Cryogenic Hydrogen Technology
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Lecture 39
Cryogenic Rocket Propulsion

Welcome to this lecture on Cryogenic Hydrogen Technology. We were talking about the applications of this hydrogen fuel. And in that respect, in this discussion we will be talking about the cryogenic rocket propulsion. In the last class we have discussed about the use of hydrogen for generating the power, but there we have directly you know converted this hydrogen and into electricity when it is combined ah with the oxygen. So, that in that process we have talked about I mean the use of fuel cell, but directly the energy was converted ah into electricity using hydrogen in presence of the oxygen or in with the help of oxidizer. So, now, in this case ah it is bit different from the earlier discussion because this is a thermal process and ah let us see cryogenic rocket propulsion ah and these are the keywords will be initially talking about the thermodynamic aspects of this propulsion and later on we will look into the propellant feed system.

As such this cryogenic propulsion system is quite complicated, it will have so many ah I mean auxiliary systems like it may have a pump, it may have the oxidizer fuel, then it has to come with a particular flow rate, we have to have different kind of valves which will monitor or generate the ah I mean flow of I mean feed as well as the oxidizer or the fuel and the oxidizer at a particular flow rate. Then it has to you know withstand ah different kind of environment sometime it will be in atmosphere sometime it may be in vacuum. So, all these things make the propulsion of cryogenic engineering I mean cryogenic propulsion system very complicated, but we will be looking into the general aspects of this cryogenic propulsion and let us have a look. So, ah it is basically ah what we are going to talk about is the thermal rocket engine or ah like any any heat engine ah it is not really work wise very different.

Say where we are changing the heat energy or the thermal energy into you know in a say for example, in a noisy the engine we are converting the thermal energy into ah work and that work is moved out and I mean transferred to a piston. But here ah it is basically this thermal energy is ah going to be transferred to a stream of exhaust gases. So, we have ah basically ah combustion chamber where in the combustion chamber you can understand that this one end of it is open and here we have the fuel which is coming in or at a particular rate we are ah injecting the fuel. Then ah basically we are talking about the liquid propelled ah in a liquid fuel rocket. So, this fuel has to come in at a particular ah flow rate.

Then we have oxidizer mainly it is oxygen and that is you know going to burn ah it is a basically a chemical reaction that is taking place and that chemical reaction is generating high velocity exhaust or ah this standard molecules are basically you know you are making it into a I mean random to ah streamlined one and that work as we have discussed that work will be either transferred to ah a piston in a IC engine. But in this case ah it is you know going to come out of this nozzle and there is a converging diverging nozzle. So, from which it will basically generate the high velocity. So, as a result what will happen? So, this heat energy or thermal energy is getting converted into this kinetic energy ah associated with the exhaust gas and it will be coming out at high speed and that will be expanding from this basically this chamber is at higher pressure. So, from that high-pressure ah if we are allowing it to you know come out ah basically it is getting expanded through this ah you know this is the throat of the nozzle.

And ah when it comes out at high velocity it is basically giving ah an opposite reaction of thrust to move the rocket in this in this direction. So, fine let us look into this and then thrust equation. So, ah we understood that there is a you know ah combustion which is taking place and because of that combustion you know this ah stream exhaust stream is moving out at the exit of the nozzle. And ah if we look at this exit plane we find that we have ah it may have you know atmospheric pressure and this is the area of the exit plane and these gases are ah moving out at velocity ue and this is the Pe that is the pressure at this exit point. So, these are the different you know I mean things that are ah associated with this combustion.

So, as a result of it you will have this force which is you know moving out with ah this is the thrust which will be generated by this ah combustion as a result of it. And if we now try to ah make a thrust equation. So, what we have to look into that if we write the momentum equation ah this is the you know the control volume. So, using the control volume and of course, there are so many ah simplifying assumptions we have to make like you know it is a steady state 1D model and to no external heat is you know added or rejected through the nozzle or through this combustion chamber. Ah Then the gases are leaving only in the axial direction that means, we have assumed the velocity like this on this side.

So, with this ah conditions we can now ah try to write the equation. So, it would be something like ah this is the force with which this you know ah I mean rocket is moving. So, here on this side we have ah the atmospheric pressure and this atmospheric pressure that is you know with this one which is acting on this direction. So, it is adding to this force thrust force, but there is something you know which is coming out at this direction. So, it will have ah this resultant part would be you know giving a ah velocity imparting a velocity with ah u v.

So, that is the exhaust velocity. So, this is what is that thrust equation and this equation will be finally, used for different you know thermodynamic calculations later on. So, what we can understand that ah this is where we have this combustion ah you know ah chamber where it is generating at a particular high pressure. So, this pressure has to come out or you know this is the convergent divergent nozzle and this is the throat area. So, how effectively we are able to ah take this gas out or basically ah what we intend to do is that we need a high velocity gas ah I mean velocity for the exhaust gases.

And if we look at this effective exhaust velocity because we can understand that it is having ah ah two terms basically if we look at this is the exhaust velocity, but there is another term where we can see that this depends on the pressure in the nozzle area as well as in the you know what is the the atmospheric pressure. So, this is the effective exhaust velocity we define the force per unit mass and this is basically comprising of ah this ue that is the exhaust velocity and the two pressure terms one is the atmospheric pressure another one is the ah nozzle pressure. So, here ah it is apparent from this equation and this Ae is the ah you know area of at the exit of this ah nozzle at the ah ok. So, now what we can understand that this effective exhaust velocity it is depending on a pressure term and ah at some point when the rocket is being fired at the sea level ah this atmospheric pressure will be present, but gradually it it will be as it moves up upward ah it will be ah having you know the less ah atmospheric pressure or this will be ah started depleting. And ah so, what will be there ah I mean what we can expect that if this Pe or the pressure at the exit of this nozzle you know if we can make it very near to ah I mean almost like atmospheric pressure or if it is you know very small pressure then this term if we can make 0 that is giving us the maximum velocity.

So, we can understand that the maximum thrust we will be obtaining ah if we ah change the ah size of the nozzle and that nozzle will be um I mean different for atmospheric pressure when we are firing it or when it is going to the space or when you have a very small vacuum at that point you know this nozzle size will be different, but it cannot be ah you know an adjustable nozzle size. So, what we have you know the nozzle size basically a short nozzle which will be required at the sea level, but as we progress or as we you know fly high and in vacuum it will be almost like an infinite ah length of the nozzle, but it cannot be again done you know. So, we need ah an optimization because ah we understand that if we are ah when we have outside this there is vacuum in you know in the upper stage this Pe if we are ah able to make it very small we can have a very high thrust. So, this is how it works and another part that we you would like to mention here that ah I mean this as we understand that all this masses ah initially say we have of course, denoted it with capital M0 that is the initial mass and this initial mass as we have fired it ah we have this velocity you know the exhaust velocity effective exhaust velocity and there is the change in the speed of this rocket and that is associated with the at a at a particular time say if it is starting from a particular velocity and or if it is starting from 0 the change in velocity can be related to the change in the mass or the this is a mass at a particular moment and this is the initial mass that when the velocity is say 0 or velocity is ah you know if we write it as say V1 minus V2. So, V1 is the initial velocity and V2 sorry V1 is the initial sorry ah I mean this is the change in velocity if we look at and then we can relate it with M0 and M where this mass ratio R is ratio R is given by M0 by capital M.

$$\frac{1}{2}M_p{u_e}^2 = M_pC_p(T_c - T_e)$$

And now let us look into ah the exhaust velocity now if we can correlate it with the chamber pressure and because we will not be able to measure many of these parameters. So, if it would be nice if we can correlate it with ah measurable quantities. So, basically what we are trying to understand here or try to write here is a say assume that there is a packet of gas and that is Mp and it is a kinetic energy of that packet of gas that is moving out with ah half m ue square that is the kinetic energy part of that ah gas and that is associated with the I mean basically the enthalpy change if we look at it can be M into Cp into delta T or Tc minus that is the chamber pressure minus the exit exit temperature. So, this is the chamber temperature and this is the exit temperature. So, we can try to put it ah using say assuming that this you know combustion is taking place ah isentropically.

$${u_e}^2 = \frac{2\gamma}{\gamma-1}\frac{R_uT_c}{M_{mix}}\ (1-(\frac{P_e}{P_c})^{\frac{\gamma-1}{\gamma}}$$

So, we can write that P v to the power gamma is equals to constant and from there we can write that ah p into ah T into p to the power gamma by gamma minus 1 equals to constant. So, this will lead to ah I mean if we use make use of this I am not going into the derivation part of it you will find that this Cp will be coming as 2 gamma by gamma minus 1. So, basically here we can write ue square is equals to ah 2 times Cp into Tc if we take common you will be having 1 minus ah Te by Tc and this Cp we can write in terms of gamma ah it would be something like gamma by gamma minus 1 into the universal gas constant divided by the M that is the ah basically the ah molecular weight of the gas mixture. So, this would be the Cp and if we replace this value in this equation and along with that if we you know put this Te by Tc it would be coming in the form like this. So, we can try to find out say all these parameters like we have the exit pressure, we have the chamber pressure and this Tc that is the temperature inside this ah I mean combustion chamber.

So, all these parameters can be related to have an estimate about ah the ah ue or the velocity at the exit of the exhaust gas of I mean at the exit of the nozzle. So, that is about the exhaust velocity, but now looking into the basically the different performance parameters. So, we understand that there are combustion you know characteristics we have we have the nozzle performance. So, we write ah there are different parameters like a thrust coefficient. So, that talks about the you know force or the thrust that we have to achieve ah and this is Pc the chamber pressure along with that this A star is the throat area.

$$I_{sp} = \frac{F\,dt}{mg\,dt} = \frac{mv_e}{mg}$$

So, basically, they talk about the performance of the nozzle and then we have the characteristic velocity which again relates to the chamber pressure and the throat area the multiplication divided by the mass that is you know used. So, it talks about when we talk about the characteristic velocity it means that how much you know ah how efficiently these masses are burning in the combustion chamber or it talks about the performance of the ah basically the combustion chamber or the combustion process and there you know finally, generating the high speed you know exhaust gas. Apart from that if it is about the construction part ah and basically the thrust coefficient and later on I mean this characteristic velocity talks about the combustion performance then we have another parameter called the specific impulse. So, we have this thrust that has been generated and it has been generated for a limited time that you know this is this of course, this F is you know trying to vary ah and F multiplied by this dt that is what is the impulsive force and this specific impulse is basically you know we are trying to see how much you know mass we are putting and what kind of impulsive force that we have been able to generate. So, that is talks about the specific impulse if you look at this value you will find that it is just nothing, but ah I mean Me ah because we have already talked about it, but ah this mve ah and divided by mg.

$$\mathrm{F} = p_c \times A^* \times C_f = mv_e = I_{sp}mg$$

So, this mass will be cancelling out basically it would be ve by g and ah you can understand easily that it is one is the velocity and this is acceleration. So, it is unit would be in second, but ah as such this will tell you what kind of thrust we are generating per unit mass of this ah you know not thrust basically the impulse that we are generating per unit mass flow of this ah propellant flow rate ok. So, that is about this ah the specific impulse. So, we can now you know try to look at this equation that thrust coefficient involving the thrust coefficient. So, F is basically Pc into ah A star into Cf the throat area multiplied by the thrust coefficient and also we have seen this F that is equals to m into ve ah the effective exhaust velocity.

So, effective exhaust velocity now we can see that from this two relations Isp into m into g, g is the acceleration due to gravity. So, these relations will be sometimes you know I mean useful for doing some ah small calculations. Let us have a look into this numerical problem where the changing of an ah you know ah 6.2 Newton or payload ah it is changing it is orbit and because of that you know there would be ah you know a change in the velocity that is I have directly given you this velocity value ah. So, ah and upper stage with liquid hydrogen and liquid oxygen ah has been used for this rocket to calculate ah the following and other things are given already the mass of the propellant that is required we have to find out then actual mass of the propellant tank and where we have been assuming that it is a spherical tank with the single ah spherical tank containing both the propellants at the mean density and ah it is a aerial density is given as 10 kg per meter square ah and then you have to ah use the thrust calculate to have to calculate the thrust and you have to calculate the burn time.

So, these are the ah values those are given here and along with that you know we will be calculating ah basically the different parameters. So, we have already talked about that equation that we are ah going to ah use. So, let us look at that equation. So, we have seen that this ve that is equals to C star already it is given and Cf this is also given.

$$\Delta v = v_e \ln\left(\frac{M_0}{M}\right)$$

So, this will give you 1.9 into 2386 these are the two values. So, this combined together this would be 4533 meter per second and then we can write that the mass that we are getting you know of the tank ah basically ah it is 6.4 this is you know that relation that we have used delta m ah or basically that delta v ah we have said that is equals to ve ah multiplied by ah log ln of M0 by M and from that relation basically we can see that it would be coming like ah this equation will give you ah the M is equals to some e to the power minus delta v by ve and then minus 1. So, delta ve delta v is already given in the problem and delta v has been given as 3.3 meter per ah meter per second this is the delta V we have given and ve is already we have obtained the value of ve that is 4533 meter per second.

So, from there we can calculate ah this M ah of the ah fuel or basically the fuel tank is 6.942 ton. So, this is the mass of the fuel tank or this is basically the mass of the fuel and if we now look into the tank volume. So, the tank volume would be ah basically ah it is this much ton.

So, 6.942 divided by 320 and this is 10 to the power 3 if it comes to a meter cube. So, this many 2169 meter cube would be the volume of the tank. So, this volume of the tank ah is basically ah you know ah will lead to the if it is a spherical volume this will lead to ah an R cube of 5.178 meter cube and from there you will find R equals to 1.73 meter. So, so if we look at this ah we can also calculate the area because we have later on you know in the problem we will see that this is 4 pi R square. So, this comes out to be 37.61meter square. So, from this area we can calculate the mass of the tank that would be 376.1 kg because we have been given ah the aerial density that is 10 meter square ah sorry 10 kg per meter square.

So, ah if it is 10 kg per meter square then you multiply it you know that will be the mass of the fuel tank. Now, ah the thrust that we can now calculate from this ah it would be coming like ah F equals to you know that is Pc into A star into Cf and that would be say 6 into 10 to the power 6 that is the pressure and then we have the throat area 10 to the power minus 2 and Cf is basically ah ah given as 1.9. So, this comes out to be 114 kilo Newton.

$$F = \dot{m} \times c^* \times C_f$$

So, this is the thrust. So, if we have the thrust we can you know ah correlate it with the ah basically this F can also be written as m dot then C star into Cf that is what we have ah learned about earlier and then we can put it as is calls to ah m dot from here we can calculate to be 114 if you put this value. So, 25.14 kg per second and from there this time would be coming as ah you know we have already obtained that fuel that is you know 6.942 into 10 to the power 3 and then we have this mass flow rate that is 25.14 kg per second. So, this will be lasting for about 276 second. This particular problem has been taken from this book ah by M. J. Turner. This is Rocket and Spacecraft Propulsion 2005.

So, you can have a look into this book for this particular problem. So, now let us look into ah quickly ah the different ah propellant feed system. ah There are in general 2 type of propellant feed system ah one of them is basically the pressure feed system where you will have these 2 tanks the fuel tank and the oxidizer tank and we carry an separate you know high pressure gas bottle with it. So, that this tank you know will be pressurized because this liquid are to be transferred and this is the combustion chamber where it has to come in the form of liquid. So, ah this is one of the way of you know feeding this ah this combustion chamber where this external gas will be pressurizing this fuel and the oxidizer to come to this combustion chamber.

Alternatively, what we have to use you know some pumps and that pump will be you know generating this ah or feeding this fuel and the oxidizer to this combustion chamber and this I mean pumps basically this need this fuel pumps they need a good amount of energy. So, that has to come from this turbine and this will be generated this will be a gas generation system and this turbine exhaust will be you know again there would be another nozzles from where this will be coming out or it may come within the this gas combustion chamber. So, depending on this you know arrangement we will have this ah you know ah pump feed system ah basically these are the two way of generating or pumping the fuel into the system. And now as you can understand that if it is the high-pressure gas ah system ah we have to use a thicker walled I mean tank. So, that it is able to withstand that you know amount of pressure ah.

So, that while pressurizing it would be able to withstand that feed gas pressure system. But whereas, these are basically can be of low wall thickness and that is basically sufficient to feed this pump and that that that is you know it is a better situation in terms of the thickness of these tanks. But at the same time, we need to look into the pump and turbine and etcetera, to generate its power and so on. So, now let us look into the different propellant feed system. Here you will find that one of them is the gas generation system where the gas will be generated ah we will be taking a part of the fuel ah this is coming from the fuel tank.

We have the oxidizer tank and both of them are you know combined together to generate this ah I mean gas and that gas will be coming over here ah to you know put it in the turbine and the turbine exhaust is getting expanded through another what is called gas I mean this system. So, ah this is one system which is called the gas generating system. So, we have talked about this gas generating system this is where we have you know taken the fuel and the oxidizer and putting it in the turbine. But this turbine has a limited capacity of handling the high temperature gas. So, we need to be careful about the mixing of this fuel and the oxidizer and sometimes we need to have you know ah separate cooling system for this turbine.

So, ah other than that we have another system where we allow this gas to come to the combustion chamber or basically what it the name suggest is that a combustion tap off. So, that means, we have the ah the turbine ah will be generated or basically it will be running from a tapping a part of that combustion mixture and that will generate the or this will run the turbine and the turbine exhaust will either come through another nozzle or it may be fed again into back into this main nozzle. So, what it gives basically all the time you know this it is basically difficult often you know to have the uniform gas mixture at this point and tap it to run the turbine. Then we have another cycle called the stage combustion cycle where we ah you know put it in the basically what we do is that we take the fuel and then it is passing through the nozzle. Then it comes over here and here again we generate ah take a basically take the oxidizer and we put the fuel enriched thing and then we put it in the turbine.

So, basically all in all these cycles we see that we are ah you know we have to ah run the turbine using some you know the fuel and the oxidizer. So, a part of that part will be taken back to run the turbine and finally, that exhaust will be either coming over here or we have to use the ah separate nozzle for that. So, this is called the staged combustion cycle and then we have the expander cycle well we will find that this fuel is directly coming to here and then it is passing over you know through the nozzle ah it is getting heated up and then ah we put it directly in the turbine. So, this turbine ah exhaust will be connected over this combustion chamber and finally, it will be taken back.

So, these are the typical propellant feed system. So, where we have the fuel pumps to you know run this oxidizer and fuel ah pumps and that will be supported by this turbine power and here this exhaust will be upcoming out from the combustion chamber. So, these are the references ah you can look into along with that M. J. Turner's book and ah. So, to conclude ah this exhaust velocity is an important factor basically ah it is telling you how efficiently we are ah generating the ah basically converting the propellant and finally, generating the thrust.

Along with that we have also ah understood that this is the pressure phase system is not you know ah suitable for ah I mean this is suitable for the low thrust engine, but for high thrust engine we should have ah you know the pump-based feeding system. So, thank you for your attention.