NPTEL Online Certification Courses
Industrial Robotics: Theories for Implementation
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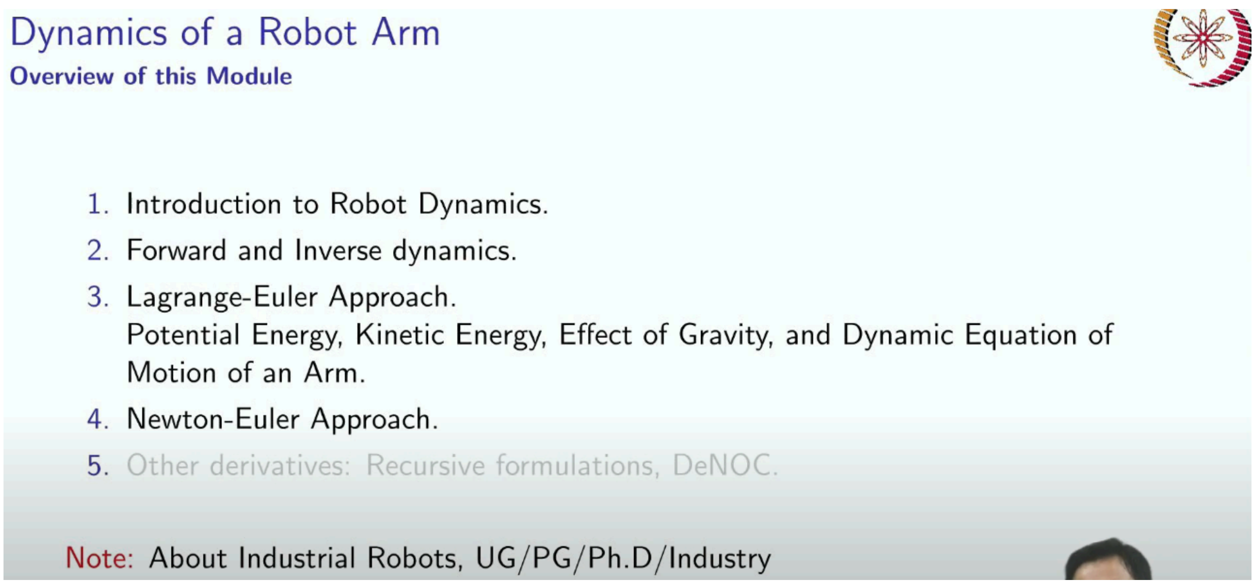
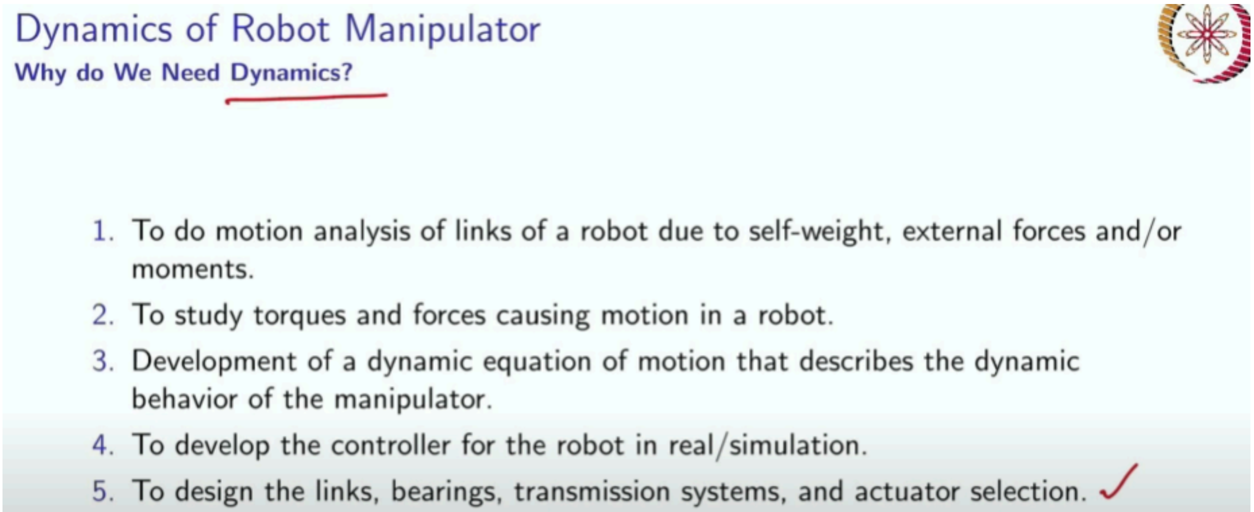
Week: 09
Lecture 36

Introduction to Dynamics: LE Approach: Dynamics of 1DoF System

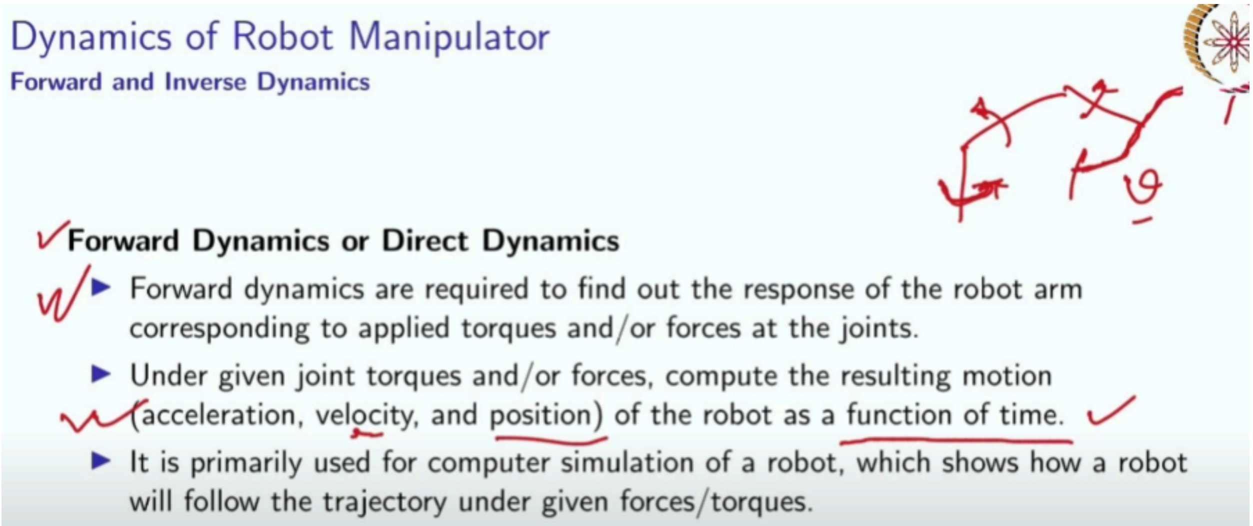
Hello, and welcome to the module, Robot Dynamics. So, before we start, let us quickly look at the overview of what we are going to cover today and in this module.



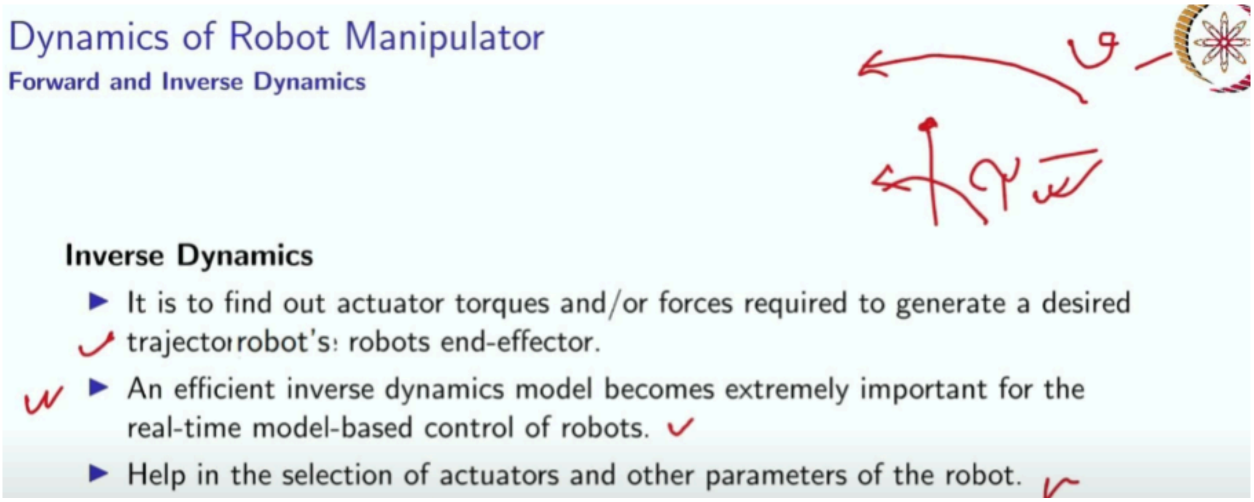
So today, we will be discussing robot dynamics. Why it is important, how it is done, various aspects of robot dynamics, how it is beneficial in robot design, controller design and all. So, I will also introduce you to forward and inverse dynamics: why they are used and, why they are important, how they are done. So, a different approach to robot dynamics is the Lagrange-Euler approach. So, we will start with this today, and potential energy, kinetic energy, and effects of gravity finally will reach the dynamic equation of motion of an arm. We will also discuss the Newton-Euler Approach in the next few classes, and there are many other approaches to dynamic equations of motion. So, that is the recursive formulation and DeNOC-based approaches that we will not be covering in this module. However, please note, this module is for academic students, who are basically in UG, PG and PhD, and research students. However, industry students or students who are from industry can quickly go through this module without going too much deeper into it if they are not interested in the research aspects of it. But, yes, that will help you to understand, still, the functioning of a robot controller and why your robot is performing this way. If at all it is performing badly, why is it so bad? If it is so good, why is it so good? So, you will be in a position to understand the behaviour of your robots. So, still, that will be useful. Sometimes, in the advanced applications and complex methods that we follow to implement our robot algorithms. This may still be very, very useful to the industry students also.



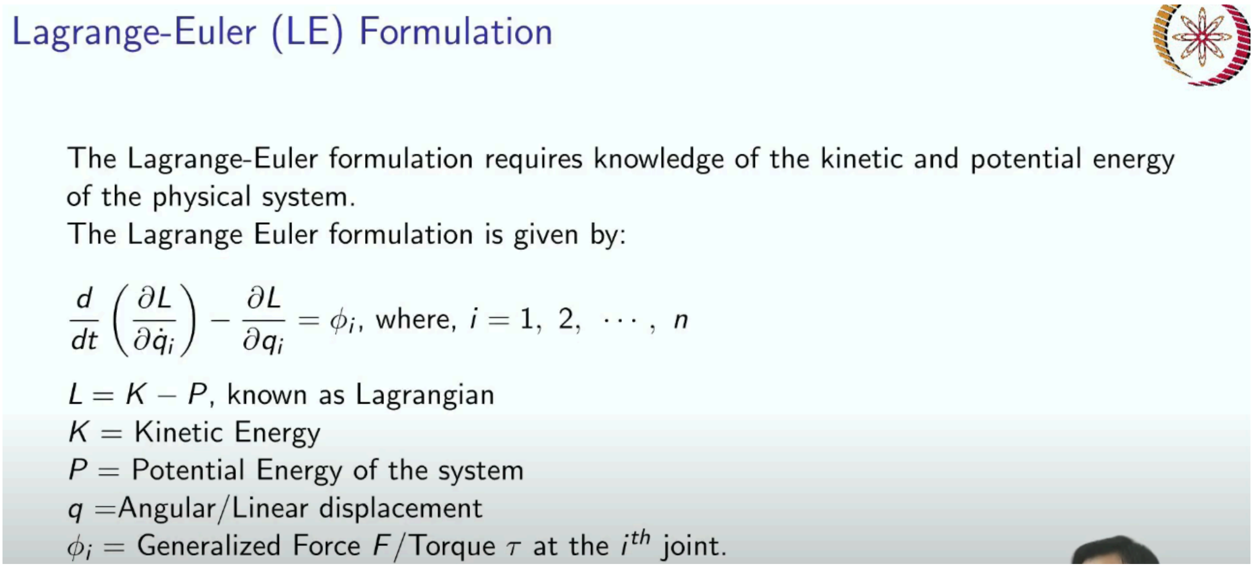
So, yes, let us just quickly start. So, yes, why do we need robot dynamics? First, So it allows us to analyse motion. Motion, in particular, links the robot due to its weight. How does it behave? External forces, how does that affect the robot? Motion, External forces or moments, if it is there from outside And we can closely understand the behaviour of a robot if it is commanded to give a particular kind of trajectory to follow. How much would that be possible? First. If your robot has got the sufficient moment of inertia and masses at particular locations, so whole of the link is fully defined. Now, with that set of moment of inertia and masses and the architecture of your robot, the whole of the equation of motion will try to derive. So, that is what is done in robot dynamics also. That we will do, we will begin today also. So, to study the torque and forces that cause motion in a robot, or if a robot is in motion, what torque and forces are going to come at the joint level? So that is what is easily understood by dynamics. So, to develop a dynamic equation of motion that describes the dynamic behaviour of a manipulator. Why is it extracted? That is, to develop a controller for a robot that works in real hardware or sometimes for simulation purposes, also even to sometimes design animation or maybe just a cartoon. Also, nowadays, with advanced features of your cartoon character design software, they use this dynamic equation of motion for various lumped bodies or sometimes solid bodies if they are using it. So, that is useful for various other reasons also, not just in robotics over here. So, to design the links to design bearings, transmission systems and actuator selection. That is very, very important because it is the actuator that is going to handle everything, ranging from your torque, for the forces, for external forces, for the payload, for supplementary loads, so everything is going to come on the actuator. So, this will help you to understand this equation of motion, what we are going to derive through dynamic algorithms. So, that will be useful to understand that kind of behaviour of your robot and to select a particular kind of actuator for a given robot design.



So, what is forward dynamics, and what are inverse dynamics? So first, start with forward dynamics. So, forward dynamics are required to find out the response of a robot arm corresponding to applied torque or forces at its joints. If at all you have a robot, you have certain torque over all the joints. So, how is the end effector going to behave? So, is it going to move with a certain velocity which is uniform all over, or is it going to have increasing velocity at the beginning, going at a constant speed, and then decelerating and finally coming to a stop. So, everything will be defined by the kind of joint torque and forces which are there at the robot joint. So, that is what is forward dynamics. So, you apply torque and force at the joint, and you study the effect of end effector motion. So, that is what is forward dynamics. Under any given joint torque and forces, we compute the resulting motion. So, what are the motions that we do? We study acceleration, and we study velocity and sometimes, if it is point-to-point motion. We study also where it is going to stop. So, that is the position characteristics. Also, we can study the robot as a function of time. This is very, very important. That is as a function of time we are going to study. It is not that it is going to start today and stop tomorrow. Everything is with respect to the time we are talking about. So, we are very particular about the kind of acceleration behaviour or velocity behaviour when, even if it goes from one point to another point. So, it is primarily used for computer simulation of the robot, which shows how a robot should follow a given trajectory under given forces and torque. That is at the joint. So, not just for simulation, it will also help you. It will also help you to understand the behaviour of your real robot if it is given by same amount of force and torque. So, simulation is nothing but offline visualisation of your real robot most of the time.



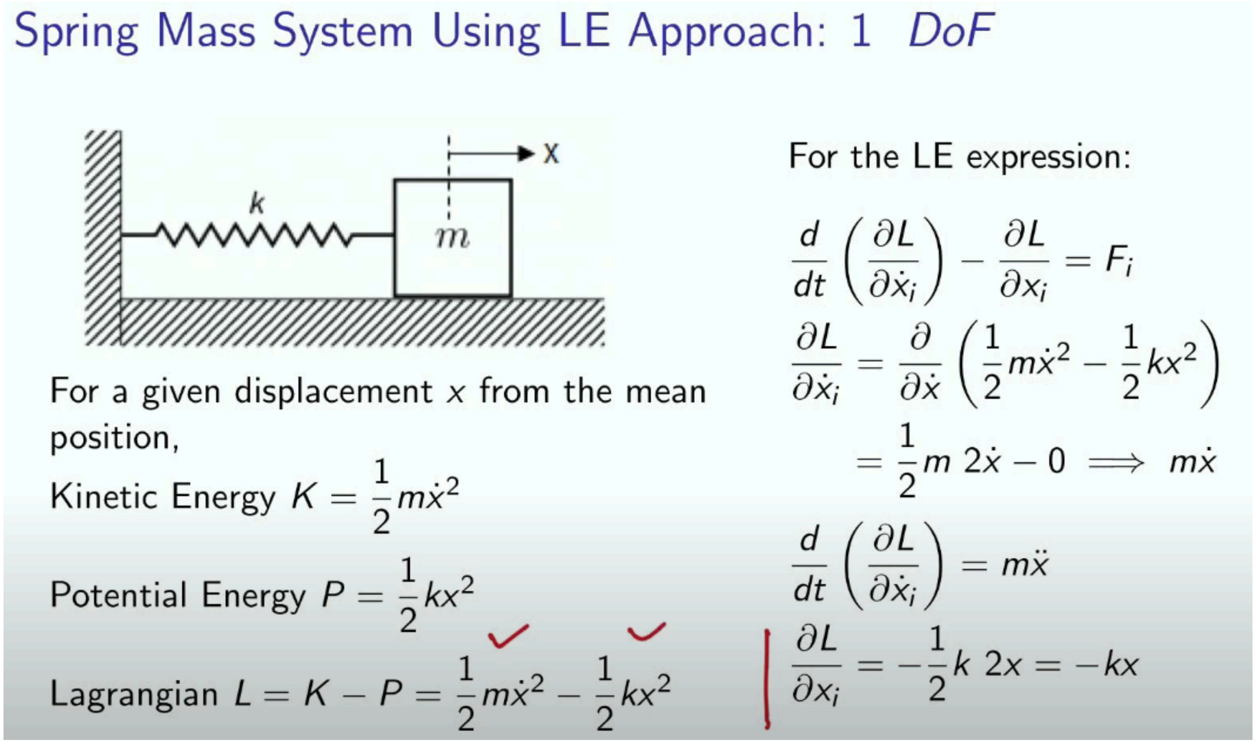
So, what is inverse dynamics now? So, it is to find out the actuator, torques and forces that are required to generate a desired trajectory. So, it is the other way around. So, in the forward kinematics, you applied the torque at the joint, and you saw the end-effector motion. So, over here, it is the reverse. So, you are given a particular end-effector velocity, and now you are supposed to find out the joint torques and forces which are responsible for giving such kind of motion. So, an efficient inverse dynamics model becomes extremely important for the real time model based control of the robot. If you are, you are using this kind of algorithm for model-based control. We will look at control algorithms later in other modules of this course, So in which we will be using forward and inverse dynamics both. So, where it will be much clearer to you, it will also help you in the selection of actuators, you see, and other parameters of the robot. Let us say you want to make your robot with a certain moment of inertia of the link and masses of your link and external forces. So, now, in order to create a certain kind of trajectory, which is the applied trajectory you want. So, what kind of actuator should you have that can give you that? So, the torque behaviour corresponds to the end-effector motion or even the joint motion. What is the requirement? So, you will understand the requirements over there. Accordingly, you will select a particular actuator for that particular robot. So, that is what inverse dynamics is. There are various kinds of algorithms that help you to get through the equation of motion for a particular robot. A few of them, as I have named earlier, are the Lagrange-Euler approach and the Newton-Euler approach. So, today, we will start with the Lagrange-Euler approach first. So, how does that help you to arrive at an equation of motion for a given robot system?



So, yes, the Lagrange-Euler formulation requires the knowledge of the kinetic and potential energy of a physical system. So, this is an energy-based approach. So, the Lagrange-Euler formulation is defined by this equation, the one which is here.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \phi_i, \text{ where, } i = 1,\ 2,\ \cdots,\ n$$

What is it? It is the time derivative of the partial derivative of Lagrangian with respect to the joint variables. So, what is different? Parameters which are here: L is known as Lagrangian. What is that? It is kinetic energy minus the potential energy of the system, So that is known as the Lagrangian which is here. K is kinetic energy, and P is potential energy. So, L is Lagrangian. Q is the angular or linear displacement of the joint. So, Q is the joint variable and psi i here is a generalised force or torque. If it is a rotary actuator, you see over here, it is torque. If it is a prismatic joint, that will become your force for any particular joint. So, this is an equation for the ith joint. So, you have to repeat the whole of this equation, this Lagrange-Euler formulation, for each and every joint. Also, we will see by example. So, it is not just this Lagrange-Euler formulation is not just for robots. It is from any other system; any other mechanical system can use this.



So, let us begin with the very beginning of our study when we did the spring mass damper system in physics. So, let us start with a small system with just one mass that can slide over the platform here, which is perfectly flat and without friction. The mean position is marked by this dotted line, and this mass is moved by a small displacement, x, along the x-axis of this coordinate system. Let us say it is having no motion along y, it is making perfect contact with the surface Sliding on that. K is the stiffness of the spring which is attaching this block over here, and it is attached to the wall over here. So, this is how it is defined: the whole system is defined. So, how many degrees of freedom is there for this system? It is just one degree of freedom, So let us start using the Lagrange-Euler formulation for this. So, for the given displacement x from the mean position, how much will be the kinetic energy? So, kinetic energy here will be for, let us say if it is at a displacement, x. So, velocity over there. Let us say it is an x dot. X dot is nothing but a derivative of x with respect to time. So, kinetic energy will nothing be anything, but it will be one by 2 m v square. So, it finally becomes one by two v. Is this if I tell so it is one by 2 m x dot square? So, this is what is kinetic energy at any instant of time.

$$\text{kinetic energy } K = \frac{1}{2} m\dot{x}^2$$

So, in an instance, you have velocity, which is given by an x dot. So, how much is the potential energy? Potential energy here is only because of the spring extension or compression. If it is on this side, it becomes compression. So, you know, by basic physics. So, 1 by 2 K is the stiffness of this spring, and x is the displacement. So, it is given by 1 by 2 K x square. Stiffness is defined by the force developed by the spring per unit displacement of the spring.

$$\text{Potential energy } P = \frac{1}{2} kx^2$$

So, that is how it is defined. So, potential energy is this. So, Lagrangian becomes: L is equal to kinetic energy minus potential energy.

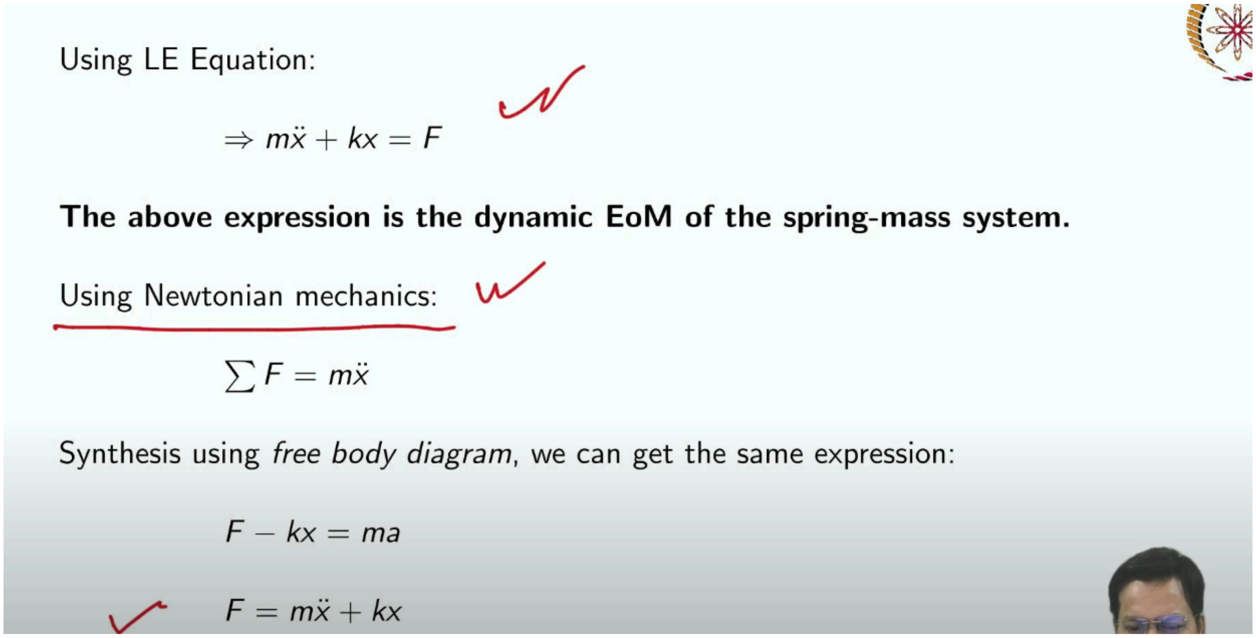
$$L = K - P$$

So, this minus this, this is your Lagrangian. Now, I will apply Lagrangian wave formulation, which is given by this as, because it is a linear displacement. So, you should see force over here. So, what earlier was psi, i, which was the generic force and torque? Now, it is confined to force only, and it is just one degree of freedom. So, i is just 1, and there is no other joint over here. So, I'll use Lagrangian now. So, talking about the first part of this equation, that is this one.

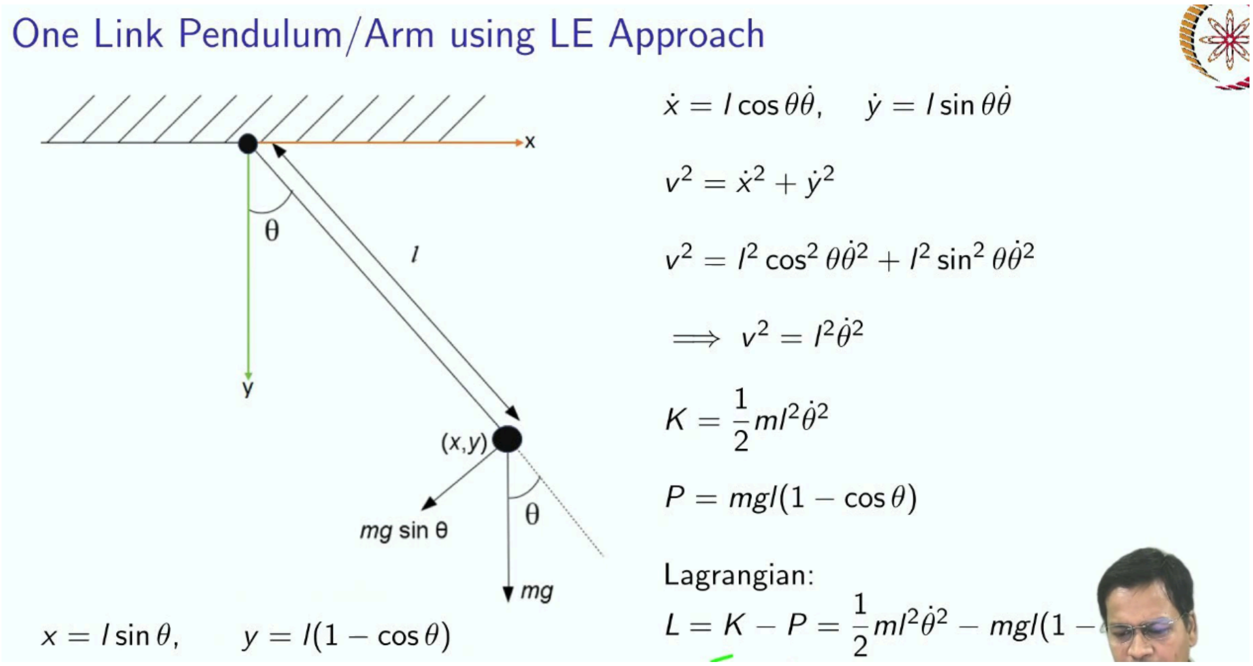
$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}i}\right) - \frac{\delta L}{\delta xi} = F_i$$

So, that is doL by dox dot. So, it is the partial derivative of L with respect to the x dot. That is the velocity. So, taking derivative of the whole of this now. So, this goes here. So, do by dox dot. So, applying it to the first term of it. So, that only remains. So, this is equal to 1 by 2, 1 x dot square. So, the derivative of the x dot square will be 2 times of x dot. So, the second part turns out to be 0. The derivative, partial derivative, of the second part, turns out to be 0 because there is no velocity over there. It is a partial derivative minus. So, that gives you 0. So, effectively, what you get here is doL by dox dot is equal to mx dot, so this part is mx dot.

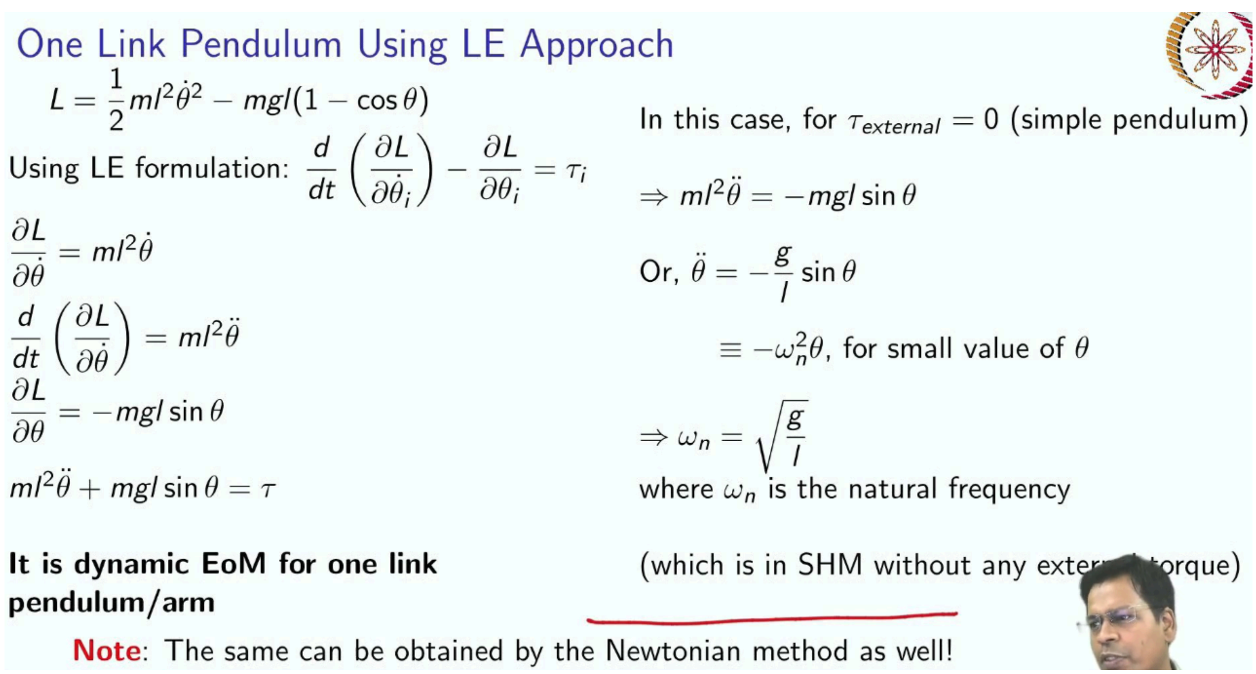
Moving ahead, taking the time derivative of that. So, what? It should give you a d by dt. That is the time derivative of this. So, it gives you a derivative of mx dot, a time derivative of this. This gives you this mx double dot. It is mass into acceleration. So, that is the first part of this Lagrange L that you have obtained. Now, the second part is doL by dox, so doL by dox. So, again, starting with this, this doesn't have any displacement over here. This has it. So, the second part will go as 1 by 2. The negative sign will remain 1 by 2 k. that is constant. So, x square, the derivative of x square, should be 2 times x. So, again, these 2 go. Finally, you are left with a minus of km. So, finally, you have both the terms in your hand, this term and this term.



So, putting them together, it gives you mx double dot plus km should be equal to the external force which is applied. So, if at all there is no external force, it becomes mx double dot is equal to minus of an x. So, that becomes your dynamic equation of motion. So, this is what is known as the equation of motion, for in this case, it is a spring mass system which has no friction and there is no damping. So, using Newtonian mechanics also, you can derive it. So, the sum of all external forces, all the forces should be equal to mx double dot because there is no other body which is there. This is the only force that you will see, and that is due to the motion of this body. Only synthesising that for a free body diagram, you can do, and you can get to the same expression, that is force minus spring force. External force minus spring force should give you mass into acceleration. If at all there is no external force, you will reach the same point. So, this is exactly similar to this. So, you can do it with simple physics. So, now the question arises: why should we use lagrangellar in this case? At least, in this case, you have just one degree of freedom. You can isolate the body. You can make a free-body diagram. It is quite trivial to do that, so you can very well follow Newtonian mechanics, the age-old system that you have been using through your school days. You have been doing it. Simple: force balance can do it. But, yes, in the case of robotics, when the bodies are lying freely in space, multiple bodies are there which are connected. So, it becomes very, very difficult to follow this kind of equation to create using Newtonian mechanics. So, in that case, energy-based techniques like lagrangellar formulation will help you. So, that is the reason we are doing it all here.



So, now let us start with the general system. We'll extend upon that, and finally, we'll reach a robot. So, this is how a one-link pendulum or an arm is given by this, so it is attached over here. This is your mass, so make things simple. I have taken a concentrated mass at the end which looks like a pendulum here, so it is nothing, but this is your reference frame about which this link: substance and angle theta. It has a mass so that it will have a gravitational pull along the direction of g. If it is like this, so it is mg which is like this. This angle will also be theta. So, you have analysed a similar situation quite a number of times earlier. So, this system is very, very simple for you. So, this will be your perpendicular component. So, that will be given by mg sine theta. So, this angle is equal to this angle. So, you can project it like this, and the one which is over here, this force is mg cosine theta. So, that is giving you just the tension in this length. So, this is a solid link. So, you will see a kind of tension because of this force only. But yes, if it is moving, it will definitely have other forces, also. So, it should be m, r, omega square. M is the mass, r instead of r you should see l over here. Omega is the angular velocity, so that is the centrifugal force. That also will be added to this. So, finally, that will be the tension in this link. So, we are not concerned about the tension in this link unless we are here to design this link otherwise in order to study the motion only. So, we are concerned, only with the force which is creating the motion. So, this is the force which is creating the motion. It is given by mg sine theta into l. that is effectively the torque that comes over here. So, that is what is creating the motion. So, let us do it: do the same whole analysis using the Lagrange-Euler technique. So, with this as a reference frame, which is here, this is your x direction, this is your y direction. So, x should be equal to if this is l, it is l sine theta. So, displacement of this bob: if it is projected on the x-axis, it is l sine theta, correct, so that is your x. So, how much is your y if it is totally at the bottom of this trajectory? So when is it here? So this was l, is it not? So this total length was l. So, when it goes to this point, it is still here. Now, the projection comes here. So, how much is it from here to here? So till here, it is l cos theta. So, the total was l. So, you see, it is l minus l cos theta is the displacement that it has done. So, when it goes from this point to this point, the displacement, which is there is l minus l cos theta. So, y displacement is given by l 1 minus cos theta. So, this is how it is derived. So, that is your y, this is your x. So, yes, if you take the derivative of that, you will get velocity along x velocity along y. Taking a derivative of these two should give you this. Squaring and adding them should give you the velocity at this point. So, this is your velocity, which is given by velocity along x square, velocity along y square, so that will be your v square. So, v square is nothing but squaring and adding these two should give you this: effectively cos square theta plus sine square. Theta is equal to one. So, effectively you get v square is equal to l square, theta dot square. So, you can reach us here directly. Also, v is equal to r omega, so our radius over here is l, and omega is theta dot. So, this is what your v. You can reach directly here. But I have to follow this way because, if at all, you have multiple links connected in series. So, this should be the approach you should go for, because. You have other links, also. One of them supports this direct approach, whereas other links you have to go through this channel. So, that is the reason to begin with you should begin like this: finally, v square is equal to l square, theta dot square. So, you can directly write your kinetic energy like this: so it is one by two: m v square. So, it is m v square. So, v is this. You got the kinetic energy. Now, how much is the potential energy? It is mg. This was your lift from the bottom-most position to the next position. It is l one minus cos theta. It is mg is the force. This is your displacement. So, it is your potential energy. So, you know now kinetic energy, potential energy is here. So, Lagrangian is kinetic energy minus potential energy. You got the Lagrangian. Once you get the Lagrangian, you can start doing your stuff.



So this was the expression. I have just transferred it here also. and this is your Lagrangian formulation. So, going through the derivation of Lagrangian is beyond the scope of this course. That should be of interest to students who are interested in doing higher studies and doing advanced studies of robot and multi-body system dynamics. So for them, this should be very, very useful. But for industrial robotics students that should not be very much of a concern. But yes, we'll be using this directly now. So, this again will start here so as because this motion was about a rotary joint, so this time, I'll use torque over here instead of force. Rest, the whole of the equation remains as it is. So, instead of generic force and torque, this time, it is for this joint. This joint is one degree of freedom system and rotary joint, so it is psi i become equal to torque. The same equation can give you force as well as torque, depending on the type of joint. It is so. Instead of Qi, which was the joint variable over here, so it is theta i. So, in the earlier case, it was xi. Why? Because it was linear displacement. This time, it is theta i, which is rotary displacement. So, if it is a rotary displacement or angular displacement, this is your torque. What it. So, the formulation remains the same. So, let us begin with this Lagrangian, what we have just derived. So, again the part which is here I am using this as a Lagrangian- taking partial derivative with respect to theta dot. Only this term has it, and this one doesn't have. Everything will be treated as constant. So, it is do1 by 2 ml square theta dot square.

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \theta^{\cdot}}\right) = ml^2\theta$$

If you take the derivative, it becomes two times theta dot. So, you can strike off two from the numerator and denominator. Finally, you reach here, is it not? So again, taking time, a derivative of that, will give you time. Derivative of this should give you ml square theta double dot. So, this is quite trivial, you know it. So, again the second part, this part. If you take yourself by your theta, this will be treated as constant. There is no theta over here. Only this part is there. And even for that, this is a constant. Only this part remains. So, the cosine derivative is sine a, with a negative sign. So, negative, negative. Finally, it is positive and again, due to cosine, it is negative. That remains. Here, mg will remain, mg, l will remain. So, this is your second part of the Lagrangian. So, now, putting them together, you get to this: the first part and the second part. Negative again over negative here. It becomes positive. So, this is your equation of motion. This is your equation of motion, got it? So, it is the dynamic equation of motion of a one-link pendulum or an arm. If it is an arm which has just one degree of freedom, arm one link system, it is driven by torque, and it can give you a motion. So, in this case, if it is torque, any external torque is zero. It becomes a simple pendulum. Otherwise, it is a one-link robot. So, that is a simple pendulum now. So, in k, in that case tau becomes equal to zero. And this is your equation. You have done it plenty number of times in your high school days also. You will remember it just now. So, theta double dot is equal to m, goes off, l square and l goes off. So, finally you are remaining with theta. The double dot is equal to minus g by l, sine theta, for a very small value, of, theta sine. Theta tends to the theta, and you can write it like this. So, now you can recall. It is what it is. Simply, omega n is equal to the square root of g by l. What was omega n? It is the natural frequency of the system. So, this becomes a simple pendulum which can make a simple harmonic motion without any external torque, so it can keep on oscillating. If there is no friction and other losses, it will keep on oscillating like this. So, this is how you can arrive at the same old physics equations using Lagrange-Euler's treatment of a simple pendulum. So, the same can be obtained using the Newtonian method as well. If you can create a torque balance equation, draw an isolated free-body diagram of the pendulum, and you can get to the same equation, So again, because it is single degree of freedom system and one link, you can think of that. Otherwise, it is very, very difficult. Suppose it is more than one link, Sometimes more than two links. You can treat as many as two links, maybe.

So, in the next class, we'll treat two link systems using a similar approach, that is, the Lagrange-Euler approach. So, that's all for today. In the next lecture, we'll be doing a dynamic equation of motion of a two-link manipulator using the Lagrange-Euler approach. We'll also study the parts of the general dynamic equation of motion. So, what does each of them signify? So that is what we'll be doing in the next class. For today, that's all. Thanks a lot.