we ignore these, where I say this I mean these intermolecular forces. For a perfect gas we ignore these forces. And when we do that there is a certain relationship between the pressure volume and temperature density etcetera of that gas. And that relationship is what we define as the equation of state, which means that if I have a sort of you know some mass of gas then that will obey this sort of a relationship provided we ignore the intermolecular forces.

Now the next question is; when I define this pressure volume temperature etcetera. Do I define it for or we considering say the pressure of every single particle, are we considering the temperatures of every single particle; when I take a mass of gas or what am I doing. Now, it is important to say that all these parameters here are all defined on a continuum.

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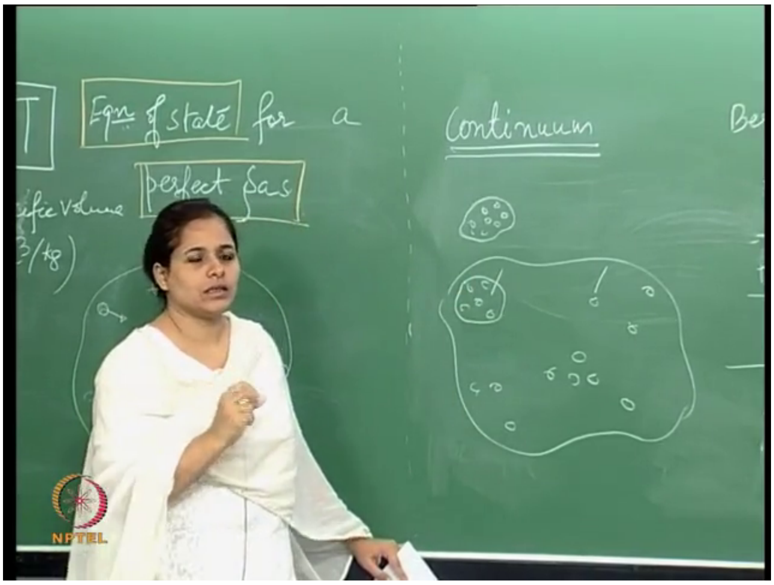

These are all defined on a continuum. Now what do we mean by that? Now what we mean by this is that let me take these a boundary like this right; now I said the number of particles is changing etcetera you know and this is just a imaginary boundary.

So, what I am going to do here is take a collection of particles, say collection of these particles more or less I take such a boundary that these number of particles is not changing. The number of particles more or less remains the same. So, then what happens is that the let me redraw this. So, I have a boundary like this, I have a boundary like this which is big enough so that I do not consider individual movements of properties of the particles. Instead however, we are concerned about the behavior of this whole collection of particles; this collections of particles as a whole. So, individual behavior of each particle can be ignored.

At the same time now this is not so large enough that it is a such a big space that you know there is agglomeration of particles here, here, here, and the particles are not uniformly distributed. Then, as a collections of this we may not be able to say that there is one particular temperature pressure or volume, because for example, if I have something like this the temperature here will probably much larger than the temperature over here. So, it should not be so large as will, instead if I have a volume like this, this would be a good continuum. So, that this I can just consider as a group of particles. So, more or less I around here the number of particles remains same and I can consider at this particular area for these group of particles a single a value of the pressure temperature density volume etcetera. So, that that is what we define as a continuum.

So, all these parameters pressure, volume, temperature, etcetera are all defined on a continuum. Now we can write this expression now in a some more ways.

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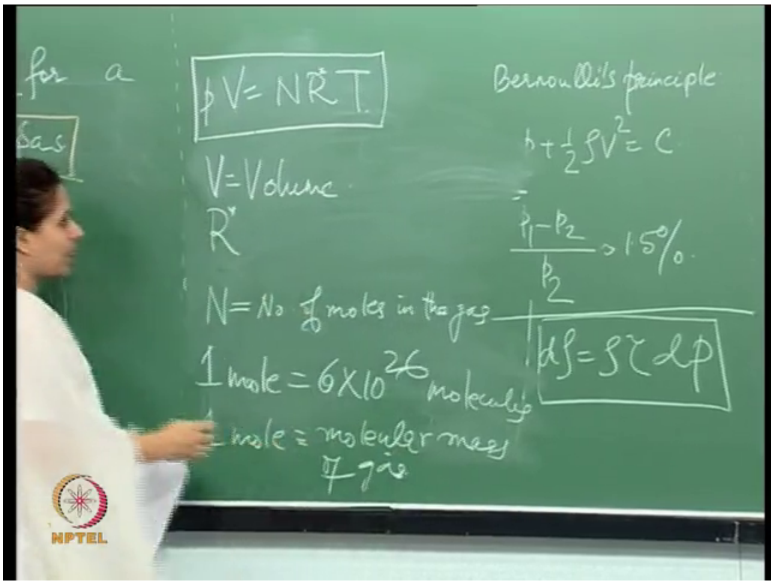

For example: so here we wrote this in terms of specific volume. So, we said for which means a the volume per unit mass, all I am done here is because this here is the volume this is volume, this is volume and this is gas constant. Let me differentiate between this and this so let me call this as a star. This one is the specific gas constant and this is the gas constant. Basically what we doing is multiply it by the mass out here the whole equation. Now what is this N here? So, the N is the number of moles in the gas, which essentially this is doing nothing but giving us an idea of the number of molecules in the group of gas that we are considering. And I think you will remember that 1 mole of gas consists of 6 into 10 to the power 26 molecules; the Avogadro number of molecules.

So, what we essentially saying here is that for a given number of molecules of that particular gas this is a relationship that it holds; this is also for a perfect gas just trying to look at it a little differently, ok. Now, again what is 1 mole? What exactly is 1 mole, what is a concept of a mole? It is nothing but the molecular mass of a gas; for example chlorine is 17 I think. So, that is the molecular mass of a gas.

So, that is a essentially the equation state for a perfect gas. And I will just give you an example of not example just give you some number; just to remember some of these numbers to be familiar with it and will do just a small example and will wind it up for today, ok.

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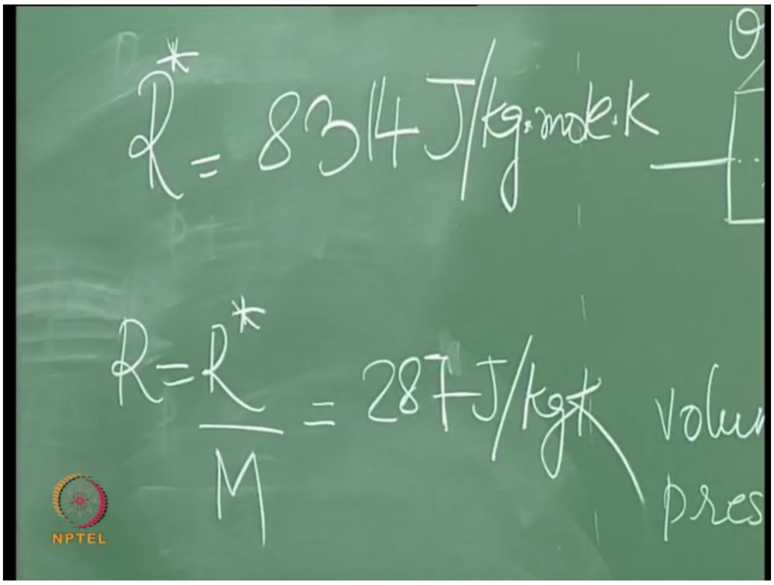

For example, this is the universal gas constant. So, this is equal to; so this is R star basically how related star. So, these is for example 8314 joules per kg mole Kelvin and say specific gas constant is for a mass this is a molecular rate, and for air for example so this is equal to 287 joules per kg Kelvin. So, these are some numbers which you should be just familiar with. What we will do now is just quickly do this and wind up for today. Now all of saying is that.

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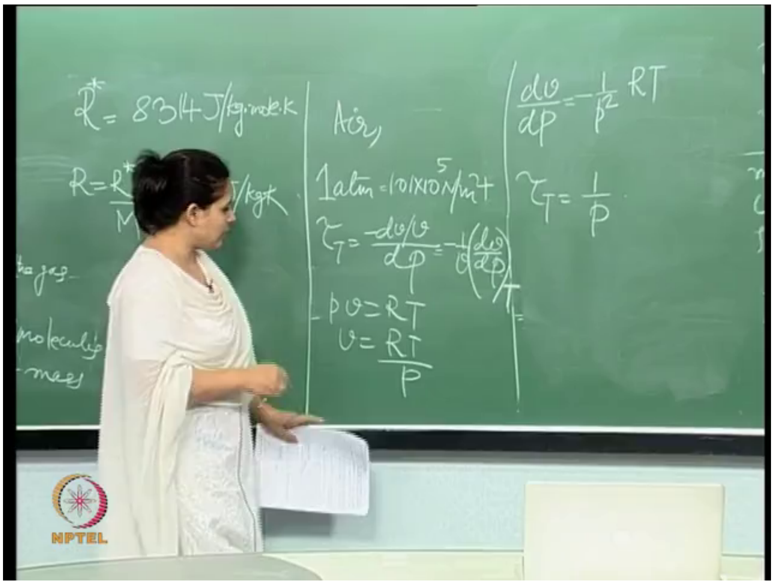

So, air is given, so I have air and the pressure is at 1 atmosphere and that I can write as: calculate the isothermal compressibility. What is the isothermal compressibility? So, compressibility is nothing but fractional change in volume per unit change in pressure. So, this I can write as, right so isothermal compressibility. So, how do I find this out? Now let us go back to the first equation of state. So, pv is equal to RT. So, if I do this then I will write the volume as RT by p.

Now if I do that. So, if I do that then all I need to do is dv by dp. So, I do dv by dp and what I do here is that; right this is what we get. So, then if I input that into this expression the dv by dp a into this expression what I get is this, hopefully you should be able to; so 1 by p and the p is given. So, the p is given.

Now what you can see here is there I said isothermal compressibility, right. So, what I mean by a saying that over here is that you remember this expression. So, essentially what we mean out here is that this change in volume and pressure is happening over a process which is isothermal, so that temperature is constant. So therefore, when I did this over here I kept the temperature constant and that is how I calculated the derivative over here. Now a temperature is a constant meaning that we are not considering any internal forces. Obviously, because then there will be changes in its internal energy and I cannot say that the temperature is a constant.

So therefore, I can only calculate the isothermal compressibility from here. For example, I have the isentropic compressibility as well it is the same formula and we have the same you know etcetera is available, so I can actually use that. But the idea is that when we use such a formula you need to understand that what exactly you using it for. So, if I said find out the isentropic compressibility etcetera you cannot use a process in which a temperature is constant. So, we will have to you need to (Refer Time: 49:19) think a about that.

So, I think we will stop over here. What we shall continue to do next is that; when we I talked about that so far whatever we did temperature is constant. So, why we will going to do a review of thermodynamics, because when temperature is not constant then I need more variables; just the pressure volume temperature is not going be sufficient t to study my gas or any process what I will need further is an expression as to how these variables relate with temperature. And to do that we will need that more variables in that, and that is what thermodynamics will provide; the concept of entropy internal energy and so on and so forth.

We will continue to do that in the next lecture.

Thanks.