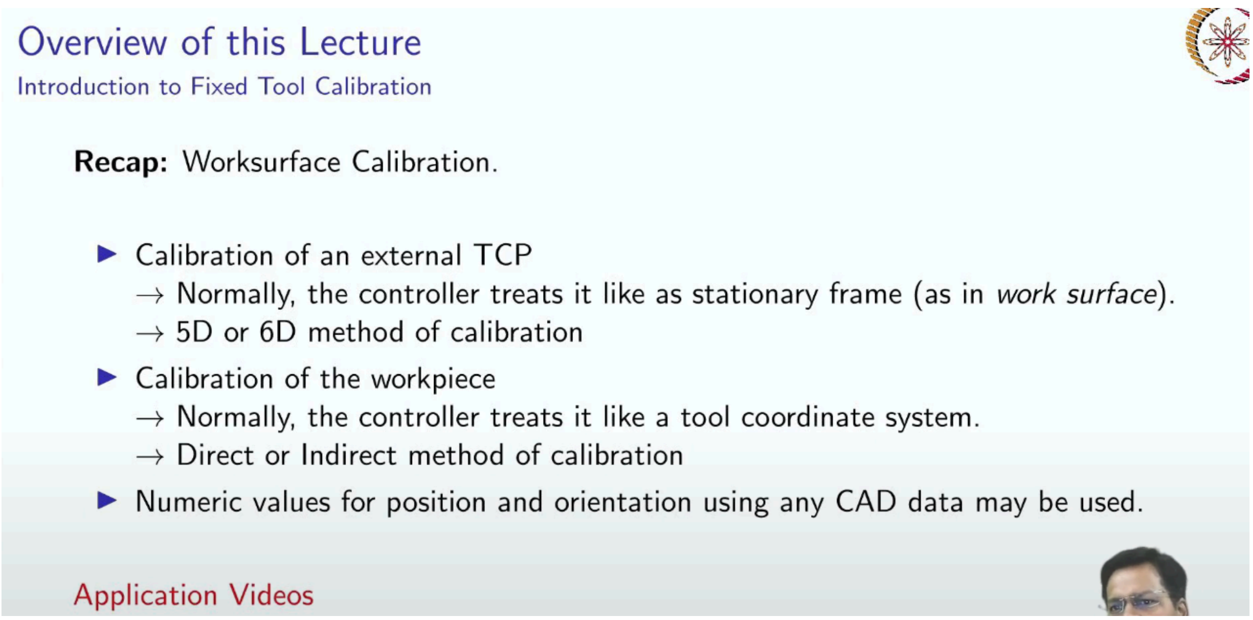
NPTEL Online Certification Courses
Industrial Robotics: Theories for Implementation
Dr Arun Dayal Udai
Department of Mechanical Engineering
Indian Institute of Technology (ISM) Dhanbad
Week: 07
Lecture 31

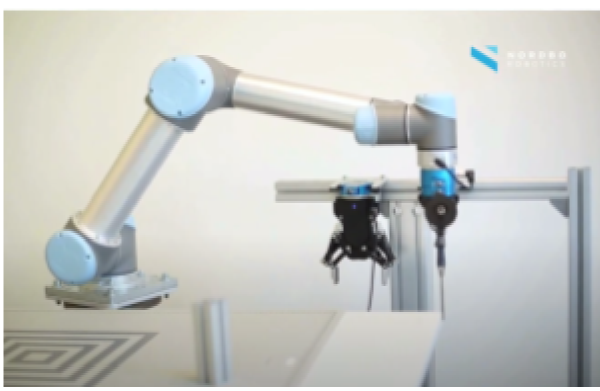
Fixed Tool Calibration: External TCP and Workpiece Calibration

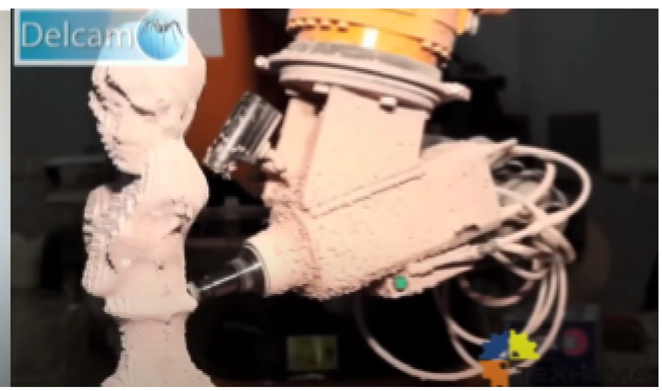
Hello everyone, I hope the concepts taught in this course, with the practical aspects of implementing, are interesting to you all. So, be with me while I continue with the lectures on the calibration of industrial robots and systems. Today, we will be doing external TCP and workpiece calibration. So, let us continue.



So yes, in the last class, we covered work surface calibration. What was that? It is about any surface on which your object circuit on which you have to perform the task. So, that surface needs to be calibrated. That means you need to have a frame about which all the coordinates are addressed. So, any object which is lying on that plane should be in the frame which is attached to the work surface itself. It will be convenient for programming. But yes, when the robot has to do it. It has to be transferred to the robot's frame so that you can do inverse kinematics and you can do your job. The robot does its job, whereas it is easy to understand, easy to coordinate, and easy to program the coordinates when it is there in the workspace coordinates, But the robot does its job using its coordinate system so that coordinate to coordinate transformation is to be done. In order to know that, you did some sort of calibration of the work surface. So, that was the job that we did in the last class. So, continuing further, today we will be doing calibration of an external

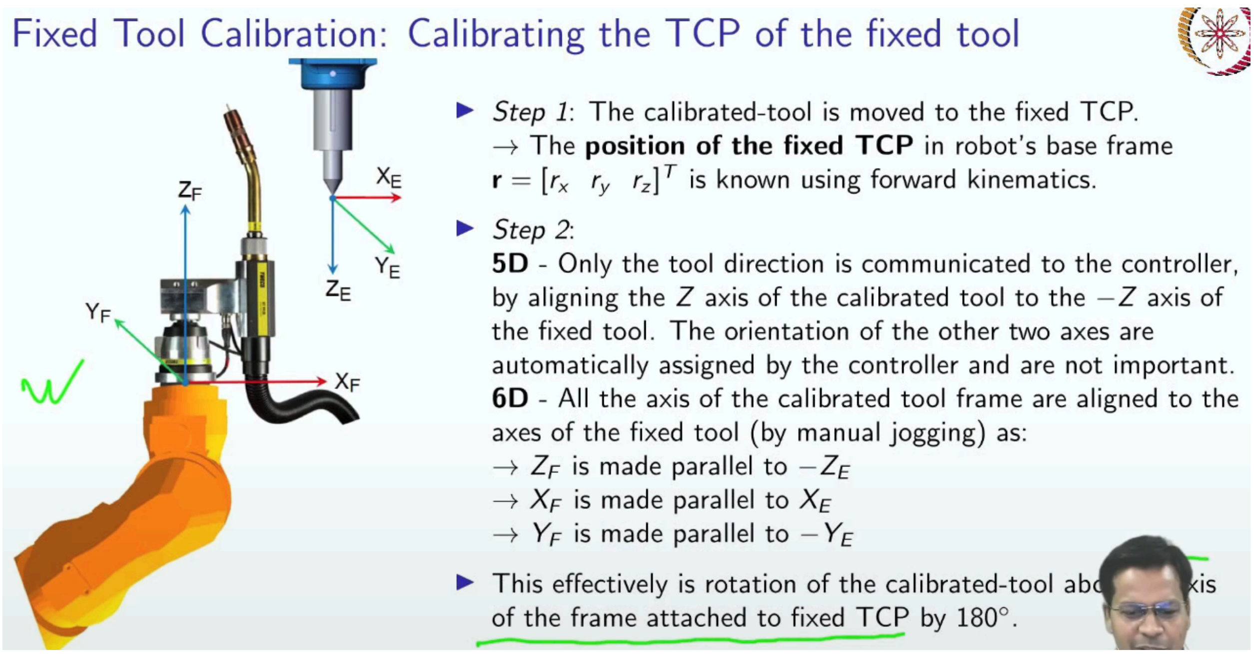
TCP, which is a tool centre point which is not on the robot. Today. We will have it on the workspace, somewhere in the workspace and your robot is trying to do a job on that, or maybe placing a tool over there and picking it up. So, that frame, the coordinates of that frame and the orientation of that frame need to be understood by the robot. So, that is the first task that we will be doing. So, normally, the controller treats it like a stationary frame. It takes it like a stationary frame that itself is not moving, as in the case of the work surface. So, similar approaches are there for calibration. One of them is a 5D approach, and the second one is a 6D method of calibration. We will be discussing that today and the second thing that we are going to do today is the calibration of a workpiece. So, that, again, the controller will treat it like a tool coordinate system. That is not just a place where you can address, but it may have a tool, a physical tool that is, maybe, spinning at a very high RPM, it may be a drill bit, it may be a grinding tool, but definitely, that is going to be statically located at that particular position, and maybe it is moving. Even if it is moving, the frame is fixed. So, that is what we want to calibrate today. There are methods to calibrate it: direct or indirect approach. We will deal with that today and, of course, there are methods like numerical values for position and orientation using any CAD data. If you have CAD data for the whole of the workspace and the TCPs which are located in the workspace, then you can definitely put them in your program using the robot controller directly.





So, let us see some videos directly before I continue. So, this is one example. When you see it is by Norco Robotics. What it is showing is picking up a tool from one object. The tool is operating on an object which is kept in the workspace, and it has picked up another tool. It does something. So, you see at least two coordinates which are there which can be programmed where you can place your tool. So, you should know the axis of the tool and how it can be placed in that frame, and you should also know the location, that is the entire frame. Position and orientation should be known. So, that is one similar video over here. It shows it is by Delcam. It is a robotic CNC machine. So, what is shown here is a robot holding the tool, and your object is stationary, on which you have to do some sort of machining. So, the object's frame should be well known to the robot's coordinate system as well, and you can approach it from different directions each and every point on the surface is known how that is known, how the frame, position and orientations are known. That is what we will be doing today. So, this is yet another video, and so see how nicely it is making. A similar one is a video that you have already seen earlier. See, we have a dispenser-adhesive dispenser dispensing the glue on top of the surface and the whole of that chassis. Part of the chassis is now held by the robot. So, the robot is handling that, and your tool is fixed. Your object is held by the robot, and you are working on it. This is yet another example, which is a similar one. This is a KUKA robot handling Mercedes-Benz's last front block. Again, it is applying adhesive and it is putting it in front of that car chassis.

So these are some applications. That shows why we are doing it, why it is important, what the scenery is, and how it will appear in the software industry. So, how a robot does this kind of job, we will look very much deeper into it.



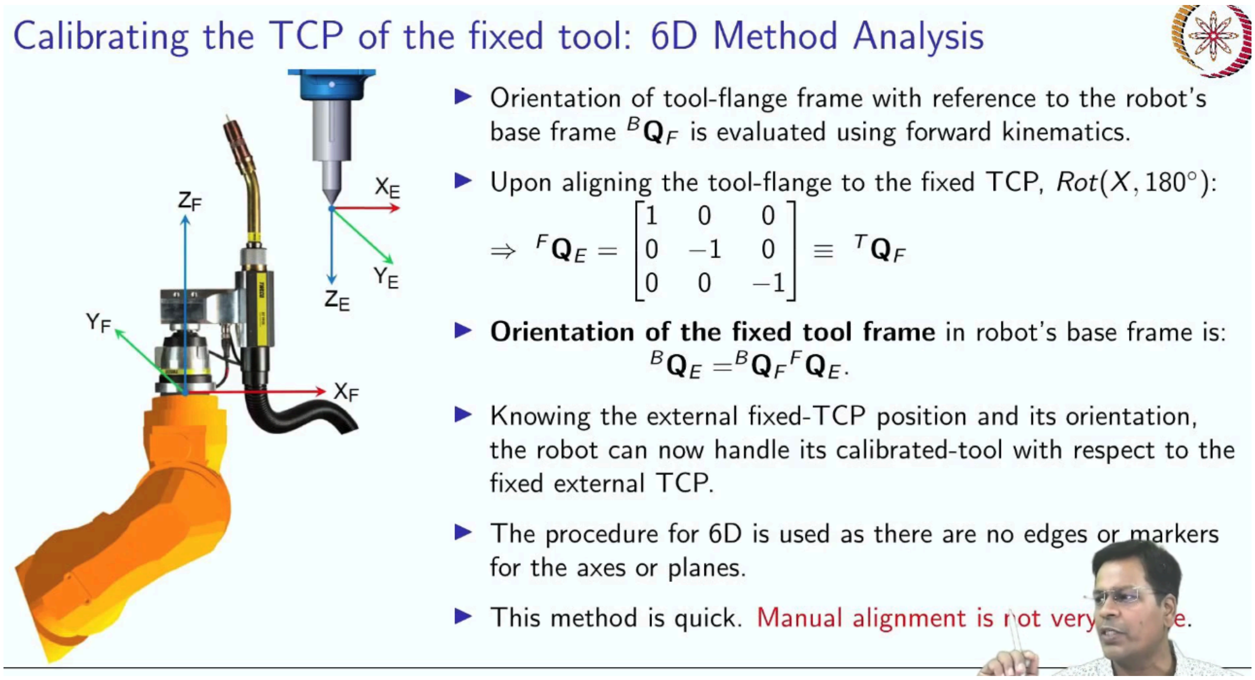
Now, let us begin with fixed tool calibration, that is, calibrating the TCP of the fixed tool. The tool is there in the workspace. You have to get to know the location of that tool as well as the orientation of that tool. So, that is our job now And what this robot is already having. It has a tool which is fitted at the end effector, That is, the tool flange of the robot.
So, step 1 is the calibrated tool that is on the robot. This one is moving. So, this is already calibrated. What does it mean? That means you know the TCP which is here at the end of this tool. That location is calibrated, and its orientation is also calibrated. So, wherever your robot takes it, you can precisely know the tip of the tool position with respect to the robot's base. So, that is a calibrated tool which is mounted on the robot. So, this is now moved to the fixed TCP. Where is this fixed TCP? It is this one. So, this one is your fixed TCP. So, what I want to know is the location of this TCP. This time, the TCP that I am referring to here is not the TCP which is on the tool which is mounted on the robot. I will call it a calibrated tool only, Whereas the TCP if I say it is on the fixed TCP, which is outside of this robot, is external. So, in step 1, I am taking the calibrated tool to the fixed TCP. So, what do you get to know in this step? The position of the fixed TCP R is known using the forward kinematic. So, if both the points coincide, that is the

fixed TCP and the calibrated tool TCP. So, the positions are the same. So, if you can calculate this position using the forward kinematics of the robot, that means you have now come to know the fixed TCP tip also. So, that is now known with respect to the robot's base. Once you are taking it, you can record the position directly from the robot controller, because both the tools are now coinciding. You can know your tip, so you also know the tip on which you are making contact. So, that location is very well known now. You get to know this location. That is the fixed TCP location with respect to the robot base. So, this is now known. I will call it an r vector, in which r_x, r_y, and r_z are its coordinates, which are known using forward kinematics. So, this is your first step.

Second step: now that I know the position of that TCP, I want to know the orientation of that as well. So, there are again two approaches: the way we had calibrated for the workspace surface. The same way we have done this also. So, you have 5D calibration. That means only the tool direction is communicated to the controller. I am just interested in the direction of this tool, irrespective of how it is oriented in the other two axes. What are the other two axes? They are the axes which are orthogonal to this. So, if this is your Z, this may be your X and Y. So, I am not bothering about how my X and Y are oriented. I am just bothered about how this Z is aligned. So, I want to know the direction of this. So, such could be the tool like the TCP that I just showed you a video of. If it is an adhesive tool or adhesive dispenser, irrespective of its orientation in the above Z axis, whatever is X and Y, it doesn't matter much. , So it will dispense the same thing at the same location. So, that could be one possible application which doesn't require the other two axes to be known, that is, the X and Y axes. , So that is one application. So, what will I do now? I will just align the Z-axis of the calibrated tool. So, if this is my calibrated tool, I have just aligned the positive Z axis of this tool to the Z axis of this fixed TCP. The orientation of the other two axes is automatically assigned by the controller and that is not important also. So, if both the axes are aligned, you can directly know the orientation of the fixed TCP with respect to the robot base. Why? Because you know the orientation of this, and now that it is aligned, you can directly record it.

So, yes, moving ahead to 6D calibration, again, a similar step will be done. The calibrated tip is now moved to the TCP. By doing so, I will know the location of this external fixed TCP tip location. , So that is known as forward kinematics because I have made contact with that. So, wherever is the tip of that, my robot goes, so that becomes. That is actually the position of that, just by sensing it from there. , So that is what we did.

Now, 6D, calibration: What it is, All the axes of the calibrated tool frame are aligned to the axis of the fixed tool, But manual jogging. You have to jog it gradually so that it gets almost aligned. How they are aligned: Z of F (Z_F), suffix here denotes the tool flange frame which is here. Tool flange F is made parallel to the Z of the external. So, this is your external TCP. So, Z_E, negative of Z_E, is a long positive of Z_F, again X_F is along X_E, both are along the same direction, and Y_F is made parallel to minus of Y_E. So, that is equivalent to rotating this frame, the frame which is here E frame, rotating that with respect to the X_F frame by 180 degrees. Is it not? So, that is the effective rotation of the calibrated tool about the X-axis of the frame attached to the fixed TCP by 180 degrees. So, if you do that, it will get it. It will exactly become aligned like this, and that becomes the key. How will we calculate the transformation matrix between this E and flange F. So that transformation can easily be estimated?



How? So, let us continue that. So, this is what we did, and orientation of tool flange: tool frame with reference to the robot base. This is evaluated using forward kinematics. So, this is quite easy and trivial now because, using forward kinematics, this frame, which is attached as the robot tool flange, you already know the orientation and position of this using forward kinematics, So you can quickly obtain. So, I now know this.

Next, upon aligning the tool flange to the fixed TCP, when you align it this way, there is effective rotation about the X-axis by 180 degrees. How does it look like, actually? That is, this is your F, this is your E. When you say your E is aligned in this way to F, that means this is your projection of E along F_X. So, that is there. So, X_E is along X_F. So, both are the same. So, 1, the rest two are 0.

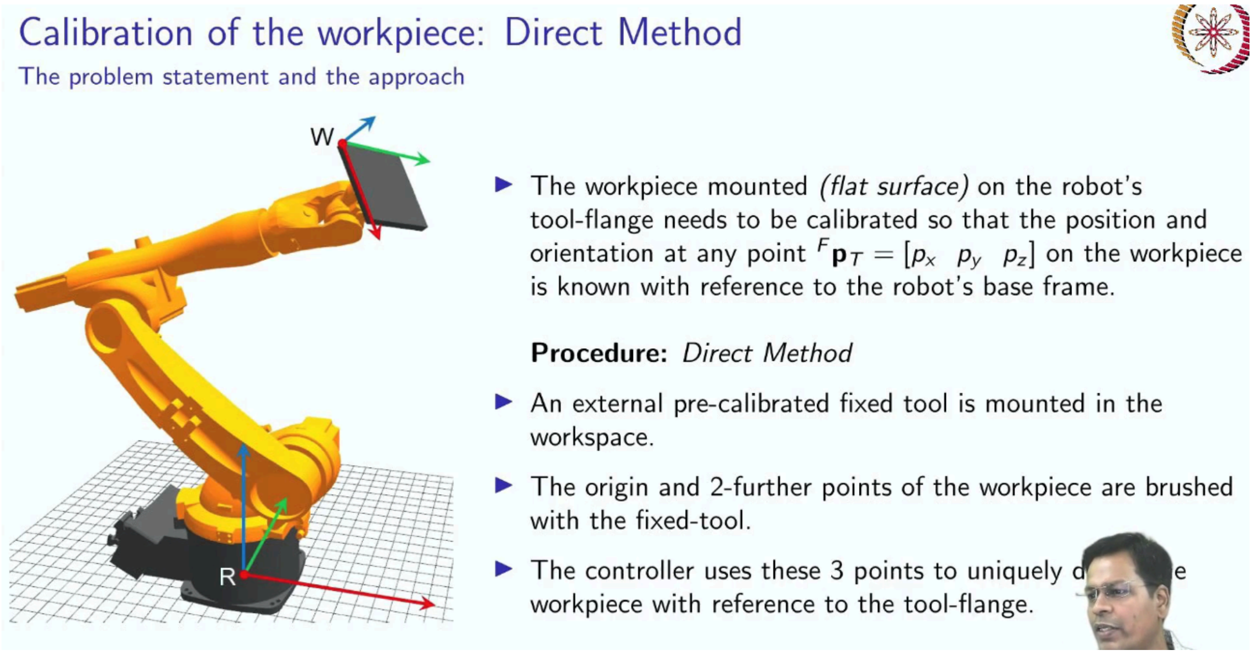
Similarly, Y, Y_E is opposite to Y_F, and along X, it is 0; along Y, it is negative; along Z is equal to 0, and again Z, no projection along X, no projection along Y. Along Z, it is opposite, it is directed opposite. So, ZE is the opposite to ZF. So, that makes it like this. You can directly look over here. It is rotated about the X-axis by 180 degrees. So, that is how it is directly calculated. So, this is your transformation between frame F and frame E.

The orientation of the fixed tool frame, fixed tool frame in the robot's frame, in the robot's frame. So, this is your fixed tool frame in the robot's base. So, wherever it is mounted, your robot is mounted. So, this becomes your base frame. So, with respect to this, this frame is given as B to E. How much is that? It is B to the robot's flange frame and flange to the external frame that is given by this. Now that both are aligned like this, one over the other. So, you can quickly know the transformation between them. So, you already have this in hand. So, B to F through forward kinematics, you know, F to E is by alignment, you know. So, you can write it as B to F and F to E which becomes a B to E transformation that is orientation.

$$^{B}Q_{E} = {}^{B}Q_{F}\,{}^{F}Q_{E}$$

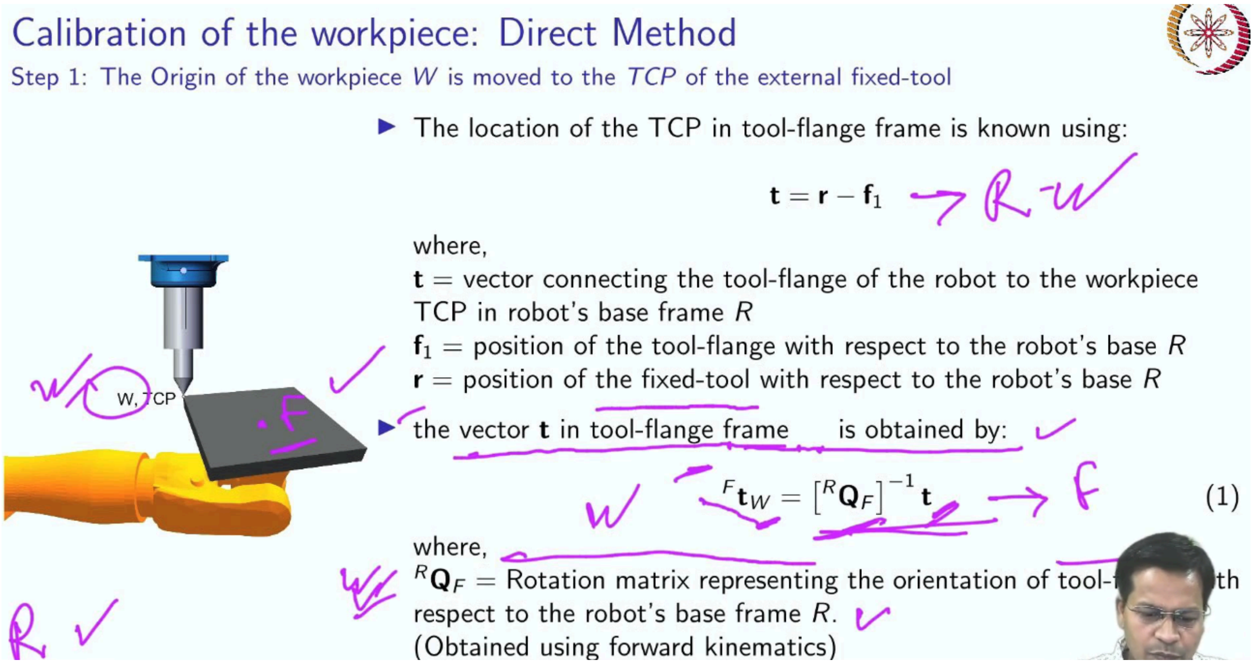
That gives the orientation of the fixed tool frame with respect to the fixed frame which is there at the robot's base. So, that is something which is known. So, you already know the location that is in step 1. You just brushed your TCP to this location, and you got to know the r vector that gives you the position of that. This gives you the orientation of that. So, the whole of the thing is now calibrated. You now know the position and orientation of the fixed TCP with respect to the robot's base. So, now everything is known. Your tooltip is known, and your fixed frames are known. So, now you can handle that. You can go and take up anything from there, and you can do any job you see.

So, in this position, the location and orientation of the tool where it is kept on the stand are precisely known, and this type of job can be easily done. You know both the locations and both the orientation. You can do this. So, this is just one example which uses a similar kind of calibration. So, knowing the external fixed TCP position and orientation, the robot can now handle its calibrated tool with respect to the fixed external TCP. So, this is the job which it can do and the procedure for 6D is used here, as there are no edges or markers for the axis or planes which are there. So, you don't find any edge about which you can take two points and find a straight line on that or maybe an axis you can fit it. So, that is not possible. So, this is the only technique, that is, the manual jogging technique, by which you can do that and this is quite good enough for a pretty good amount of robotic tasks in which bits of compliance are there. So, this method is very, very quick, but definitely, because it is a manual alignment, it is not very, very accurate.



Now, we will go for the calibration of a workpiece. What is a workpiece? It is an object which is now held by the robot. So, at any point, let us say I am holding this mobile. So, if I am holding it, any point which is on this plane, if this is your XY plane, if there is any point which is addressed with respect to the frame which is attached to this object, if this is known with respect to this object, I should know the same location in my frame, my body frame. So, in the case of a robot, it becomes the robot's base frame, that is, R, which is shown here. So, any point which is on this, now I want to know that point in this frame so that I can handle that. I can take that point to any fixed TCP. Let us say I can take that point to any location, the way you did while dispensing glue from a tip, and you can take that point, and you can manoeuvre your robot and do that task. So, that is what a typical application can be done with this. So, this is your problem statement now. So, what are the approaches to it? The workpiece mounted over here it is a flat surface. I won't make things very complicated now. So, I want to know the position and orientation of my work surface, that is, the workpiece frame. It should be quite good enough if I have any point with respect to that particular frame. So, I will just try to find out the position and orientation of the frame, which is at W, So it is a flat surface. The two flanges need to be calibrated so that the position and orientation of the point F to T, that is, frame location and robot's flange location. So, this is your flange location to the object's frame. So, that is what you want to know. So, the workpiece is known with respect to the robot's base. So, that is what our job is.

The procedure is a direct method in which an external pre-calibrated tool, as we had in our earlier calibration, had a fixed tool in the workspace. We already know the position and orientation of that fixed tool through our earlier calibration technique. So, I want that tool to be in place again for this calibration. So, an external, pre-calibrated fixed tool is mounted in the workspace. This is done. So, the origin and two further points on the workpiece are brushed with the fixed tool. So, that tool, that is the fixed tool, will now be made to contact this particular workpiece. So, the origin will be contacted, and two further points will be contacted. Using this, the controller can now calculate the position and orientation of frame W with respect to the tool flange, that is, the robot's tool flange. So, that is our job now.

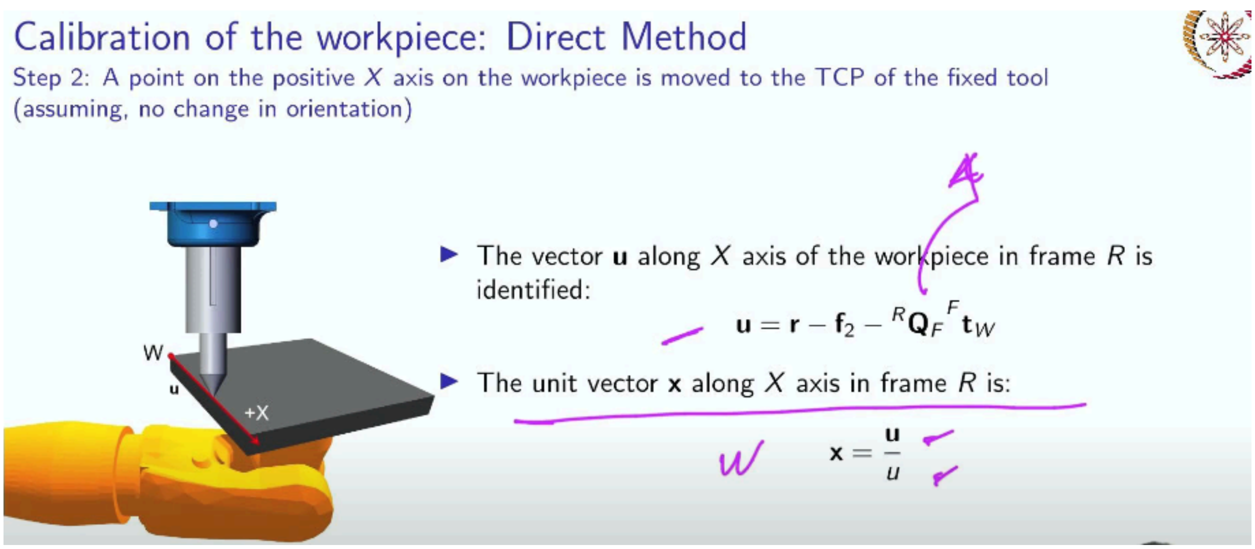


So, let us begin with the details of this procedure. So, step one: what I did is the origin of the workpiece is moved to the TCP. TCP, that is, the fixed TCP, is an external fixed tool because it is already calibrated. This position is known with respect to the robot's base. So, that is what is known. So, r is known. That is, the position of the fixed tool with respect to the robot's base is known because that fixed TCP is a calibrated TCP F1, and F1 is the flange centre position. Whatever the centre on which this is mounted, this position is your F1. F1 because it is step one, So the flange centre position is now known. Using the robot's forward kinematics because the whole of the D-H set joint angles are known once you take your robot to any place. So, using that, F1 is known. So, F1 is known. r because this is also known. So, taking the difference using a simple vector triangle, you can calculate what You can calculate: T. What is that t? t is a vector that connects the tool flange. This is your tool flange of the robot- to the workpiece TCP. I will draw it here. It is this one, So this is what is identified. So, this is your t. this was your F1, this is your r. So, these vectors are known. So, f and r. both of them are position vectors of tool flange and TCP. So, both start from the same location. That is the robot's base frame. So, that makes it a triangle. So, the triangle looks like this. So, here goes your robot's base. This is your F1, this is your r, and this is your t. So, that is making it here. So, that is known. So, now you know this t. This t is now known, and how is it oriented? It is oriented in the robot's frame in a way which is given by the orientation of the flange, robot flange-at this particular pose. So, it is a variable, orientation is a variable, whereas the magnitude of t is constant. So, can I make it constant as well? If I can express my point, which is here, this TCP point in this frame, that becomes a constant. So, if my robot tool flange is rotating, this also is rotating. So, any corner of this workpiece remains stationary with respect to your tool flange, So that does not change, That orientation does not change, and that position does not change. So, if that is known, the whole of this is now known.

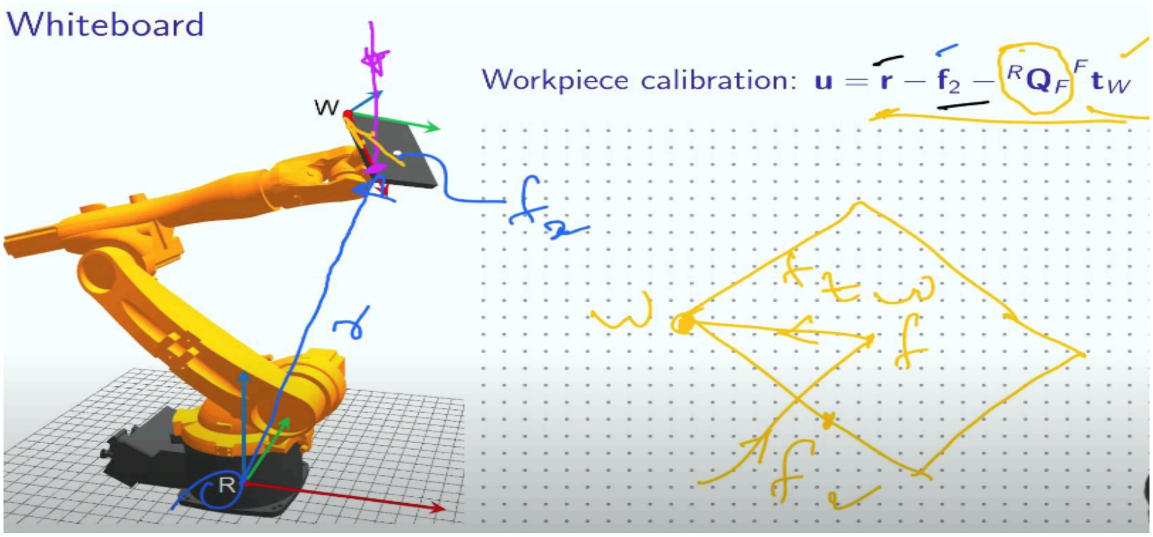
So, let us move ahead and use that. The vector t in the tool flange frame is obtained by this.

$$^{F}t_{W} = [\ ^{R}Q_{F}]^{-1}t$$

So, how was this? This was in the robot's base frame. So, this was in the robot's base frame, R frame, one that I showed in the earlier slide. So, this was in the R frame. Now, this becomes in F frame. Vector magnitude remains the same. Now, it is expressed using the orientation matrix. I am taking the inverse of that. What is R to F? It is the robot's frame to the F flange frame, which is the rotation matrix that represents the orientation of the tool flange with respect to the robot's base frame. So, that is known as forward kinematics. So, this is known. So, the orientation of this with respect to this is given by Q, R and F From R to F. So, that is known. So, this vector in the R frame is already known. So, taking this, we can calculate this in the W frame. What is W? W is with respect to the workpiece itself. So, now you know the location of W with respect to your frame, which is attached here. So, in this frame, if the object is rotating, your frame is rotating. So, with respect to F, W is always a constant, So this is a constant, you know. ,



Now, can I know the orientation of the frame which is fitted over there at W? I already know the location of this with respect to this That we obtained earlier. So, now I want to know this. So, the first thing that I will do that will be in step 2 a point on the positive x-axis that we intend to make positive x-axis. One of the edges is convenient. So, I have taken one of the edges. So on, the workpiece is moved to the TCP. So, this is your external TCP. I will take that edge. I will establish a contact like this. So, this is held by the robot. This is your fixed TCP. You are making a contact like this. So, that contact is made Upon doing so. So, this is something that you have to understand now. So, I have made a contact somewhere over here i.e. W. This was the location, which is known. This vector is now known. That was t already known. This was known. This is a new orientation matrix because you have moved your system, so this has changed. So, this is your new orientation matrix. So, that is again obtained from forward kinematics. F2 is the location of the flange frame. This is your F2 with respect to the robot's base. r is the location of this, your fixed TCP with respect to the robot's base.

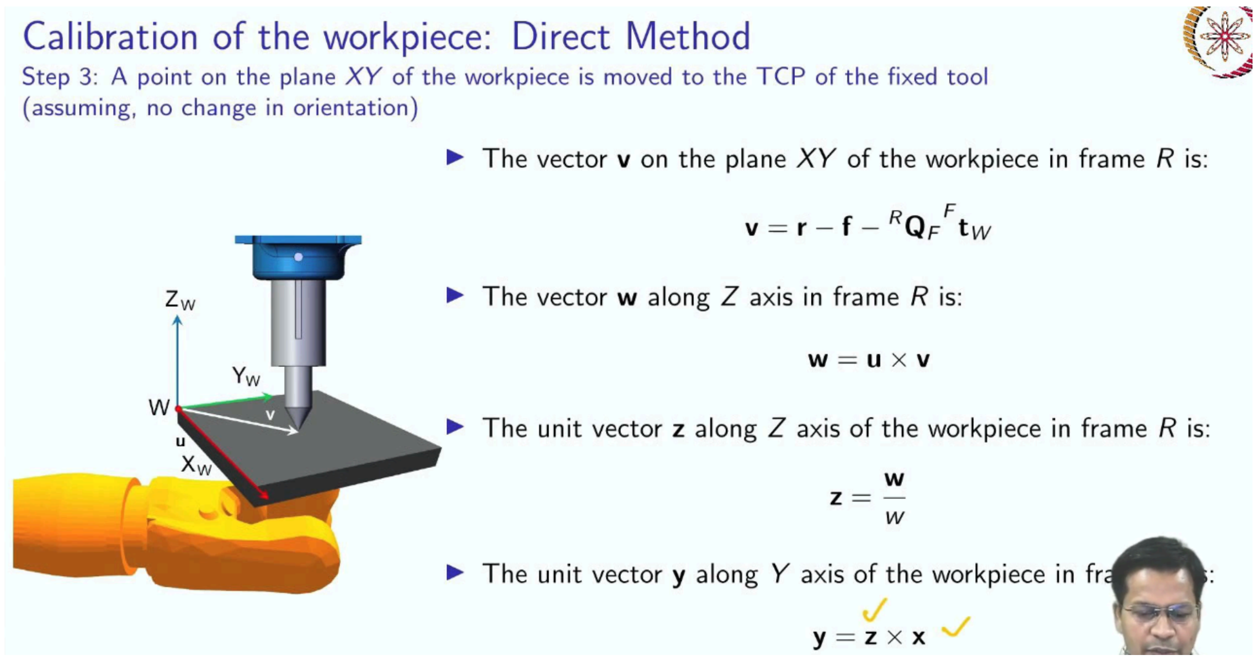


So, let us closely see this over here. So, where are the points? How are we making contact? Let us draw it here. So, yes, you are making. Here goes your fixed TCP that is making contact over here. So, what are the things that are known? It is the centre position of this which is holding. So, this is your F2. That is known, So that is known. Again, what else is known? r is known. So, again, what you know is this vector, a vector which connects to your tool TCP. So, your tool TCP is not making contact over here. But the position of tool TCP-fixed tool TCP-is not changing with respect to the robot's base. So, this R is known, and it is a constant over here.F2 is known. That is with respect to F2 because it is a new position of flange. So it is F2. So, F2 is also known with respect to the robot's base, Got it? So F2 is known, r is known, and you already have obtained this vector, the vector which is here. So, that goes here. This is your orientation matrix that can be obtained from forward kinematics. This was obtained in the first system. So, using this, you can calculate the vector which is aligned. So, this was your base. So this is your W location. This is your flange location. This vector is already known. That is, T of W with respect to F. F2 is known. So t point on this is also known. That is given by R now, So you know this. You know this, so you can create a vector which is u. that comes here. , So that is what is evaluated in your previous slide. So, u is given by r minus f2. So, this is formed by that vector triangle. So, f2 is here, r is here. This vector is known. So, you create a closed vector, and I will just draw the whole of this triangle, the set of vectors which are there, and the set of triangles which are there. So, this (r) is 1, this (f2) is 2, this (q) is 3, and this (u) is what you want to find out, Got it?

$$u = r - f_2 - {}^{R}Q_{F}\,{}^{F}t_{W}$$

So, using that, this is the orientation matrix that just changes the frame above which this vector is to be represented. I want all the vectors to be in the R frame. That is the reason I multiplied it with the orientation matrix of the flange. So, then I have changed this vector to the robot's base flange. That's all. So, now, u is known. So, once u is known, the unit vector along the X axis will nothing be anything but a vector by magnitude of that. So, it is the vector and its magnitude. You got a unit vector along X. Now, I am not changing my orientation. I am just moving in Cartesian space. So, I want to make this a constant once, Every time. I don't need to use a new orientation vector. It is, most of the time, quite convenient to touch all. The locations without changing the orientation. So, if this does not change its orientation, I can make it go anywhere on this plane, As long as it is a flat surface. It is easy to do so. So, I am not changing my orientation, so it remains the same. So, x is now obtained. Unit vector along the X-axis is obtained.

$$\mathbf{x} = \frac{u}{u}$$



Now, in step 3, a point on the XY plane on the workpiece is moved to the TCP of the fix tool. So, now I am making a brush on that surface somewhere over here. This is your fixed TCP. It is making contact somewhere on the XY plane, Not necessarily along the Y axis. It is on the XY plane anywhere. So again, no change in orientation. So I can quickly obtain what vectors. I can obtain it if my frame location is somewhere over there. So, this vector is known. What is this? It is this one, So this is in the frame of F. I have converted it to the frame of the robot, Which is the R frame. So, using this multiplication. So, using this, I have converted it to a robot's frame. So, this vector is now in the robot's frame Done. So, that is known.

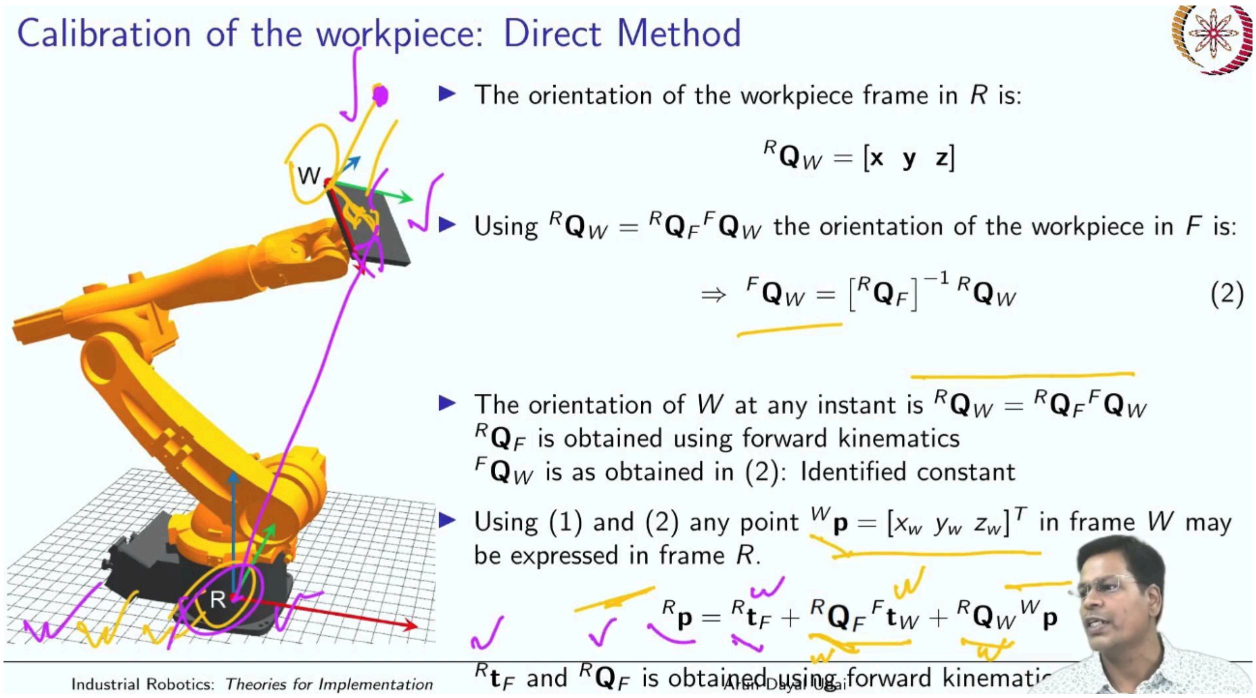
F location, That is, the flange location, Is again known using forward kinematics. You can obtain that. This also is obtained using forward kinematics, But that is not changing, Only the location of the robot is changing. I want to move it to different locations on the workpiece. r is also known. That is, the location of the fixed TCP is also known. So, r is known. f is known, and your t in this frame is known. So, everything is now known. So, again form a similar four vectors, and you can find out the unknown one, That is, the v vector. Vector v. Where does it go? It is hidden inside, so I will redraw it. So, it is this vector. That is. The v vector is now known using a set of all the vectors which are known, So now, once v is obtained, u was already known in the first step. Taking a cross product will give me a vector W which is aligned along Z. So, taking a cross product gave me W.

$$w = u \times v$$

If I take it to make it a unit vector, it becomes a Z vector, that is, a unit vector along the Z axis. It is nothing but W by the magnitude of W. So, this gives you Z. x was obtained earlier. Taking the cross product of X and Z should give you the y vector. So, what is Y? It is this vector.

$$y = z \times x$$

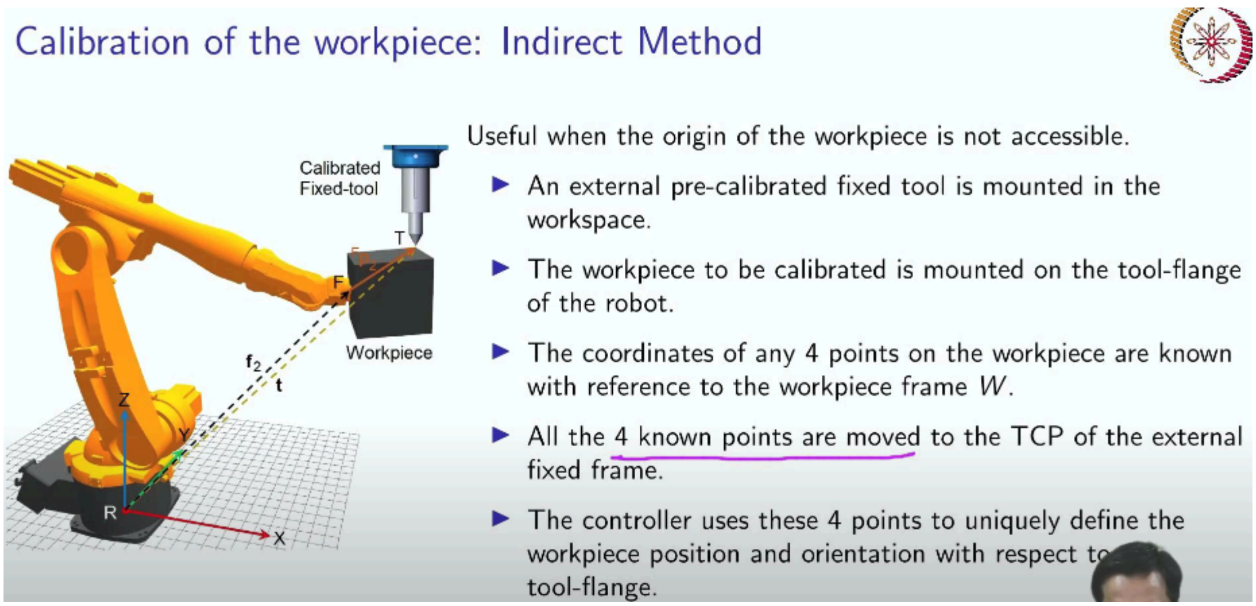
So, all three vectors along the frame, along the frame axis, which is placed at W, are now known.



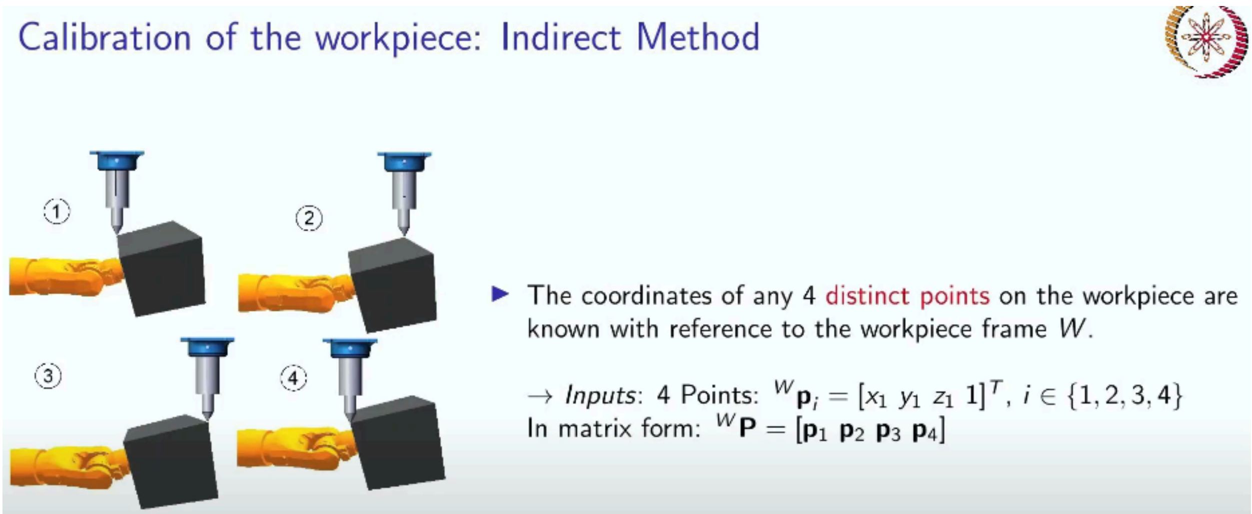
So, that can be expressed like this: All of these are unit vectors. So, the orientation matrix can be written like this because you know all X, Y, and Z were in the robot's frame. I still haven't changed the orientation still. So, those vectors are still there. , So I quickly now know W, QC in with respect to R. This is now known. Now, I will use it to express the orientation of W with respect to the flange frame, which is here. So, I can write the W with respect to R as two rotation matrices. First, we will take it from here to F. That is the first one that is obtained using forward kinematics F to W. From F to W is something that I want to know. R to W is already known. We have just obtained, using X Y, Z. I have obtained this one. So, now, taking the inverse of this, I can obtain F to W. This is the orientation of W with respect to F. It is firmly fitted, so it is not going to change. So, as long as that workpiece is mounted on the flange, it will remain there. It will remain there, and that orientation is not going to change with respect to the frame. The flange is moving, but with respect to the flange, W is not moving. So, this is a constant. So, the workpiece is now defined with two things. First, a vector that connects this in a flange frame and also the orientation matrix that expresses the orientation of W with respect to F. So, everything is now known. So, the orientation of W at any instant while this robot is moving with respect to R can be given as R to F, F to W that is what we have obtained just now.

$$^{R}Q_{W} = {^{R}Q_{F}}\,{^{F}Q_{W}}$$

So, you can obtain the orientation of W with respect to R at any instant of time. This is the orientation, and Q of W is obtained just now we have obtained It is an identified constant. Using one and two, this and the first one, any point which is in the W frame now, that is in this frame now, can be expressed in this frame using this. What does it use? It quickly uses the vector which we have obtained. This vector, this was there in, that vector is used here. It is this one. So, using this, this can be obtained right from here. So, you can convert that to a robot's frame. So, this vector can now be converted to the robot's frame and used over here again. Any point P which is in this frame-, so that also is to be converted to the robot's frame. So, that also is there. So, this vector is also expressed in the robot's frame, this in the robot's frame: T, R to F. So, R to flange, this is already known using forward kinematics. So, you can obtain this very point which is here in the R frame. So, it is just a sum of vectors, and I have converted some constants to the constants which are expressed in the W frame to the robot's frame using some dynamic parameters, that is, tool frame position and robot's tool frame orientation. That is obtained using the robot's forward kinematics. Got it. So, this is your direct method.

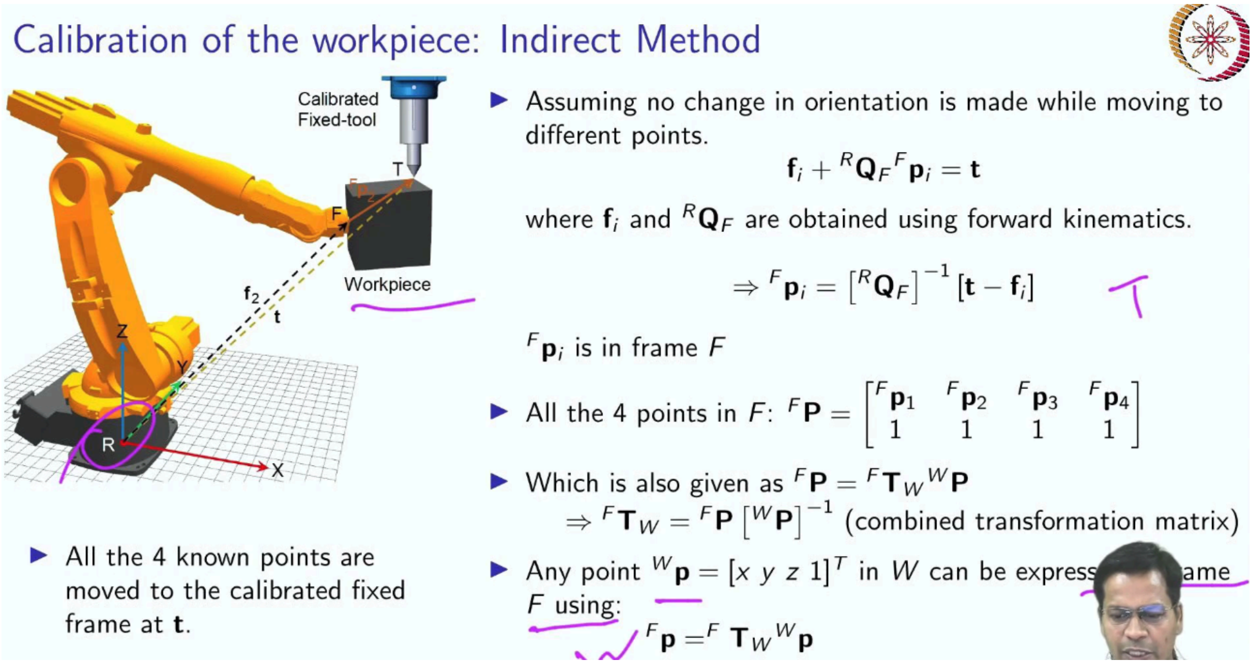


Now, calibration of the workpiece. This is different from a simple workpiece which was a flat one, and it was mounted there. So, we just calibrated that. This one is an indirect method. Why is this important? Because it is not always possible to reach to the origin. What we did in the first step we make contact with external TCP was moved to the origin. If that is not accessible, how can you do that? So, now what you can do is these: the external pre-calibrated tool is already there in the workspace of the robot, and you have the workpiece. Also, it is mounted on the tool flange. So, a workpiece is there, a calibrated fixed tool is there, that is in place. Now first step will be all the coordinates of four points. I have to use four points, which are defined in the workspace frame. So, I have four inputs if it is a surface. So, I know all the points with respect to the frame, which I cannot see. So, those points are well known in this frame, in the workpiece frame. So, that is now known. So, that becomes the input. So, using all those four known points, I am moving all the known points, all the known points. So, I am holding this workpiece now at the tip of my robot, at the robot's flange, and taking it to all four different locations. So, that is what I am doing. Using this, I am able to calculate the position and orientation of the tool flange of this workpiece with respect to the tool flange.



So, how that is done, this is the coordinates of four distinct point-minded. Those points should be very, very different. Otherwise, it will cause some singularity issues because I am going to take the inverse offset of points. So, if those points are flat on the flat surface, it is quite possible that while inverting, it will create a non-invertible matrix that becomes singular. So, you have to be very careful. While choosing those four points, it should be very distinct with respect to the workpiece. So, four distinct points on the workpiece are known with respect to the workpiece itself, and that becomes the input. I am just expressing those four points in a homogeneous matrix way because I know I have to use a homogeneous transformation matrix. I have to find out that homogeneous transformation matrix here. So, I have written it in this way.

Furthermore, I have written all the four points like this, so each point is now x1, y1, z1, 1. Furthermore, I change from 1 to 4 for all the four points. All of them are in a workpiece frame, mind it. Now, I am taking them to a TCP location.



So, this is the job. So, all four known points are moved to the calibrated fixed-step. So, this location I'll take it to. So, that is already known. t vector is known. You see, it is in the robot's base frame. So, that vector is known because it is calibrated. So, assuming no change in orientation again, I am able to take all four points without changing my orientation. Just by making some Cartesian translation motion, I am able to establish that contact. So, that is why orientation is constant. So, for all the points, this remains the same. So, f_i is all known. What is this f_i?

f_i is obtained using forward kinematics. That is the location of the flange Center, which uses forward kinematics. You can quickly calculate that. Also, flange orientation with respect to this can be calculated using forward kinematics. So, t is already known. t is the location of TCP with respect to the robot base. So, t is known using the calibrated tool. f is known for using forward kinematics. Take the difference, bring Q, inverse, and you can obtain what you can obtain: a vector which is in with respect to f. So, that is what you can obtain. So, it is this vector I have obtained here. So, using a vector triangle, I have obtained this. Why do I need this orientation? Here it is. It is just here it is. This one is something which is a constant. That is going to be a constant one because I want to get to the point where I have made contact with respect to the flange. That is a constant that is not going to change with respect to your frame. That is, it becomes the same. It is mounted on the flange. So, with respect to the flange, but point of contact is not going to change, but the orientation of that is going to change with respect to the robot's base. So, multiplying with robots, transformation matrix, and orientation matrix, you get that in the robot's base frame. So, that is why it is done. So, I have expressed I have brought all the vectors in the same frame. That is the R frame. Initially, it was not, it was there in the workpiece frame only. So, I have converted using this. So, this is updated now for all four points in f. Now, I am expressing it as putting one at the bottom. So, if it is XYZ, which is here, I have put it as XYZ one so that this becomes a set of four points, four crosses, and four matrices, and it is homogeneous. It can be handled by a homogeneous transformation matrix set of four points. So, now, what was this?

$$\text{All the 4 points in } F\text{: } {}^{F}\mathbf{P} = \begin{bmatrix} {}^{F}\mathbf{p}_1 & {}^{F}\mathbf{p}_2 & {}^{F}\mathbf{p}_3 & {}^{F}\mathbf{p}_4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

This is all the points in the W frame. That was already known. We knew all the points this is now we have created, and how is that? It is with respect to the flange frame. It is with respect to this. We have all arranged it here, so only those locations are changing. What we want to know is our transformation. That can take me to this. So, what is done here? It is points in the W frame. By multiplying with the transformation matrix, that is, F to W, we get it in the F frame. That is the flange frame. So, now that both point sets are known, I have obtained the combined transformation matrix that can do so. So, it is this transformation matrix which can convert any point which is there in the workpiece frame to the flange frame. , to the flange frame, once you know it. In the flange frame, you already have worked through forward kinematics. It is the transformation matrix of the flange. With respect to robots, the base is already known. That is using forward kinematics, so this transformation is known. This is what is a constant. Once that is formed, all the points are set. So, this becomes a constant. It is a constant transformation matrix between the frame, which is fitted at W, to the frame, which is at F, that is, the tool flange. So, that is constant. So, using that, any point which is in the W frame, that is, the workpiece frame, can be expressed in the flange frame, tool flange frame using.

$${}^{F}p = {}^{F}T_{W}\,{}^{W}p$$

and again, if you have to bring it to the robot frame, you can already use its transformation matrix of the flange with respect to the R. So, multiplying it further with this pre-multiplying with this, you can express that set of workpiece points with respect to this. Now, you can take that point to any location. Got it? So this is what can be used.

So, that's all for today. So, you have calibrated your workpiece. You have calibrated your workpiece, which can be mounted on top of the robot, using different techniques today. You saw direct techniques, and you saw undirected techniques, you saw external fixed frame calibration, and also 5D and 6D methods. So, with this, I'll end for today, and in the next class, I will be dealing with some more external accessories which a robot can have. A robot can be mounted on top of a linear rail, so those are linear units on which you can place your robot. So, a robot can increase its workspace by placing itself on top of that, and now all of that can be served together. So, those details will be discussed in the next class. What, additionally, we will be discussing is a rotary turntable, so we'll be calibrating that also. That's all. Thanks a lot.