Cryogenic Hydrogen Technology
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Week - 06
Lecture 28
Cryogenic Liquefaction - Numerical

Welcome to this lecture on Cryogenic Hydrogen Technology. We were talking about hydrogen liquefaction, in that connection in the previous two classes we have talked about the simple Linde cycle, then we have talked about the precooled Linde Hampson cycle followed by Claude cycle. Now, in continuation to that we will also in this lecture we will be talking about cryogenic liquefaction process and is basically continuation of the in continuation of the cryogenic liquefaction processes. And in this lecture, we will be primarily talking about the dual pressure Claude liquefaction cycle. So, in that Claude liquefaction cycle we have seen that there is a single compressor and that compressor is compressing the hydrogen gas and in the numerical problem we have seen that it is compressing it directly from 1 atmospheric pressure to 40 atmospheric pressure and accordingly we have solved the numerical problem. So, this is a dual pressure Claude cycle where we will see that the compression is done in two stages ok.

And so, how do we do that we will learn about it. So, here this is the Claude cycle that we have already you know studied in the previous lectures and there is no precooling that is mandatory, but we do use some precooling to enhance the liquid yield. At this moment we are not going to discuss about that part we will come about that later on. So, here this is a single stage compressor as we have said that the state point is you know we are compressing the gas from state point 1 to state point 2 directly.

So, that means, if the gas is coming say from 1 atmospheric pressure to I mean 1 atmospheric pressure we are compressing it to directly to 40 atmospheric pressure though it is not very practical ok. And we have seen these are the two I mean expression one is for the liquid yield the other one is the work requirement per unit mass of gas compressed. And here this term takes care of the work that has been derived from this turbine. So, and that is assisting the compression process, this compression process is you know being assisted by this work and naturally there would be some reduction in the compression work done. So, this is what is about the Claude I mean singles pressure Claude cycle, but this Claude cycle when it is a practical cycle of course, but often we go for a dual pressure Claude cycle.

So, let us look how it works. So, here there are two compressors ok. This is the first compressor that will compress the gas from state point 1 to state point 2. And then you know it will be a part of the compressed gas will be you know taken out from here itself and that will be cooled through this three-stream heat exchanger. And then it will directly you know come to this turbine.

So, earlier it was passing through I mean I mean this entire gas was coming through the first exchanger that was a two stream exchanger if you remember. And then we were taking that cold gas through this turbine, but at this I mean in a dual pressure system. So, if this gas is being you know this total mdot gas is being compressed you see here we will have m minus mdot amount of gas and there will be replenishment of mdotf from there and that will constitute I mean if it is combined here at this point you will find that we have mdot amount of gas that is coming to compressor first stage. So, state point 1 to 2 is between one pressure level to the second pressure level and from state point 2 to 3 this is taking us to the final same pressure. Say maybe you know if we take the example of that 40 atmospheric pressure that numerical problem that we have solved.

So, this is one atmospheric pressure there is some intermediate pressure we are pressurizing and from you know this intermediate pressure we are pressurizing it to that ultimate pressure of 40 atmosphere pressure. But here a portion of the gas that is being taken through this heat exchanger that is being cooled before we pass it through this turbine and rest of the things are same as usual. So, this expanded gas is mixing with the return stream and this high-pressure remaining fraction you see this remaining mdot minus medot fraction is passing through the second exchanger third exchanger and then it is getting expanded through this JT valve and then we are getting the liquid in the steady state. So, ah now if we try to draw the TS diagram for this process you will find that it would look like this ah it is on the this scale you have the temperature on this side you have the entropy and we have this kind of pressure lines. So, this is say if we tell that this is the state point 1 we have another line like this ah which is you know taking us to state point 1 to state point 2 and then from there ah you know it is coming to state point 3.

So, this is state point 3 according to there has to be 1 is to 1 correspondence here. So, before it goes to ah, but you see ah this state point 2 is not coming to you know state point ah 3 ah a fraction of the gas is of course, coming to state point 3, but the remaining gas is passing through the turbine. So, this 2 to 4 it would be cooled down and then from here it will come to this e state whereas, this rest of the gas is coming to state point 3 and then it will you know come to this ah some ah 3 say let us call it 4 prime. So, this will come to 4 prime and so on 5, 6 and finally, it will come to state point 6 before it is expanded to what pressure that is the lowest pressure it is getting expanded. So, it will come over here.

So, this is getting expanded to the lowest pressure and we are getting a portion of the liquid and vapor mixture in this ah the liquid vapor dome inside this liquid vapor dome. So, we have some intermediate pressure and from that intermediate pressure we are expanding the gas which in the earlier case was like you know this was compressed to the highest pressure and that highest pressure was ah you know from that highest pressure we were you know ah expanding the gas to the atmospheric pressure or the lowest pressure. So, here now what happens you can understand why should we do this ah dual pressure Claude cycle like this. The point is that the one the gas which we are expanding through this expansion valve ah there we are putting the maximum pressure. So, that means, we have this maximum pressure line this ah you know we have achieved this maximum compression through this process this state point 3 is you know coming over here and then we are expanding it.

Because ah and this the part which we are expanding through the expansion engine that is from here to this in this part you will find that we are doing it from a ah relatively lower pressure. So, as we know that this turbine or this isentropic expansion is ah I mean basically ah reversible process ah in reality of course, there are certain irreversibilities that we will look into later on. But as such this is reversible process whereas, this is irreversible process the temperature drop that we obtained from this isentropic process is much larger compared to the ah relative to this isenthalpic process. So, if it is beneficial if we expand we for if we could expand it from a higher ah temperature ah sorry higher-pressure level like from here we are come I mean you know expanding the gas from this point to this point. Whereas, you know this is the relatively lower pressure from which we are expanding the gas from state point 4 this is state point 4 to state point e to have ah ah I mean relatively ah larger temperature drop still larger temperature drop would have been possible if we are expanding it from this ah you know higher pressure.

But we are not going to do that we are you know compromising with ah that part and because of that we will see later on when when we analyze this system that we are losing a little bit of you know the liquid yield, but at the same time there is ah you know ah advance I mean ah relatively advantage will be ah obtained in terms of the work done. So, ah if we now try to analyze this ah first of all we will try to calculate the liquid yield how this liquid yield is ah reducing or you know increasing with ah this ah part and later on we will also try to calculate the work done part. So, here comes ah the control volume and ah what are the ah terms that those are going in and out ah we can try to calculate ah. So, here we have ah one part that is going in medot with ah the enthalpy of h2 then there is ah you know since already medot has ah come out of the system. So, we have mdot minus medot flowing in to this heat exchanger and mdot minus mfdot ah you can easily find out that this stream is leaving this control volume this is the control volume and mdotf that is what is going out of the system.

And ah what else is there ah of course, this you know turbine part medot mass is ah going out of the system and medot ah at enthalpy he is going into the system. So, if we now apply the ah first law of thermodynamics ah for this ah open system ah we have we do not have any work ah or heat input ah in this ah system. So, Q minus W is ah this W is basically the net work and for this control volume there is no work there is no heat in loop heat in leak. So, we have the summation of the outgoing enthalpies and the corresponding mass. So, let us ah take what are the outgoing enthalpies.

So basically, we will find. So, mdot of the outgoing enthalpies hout and minus already I think you have learned by this time this is the incoming enthalpies. So, let us try to identify the terms ah what are going out. So, mfdot is going out at hf then we have this stream leaving the system is mdot minus mfdot with an enthalpy h1. Then we also have ah this term h ah sorry this is not leaving the medot that is leaving the system at h4.

So, this is h4 these are the streams those are leaving the system and the streams entering the system is ah first of all mdot minus medot multiplied by h3. Then we have the stream leaving ah sorry the stream entering this system ah is h medot into he and then we also have ah another term. So, h3 has been included h2 medot into h2 that is also getting in. So, these are the terms ah that we have you know ah find those are going in and out. So, this is equals to 0 on the left-hand side.

So, we have to now ah you know call it this part mfdot if we take. So, we will have ah mfdot multiplied by ah say h1 minus hf with a negative sign and if I am taking it to this side right on the right-hand side this is h1 ah and we are taking it on the other side. So, this is h1 minus hf. So, then you have on this side mdot and mdot h1 minus ah the other mdot term is h3. Then we have ah plus medot term there are quite a few terms.

So, one of them is related to the turbine and this is say h4 minus he that is the enthalpy drop we are getting. Then you have an additional term ah me dot with related to ah this was h4 minus he. So, medot this is basically h3 h3 minus h2. So, this is a negative term and this is a positive term ok. So, that if we try to calculate the liquid yield that is mfdot by mdot you will find that we have h1 minus h3 by h1 minus hf.

$$y\left(\frac{\dot{m}_f}{\dot{m}}\right) = \frac{h_1-h_3}{h_1-h_f} + \frac{\dot{m}_e}{\dot{m}}\left\{\left(\frac{h_4-h_e}{h_1-h_f}\right) - \left(\frac{h_2-h_3}{h_1-h_f}\right)\right\}$$

So, this is same as the term that we have obtained earlier the state point 1 and 3 ah these were also there in the ah previous simple Claude cycle ah Claude cycle where the state point 1 was corresponding to 1 atmosphere and if we talk about that numerical problem and this was the highest pressure that is 40 ah atmospheric pressure. So, here this term corresponds to that exactly, but there are additional terms we have to you know now take into account. This is medot by mdot and we will have h4 minus he divided by h1 minus hf and if you look at ah this 2 terms h2 minus h3 there would be a basically ah this we can write we have taken common. So, this would be a negative contribution h2 minus h3 by h1 minus hf ok. So, these are the ah terms that would come in the ah liquid yield part.

Again this is the expression for the liquid yield it is changing a bit, but let us try to see what happens to the work done whether really, we are getting any benefit out of this dual pressure system. So, we have already understood that this will be a slight there would be a slight reduction from the single ah stage cloth cycle, but let us try to look into the work. So, you have to ah it is not very easy I mean it is not easy to remember this expression either in the exam part you have to either derive it or you know you have to I mean it is better you try to derive it and use it for solving the problem. Now let us go to the work requirement part. So, here comes this work requirement where you will find that this is the compressor 1 where we have you know m it is handling mdot mass and we have given some work input of wc1.

And in the second stage we have mdot minus medot because already m e dot mass has gone out and through the exchanger you know it is coming to the turbine. So, it is handling ah you know mdot minus medot and we have given you know wc2dot is the work inputment work input and this is the heat rejected from this process. So, ah if we just take a control volume around this ah first stage you will find that ah the here the work ah and heat and you know mass going in and out all are there. So, this is the enthalpy at h1 this enthalpy is at h2 and we can now corresponding to this we can write this expression as. So, q ah r 1 dot minus ah I mean wc1 ah dot that is equals to the outgoing enthalpy what is the outgoing h2 and this is mdot minus incoming enthalpy is h1 and mdot.

$$\frac{-(W_{c1})}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2)$$

So, that gives you ah simple I mean earlier also we have solved this ah expression wc1 by mdot comes to be ah h um ok h2 minus h1 h r h1 minus h2 sorry h1 minus h2 with a negative sign and minus QR1 ok. So, if we assume this is the compression process to be reversible and isothermal ah we can you know evaluate this term as equals to T1 and then you know s1 minus s2 and this is already mdot is taken into account. So, this is minus h1 minus h2. So, this is the expression for the work ah requirement in the first stage. So, this first stage work is like this ok.

So, for the second stage what we have is we can you know if we ah take a control volume you will be able to write it as. So, the mass it will be almost similar expression, but this would be wc2 by ah mdot minus medot I mean just drawing the similarity ah I mean exactly it would be the similar and since we have said that this is isothermal. So, we have ah T1 and then ah I am sorry this is state point 2 and this is state point 3. So, the temperatures are the same. So, this is state point 2, this is state point 3 and ah this is going from s2 minus s3 minus the corresponding enthalpies now let us put h2 minus h3.

$$\frac{-(W_{c2})}{\dot{m}} = \frac{(\dot{m}-\dot{m}_e)}{\dot{m}}\{T_1(s_2-s_3)-(h_2-h_3)\}$$

So, this is h2 and this is h3 and ah you know now if we try to put it in the same unit or same denominator mdot. So, we have to ah you know put ah mdot minus medot divided by mdot. So, this is that additional part we have to do s1 minus s2 minus h2 minus h3. So, you can understand that this would become 1 minus medot by mdot and then comes this ah whole part T1 into s2 minus s3 minus h2 minus h3. So, the total work done per unit say now if we combine this 2 say minus wc1 plus wc2 this is the total work per unit mass of gas compressed in the first stage.

So, this is is what that is you know. So, this is T1 into s1 minus s2. So, this term will be there minus h1 minus h2 and along with that we have this term. So, you can understand that there would be a term related to this medot by mdot and this whole part would be there, but rest of the things will be combined with this part ok. So, which is T1 s1 minus s2.

So, I will take that part common now. So, here this would be ah I am sorry let this not be there ok fine T1 and this is T1 into s1 minus s2. So, this is equals to T1 s2 minus s3 ah minus h2 minus h3 that would be one part the other part is minus medot by mdot this term multiplied by that bracketed term T1 into s2 minus s3 minus h2 minus h3 ok. So, if we now combine these two parts you would now find that this term this T1 s1 would be there s2 this this is negative this is positive. So, they will cancel each other. So, this total work done per unit mass of gas compressed in the first stage will have T1 I am writing it here T 1 into s1 minus s3 then you have this is minus h1 plus h2 and here you have ah minus h2 plus h3.

$$\frac{-(W_{c1}+W_{c2})}{\dot{m}} = T_1(s_1-s_3)-(h_1-h_3)-\frac{\dot{m}_e}{\dot{m}}\{T_1(s_2-s_3)-(h_2-h_3)\}$$

So, in total it would become minus h 1 if I am taking this negative common you will find that this is negative of course, here. So, h1 minus h3. So, this is this part will now come over here and then you have this is equals to minus medot by mdot along with that you have T1 into s2 minus s3 minus h2 minus h3. And if you are using that cold ah sorry the work produced by the you know the turbine then there will be additional you know lessening of the work requirement in the compressor. So, that part is always there ah I mean if it is this expression is for the work requirement per unit mass of gas compressed in the first stage excluding the ah turbine output ok or turbine work output rather to be very specific.

So, if you want to include that I mean if you are if you are doing it like this that this work output of the turbine will also be utilized. So, there will be a proportional you know medot by mdot and h 3 sorry here it is you know h4 basically h4 minus he by ah there would be ah this term this will be lessened by that factor. So, ah I think it is correct because this is the work output from that turbine. So, that will be lessened by that term. So, now this is the work requirement now if you look or compare it with the previous situation you see this was the work requirement when you were compressing the gas from state point 1 to state point 3 directly in a single stage Claude cycle and the work requirement was T1 into s1 minus s3 minus h1 minus h3.

Now that is you know reduced by this factor and of course, this is you know is common part both the cycle. So, there also it was ah h4 minus he and here also it is h4 minus he. So, you can understand that there is a reduction in the work requirement, but at the cost of lessening in the work requirement sorry in the ah I mean proportionate ah reduction in the liquid yield. So, now let us try to ah solve a numerical problem based on this dual pressure system ah this is the problem statement this is exactly the same problem that we have solved in the ah Claude cycle earlier, but this Claude cycle this cloth liquefier is a dual ah you know pressure Claude cycle this is this is ideal dual ah pressure dual pressure ah Claude cycle and the compression is done in two stages ah ah the same temperature everything remains same the I mean the first stage it was one atmospheric pressure it is getting compressed to you know 4 MPa or 40 atmospheric pressure and intermediate pressure we will try to take it you know in two ways ah first we will solve the problem with when you know the intermediate pressure is 1.5 MPa and later on we will solve another problem where it is compressed to 2 atmosphere and 20 atmospheric pressure.

So, there are two problems here and there is no ah LN2 pre cooling that we have ah talked about the entry to the turbine is ah here also it is the same that is ah in earlier it was h3 now in our case this is h4 and I mean this corresponding to this temperature is T4 rather we should say the h4 is different because once it will be ah corresponding to this pressure and the other time it will be corresponding to this ah you know 20 atmospheric pressure in the previous problem when it was a single stage it was at 40 bar ok. So, this ah inlet to the turbine is ah I mean enthalpy is changing and accordingly this ah I mean it is liquid yield will also change this part is remain constant. So, the mass that is passing through the turbine is ah again ah the mass of the gas compressed in the first stage due to the ah mass which is passing through the turbine is ratio is medot by mdot that is 0.5 and you are supposed to find out the liquid yield the work required per unit mass of gas liquefied figure of you know merit and of course, assume that the expander work is utilized in the compression everything remain same except that you know we are compressing the gas in two stages and I mean ah we have this is problem number 1 and this is problem number 2. So, in first stage we will say that it is intermediate pressure is ah 15 atmospheric pressure in the second problem we will try to see if we are doing it a compressing the or putting the intermediate pressure as 20 atmospheric pressure.

So, that is how we will try to solve and let us go to the ah solution technique this is the ok ah we will come here from here to the excel part. So, now, ah this was the problem ah you know that was the simple Claude cycle you might have ah done it and these are the values ah we have taken using that ah coolprop software. So, in the previous lecture we have done this and the liquid yield was 0.197 with the work requirement per unit gas compressed is 3709.5 and the work requirement per unit I mean gas liquefied is basically this work requirement divided by ah this liquid yield and it came out to be 0.638. So, this is that has already been solved ok. So, now, let us go to the problem ah 1 in the dual pressure system when we have said that the intermediate pressure is ah only 15 atmospheric pressure from 1 atmosphere to 15 atmospheric pressure and from 15 to ah 40 atmospheric pressure. So, these are the ah temperature and these are the state point 1, state point 2 is 15 atmospheric pressure, state point 3 is ah 40 atmospheric pressure. So, state point 4 that is ah corresponding to 1.5 MPa that is you know 15 atmospheric pressure and the temperature is 180 Kelvin. So, you have to look into this part and the corresponding enthalpy you have to calculate is using the coolprop.

This is the corresponding enthalpy-entropy and the entropy corresponding to state point E is basically the same entropy ah you can see this entropy and this particular ah part is same. So, we know this and the atmospheric pressure it is getting ah you know reduced to. So, this is also known to us. So, we can calculate the temperature and the enthalpy and finally, this is the enthalpy of the fluid part ok. So, now, based on this ah we can try to calculate the liquid yield.

Now, we are finding it that it is liquid yield is 0.168. Earlier it was 0.197 and now it has reduced to 0.168. Now, let us look into the work requirement per unit mass of gas liquefied. Now, it is it has reduced to 192.07 ok. So, earlier it was ah I am sorry 1800 something ok ah this is ah I am sorry the single stage it was work requirement was 3709 and this is the work requirement has gone down to ah 3225 work requirement per unit mass of gas compressed, but ah this has gone up because you see there is a proportionate lessening in the liquid yield. So, that is why it has gone high. So, now, we will go to the ah third part the dual pressure system. So, we will find that ah in the single stage we had 1800 ah you know 18836 per unit mass of gas liquefied in the dual pressure system when the intermediate stage is ah 20 ah it is coming to 18909 that is the per unit mass of gas liquefied, but the liquid yield has slightly improved if we take it the intermediate stage to ah you know 20 atmospheric pressure and from intermediate pressure is 20 atmospheric pressure and from 20 atmospheric pressure and 180 K if we are expanding it we have ah slightly ah ah I mean improvement slight improvement in the ah liquid yield.

Earlier in the 15 atmospheric pressure it was 0.168 now it is coming to 0.178 and ah the work requirement with respect to the the first single stage it it is 3709. So, now, it is it has gone to 33.61, but in terms of the gas liquefied you can still you can see that they are ah comparable ok and 18836. So, here 18909, but you can see that ah from this calculation there are quite a few number or parameters this has been taken arbitrarily ah, but there are parameters like medot by mdot how much mass we should ah pass through the turbine what should be the inlet temperature of the turbine all these factors ah will determine what is the optimum mass that has to be sent and what is that intermediate pressure that should be chosen, but ah overall you can understand that the work requirement per unit mass of gas compressed is ah you know ah reduced with a proportionate reduction in the liquid yield.

So, we have to come to an ah I mean conclusion or compromise with the liquid yield and the work requirement. So, this is the numerical problem that we have solved ah I mean you can try to solve it these are the references we have ah we have already told about and you can go through it. So, conclusion is that well pressure Claude cycle there is a reduced liquid yield and there is of course, the work requirement per unit mass of gas ah I mean ah compressed that is also going down and ah, but the there would be finally, ah if you look at the per unit mass of gas liquefied there may be you know that we have to ah come to an optimum value depending on the mass flow rate that would be passing through the turbine and that intermediate pressure we have to determine judiciously. So, thank you for your attention.