NPTEL Online Certification Courses
Industrial Robotics: Theories for Implementation
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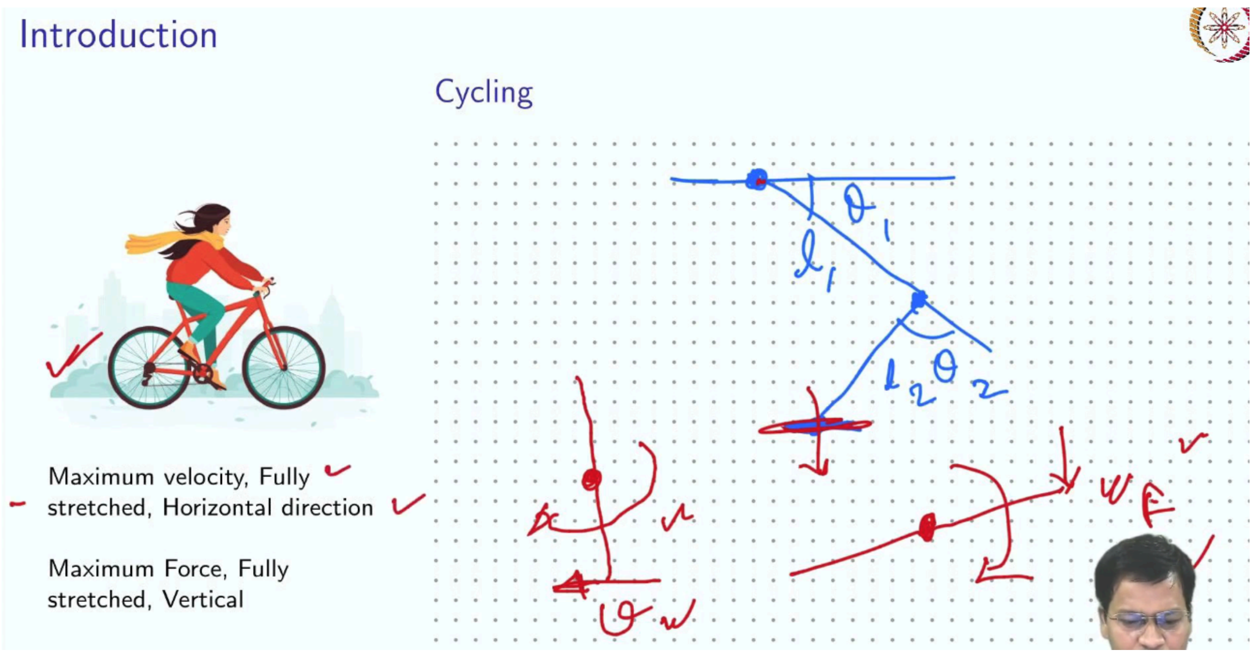
Lecture 35

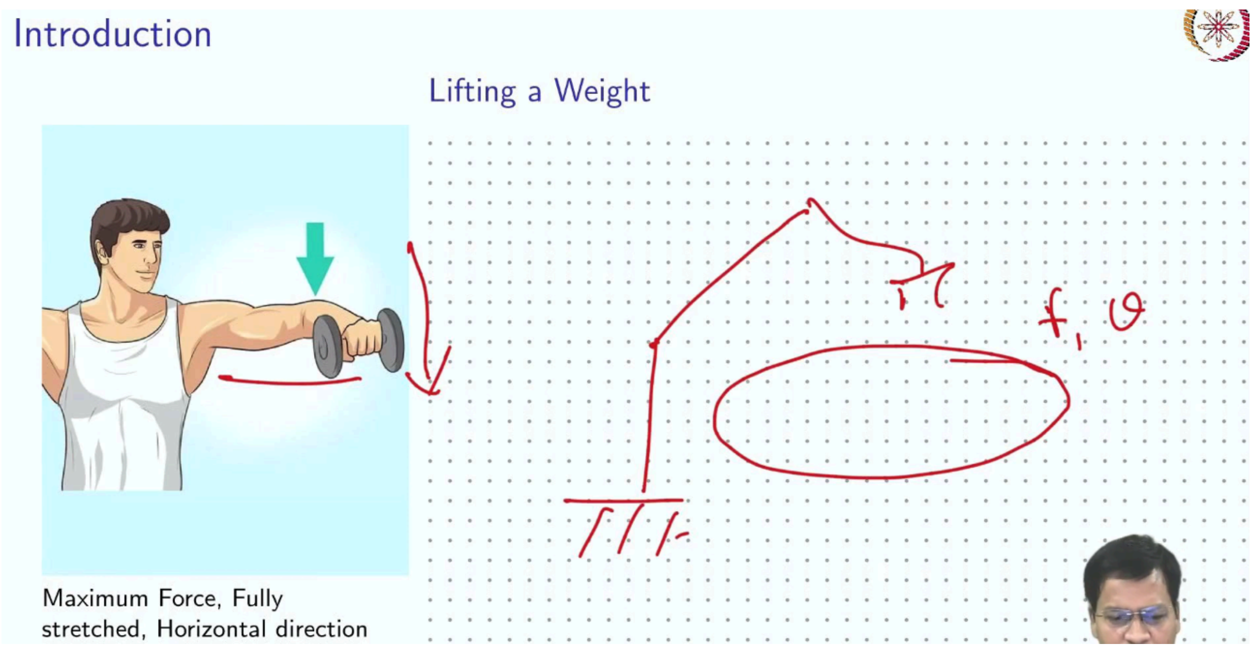
Kinetostatic Measures for Robot Design

Welcome back. So, moving ahead to the last part of the robot statics, today we will discuss kinetostatic measures for robot design and its selection. Apart from the performance characteristics that we discussed earlier in module 1, these measures will allow us to compare different robots based on the optimum workspace that a robot can offer.

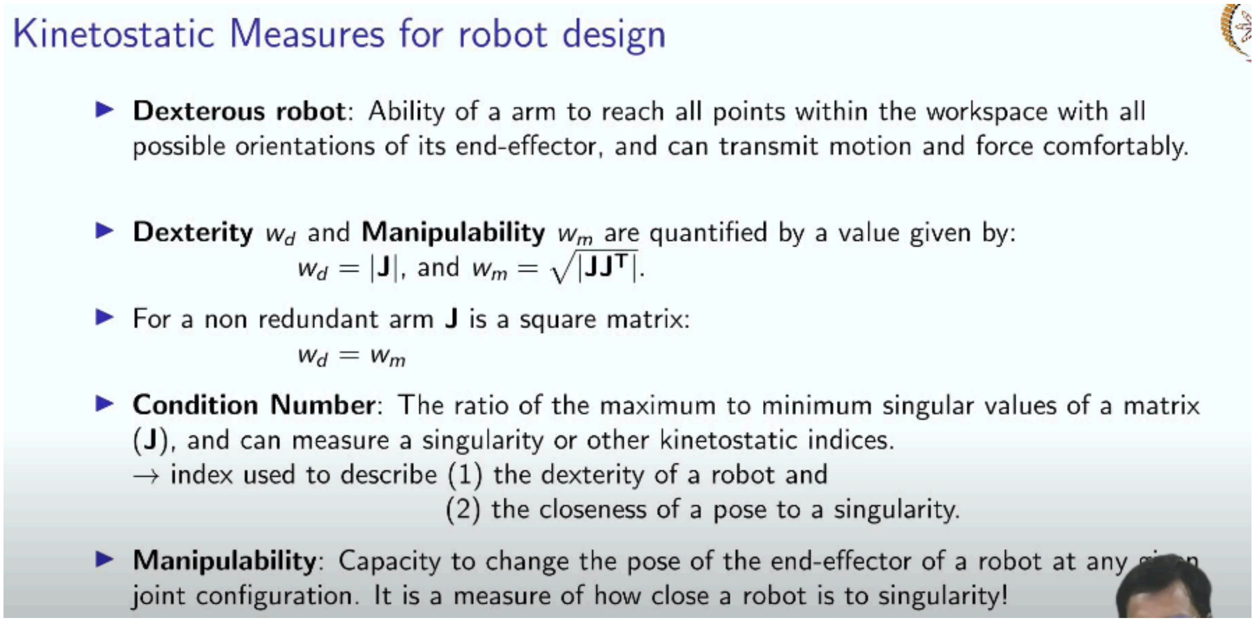


So, let us begin with today's module. So, yes, let me just introduce you to a small day-to-day problem that we normally handle. So, one such activity that we do normally is cycling. Hope you remember how you do cycling. So, you see your leg when it is acting on your pedal. It is something very much similar to a 2-degree of freedom arm that you have worked out earlier. So, this is your foot. This acts on the lever that is your pedal. This is I consider this as link 1 and this as link 2, this is your joint 2, this is your joint 1, this is theta 1, this should be your theta 2. So, it is very much similar to your 2-degree-of-freedom robot arm, which is mounted like this.

You see, you are applying pressure over here, which is your pedal. That comes here, and you have to push it downwards. When your pedal is somewhere like this, and you are pushing it at this location so that you can make it rotate like this. So, you require a maximum amount of force over here, whereas when it is almost vertical, you don't want to push it further down; rather, you want to make it go like this, is it not? You want to make it go like this so that it keeps rotating like this. This is your pedal structure. I tell you, so red one is your pedal structure. So, this is the place where you want maximum force, whereas this is the place where you want maximum velocity. These are two things. That is normally handled by the robot. Also, It has to pick up a weight or it has to move, So these are the two things that it encounters most of the time. So, you see, even with your leg. So, if you are oriented like this, and you want to move this pedal, two things are required. So, you want your robot to work best in these two parameters, that is, force and velocity. So, you need to find an optimum workspace where you should see this velocity, which is maximum, as well as force, which you can put best. Obviously, velocity will be maximum, best at a certain location, whereas force will be best at a certain location. So, there is some optimum workspace within the robot workspace where you have the best that is required for that particular application. So, this is just the peddling operation that we do, and you see, maximum velocity is at a fully stressed condition along the horizontal direction, and maximum force is at a fully stressed condition, that is, along the vertical direction. So, that is one application.



Similar is another application. So, you want to build up your body. So, when you go to the gym, and you want to exert maximum, you put your arm fully stressed where you have to apply the maximum amount of effort to maintain that position. That is the maximum effort that you are applying So your body can apply the least amount of force along this direction when it is fully stressed. So, this is a typical gym sort of application. So, your robot, when you have a robot in a workspace, your robot end effector, should work in a region where you have optimum performance in terms of force as well as velocity. So, there are measures which can measure this mathematically. So, where is your best location in order to work out numerically and get a value so that you can compare two different robots? So, that is what we will be doing in this module. So, hope you understand using these two examples. A much better explanation is there with mathematics.

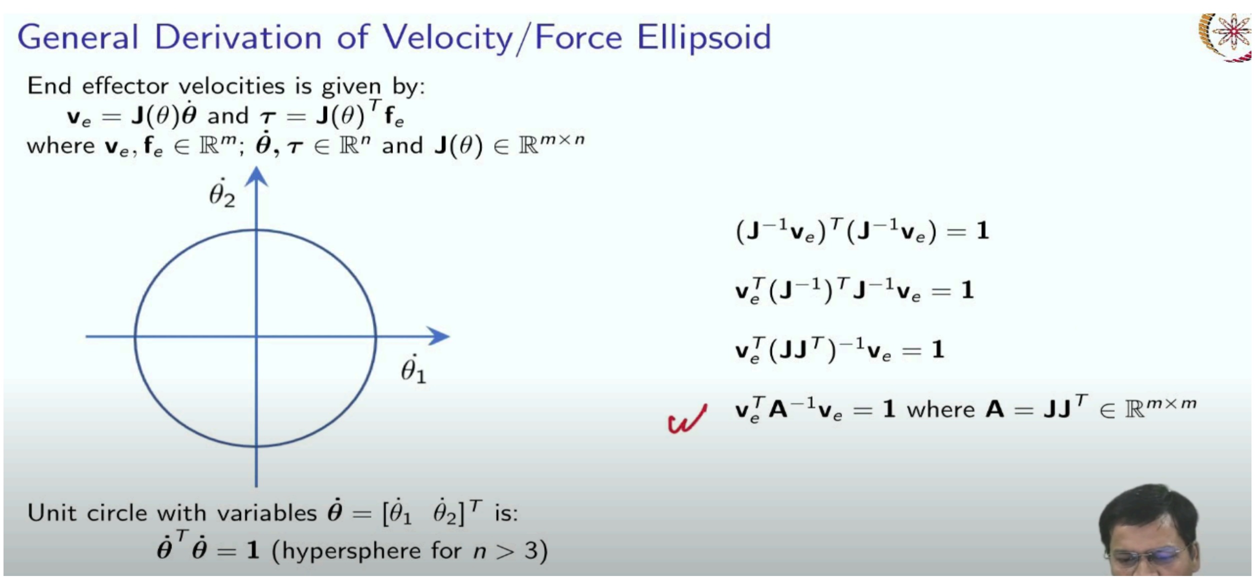


So, yes, kinetostatic measures for robot design. Also. The same is used for the design as well as when we select any robot. So, Dexterous robot that is quite often you must have heard a robot is dexterous. What it is actually is the ability of a robot arm to reach all the points within the workspace with all possible orientations of its end effectors and can transmit motion and force-that is, the velocity and force comfortably. So, the robot should be comfortable when it is working for motion or even applying a certain force. So, that is what is a measure. So, how will we measure it mathematically? There are terms, and there are many terms. Over the years many similar terms have come forward through different researchers. So, one by one, we will discuss some of them. So, dexterity and manipulability, w_d and wm are quantified by a value given, as wd is equal to the determinant of Jacobian, J is Jacobian over here- whereas manipulability is given as JJ Transpose determinant of that square root of that will give you wd and wm. Both are effectively the same. If you work out for a regular robot with 6 degrees of freedom, when the determinant is six cross 6, so this becomes equal to this. So, for a non-redundant robot, J is a square matrix.

$$\text{Wd} = |J|, \text{ and wm} = \sqrt{|JJT|}$$

In that case, wd is equal to wm, whereas for 7 degrees of freedom or more, so you may have to use this one where JJ transposes, becomes a square matrix, and you can still find out your manipulability. So, both are equivalent terms that are used in the context of robots to find out the manipulability. So, which one is best. Higher is best or lower is best. So, that is what we will see now. So, yes, there is another very similar term. So, you see, when you are in a fully stressed condition, when you could apply maximum velocity and minimum force, that is one case. There is an inner position also when it is fully folded. In that case, you can apply velocity in another direction, whereas you cannot apply forces in one direction. So, yes, both are orthogonal. Normally, we will see it mathematically also. So, both the positions were singular, you see. So, that is, there is a number which is known as a condition number. It is the ratio of maximum to minimum singular values of a matrix, J. So, we are using J over here because J is something that relates to velocity also. V is equal to J theta dot. If you remember, this is the Cartesian velocity at the end effector is related using this.

Similarly, forces are also there. J is equal to, and tau is equal to J, transpose of force. At the end effector, tau are the joint torques. That is something which is related to the forces as well as torque. That goes at the end. So, we are using J over here. We will try to formulate it better now. So, for now, you just take it: J is the measure that can give you that. So, the determinant of J used to give me this one. Now, determine if it is close to singularity. You know you cannot relate these two Because in order to find theta dot, you have to find the inverse of J, and you can do this. So, the inverse is possible only when a J determinant is possible. If it is close to singularity, you cannot do this. Your robot gets stuck, If at all. You are using any control algorithm, so definitely that uses this, and your robot gets stuck. So, that is known as a mathematical singularity. So, condition number is a measure that measures how close your robot is to the singularity. It should not be singular. So, the closeness of a pose to singularity is what is measured by condition number and whether a robot is dexterous. The dexterity of a robot is also measured using this condition number. So, these parameters are very, very important. So, manipulability is the capacity to change the pose of the end effector of any robot at any given joint configuration. It is a measure of how close a robot is to the singularity again. So, that is what is dexterity, manipulability and what is condition number. So, you now know these things very closely.



Now, let us move ahead and try to formulate it more mathematically. So, end effector velocity, not just velocity, as well as the torque at the joint are given by these two relations, which are this:

$$\mathbf{v}_e = \mathbf{J}(\theta)\dot{\boldsymbol{\theta}} \text{ and } \boldsymbol{\tau} = \mathbf{J}(\theta)^T \mathbf{f}_e$$
$$\text{where } \mathbf{v}_e, \mathbf{f}_e \in \mathbb{R}^m;\ \dot{\boldsymbol{\theta}}, \boldsymbol{\tau} \in \mathbb{R}^n \text{ and } \mathbf{J}(\theta) \in \mathbb{R}^{m \times n}$$

What is J? J is a function of theta. It is known as Jacobian, and you know that already. Whereas ve and fe are end effector forces and velocity they have, they are m dimensional, real numbers. So, that is given by, it is set of this and similarly, theta, dot and tau is n dimension real number. So, it is an element of this. So, yes, again, it depends on n. The n is degrees of freedom, that is, the number of joints a robot has. So, the Jacobian of theta, that is a function of theta, is R m times n. It is m times n. that is your J, that is the dimension of J. So, if you want to study the effect of joint velocity, the maximum possible combinations of joint velocities can give you different kinds of end effector velocities. So, let us say you have two degrees of freedom to begin with. So, it gives you this particular combination, which is very, very useful in studying robot behaviour in terms of velocities. So, let us say you have theta one dot and theta two dots, which is plotted in a unit circle, which is given by this: So theta bold dot will be an array of theta one dot and theta two dots. So, this is what it is. So, if you say theta dot, theta dot, like this, so it is nothing but theta dot, theta one dot square plus theta two dot square is equal to 1. What is that? This represents? What does a unit circle like this? What does it actually have, If at all? You have a set of points which lies on this unit circle. So, you see, you have different combinations of theta one dot and theta two dots. So, all the directions of theta one dot and theta two dots are under study with a certain value that goes on this radius. So, you see, it has both positive here, positive and negative here, and negative, negative here and positive, negative, like that. So, you have all the combinations of theta one and theta 2. This can be a very good combination to study the effect. So, this is why it is chosen. So, you see, if you substitute this over here, you have just written: as theta is equal to J, the inverse of and effector velocity. All are vectors, got it?

$$\Theta = J^{-1} v_e$$

So I substitute that here and here as well in this.

$$(J^{-1}v_e)^T(J^{-1}v_e) = 1$$

So, I get this moving; further, this can be written as this:

$$v_e^T(J^{-1})^T J^{-1} v_e = 1$$

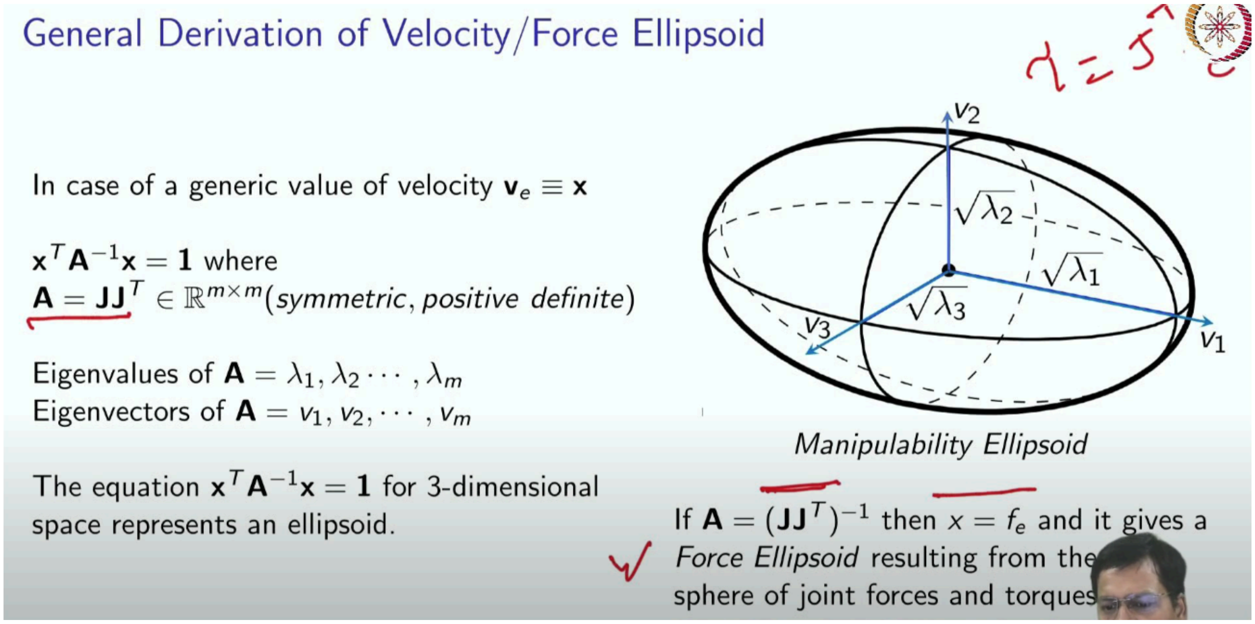
Moving further ahead, you can write it like this:

$$v_e^T(JJ^T)^{-1} v_e = 1$$

So, you have what you have: Ve transpose. Ve is equal to 1. In between, you have JJ transpose. So, what is that? This is equivalent to an ellipsoid equation. I will tell you how. So, this gives me a relation which can be explained geometrically also. But yes, over here, if it is a 2-DoF planar robot, it tells you a circle. Suppose it is a 3-degree-of-freedom robot. So, it gives you a sphere for a higher degree of freedom robot. So, it gives you a hypersphere. For n greater than 3, and similar is the case over here. This gives you an ellipse ellipsoid or a higher dimension ellipsoid. So, both are similar. So, yes, if I can write it like this: Ve transpose A inverse Ve is equal to 1, so A is nothing but JJ transpose.

$$v_e^T A^{-1} v_e = 1$$

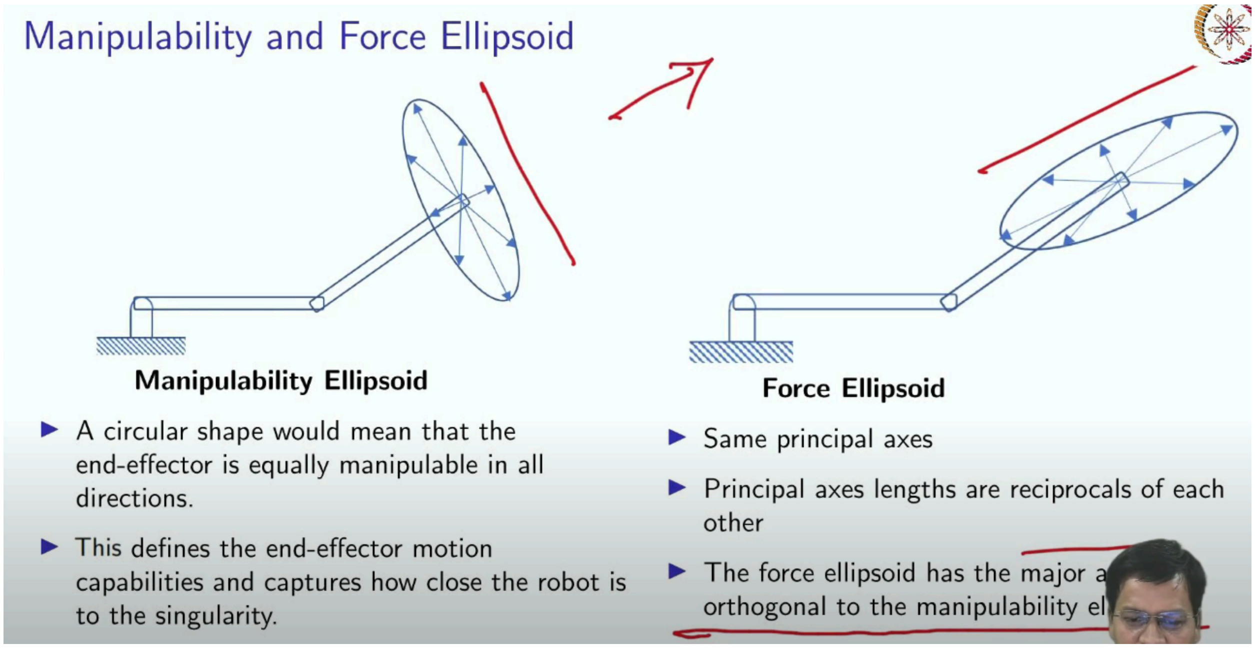
This is m cross m, which is a square matrix which is also inverted invertible.



So yes, in the generic case, mathematically, if I say it is X. So, I can write it as X transpose A inverse X is equal to 1. A is JJ transpose, which is a symmetric, positive, definite matrix. So, eigenvalues of A are lambda 1, lambda 2 till lambda n. Similarly, eigenvectors are V1, V2, and Vm. So, these are eigenvalues and eigenvectors. So, effectively, this equation represents a three-dimensional space that represents an ellipsoid. So, basically, this equation represents a three-dimensional ellipsoid in space. If it is a 3-dof robot, remember that. If it is a 2-DoF, it is an ellipse. It is n degree of freedom. So, it is something very much analogous to that is extended in an n dimensional space. So, manipulability is an ellipsoid. It looks like this. So, you see, lambda 1 square root, lambda 2 square root, lambda 3 square root. They are nothing but the axis of this ellipsoid. This is a mathematical relation. It has got not much to do with robotics. So, this expression can relate to a similar space in mathematics, which is over here. If I say it is Ve, it is a velocity ellipsoid. If it is a force, it becomes a force ellipsoid. So, I will tell you how and how they are related. So, this is what you have to understand here. These are your eigenvalues. That goes here, and if A is equal to JJ, transpose inverse over here, A was this.

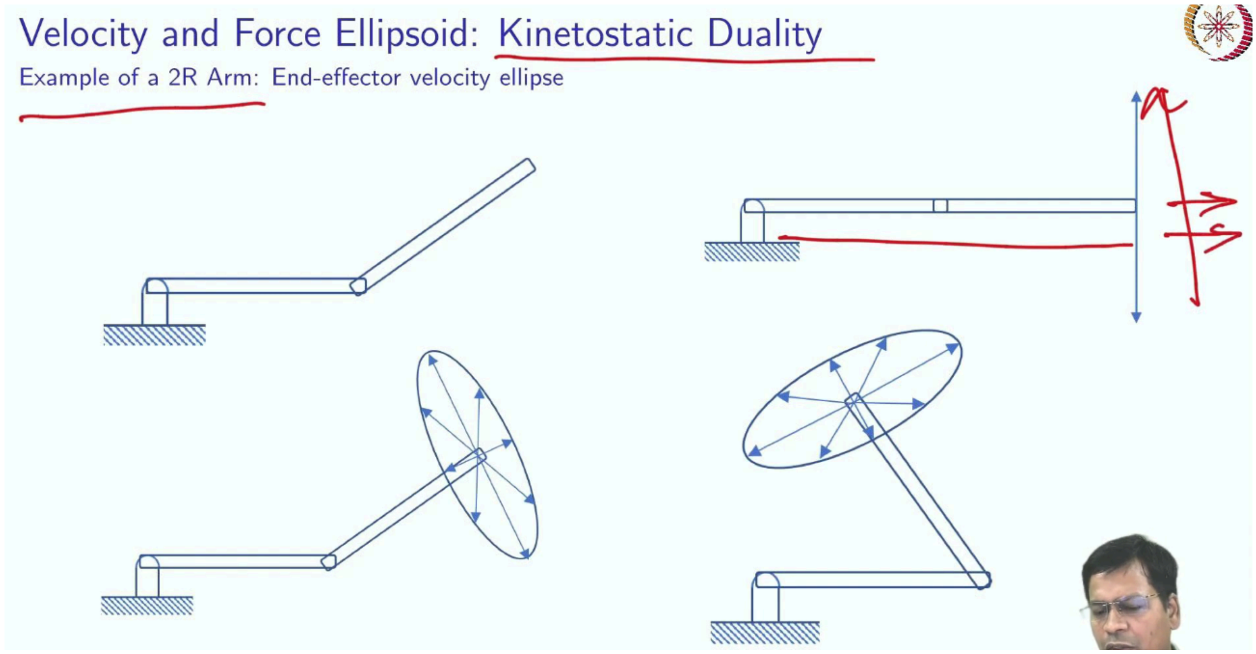
$$A = JJ^T$$

If it is JJ, transpose inverse, X is equal to fe. It gives a force ellipsoid that results from a unit sphere of joint forces and torques. So, just now, we have considered joint velocities. Now I have to consider joint forces and torques, and I can use the relation which was here, that is, tau is equal to J transpose of f. f is an end effector force. So, end effector force, joint torque and force. That comes here. If I use that over here in this equation, you get to a similar one, which will give you an ellipsoid of force, and you see both are orthogonal to each other. If the velocity ellipsoid is like this, the force ellipsoid will be like this.

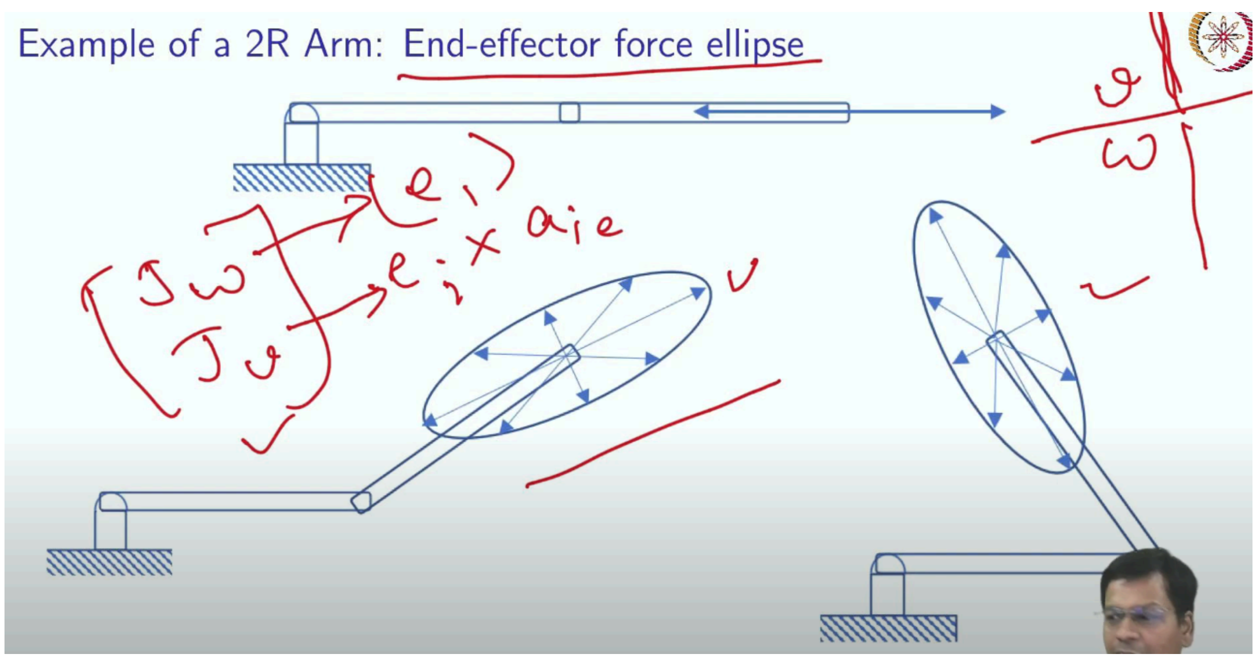


Now, I will compare both of them: manipulability and force ellipsoid. That is, velocity and force ellipsoid. So, a circular shape would mean that the end effector is equally manipulable in all directions. So, that means, if at all, this arm is fully extended like this. The shape of this is something like this. Somewhere in between, it will have a shape which is something like this: over here, it becomes very much like this: okay, the shape of this manipulability ellipsoid, it becomes like this: okay, so it can only have velocity in this direction. Over here, it can only have velocity in this direction. Over here, you can have velocity in this and this direction equally on both sides. So, that is what is changed in velocity, ellipsoid size when it goes from here to here, these two locations, the circular shape is something that is very much desirable. It is the workspace of this robot where it works best. You can have velocities, equal velocities, in almost all directions, so it is known as equally manipulable in all directions. This defines the end effector motion capabilities and captures how close the robot is to singularity, if at all.

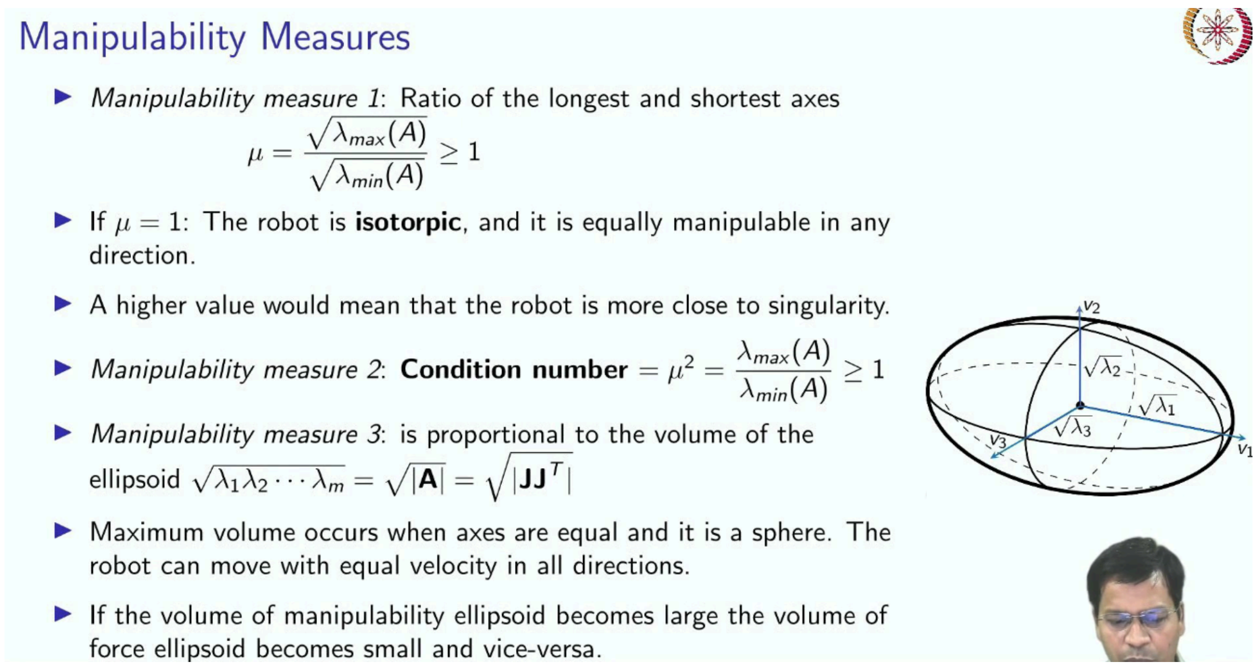
You see this or this that is at the end and the outside, complete outside and complete inside one boundary. So, it is almost singular, you know. So, this also captures that essence. So, that is what is here. And force ellipsoid, if at all. It is shaped like this. This will be orthogonal to this, with the same principal axes. The size of the principal axes will be the same. The Principal axes are reciprocal to each other also, and the force ellipsoid has a major axis orthogonal to the manipulability ellipsoid. So, you see, if this is like this, this has to be like this so that you can apply maximum force in this direction, which means you have a minimum for velocity in the same direction. So, that is what is manipulability and force ellipsoid.



You see, there are various examples that I have given here for a 2R manipulator or a robot arm, you call it. So, you, there is some kinetostatic duality; it is known. As you see, both are directed in this way. So, one of them, you see, if it is fully extended. In this case, it is like this: you cannot have motion along this direction further; only you can move in this direction, whereas you can only apply force, and along this direction, you cannot apply you or a minimum force that is applicable in this direction when it is oriented like this.



So, it is one situation which is shown here and a similar one. You see the end effector: force ellipse. It is directed orthogonal to each other. So, for the same two situations as it was here. So, this time, it is directed orthogonal. So, in case of force, it is directed lightly, like this. In the case of velocity, it is like, it is like this. So, this is just an example. So, you see, you should expect something like an ellipsoid if it is a three-degree of freedom system, even for six degrees of freedom system. It has two partitions which are there. It can be linear velocity, it can be angular velocity. So, three for. So, three for velocity, linear translation velocity and three for angular velocity spaces it will have. Jacobian will be six cross 6. You know, in that case, j omega and jv are there. You know that very well. ei, ei cross aie., so this part was taking care of the angular velocity component, and this part was taking care of the linear velocity component. So, for even degrees of freedom robot, you get to see angular velocity, ellipsoid and linear velocity. So, that is what is the case if it is a six-robot. So, at least for six o.f. It is quite trivial and almost easy to understand. But if it goes very high, it becomes a hyperspace. In that case, it is difficult to visualise it geometrically, but the idea remains the same.



So, yes, there are many other manipulability measures which can measure the performance of a robot in terms of kinetostatic measures. So, one of them is the manipulability measure one, which is the ratio of the longest to the shortest axis of that ellipse. So, it is given by μ, so it is lambda max by lambda minimum of a for that.

$$\mu = \frac{\sqrt{\lambda max\ (A)}}{\sqrt{\lambda min\ (A)}} \geq 1$$

You know this is nothing but eigenvalues. So, square, that is nothing but the axis of your ellipsoid that is given by the square root of lambda that you remember just now I have shown you. So, it is the ratio of those two if it is greater than one. So, that is what is a manipulability measure. Normally, it is greater than one. Yes, it should be. If it is equal to one, that means it is equally manipulable in both directions. So, you can have various notions of this for various values of this. So, if μ is one, it is known as isotropic, and it is equally manipulable in all directions. A higher value would mean that the robot is more close to the singularity. If the value is very high, you know lambda max, and if it is maximum, the other side is minimum. That means the robot is almost singular. So, just now, you sorted by examples of a 2R robot, so geometrically, it is easy to visualise, at least. There is another manipulative measure, that is a condition number. It is nothing but μ square, so you just have to take a square, and this also has a similar meaning. It is known as a condition number. Just now, we saw that definition of that. So, it also measures how close it is to the singularity. So, these definitions have evolved over the years and are given by different researchers parallelly at different instances of time. So, yes, there is another very good measure that is proportional to the volume of the ellipsoid. If you remember, the volume of the ellipsoid is given by lambda 1, lambda two and the product of all the lambdas that you can get. So, if it is taken a square root of that. What you get is something like this, and it is nothing but the volume of the ellipsoid, for in the case of the three degrees of freedom system, it is easy to visualise. So, let me just show you that figure. So, it is this one. So, you see this into this. It is nothing, but it is the volume of this ellipsoid. Got it, so mathematically, it is given. It has got not much to do with robotics here. So, that is what gives you the volume of the ellipsoid. So, if it is higher is better, so that you understand why it is better. It is closer to the sphere. So, if it is near an ellipse or a stressed ellipse. So, the volume becomes very, very less. So, you have less manipulability. That means it is very near to the singularity, also okay. So, if it is a sphere, volume is maximum. So, that is the best situation that you want. So, maximum volume occurs when axes are equal, and it is a sphere, the robot can move with equal velocity in all directions. The same will be the situation for force also. It will apply an equal amount of force in all directions. So, that is what is very good physical meaning out of something which is mathematics here. So, if the volume of a manipulability ellipsoid becomes large, the volume of the force ellipsoid becomes small. So, that is again it is true mathematically.

Notes on Types of Ellipsoids

The Jacobian $\mathbf{J} = \begin{bmatrix} \mathbf{J}_\omega \\ \mathbf{J}_v \end{bmatrix} \in \mathbb{R}^{6\times n}$

$\mathbf{J}_\omega \in \mathbb{R}^{3\times n} \rightarrow$ angular velocity/moment ellipsoids

$\mathbf{J}_v \in \mathbb{R}^{3\times n} \rightarrow$ linear velocity/force ellipsoids

So yes, in the case of a higher degree of freedom robot, like a six-DoF system, as just now I have told you, it is j omega and Jv. j omega is again three cross n, so it is angular velocity and moment ellipsoid that you will get out of it. J if you use Jv, you get linear velocity and force ellipsoid. So, these are some measures with which you can measure the performance of the robot. Kinetostatic performance characteristics can be seen.

So, with that I will end here and this module also. So, in the next module, we will start with dynamics, and we will do dynamics of various robotic systems, basically serial robots that we are dealing with. So, that's all for today. Thanks a lot.