NPTEL Online Certification Courses
Industrial Robotics: Theories for Implementation
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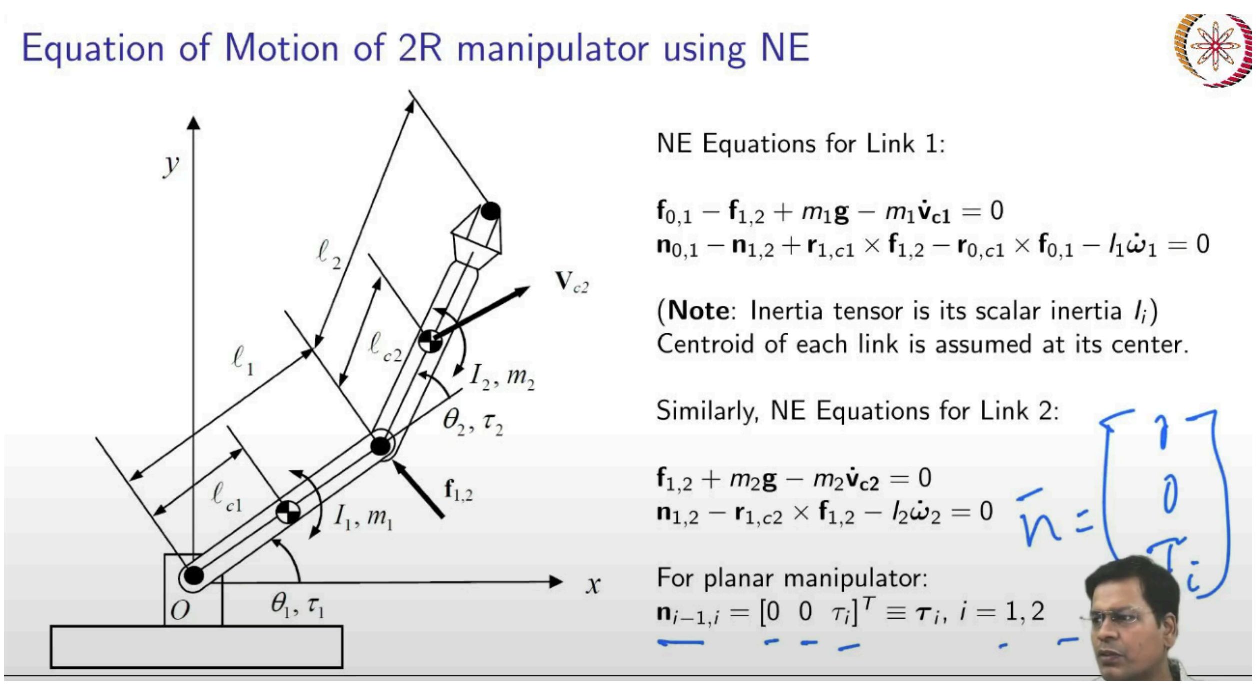
Lecture 39

Equation of motion of a Two-Link manipulator using NE Approach

So, now that we have covered Newton Euler formulation, So, and we also did one of the examples of one linked manipulator using the recursive Newton Euler approach. So, we now know the general Newton-Euler approach to derive the equation of motion as well as the recursive Newton-Euler approach, where we have a formulation with which you can do the calculation of forces and torques recursively, starting from all the joints right from the end joint to the first joint. That is the backward iteration we did, and we also did forward iteration when we calculated all the velocities and acceleration of each link, using the forward iteration of Newton Euler's approach. So, that was the approach of recursive Newton Euler. So, today, continuing further, we will do two linked manipulators using the recursive Newton-Euler approach. So, now we will do a 2R manipulator using Newton Euler's approach. 2R is two revolute joints, that is, the planar manipulator that we will be doing now.



So, let us begin. So, yes, this is the planar manipulator. We have been dealing with it for a long time, even with forward kinematics, inverse kinematics-this is the one which is very, very important, you know already. But in dynamics, if it is planar, it becomes very, very simpler. So, because the recursive Newton Euler approach, or Newton Euler approach in general, involves plenty of equations to deal with, it is normally comfortable we can do use programming, but in order to do it analytically on pen and paper, it is very, very difficult to move ahead to. So, in order to demonstrate the basic principle, I am proceeding with a 2R planar manipulator only so that we can handle the equations very well. So, yes, this is your two linked manipulator. The moment of inertia of the first link is given by I1, and you have mass m1. lc1 is the centre of mass distance from one of the ends that are attached here, you see, and the total length is l1. Similarly, lc2 you have. That is the distance of the centre of mass location with respect to this end of the second link, and the total length is l2; Vc2 is the velocity of the centre of mass of the second link. Similarly, Vc1 is also there. So, yes, you have tau1 and tau2 which is required to be found out. Forces, interaction forces are F12. You see, it is here. That is the interaction forces between the links. So, this is how it is all defined. And you have the x and y axes are fixed like this. So, you have the z axis which is rotating out of this plane, and that is those are placed here and here. So, let us continue. So, yes, the first Newton-Euler equation for link 1 is for the force and force of interaction. It is 0, 1 and 1, 2. So, it is for link 1, you see, for this link. So, $f_{0,1}$ acts at this end. $f_{1,2,}$ you see, it is acting over here, and m1g is the force due to the gravity that you see m1g, and it is directed along the z direction, and you have m1Vc1 dot. So, why is it a Vc1 dot if it is the centre of mass velocity given by Vc1? So, the Vc1 dot is the acceleration of the centre of mass. So, that is here. So, this is inertial force; this is gravitational force.

$$f_{0,1} - f_{1,2} + m_1 g - m_1 \dot{v}_{c1} = 0$$

These are the two forces externally acting on link 1, the isolator. So, this is the force balance equation. Similarly, we can write a moment balance equation. So, this is the moment that acts here, and similarly, you have the moment that acts here, and 1, and n01 and n12, and these are the moments due to the reaction forces, the joint reaction forces.

$$n_{0,1} - n_{1,2} + r_{1,c1} \times f_{1,2} - r_{0,c1} \times f_{0,1} - l_1 \dot{\omega}_1 = 0$$

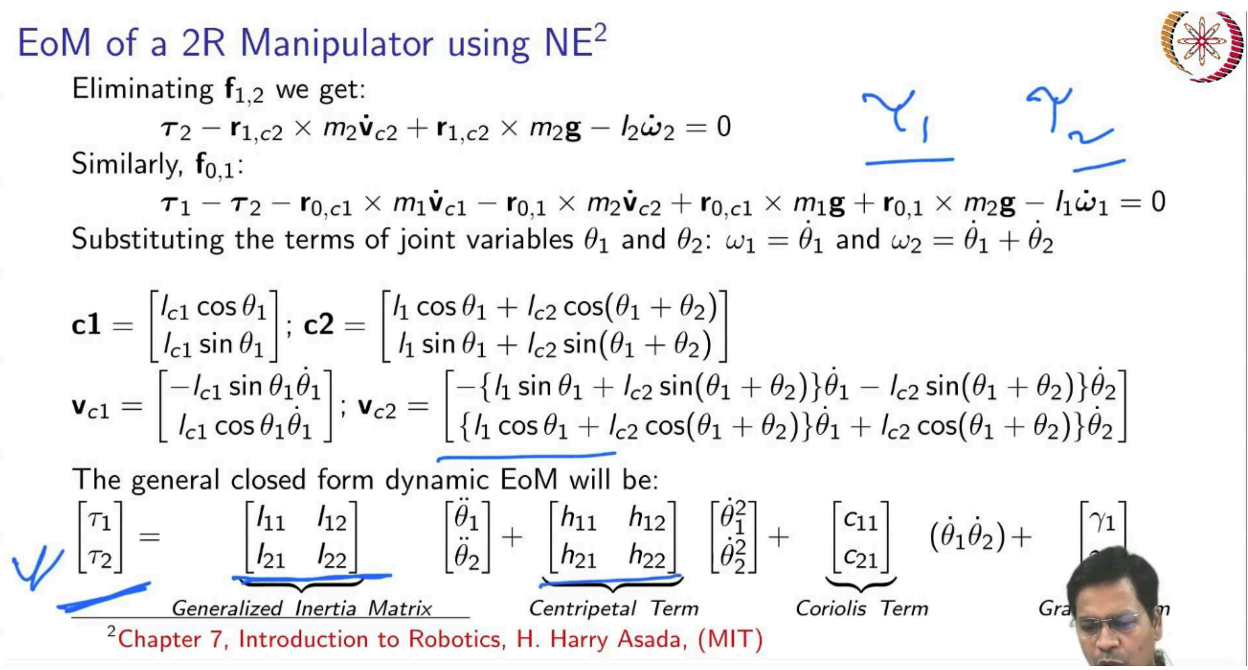
So, this is one force, and you have the distance, which is given by C1. That is this one, and that is the vector, actually. And similarly, 0, C1 is this one. So, you have this force, so these are just cross products, which is giving you clockwise and counterclockwise moments so that those go here, and this is the. That is because of angular acceleration. So, finally, it is a moment balance equation, so it gives you 0, as because it is a vector equation, this should be 0 vectors, that is, 0, 0, 0. It should be like this. So, this is for the link 1, similarly. So, note that for inertia tensor it is a scalar in this case because if it is planar, it is only in one direction. You are interested in, and more so. We have simplified this link. It is a slender bar, and you have a centre of mass situated at the centre. The centroid of each link is assumed to be at its centre. Similarly, Newton Euler's equation for link 2 is f1,2, which is the force which acts over here, and there is no external force that is acting from here. So, you don't see the other force you have, though, the gravitational force, that is, m2g, and this is the inertia force. It is mass into into acceleration. So, this is the velocity of the centre of mass. So, that is what you know already. So, this is the force balance equation:

$$f_{1,2} + m_2 g - m_2 \dot{v}_{c2} = 0$$

and this one is the moment balance equation,

$$n_{1,2} - r_{1,c2} \times f_{1,2} - l_2 \dot{\omega}_2 = 0$$

and again, you have a moment to act only at this end. So, that is getting into this second link. This is your second link, and this is the moment generated because of the force balance. So, that is what you are getting. So, this is a moment because of force. So, that comes here, and you have inertia moment, moment due to the inertia. So, because this has an angular acceleration of omega 2 dots. So, if it is omega 2 dots, so inertia of the second link is I2. So, that comes here. So, these are the two equations, Newton Euler equation, that is for link 1 as well as link 2. For the planar manipulator moment, it can be given as 0, 0 taus. Tau is nothing but the torque at the joint. It is exactly lying along the z-axis: 0, 0 taus. So, this is your n vector. So, that can be. Similarly, for both the links for 1 and 2, it is tau 1 and tau 2. Those are the joint torqued which you want to find out. So, that is what can be written like this.

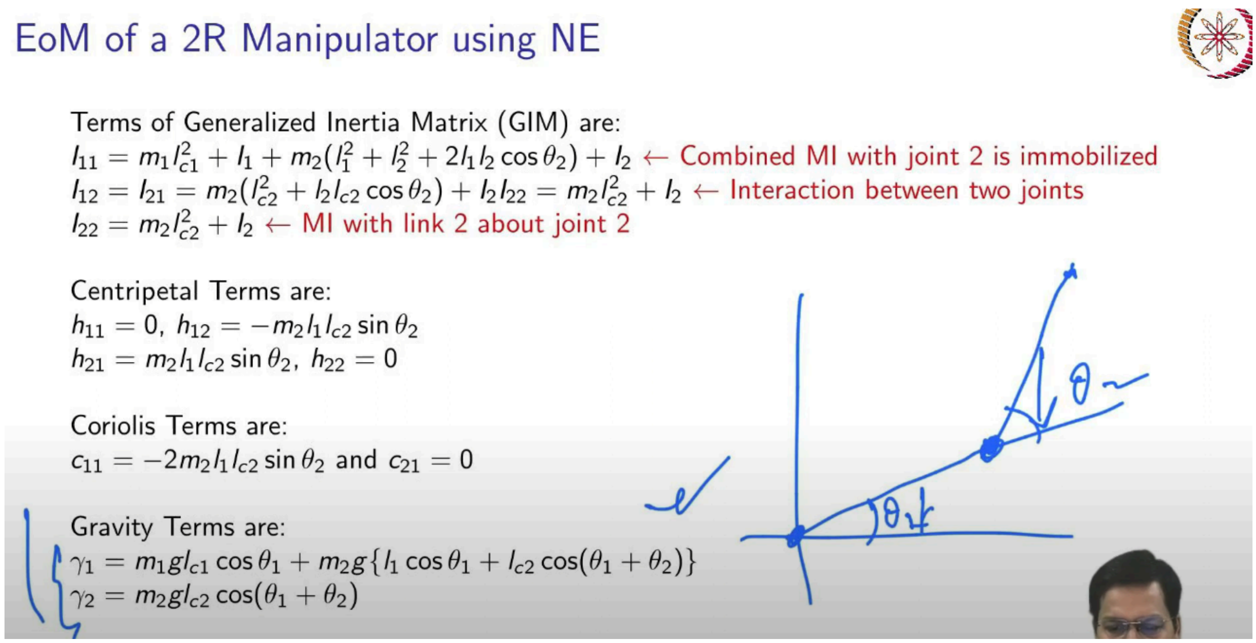


So, now, eliminating f12 what you can see here. We want to extract f12, which is here f12. So, we have to substitute f12 from one of the equations to the other one. So, just look carefully, it was there somewhere in this.

$$\tau_2 - r_{1,c2} \times m_2 v_{c2} + r_{1,c2} \times m_2 g - l_2 \dot{\omega}_2 = 0$$

So, f12 from here I am substituting it here so that that gets eliminated. So, as soon as you substitute here, the whole of this will come here in the bracket, and then you expand this. So, that is what is obtained. So, you got this and moment. You can replace it with the torque, like this:

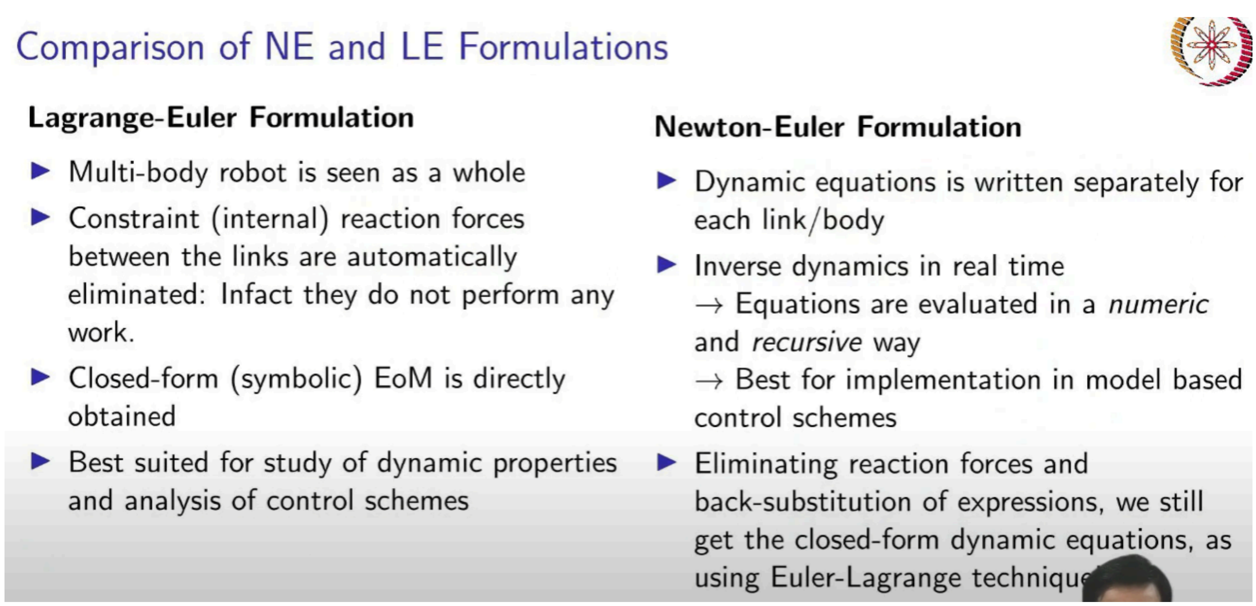
torque here is exactly the moment, and that is along. So, the moment can be written as torque 2 along z. If it is z, it can be written like this: so both are equal magnitude-wise. So, I can write it in a vector form like this also: so both equivalents I have taken. So, torque 2 vectors is nothing but 0, 0 taus 2. So, this is the magnitude, and in terms of the vector can be written like this. Again, so, here goes the original thing. So, both are the same, and I have just substituted f12 from the moment equation of the second link. So, that comes, those terms come here. So, that is an expanded form. I can write it like this: the last term was exactly this one. So, again in for the link one. I did the same thing again. I should show you this, so you see it is here so that I can substitute. This time I want to eliminate f01, so I can substitute f01 directly. f01 and f12 from the second link equation. Everything goes to the second equation of link one. So, f01 and f12, f01 from here and f1,2 from the second link equation. Both are substituted to the moment equation of link one. Got it. So, now, what do I get? I simply can write it like this. So, it looks quite expanded and difficult, but actually, they are derived out of two fundamental equations: Newton's equation and Euler's equation, that is, the force balance and the moment balance equation for link one and link two. So, immediately you can write it like this. Got it? So now, solving this and using some terms are like this: theta 1 and theta 2 are the joint variables. So, you can write omega 1 simply as theta 1 dot. Similarly, omega 2 will be the sum of angular velocity if it is written in the 0th frame, that is, the ground frame. So, angular velocity is theta 1 dot and theta 2 dots, the sum of those that become the total angular velocity for the second link with respect to the ground. and c1 and c2 are the locations of the centre of mass with respect to the base frame, that is, the 0 frame, the first frame. Let me just show you once again this figure. So, it is this frame. It is about this frame, you see. So, these are the two centre of gravity locations for any angle: theta 1 and theta 2. So, these are variable locations. Now, I can calculate the velocities by taking differentials of this. You take derivative with respect to time. So, from this you can obtain this, and from this, you can obtain this, got it? Similarly, I can also obtain the acceleration by taking a double derivative of this. So, yes, now that you have obtained the vc1 dot and vc2 dot, you can also obtain, by taking a derivative of this substitute, that here. You see, it is quite lengthy even for just two links; it becomes a very, very lengthy equation, and you have to handle analytically with symbols for such a long equation. It becomes really, really difficult, even for a planar trace like this. So, now I'll substitute this all here; omega 1 and omega 2 will also go there, from here and from here, so that can be written like this. So, omega 1 in the case of vector, you can write it as omega. If it is a vector, you can.Write because you know it is along z axis, so it becomes 0, 0, omega, the value that you have obtained, omega 1 and omega 2, inertia-here you can write it in terms of inertia tensor over there. So, that is, that can be quickly written over there. And this equation is as simple as that. So, g is simply 0, 0. It is along the negative y-axis so that it can be written as 0 minus g and 0. You see if you just check your frame, it is like the negative of the y-axis. So, that was your direction. And all those substituting: here you can obtain the generalised closed form dynamic equation of motion like this. So, now you have to organise all those elements in terms like this. So, this becomes your generalised inertia matrix. So, you have to write the scalar equation of tau 1, similarly for tau 2, and pack them together the way we did it in Lagrange Euler's equation. So, in the same way, you can arrange them in this manner. So, you get the generalised inertia matrix, centripetal term, Coriolis term and gravity term. So, you now know the physical significance of each one of them we have discussed earlier.



So, explaining them further. So, the terms of generalised inertia matrix are: so if you write inertia matrix as I11, I12, I21 and I22, this term can be explained like this: so this is basically the moment of inertia of combined both the lengths if you put them together. So, if you freeze them, this is your theta 2. This is your theta 1. So, at any instant, if it is frozen, you have to find out the moment of inertia of this combined structure with respect to this. So, that is what this is. With the joint immobilised at an angle, theta 2, this is theta 1 if you for any particular angle, so this becomes your moment of inertia about this. So, because you want to find out the moment torque at this joint. So, this term exactly tells you a combined moment of inertia when the robot is like: this got it.

Next term is I12, again I11, I12. With this, basically, I12 and I21 are interactions between the two links. So, because of the combined action between the two links in motion. So, this is the moment of inertia. So, this, this one, is the pure moment of inertia, and this also will be the pure moment of inertia, but this one and this one is due to the interaction between the two joints and that is given by this moment of inertia. This is what was a physical significance we have seen earlier also, and I22 is nothing but a moment of inertia of link 2 about joint 2. This is the simplest one. If this is your link 2, you can quickly find them. This is your x, and this is your y. You can find out the moment of inertia of link 2 about its own joint, this one, so it is nothing. But if this is l2, and you already know the moment of inertia about this, that is I2. So, I2 and ml2, square. So, that is I22. So, now you know you have all four terms in the diagonal elements and the interaction terms.

So, what centripetal terms are these? Similarly, Coriolis terms come out to be the same, the same, as we have obtained using the Lagrange-Euler formulation. So, the terms are similar and the gravity terms also, we have seen earlier. So, this is the effect of the centre of mass that is getting pulled due to gravity, and this is the torque that you will see. Even when the system is not in motion, it is static at any particular location, given by joint angles, theta 1 and theta 2. So, at any particular instant, this should be the torque that acts at joint 1 and joint 2. So that is the gravity term. So, this is all we can obtain, even using Newton Euler's formulation. You see,



So, there are some pros and cons of each of these algorithms. So, in the case of the Lagrange-Euler formulation, you have seen it is a multi-body robot. It is seen as a whole. You have calculated the total energy, kinetic energy and potential energy of the whole system. You have considered together, and so the whole body is considered, even to calculate just the joint torque at one of the names. So, that is how you have considered, and you have calculated. It is an energy-based approach. You have also seen that. Constrained inert internal reaction forces between the links are automatically eliminated. You see, it is automatically eliminated. In fact, they do not perform any work, so that is not important as well. So, it is not calculated on the way when we calculate the joint torques. So, yes, this is not important, but sometimes they are important. If you want, let us say you want to find out the joint interaction force, you want to design the shear load on one of your bearings or one of the shafts that go here. So, you definitely need to know the interaction force that is coming onto them. So, in that case, Newton Euler formulation will help you if you want to design that joint; otherwise, getting rid of that becomes makes things much simpler. So, you can follow the Lagrange-Euler formulation. Closed form, the symbolic equation of motion is directly obtained. You see, you have obtained the equation of motion directly, and that too in symbolic form, very easily, whereas in the case of Newton Euler also, you could obtain, but it becomes very, very lengthy. You have seen the process for tooling just now, so it is not that trivial. Best suited for the study of dynamic properties and analysis of control schemes when you want to analyse how your robot will behave. So, now that you have a symbolic equation in hand, you can quickly obtain the behaviour. You can obtain the transfer function, taking the whole of the robot as a single unit. You can make a model-based control. You have the whole equation in hand, and you can calculate torque for any given joint, velocities and accelerations. So, provided you have the symbolic equation in hand, these things become very, very simple in the case of Newton Euler's formulation. Dynamic equations are written separately for each of the body. You see each link. You have made a simple force and moment balance equation and attempted all of them approached from one end to the other end while calculating your velocities and in a backward manner when you calculated the torque at each joint. So, that is what you did. Inverse dynamics is done in real-time equations are evaluated in numeric and recursive ways. If you are doing in the program, so you are doing all of them in a numeric way. You directly get the values, and you don't get anything in symbols. And recursively, you do forward, you do backwards, and you get the velocities and the torques you see. So, everything is calculated at every instant of time, and it is best for the implementation of the model-based control scheme. You don't need a forward solution. When tau is equal to i theta, double dot, c theta, theta, dot and gamma theta. For a complex system like a robot, if it is multi-link, it is really complex. So, you can handle things, even numerically. That means that doesn't mean you are solving with some iterative technique, definitely so you here, numerically, I mean the values you get directly in hand. You do for every moment, every instant of time, and you use them. So, elimination of reaction forces and back substitution of expression. We still get a closed form, dynamic equations, as in Lagrange Euler or Euler-Lagrange technique. So, we got these to the same place, you see. But this, this Newton-Euler approach, you see, is the way it is formulated. It allows programming very easily. So, that is the reason I personally have preferred this one. But yes, there are. I won't say Lagrange Euler doesn't have any recursive or programming approach. Yes, it does have. There are books which are, which is covering even Lagrange Euler in that manner. So, take derivative you have a matrix which is defined. If it is pre-multiplied, you directly get the derivative of any transfer function, like a transformation matrix. So that is how even Lagrange Euler is dealt with quite a lot of times. But yes, depending on the situation, you can use any one of them. But this is quite okay. If you have to do some control scheme, like model based control scheme, this is quite, quite okay.

So with that I would end dynamics, and in the next module, I will be covering directly your load data calibration, that is, the payload and the supplementary load that needs to be calibrated before you put your robot to use. Because you know, nowadays, robots are huge sometimes. For example, KUKA Titan is a class robot of something around one-ton capacity. It can carry nearly one ton, and even fans have it. So, you see, it carries a huge payload. So, as soon as the robot picks up the load, the dynamics change. You need to know where is your, where is your new centre of gravity location. So, you have to calibrate your load where it should be picked up. If your gripper, how long is your gripper? How is that going to affect your system? So, the whole of the dynamics. Now, you know dynamics. You know how it changes your system if the moment of inertia and mass centre of location changes. So, yes, it is going to change your torques drastically. That is going to come onto your links, onto your joints, so payload and supplementary load correction is to be done. So, identification experiments are required. So, all those we will be doing. You see your link as soon as you load with some supplementary load or with cables and some few things which are not actually meant to be there, but now, after the design, they are loaded, so this information is missing to the controller. So, we need to do some sort of experiments. So, we do those experiments to identify the mass centre location and actual value of mass even and the moment of inertia of your robot, so that you exactly can calculate the complete dynamics of your system. So, these things are very, very predominant in the case of a huge robot. So, with that, I'll end here and this chapter in this module, which is dynamics. So, see you in the next module, which is load data calibration. That's all. Thanks a lot.