NPTEL Online Certification Courses
Industrial Robotics: Theories for Implementation
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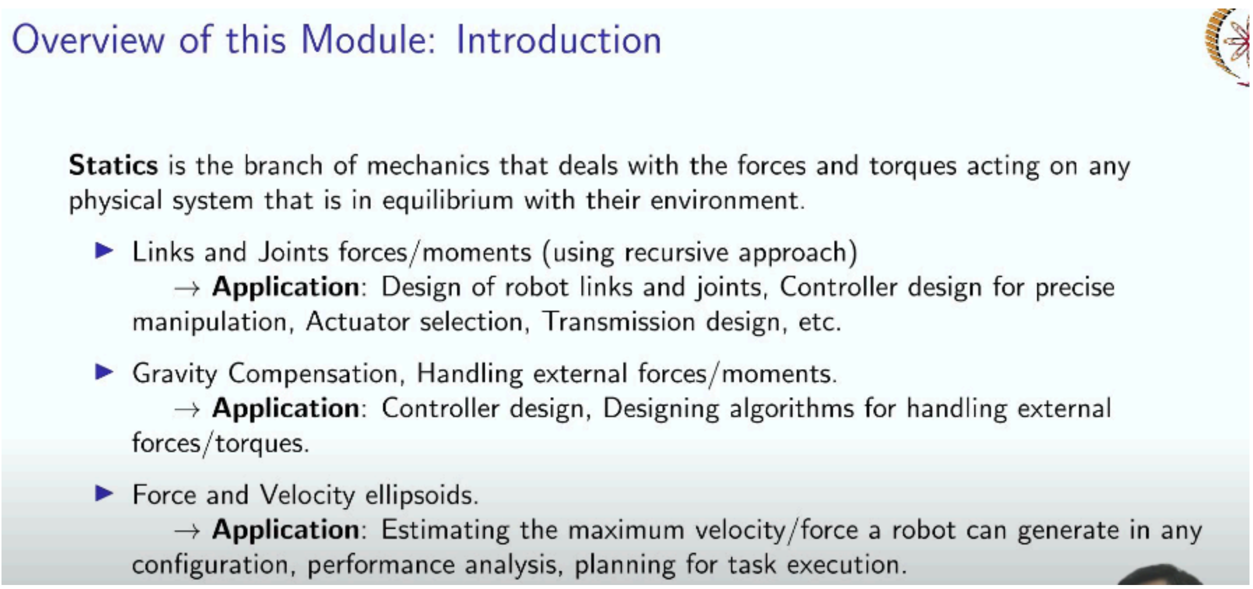
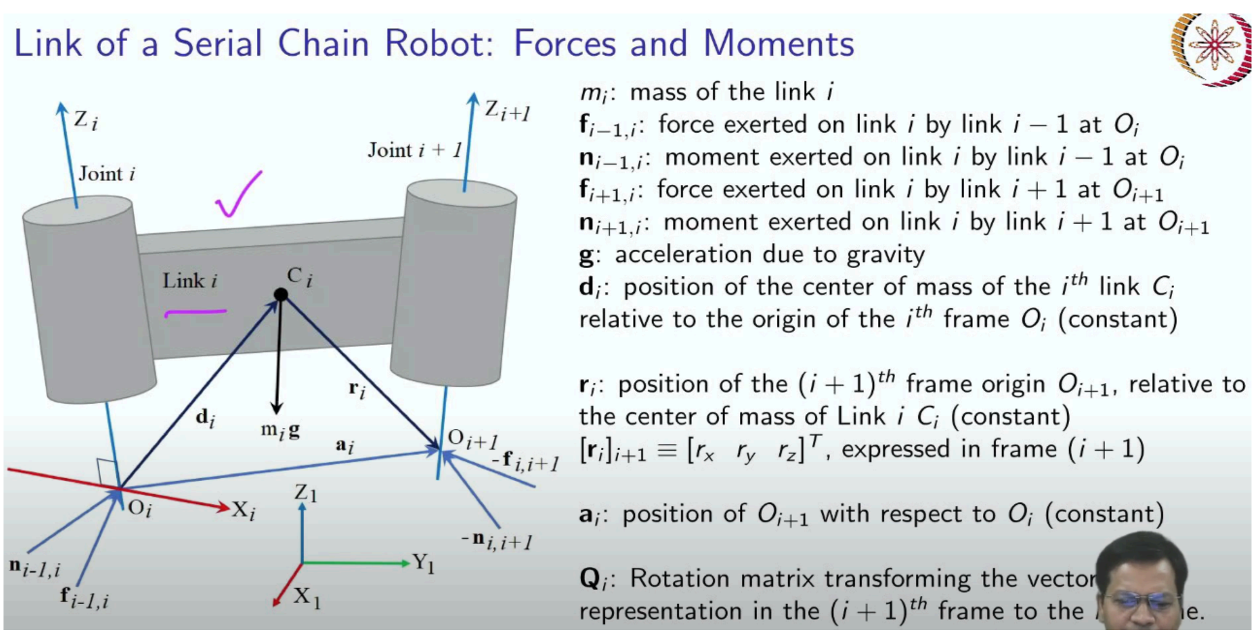
Lecture 33

Link Forces and Moments

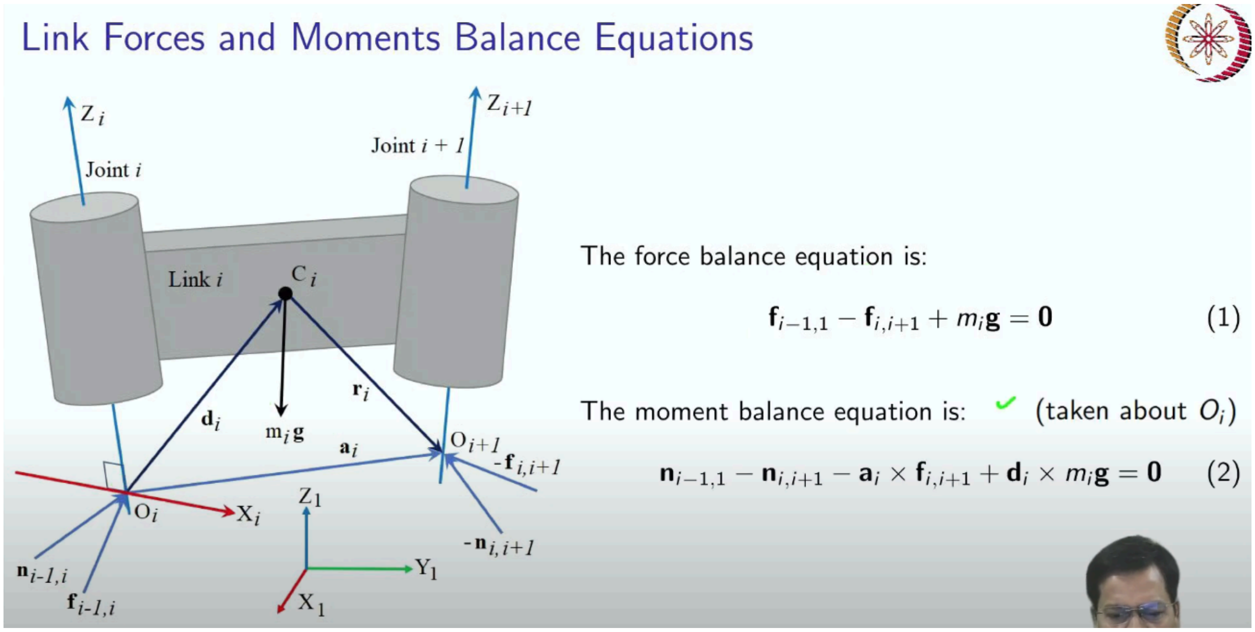
Hello and welcome to the module Robot Statistics. We are now ready with sufficient background on the structure of the robot and its kinematics, both forward and inverse. We have covered it in earlier modules. Furthermore, we have now calibrated the robot and its peripherals to do any task. Furthermore, we now know how it is installed and put into any application after installation. So, let us now gradually start moving the robot to do any task. To begin with, let us understand some static and quasi-static tasks and its mechanics. So, before we begin, let us quickly summarise the contents that I will be covering in this module.



So, yes, statics, as such, is a branch of mechanics that deals with forces and torques acting on any physical system that is in equilibrium with its environment. So, this is an age-old definition of statistics. So, when it comes to robotics, it is no different. It is very much the same, Only the source of forces is different. However, we are not going to study the source of force when we do statistics. So, where are those forces coming from? Mostly they are due to the interaction forces, what the robot can encounter while it is performing any task, or due to its masses or whatever load that it is carrying or whatever task it is doing. So, without considering any acceleration forces and dynamic forces. So, we will study the robot in this particular module. So, yes, to begin with, links and joint forces and moments, a recursive approach that I will be using that is quicker and more reliable and faster when it comes to applying those algorithms to find out the link and joint forces and moments. A few of the applications include the design of the robot, links and joints, If at all. If you intend to design any robot, you should be familiar with the mechanics that actually go in when you have to calculate the links. What are the forces that are going to come on it so that you can put boundary conditions well when you actually go and analyse? Those links, whether it will fail or it won't fail. So, this is very much useful in doing so. Controller design for precise manipulation, if at all you have knowledge of forces that are going to come. Even when the robot is static, it is not moving. So, we will see very much in detail how that is used. This kind of mechanics that we are going to do today will be used in controller design, Actuator selection, and transmission design. All these require knowledge of links, joint forces and moments. So, we will be doing this in the first lecture of this module. Next lecture, I will be covering gravity compensation handling external forces and moments. While performing any task, if at all, it encounters any sort of interaction with the environment. So, what kind of forces and moments or torques are going to come to its joint so that you can have sufficient torques through the actuator developed so that you can handle the task? So, that is all we will be covering in this module, and particularly in this lecture, we will look at gravity compensation. So, that is the second one. Force and velocity ellipsoids estimate the maximum velocity and force a robot can generate in any configuration and different directions, how much force it can apply in different directions, how much velocity it can go, and whether this is maximum or that. So, we will have a particular relation between them. Performance analysis planning for task execution all will require our knowledge of force and velocity ellipsoids. So, with this we will start today's lecture, that is, link and joint forces and moments.



So, let us begin. So, yes, before we start with links, forces and moments, let us formalise a link. What does it look like? And let a link be drawn here. So, you see, this is a link, this one, ith link, any link- 1, 2, 3, 4, 5 till n. So, whatever the number of links, you have any one of the links if you extract it out. So, it is acted upon by the forces and moment.S by a link which is prior to this, as well as from the link which is coming after this. So, it has all those. So, let me just start with the moment which is here. So, you see, this one, this is the moment that acts at Oi, that is the frame which is attached to the joint I, the bottom of that. So, about this joint, this link is able to rotate. You know that already. So, this is your Oi, on which this moment is acting. So, it comes at the one end of the link, that is, this end of the link, from the link which is attached before to this and then this is your force $f_{i-1,\ i}$. This was $n_{i-1,\ i}$. So, the moment acted upon by i-1-th link to the with the link. That is what is meant by this. This is Xi, the axis of this link. That is the coordinate system, which is attached here, and then a similar end exists at the other end of this link. So, this is this point, which is Oi plus 1 (O_{i+1}), that is, the frame which is attached to the end of this link. You have another link that comes after this that can rotate. Like this, rotate or translate, whatever it is. So, this is also acted upon by a reaction force, by the link which comes after this. So, you have ni,i plus 1 ($n_{i,i+1}$), so if this link delivers a moment, i-i plus 1 to that, so negative of that will come on it also same with force. So, it is minus fi, i plus one ($-f_{i,i+1}$) that comes on this link by the next link and you have something which is here, that is Ci. What is that? That is the centre of mass location of this link. mi is the mass of that link i; these we already know. This is over here. These two come over here. g is the acceleration due to gravity, and it is represented in the base frame. It is aligned with the wall frame which is here. di is the position of the centre of mass of ith the link. Ci, related to the origin of the ith frame, Oi. This is virtually constant because, with respect to Oi, Ci, i is not going to change as long as the robot is assembled, and retains its structure, so that Ci location with respect to Oi is not going to change. So, this is here and similar is your ri position of the i plus 1st frame. Origin: Oi plus 1, that is, this one from Ci relative to the centre of mass of the link i, that is Ci location. Again, it is constant because the structure is no more changing once it is assembled. So, this is also constant, mind it. The frames are different in which both of them are represented. So, ri is represented in this frame, whereas di is represented in this frame. So, me vectors, if it is in the same frame, should result in a_i. What is that? a_i is the position of the O_i plus 1st frame, this frame with respect to O_i. So, that is the position vector that connects both of them. So, di plus ri effectively should be equal to ai. If ri is also expressed in the Oi frame, then there is one more thing to it, that is, the Qi vector, which is a rotation matrix, transforming the vector representation in the i plus 1th frame to the ith frame. So, it is nothing but an extract of the rotation matrix that comes out of the link transformation matrix. You studied well while you were studying DH parameters and link joint parameters. So, you know the transformation matrix that takes you from Oi to Oi plus 1, so that rotation matrix that exists 3 plus 3, rotation matrix that exists as a part of that is Qi. So, it will take the orientation change from this frame, Oi to Oi plus 1, so these are some of the notations that I will be using when I will analyse this.

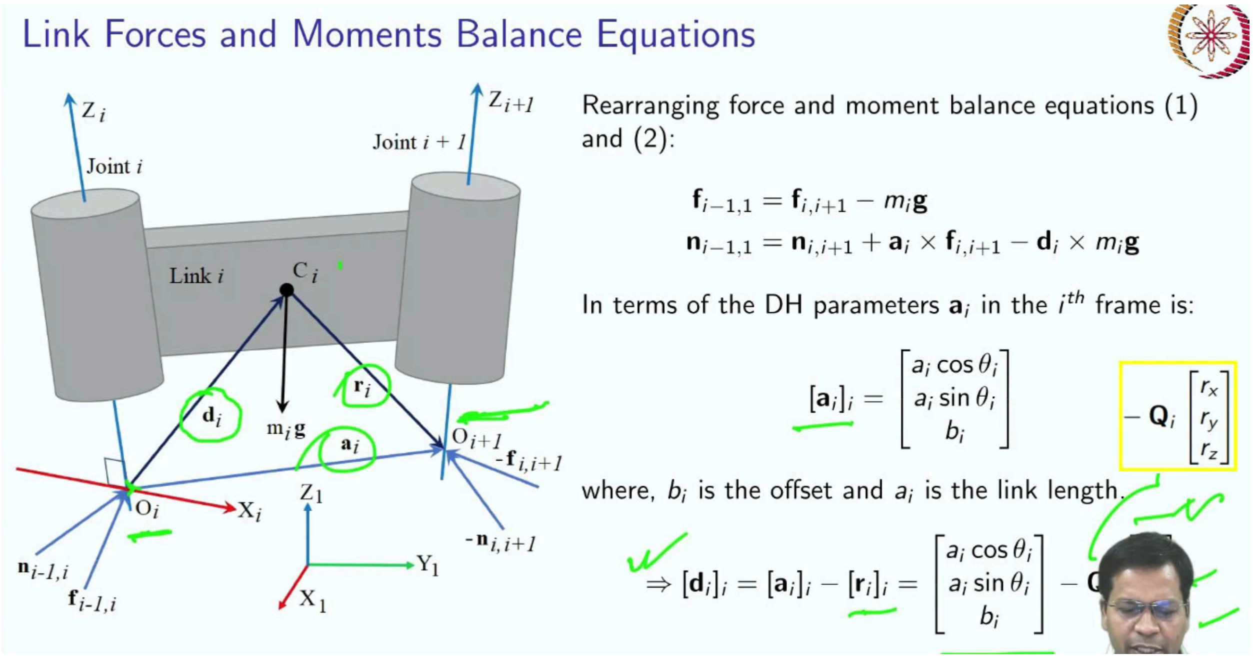


So, let us quickly see what are the forces and moment balance equations that exist. So, overall, I am going to study the statistics of it, so what I should see. This link should be in equilibrium condition, provided it is acted by the forces from the input and the output side of it. So, these two forces are acting over here, moment and force. Over here, you have a moment and a force, and additionally, you have a force that is due to the gravitational pull at the centre of mass of this link. So, all these forces should make it in equilibrium. So, the force balance equation should be. So, fi minus 1i ($f_{i-1,\ i}$). So, that is this one minus fi, i plus 1 ($f_{i,i+1}$), and it is a negative symbol because it is acted upon as a reaction force; that is, this link itself applies to the next link. So, as a reaction, it sees a negative force and mig that acts downward along the axis of G. So, that comes here. So, the sum of all those forces should be equal. Mind it, this is not just 0, and it is a 0 vector. So, this is a 3 cross 1 vector. This is 3 cross 1, this is 3 cross 1, same is this one also. So, that is the force balance equation.

$$f_{i-1,i} - f_{i,i+1} + m_i g = 0$$

Similarly, this will be your moment balance equation. So, you have a moment that comes from $n_{i-1,1}$ end of it. You have another moment that comes from $n_{i,\ i+1}$, and there are moments which are created due to the forces also. So, that is nothing but this one, ai. that is this vector and the force that comes here, that is over here with the negative sign. Similarly, you have di and this force, which is here, di and force that will be positive, and again, it is a 0 vector. So, you have the sum of all the moments equal to 0, so this basically defines the equilibrium condition. Let us name them 1 and 2, which we will be referring to later also. So, this is your forces and moment balance equation. So, this is the only set of forces and moments that comes on this link and makes this link start in space. So, this is nothing but just like a free-body diagram in space.

$$n_{i-1,\ 1} - n_{i,\ i+1} - a_i \times f_{i,\ i+1} + d_i \times m_i g = 0$$

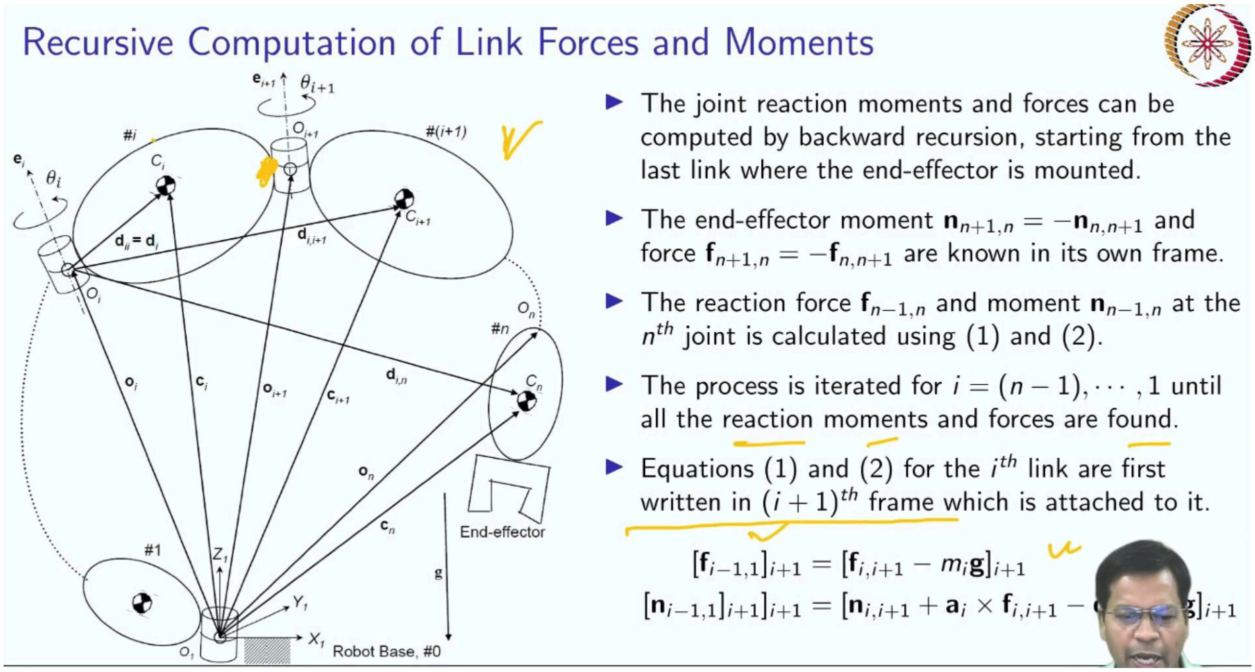


So, now let us just rearrange the force and moment balance equation as we just saw. So, I am expressing the force that comes here in terms of the force that is here. So, if you know this one, you know this one, plus this will come in between. So, you see what I am trying to do. I am trying to create a chain sort of thing so that I can do some iteration of that kind of thing. We will come more precisely later now.

Similarly, ni minus one i. this one is equal to you-have a moment which comes here and a moment that is created due to the forces, so one of the forces is over here. So, that is this and this. Again, you have another moment that is created due to m_ig with di. So, that is all. So, you have expressed the same one and two equations in this form, so in terms of DH parameters, you can write ai, how do you see? Here is a link length. What was that? It was the shortest distance between a perpendicular distance between Oi and i plus 1, so if at all you are able to draw something like this. So, you should see something like this. And there is a perpendicular offset also. That was existing, that is from along Zi, starting from Oi. So, that is your ib. that is also there. So, if at all this link makes an angle theta, I am from here. So, you see, you can write ai in an ith frame as ai cos theta, ai, sin theta and ai. So, that is what are the three offsets that you see from your origin, Oi. that creates the position vector ai in an ith frame, so that is how it is written.

Next is your closed vector triangle that is made out of di, which is in the ith frame already, and ri in the ith frame. It is nothing but rx, ry and rz. Let us say these are the three components which are there for this ri. This is a known vector. You know the centre of gravity location with respect to Oi plus 1th frame. This is your ai in the ith frame. So, di plus ri gives you ai. So, it can be written like this: quite trivial.

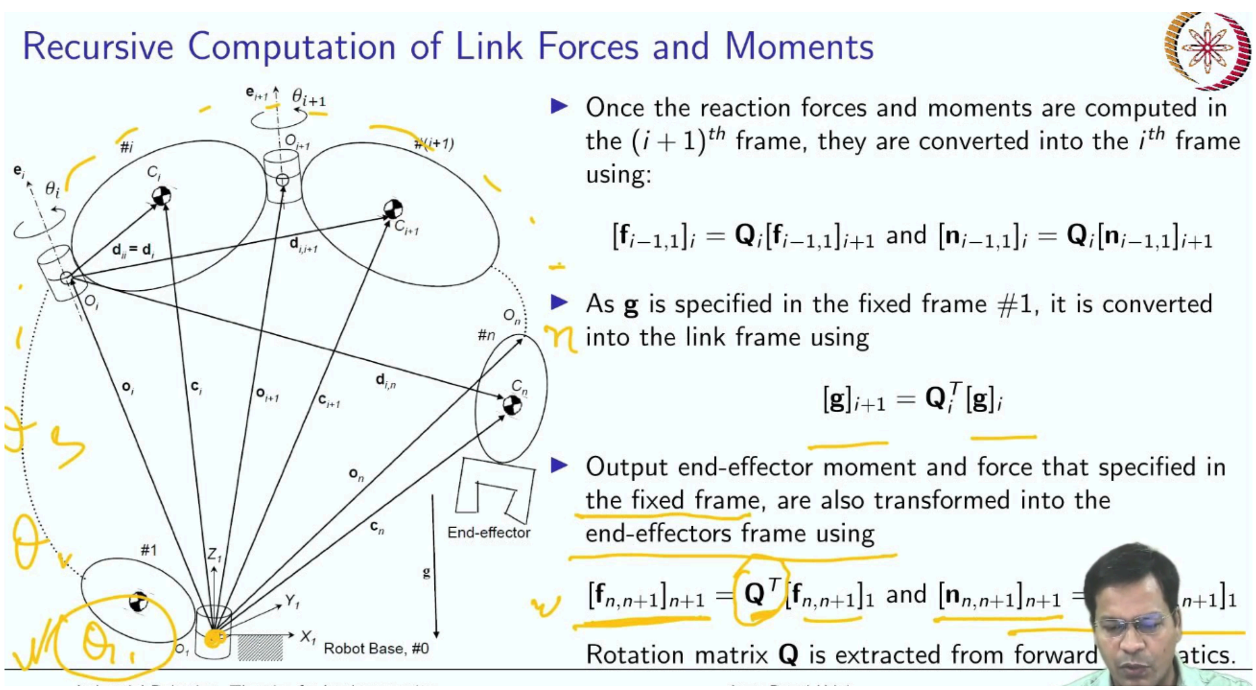
Additionally, what I have here is Qi. Why Qi is nothing but an orientation transformation that takes you from Oi to Oi plus one because ri was written in the frame Oi plus 1. So, I have to transfer this, the same vector with the same magnitude, in the O_i frame. So, that is the reason why I am multiplying with Qi, so that it makes everything in the same frame. That is the ith frame. So, this is how it is finally written in ith the frame.



Now, let us start the Recursive Computation of the Link Forces and Moments. So, the joint reaction moment forces can be computed by backward recursion. Starting from the last link where the end effector is mounted. So, what is known, fully known, is everything we know about the end effector. We know how much is the mass of this end effector. We know the gravity g direction with respect to this also, so we can quickly calculate how much the force will be that this end effector is applying. On the nth link, that is the last link. So, we have all the information about this end effector. So, that is what will be used. So, starting from this, we know what this is applying here. We know the link, mass and centre of gravity location for this also. So, now this end is calculated, whatever force and moment that is going to come here will be calculated. So, that is how we will go backwards, like this: we will start from here, which is fully known. We know nothing is attached after this, and we know everything about this link. So, we start from this one, and then we will go back. So, that is the reason why I wrote the force and moment balance equation in a backward manner. So, the end effector, moment n plus one n, is equal to minus of n plus on, is it not?

$$n_{n+1,\, n} = -\, n_{n,\, n+1}$$

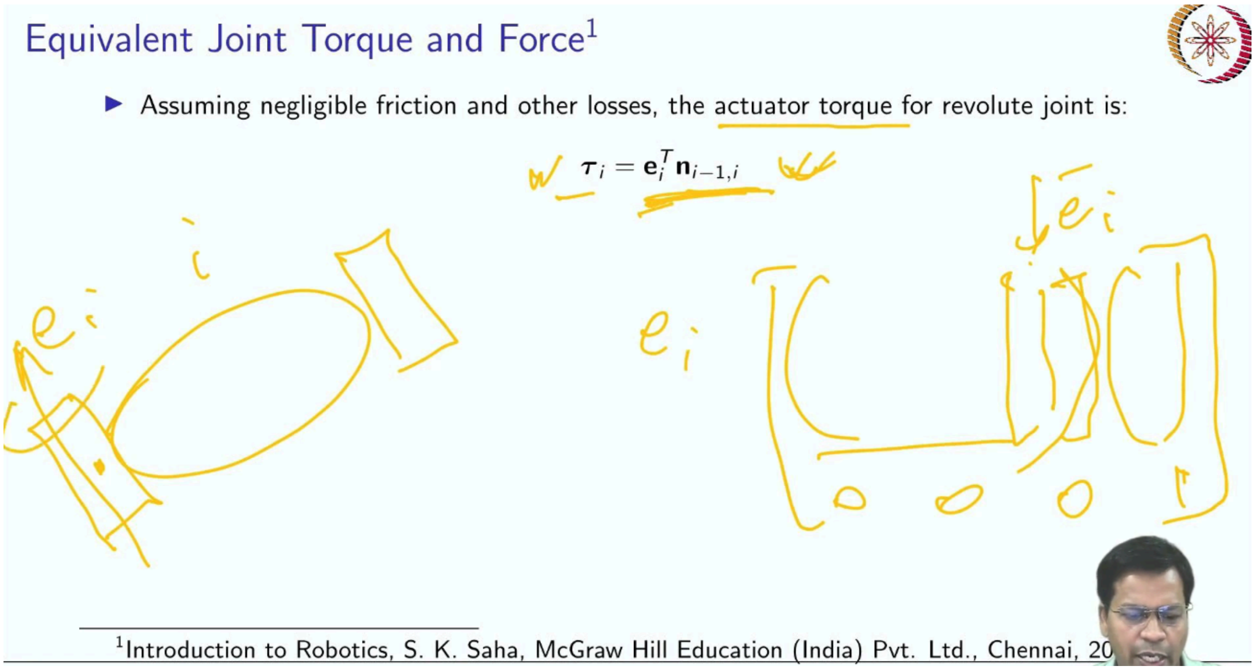
So it is the reverse. Whatever this applies forces and moments to this, this applies in a reverse manner. So, that is what it is: nothing but action, a reaction pair for the force and the moment, both that are known in its frame for the last link. It is always known. We know a hundred percent of it. For this link, you know this is always true action and reaction pairs; they balance, only the directions are inverted, and then the reaction force, n minus one, n that is over here, and the moment at the nth joint, which is here, is calculated using equations one and two. If you know what is going to come here, you already have information about the centre of mass, its location and its own mass. So, you now can calculate this one using equations one and two, as we have seen. Let me just show you once again this one. So, these are the two equations. If you know the mass and centre of gravity location, you know this, you already know this, and you can calculate this in a backward manner. The same goes for this. That is what is done here, OK, so you have now used these two equations to find out what joint force and moment. That is going to come here. The process is iterated backwards now from I is equal to you starting with n-effector, which you did first for the nth link, then you do for the n minus one, n minus two, and so on, in a backward manner till the first link. until all the reaction moments and forces are found, is it not? So, you have full information about this using this. The mass moment is calculated from the C_n location, and that is added over here. Finally, you got everything over here. Then, you proceeded backwards again for this link. You know now this one. You know this centre of mass location and its values you can calculate here. You proceeded with everything in a backward manner from this to this. You got it. So, this is how it moves in backward iteration. So, equations one and two for the link are first written in the i plus one frame. So, if it is, is there any with the link? So, you have to write everything over here so that you can do all the vector sum that comes from this side of it. So, you have expressed everything over here. So, whatever equations one and two you have for the force and moment backward iteration formula that we have written over in equations one and two, both of them are expressed in i plus one ith frame, same for the moment, same for the force. You do all sorts of vector addition, subtraction and balance over here, and then you transfer that to this location; how?



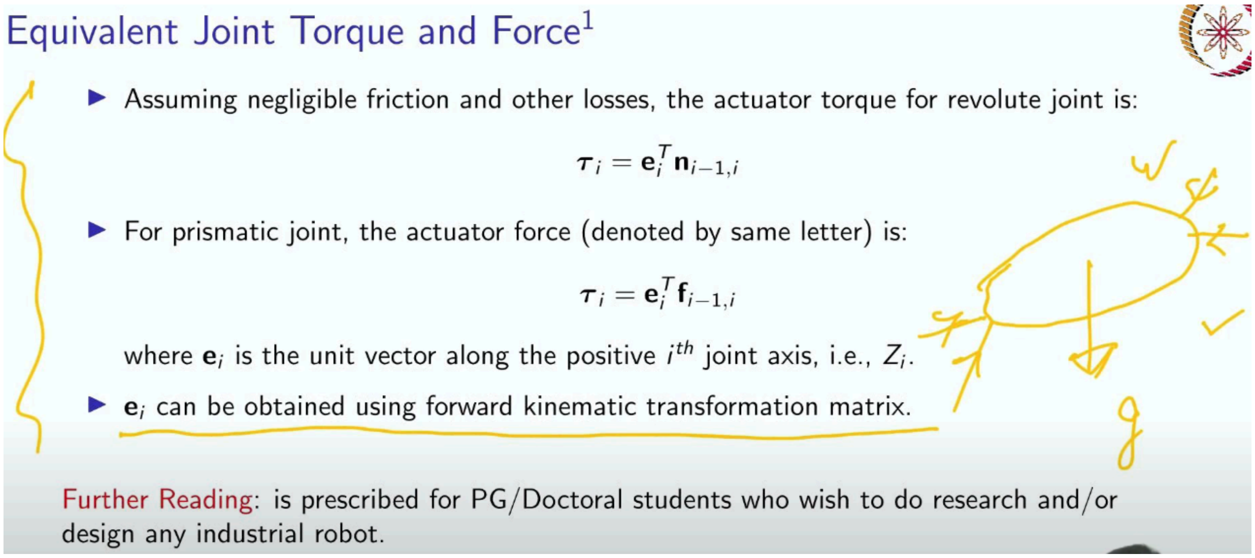
Using the link transformation matrix. That is the orientation change that is happening from here to here. So, once the reaction forces and moments are computed in i plus one with the frame that is over here, they are converted to the with frame. I bring all the forces and moments to this frame, and then I will handle the link which comes over here. So, again, all the forces and moments that will be expressed here do everything and bring it here likewise. So, you have to keep moving backwards. So, as you know, all the vectors and moments can only be added if they are in the same frame. So, this is why this value is used. You know qi for all the links. Through your DH parameters, again, as g is specified. You know, g, it is always specified in the ground fixed frame, that is, the whirling frame or the force frame of the robot, which is attached to the ground. So, that is fixed to frame one. So, that also needs to be converted to the link frame using this g_i plus one, and i will again be related to this using Qi.

$$[g]_{i+1} = Q_i^T[g]_i$$

So, if you know g in this frame, in this frame you have to convert it back. So, again, it has to be converted one by one from one frame to the other, like this. You have to keep moving backwards again. So, whenever you consider this frame, g should be expressed in this frame. When you consider this frame, g should be converted to this frame. So, whatever, again g also should be compatible to be added and subtracted. So, everything should be in the same frame. So, that is the reason I am using this, and Qi is well known. This is nothing but Qi transpose. What does it say? Is it Qi? It is a rotation matrix. Instead of taking the inverse, I can quickly find it out by taking the transpose of this. You know, rotation matrices are orthogonal in nature, so that you can do this. So, that is what is done over here. So, it was already in the i plus nth frame. So, in the previous frame you can calculate by using this, so the output of the end effector movement and force that is specified in the fixed frame are also transformed into the end effector frame using this. So, this was there already, you know. This is what you have found out: force and the movement using Q transpose or Q inverse. It can be converted to the fixed frame which comes here now. What is this? Q transpose rotation matrix Q is extracted from the forward kinematics. That is nothing but the transformation matrix, Q1, Q2, and Q3, so you have to multiply all of them together so that you can express Q1 to n. So, what is this one? So, it is the product of all those wherever you want to go. So, this is nothing but a transformation that takes you from the first frame to the next frame. So, that is what goes on here. So, that can quickly be extracted from the forward kinematics transformation matrix. It is a 3x3 rotation matrix, which is a subset which goes inside this transformation, a homogeneous transformation matrix of the robot itself. So, whatever forces and movements you get can be converted to the first frame by multiplying that by the link transformation matrix, a series of link transformation matrices. That goes till here.



So, now that you know all the links, joint forces and movements, you can obtain the joint torque. That is the joint torque. So, if you already know the moment, you know the moment is a vector that can be projected onto the axis to get the torque. So, projection can be taken by taking an axis vector. So, ei is nothing but the axis vector. So, let me show it in the figure somewhere. So, this is your axis vector. What is that? If you can recall, a transformation matrix has got a 3x3 orientation matrix. This shows the position of that frame. This is 0,0,0, this is 1, you know. So, this is your orientation. The third column, 3x1, this one will tell you ei. If you recall, we have done it earlier. We have extracted all the axis frames and the axis directions using ei. We have captured that using the column that comes here. So, that is what this is for the ith link. It can quickly be extracted from forward kinematics. So, ei. So, if you know ei if it is a link with a joint that comes here, a joint which comes here, so you know the moment, which is coming here, you already know the ei vector which is here for the ith the link. So, the moment, projected to the axis vector, can be a dot product which is there. So, it is nothing but a scalar value that comes in the axis direction. So, that is it. So, this gives you the torque value that will go on to the actuator. That comes here. Assuming there is negligible friction and other losses, this should be exactly equal to the torque that comes onto the actuator. So, this is how it helps in selecting any actuator.



If it is a prismatic joint, you have to convert the force again. Take the projection of forces that are coming there along the axis vector. So, the axis of the prismatic joint. You have to project that, and you get what is, again, the tau, the same symbol I am using here for the torque as well as force. So, it is a generalised torque and force that is there where ei is the unit vector along the positive joint axis, that is, along Zi. got it. So, ei can be obtained using forward kinematics of the transformation matrix. You know very well now. So, yes, this is how you obtain the joint torque and force a link moment, what all moments that are going to come, what all force that is going to come on this, so you know all the boundary forces that link sees, including the mass forces that it will see also, if at all, g is acting. So, now you can analyse your body-solid body, which is your link to all sorts of forces that come on it, at least the static forces; you know, you can design this link using those static forces, at least. Taking the factor of safety, it should be quite good enough if your robot does a quasi-static job if it is moving very slowly or if it is static so that it can be done.

So that's all for today. I think that should be enough to understand the joint torque and force as well as link moments and forces which are there. You can now select the actuator, you can now analyse your link very well. For any further reading I have suggested a book here which further covers this in very much detail, including the examples. If you are interested, you can go here for UG, PG and doctoral students who wish to do research and further study to do the design of any industrial robot, and you should go through this book, or any book which is even bibliographer in this book also. So, that's all for today. Thanks a lot.