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Industrial Robotics: Theories for Implementation
Dr Arun Dayal Udai
Department of Mechanical Engineering
Indian Institute of Technology (ISM) Dhanbad
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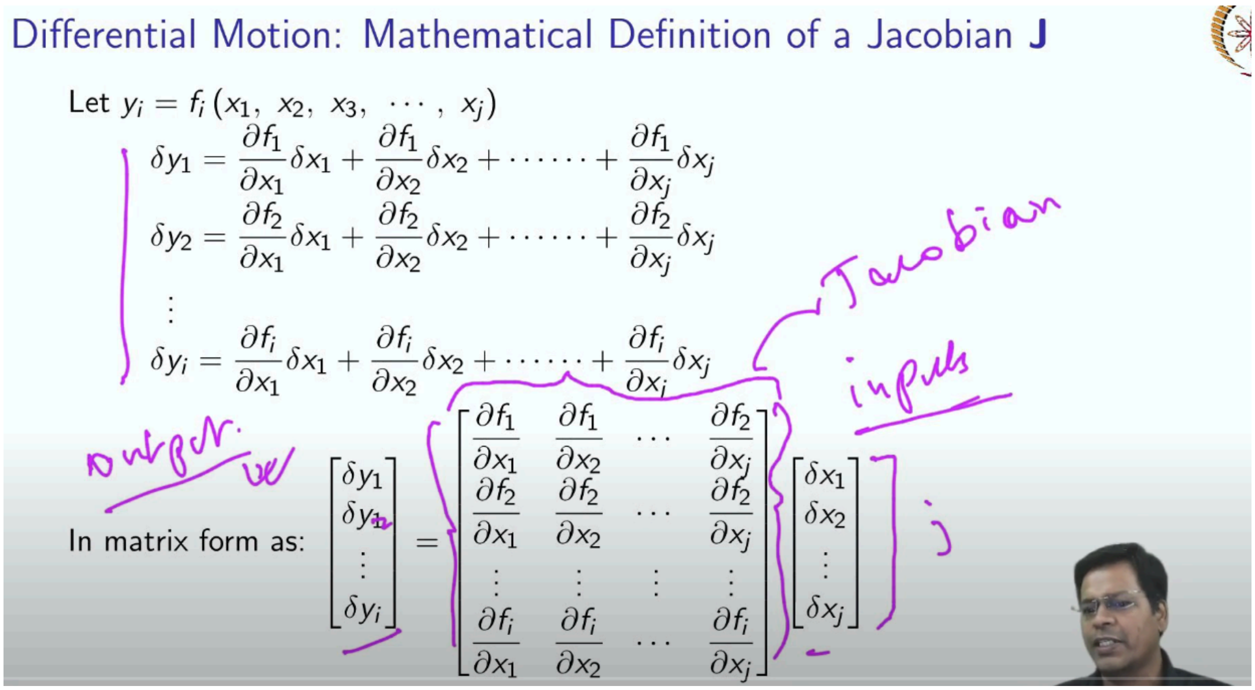
Differential Motion Analysis, Velocity and Robot Jacobian

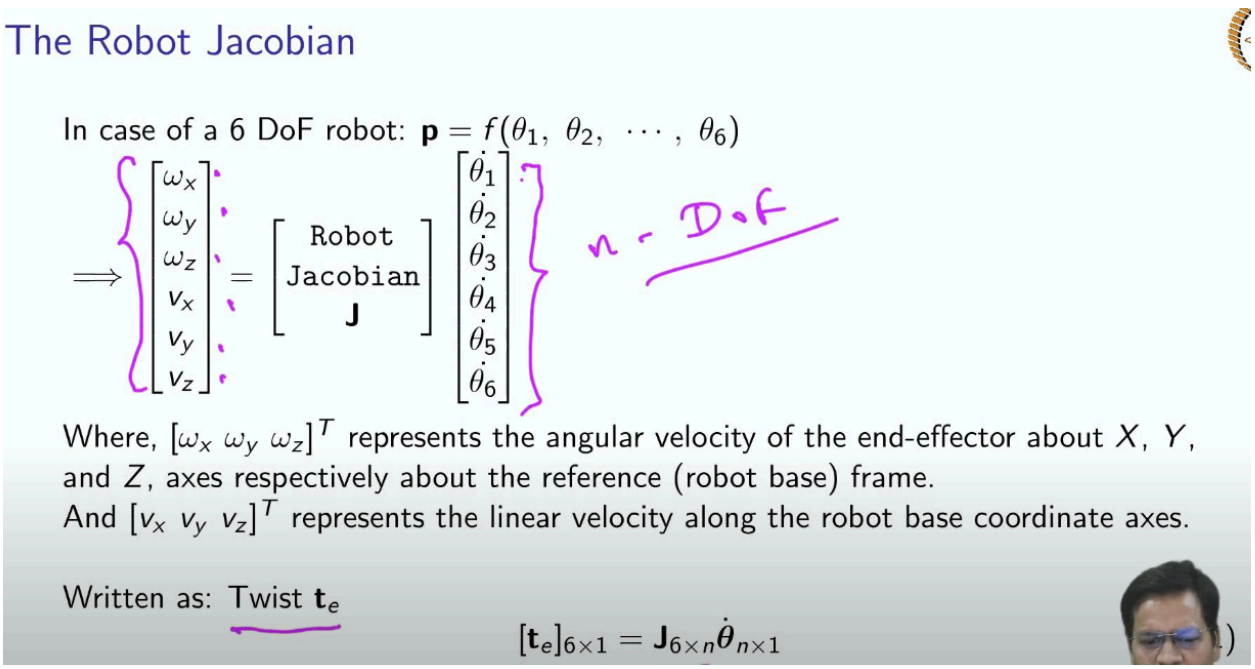


Welcome back. So, in the last class we discussed inverse kinematics, inverse kinematics using analytical techniques. So, we did the inverse kinematics of 2R, 3R, SARA and industrial serial robots. So, most of the tasks that you see in the industry are pick and place type of tasks, when a robot can pick an object from a definite place or a pre-programmed place, and it can put it to a particular location, So that type of task can be easily performed by just programming the robot from point to point, irrespective of the path that it follows While going from point A to point B. it can follow any path. But there are quite a lot of applications when you need to continuously servo the robot. That means continuously. The robot is fed with a new position without having knowledge of the next location, Like, for example, visual serving. You are continuously putting in the robot with a new set of points, and the robot goes there. So, can we do programming for these kinds of applications also using the inverse kinematics approach, or are there a few other approaches that we will discuss in today's class? So, there are a few terms which are to be known before you actually start with that. So, today, we will be discussing differential motion analysis. So, let us start with Jacobian. We will do velocity analysis, So moving ahead.

So, yes, before we actually begin, let us just discuss differential motion, what it is, and the mathematical definition of Jacobian. As such Jacobian is a mathematical term, it has got not much to do with robotics. Robotics is just one of those application.

So, let us just start with Jacobian. So, let us take a function of x, which is y, given by yi, is equal to f_i of x1, x2, x3, and these are the input variables till xj. So, this is your function. If you take the partial derivative of y, you can quickly write it as y1 is the first output variable, and yi is the ith variable, x1 to xj are the j input variables. So, taking with respect to the first output variable. You can write it like this.

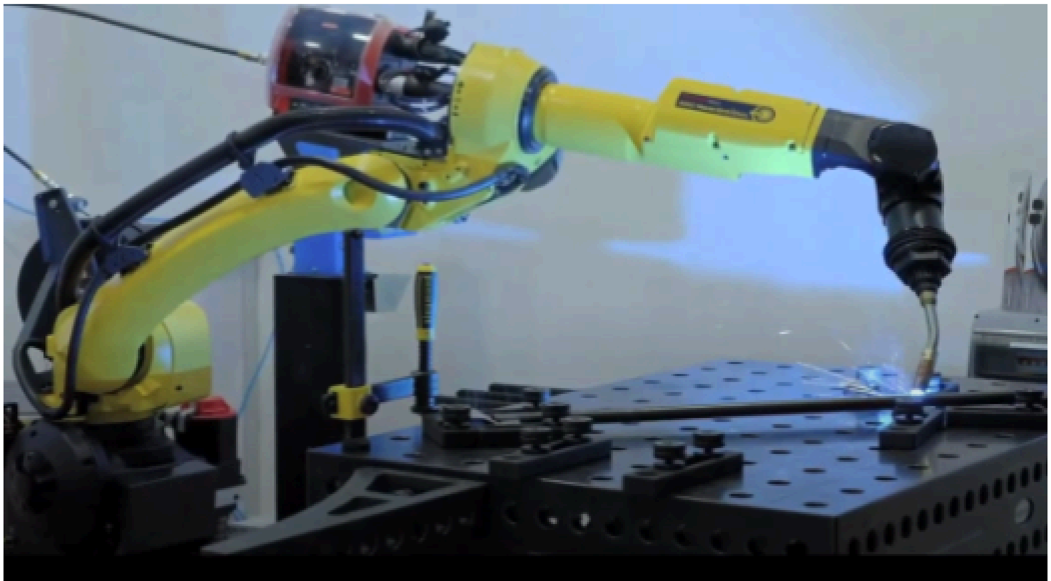
Next, going similarly, you can directly write it like this: If you can pack it together in the matrix form, so the vector of delta y1, delta y2, delta y3, delta y4 and so on, till delta yi. y1 to yi are i different output variables, and this becomes your input variable vector. So, it is delta x1, delta x2 till delta xj. So, they are j input variables. So, you see there is a pack of differentials which are there in the form of the matrix. So, this is a matrix which relates the input vector to the output vector and both are the rates. So, the rate of change of input variables comes here. Rate of change of output variable, that goes here. So, this particular matrix basically relates the input rates to the output rates. So, this is known as a mathematical Jacobian.

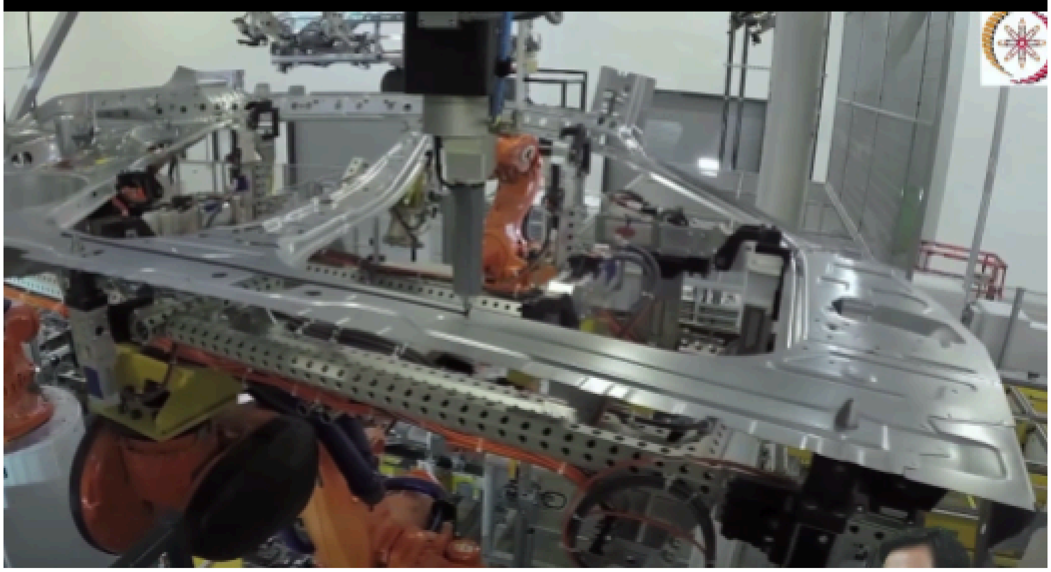


In the case of robotics, let us say it is a 6-degrees-of-freedom robot. So, the end effector position is a function of joint angles. In the case of the 6 degrees of freedom robot, it has 6 joint variables, theta1 to theta6, at least in the case of the revolute jointed robot. So, it is like this: Output is a force, so it is basically position and orientation. So, what we are seeking here is the angular velocity rate of the end effector. These are linear velocities of the end effector. So, this is the input. Joint angle rates. So, input, joint angle rates, this is the input and this is your output, and both are related by a term which is known as a robot Jacobian. So, this is what we are looking for. This is very, very important in the case of robots. You see, there are a few applications that we have seen earlier. Also, I will just repeat them once again.

So, you see, this is an application where a robot is programmed to pick an object from a definite location and put it on the conveyor at a fixed location. You see, there are n number of robots. All can be programmed similarly. So, this is a pick-and-place operation. There are many similar applications which you see which find similar programming techniques. When you are not bothered about the intermediate points, you are just concerned with the picking spot and the placing spot. A similar one, you see, is when a robot is picking an object from a fixed location and hangs it onto a moving conveyor. So, these types of applications- they don't need any robot Jacobian in between.

That may be required for control purposes, but definitely not for the programming, the path that is achieved in this type of task. This can be easily achieved using an inverse kinematics-based technique. When you know a location, you just find out the joint angles. You know the other location. You find out the joint angles. Take your joints from this to this without any collusion in between that you can verify while checking your program by pre-rearing your program before actual execution. So, that is one of the ways that you can do this kind of task.

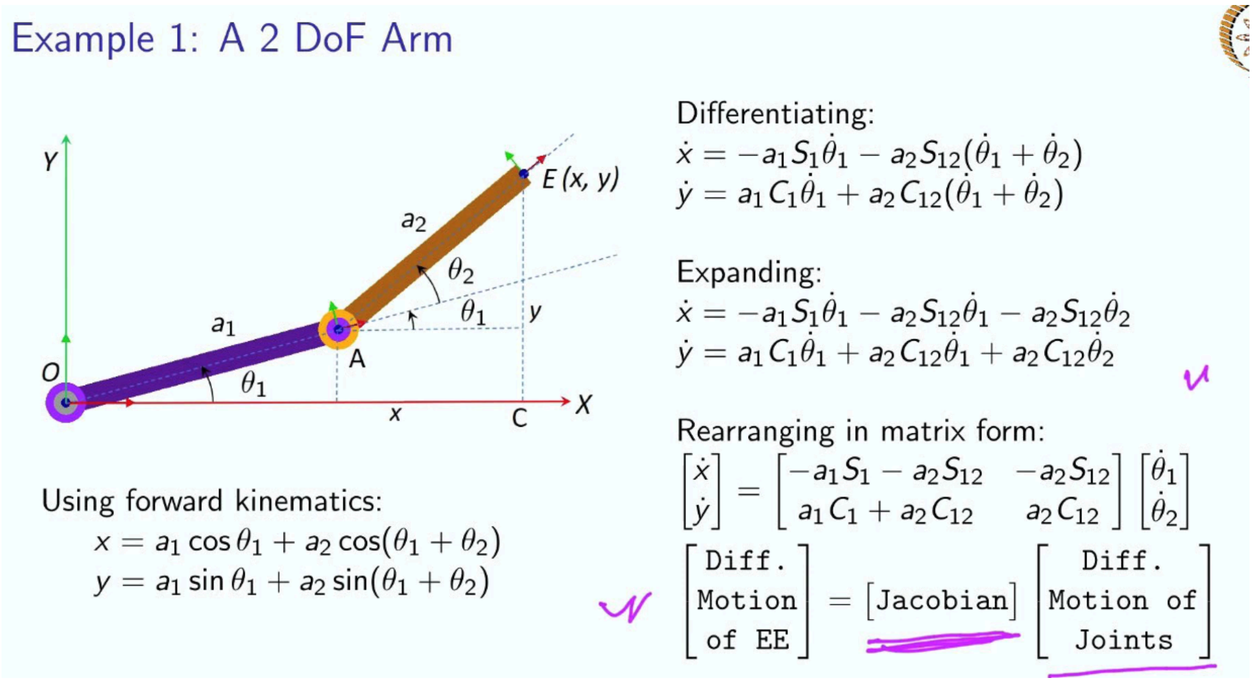




Now, let us see a different type of task. So, this is a welding task, you see. It is continuously moving along a path, and it deposits the weld. It continuously moves. So, what you see here is the robot continuously deposited the material, If at all. It has moved with different velocities in between a lump of material will be deposited in a particular place. So, while moving along the path, it has to move with uniform velocity. Now similar was the task, which is a glowing task. I will show you that. You see, the robot is continuously depositing the adhesive along the path, and if at all, it stops or goes with a different velocity. You will find adhesive getting deposited at a particular location. It is quite uniformly moving along. These are a few applications.

What you see is of a special kind when a robot is required to move in a Cartesian workspace at uniform velocity. So, yes, now coming back to my slide, when I wanted a relation which can relate the joint rates to the end effector rates. So, is there any such relation that exists? Can we find out that? So, this will be known as robot Jacobian. So, this is known as a twist vector, which is here. We will call it as twist, which is a 6 cross 1 vector and input over here will be n cross 1, whereas n is the degrees of freedom of the robot. In the case of a 6-dof robot, it is six variables which should be input. So, we are looking to find out this Jacobian matrix.

$$[t_e]_{6\times1} = J_{6\times n}\,\boldsymbol{\theta}_{n\times1}$$



So, let us proceed with our standard 2R robot. What we know here is X. that is the end effector position. X for this was a1 cos theta1 plus a2, cos theta1 plus theta2.

$$x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$$

Y is equal to a1, sin theta1 plus a2, sin theta1 plus theta2.

$$y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)$$

So, this was the position forward kinematics, and this is the end effector position given by E(x,y). a1, and a2 are the link lengths. theta1 and theta2 are the joint angles. So, by differentiating these two equations, what do we get? We get X dot is equal to this.

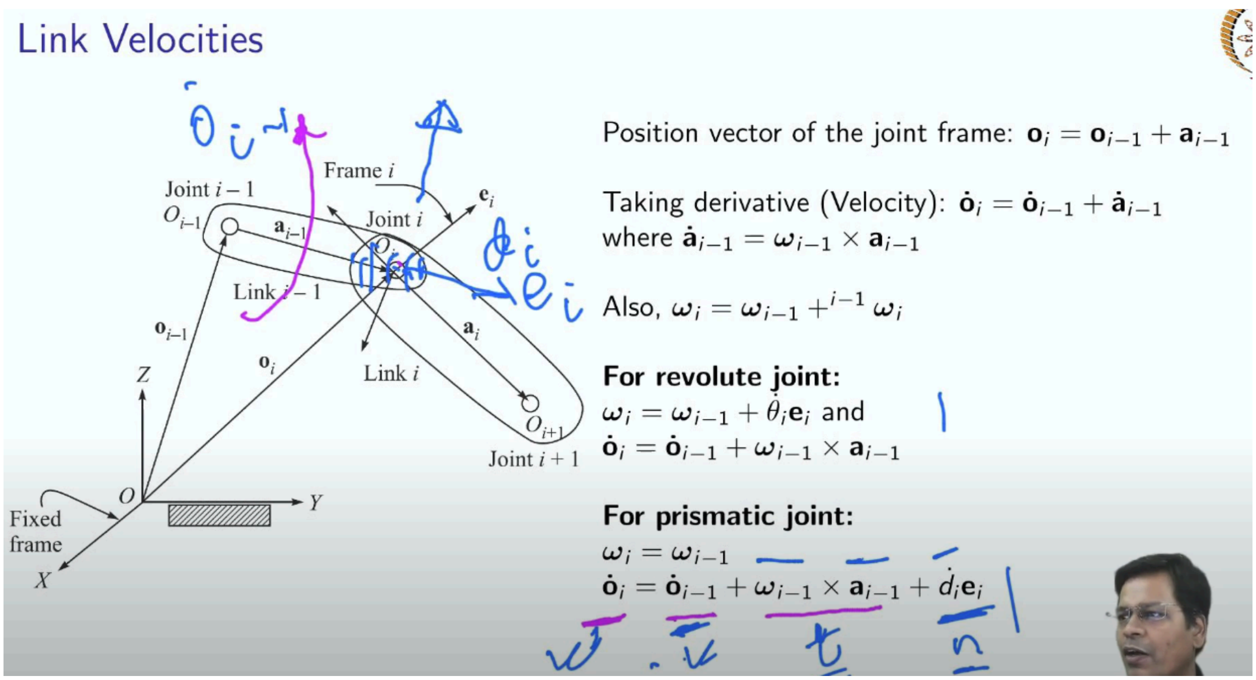
$$\dot{x} = -a_1 S_1 \dot{\theta}_1 - a_2 S_{12}(\dot{\theta}_1 + \dot{\theta}_2)$$

Y dot is equal to this.

$$\dot{y} = a_1 C_1 \dot{\theta}_1 + a_2 C_{12}(\dot{\theta}_1 + \dot{\theta}_2)$$

So, you find theta1 dot, theta1 dot, theta2 dots, and theta2 dot all over here, whereas sin theta1 is here cosine theta.1 is here cosine theta1 plus theta2 is here sin a theta1 plus theta2 is here. So, it contains both, that is, the joint angle and joint rate. So, this is the relation that we could obtain by differentiating the forward kinematics equations

So, expanding that I can quickly obtain this If I can pack them together by rearranging them in matrix form. So, what I see is an X dot and a Y dot that is the output matrix. What it is, This is basically the twist matrix which says velocity of the end effector along X, velocity along Y is clubbed like this and this becomes here a matrix which is of 2 cross 2 type. You have a matrix here, which is the input joint rates, and within the matrix, all the joint angles are gone into sine and cosine variables, which are inside. So, it is a 2 cross 2 matrices, If at all you have more than 2 variables as input. So, this matrix will take the shape accordingly. Output: in the case of a planar robot like this, you have just X rate and Y rate. So, it is exactly this. So, This will call it as the differential motion of the end effector is related to Jacobian to the differential motion of the joints And both of them are related by Jacobian. So, this is what we are looking for. Can we derive this for n degree of freedom robot? Is there a better way of doing it? Because doing it in a way like this is very difficult for, let us say, n degree of freedom robot when you have to take derivative, take the common and get it to this kind of result. So, there are various elegant approaches to do this so we will talk about that now.



So, let us do a velocity analysis first. So, the position vector of any joint frame, OI, is given as OI minus 1 plus AI. What are these terms? Basically, o_i is the joint i vector that takes you from the reference frame o to oi. That is the with joint, which is there, and this is your link, i minus 1, which is here. And you have another frame, oi minus 1, so you can draw this vector. Now It is like this. So, it is vector oi minus 1. And this is your link. Length vector for i minus 1st link. So, that is what is written using this vector triangle like this: oi is equal to oi minus 1 plus ai minus 1. So, this is your position vector for the frame. Now, taking the derivative of this. So, the oi dot is equal to OI minus 1 dot. That is the dot of this. That is the velocity relation plus ai minus 1 dot.

That is the rate of change of link length. Vector ai minus 1 dot is equal to omega i minus 1 cross ai minus 1. How do you know? You see, you have a vector, which is ai, Got it.
We know that the magnitude of this is constant. That is the link length between the frame, OI and oi minus 1. But a vector can be changed in two different ways. First, by changing its magnitude, it can also change its direction. So, two different velocities. It can acquire One along the direction which is along the axis of this vector. So, this could be one velocity if a is changing in magnitude. But there is another velocity if this is rotating with an angular velocity, omega. So, an ai is its magnitude. So, you can take the cross product of these two vectors, and you can find an orthogonal vector which is like this: So this becomes your tangential velocity. This is your normal or axial velocity. This is your tangential velocity. So, this is the direction of this vector, and this is due to the rotation of this link. And even if this is absent, this is always there Because your joints are moving.
So, if it is rotating, you see this vector. And also you have omega i is equal to omega i minus 1 plus i minus 1 Omega i. What does it mean? This link, with the link, is riding upon a link which itself is rotating by some angular velocity. So, whatever is the angular velocity of the link which is prior to the with link, that is, omega I minus 1, this becomes your angular velocity of the with link. In addition to that, you also have an angular velocity, which is a relative velocity. This is the relative angular velocity of ith link with respect to i minus 1st link. Let us say you have a motor which is fitted here.This will rotate this link with some angular velocity. So, this becomes the angular velocity of ith link with respect to i minus 1st link. So, this is your relative velocity, and this is the velocity with which the link which is prior to this is rotating, on which ith link is also rotating along with that. So, this is your total angular velocity of the link.

$$\omega_i = \omega_{i-1} + {}^{i-1}\omega_i$$

So, using both of these, now you can write for re-volute: joint: omega i is equal to omega i is equal to omega i minus 1. Omega i minus 1 plus this is your relative angular velocity. You know this is rotating at an angular velocity, which is given by theta i dot. This is the magnitude of that. What is the direction of that? Let us say e_i is a unit vector above which this link is rotating. So, that may be your z-axis that is placed at the joint. So, this is your ei vector. So, if that is a unit vector, if you multiply that with your magnitude, what do you get? You get omega i i minus 1. Got it. Both are the same. So, I have written it out here is this. So, this is the first relation is your angular velocity relation, whereas the second one is your linear velocity relation that comes from here. So, the y dot is the linear velocity of the frame. That is, the frame which is over here is equal to the linear velocity on which this link itself is riding. So, this is your linear velocity and the velocity which is generated due to the angular velocity, Omega i minus 1. So, that is what we have seen.
So, ai minus 1 dot is equal to omega i minus 1 cross, ai minus 1. So, this, we have simply put it here, Got it And we got to this relation. So, this is quite trivial. In the case of the revolute joint, it is very easy to understand.
In the case of the prismatic joint, you know, there is no joint which is here which is able to rotate like this. So, this is absent in the case of prismatic joints. So, omega I is the same as that of

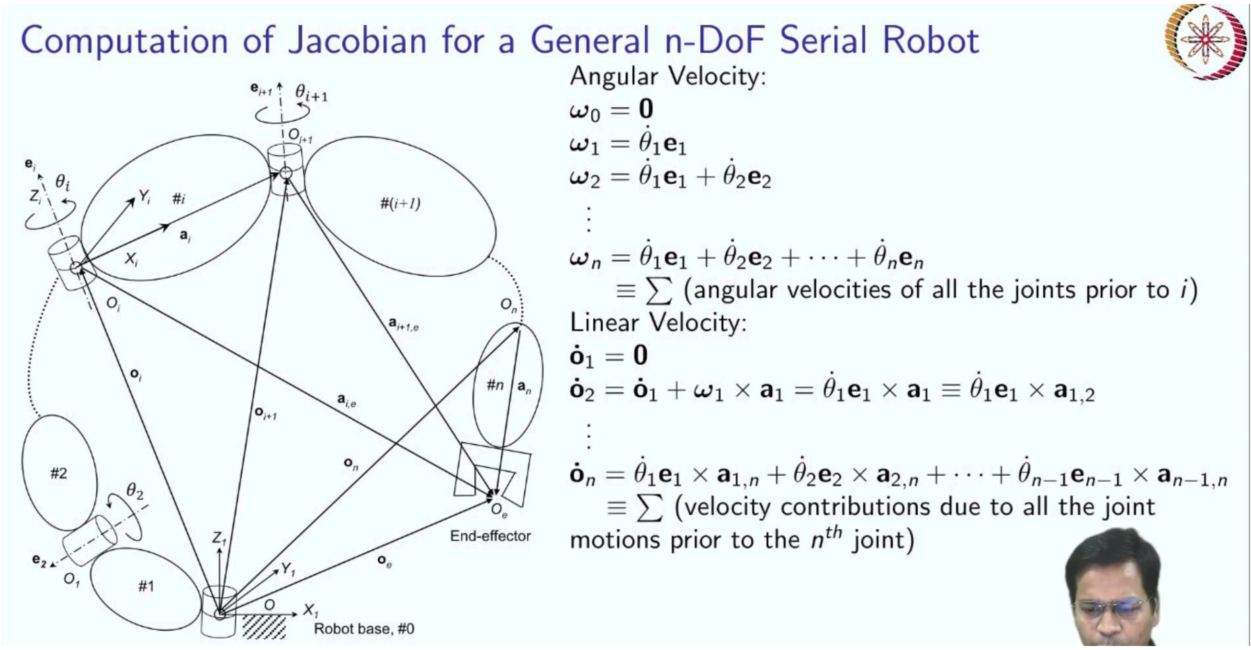
whatever is the angular velocity of I minus 1st link. So, whatever is the angular velocity of this, the same will be the angular velocity for this link as well, if it is a prismatic joint.

Now let us look at what would be the linear velocity in that case. So, linear velocity will be again similar to this. You will find the oi dot is equal to the oi minus 1 dot. This is the velocity of the previous link and there are two components this time. This is due to the rotation of the link which is prior to this. So, if this joint is frozen, this is frozen. So, what will you see? It is ai minus 1, taken cross-product with omega i minus 1. So, whatever is the velocity that comes at this point. So, you have to add this. So, this is your tangential component of the velocity that comes to oi. This is the tangential component.

$$\text{Tangential component- } \omega_{i-1} \times a_{i-1}$$

$$\text{Axial Velocity} = d_i e_i$$

This is the component which is due to the velocity on which it is riding. So, oi minus 1 dot. So, this is the velocity, this is the velocity which is here. Apart from that, this is the tangential component and this is your axial velocity term. In that case, you have a Z axis that is along this direction. Whatever is your Z axis, let us say this is your e_i in that case. And you are changing di. You know, in this parameter, you know it is di that is the offset along the Z axis. That is what is changing in the case of prismatic joint. So, what happens? This is changing. So, you have a velocity which is exactly like this: This is due to the change of di, which is along ai and the direction of that will be along the axial direction or the normal direction. So, there are two velocities: The first is due to the angular velocity with which that link is rotating, and the second one is due to the rate of change, of extension of that link, extension or retraction, Whatever it is. So, it is along the axis of that ai, That is, the axis of the prismatic joint. So, these are the two. So, the sum of all the vectors, tangential, normal and relative velocity, which it is acquiring because of the earlier joint motion. So, all these together will give you the Y dot. So, this is your prismatic joint. So, these two relations are very, very important in order to find out frame velocity and the angular velocity of the link. This is very much utilised in order to find out Jacobian later on for n degrees of freedom robot and also while we will be doing energy analysis in case of dynamics later on.



So, let us start computing Jacobian for a general n degree of freedom serial robot. n can be 6, and n can be anything. So, let us first understand the structure of this link. So, this is your link 1, which is attached to frame 1, where you have an axis of rotation. theta1 is the angle with which it is rotating. This is attached to the frame oi O_1, which forms the axis of rotation. For the second link, theta2 is the angle. E2 is the axis about which this angle is measured. So, this is basically your Z axis, which is normally in the case of the DH parameter. You know, all the Z axes are placed along the axis of rotation or displacement. So, exactly, these are the Z axis which are there for that local frame. And then your second link comes like this, and the same goes on. Now you have ith link, which is mounted on the frame, is oi, and you have theta I, which is measured here, that is, along the zi axis, you know, and ei is the unit vector, which is along the axis of theta i, about which this link is rotating. So, this vector, which we will call as oi vector, basically connects, that is, the position vector of the oi frame, which is attached here. This is your With link, and this is your oi, which is the frame. And then you have oi plus 1, which goes here. You have ei plus 1, which is the axis of rotation about which theta i plus 1 is measured, and this link is also rotating. What is this link? This is i plus 1-the link, and so forth, till the last link, which is the nth link of link length An. This was the link length ai frame here is O_n, and the end effector frame will also have a position vector now because I am concerned about the end effector now. So, we will attach a position vector here also O_e. So, you see, angular velocity 0 is the base velocity on which the whole of the robot is fitted. So, that is equal to 0. So, omega 0 is always equal to 0.

Now you have the first axis about which the first link is fitted, so you can write it as omega1 is equal to theta1, dot. e1. e1 is somewhere over here, got it somewhere over here. That passes like that, and theta1 is the angle along that vector, the ei vector.

$$\omega_1 = \Theta_1 e_1$$

So, that becomes your omega1. So, all the angular velocities can be written similarly. So, what will be the velocity? omega2 is because of 2 rotations. First because of relative rotation, that is, theta2, dot. e2, that is here, and there is one more rotation which was given to the first link. So, because you have a link, you have another link. So, this link, the next link, will automatically have the angular velocity with whatever is there, with the link which is before that. So, this is the reason why this velocity is always a part of omega2. So, this is your omega1 plus this relative angular velocity.

$$\omega_2 = \Theta_1 e_1 + \Theta_2 e_2$$

So, similar could be the third one, omega3 would be omega2 plus theta3, dot e3- got i.t. So, this will be your omega3 and omega2.

$$\omega_3 = \omega_2 + \Theta_3 e_3$$

The whole of this will come here, got it? So this is how we will keep on adding that, So effectively what you will get omega n that is the last link. Angular velocity will be the sum of all the angular velocities till this point, So it is the summation of angular velocities of all the joints prior to i. So, I am taking it till i. So, whatever the angular velocity of all the links prior to, i will be giving the angular velocity to this link.

$$\boldsymbol{\omega}_n = \dot{\theta}_1 \mathbf{e}_1 + \dot{\theta}_2 \mathbf{e}_2 + \cdots + \dot{\theta}_n \mathbf{e}_n$$
$$\equiv \sum \text{(angular velocities of all the joints prior to } i\text{)}$$

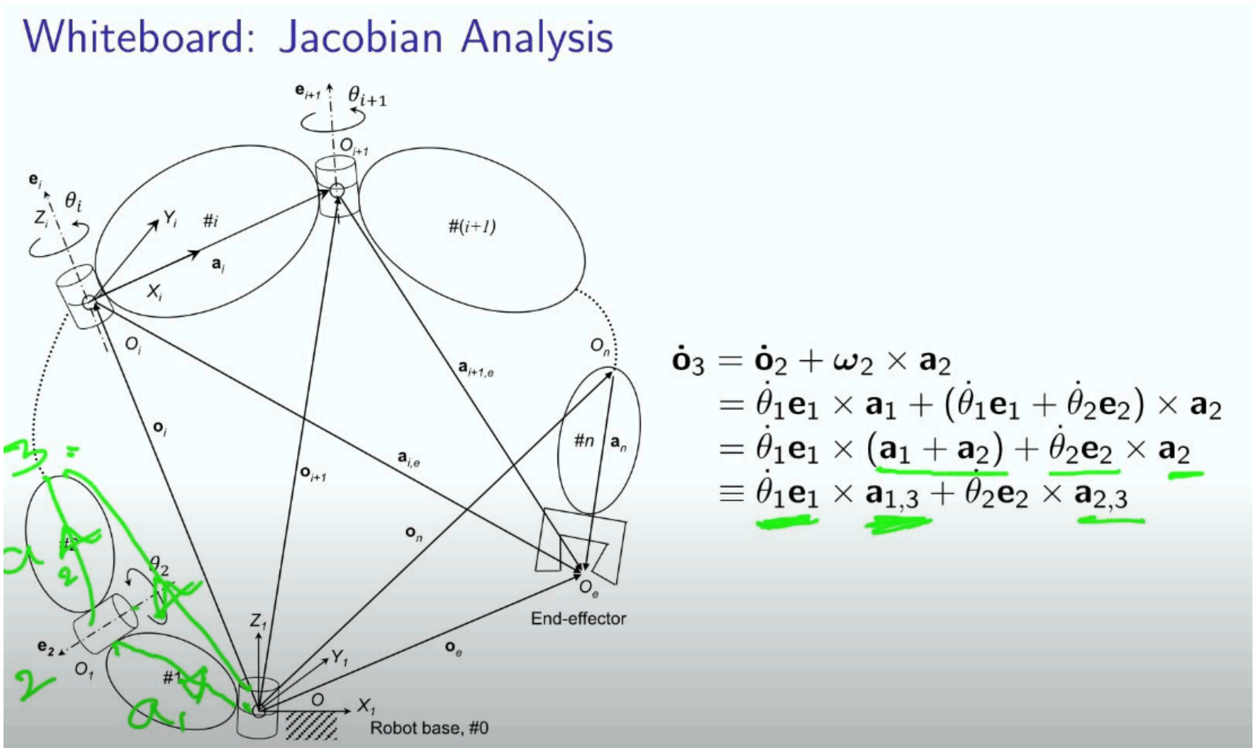
So, if this has omega i, all the omegas that are here will be added together to give the angular velocity for this link. We will simply ignore whatever is coming next to us. This does not influence this.

Now, let us talk about linear velocity. What is the o1 dot? Again, because it is attached to the ground about which your link one is moving. So, this is equal to 0. Quite trivial, very easy.

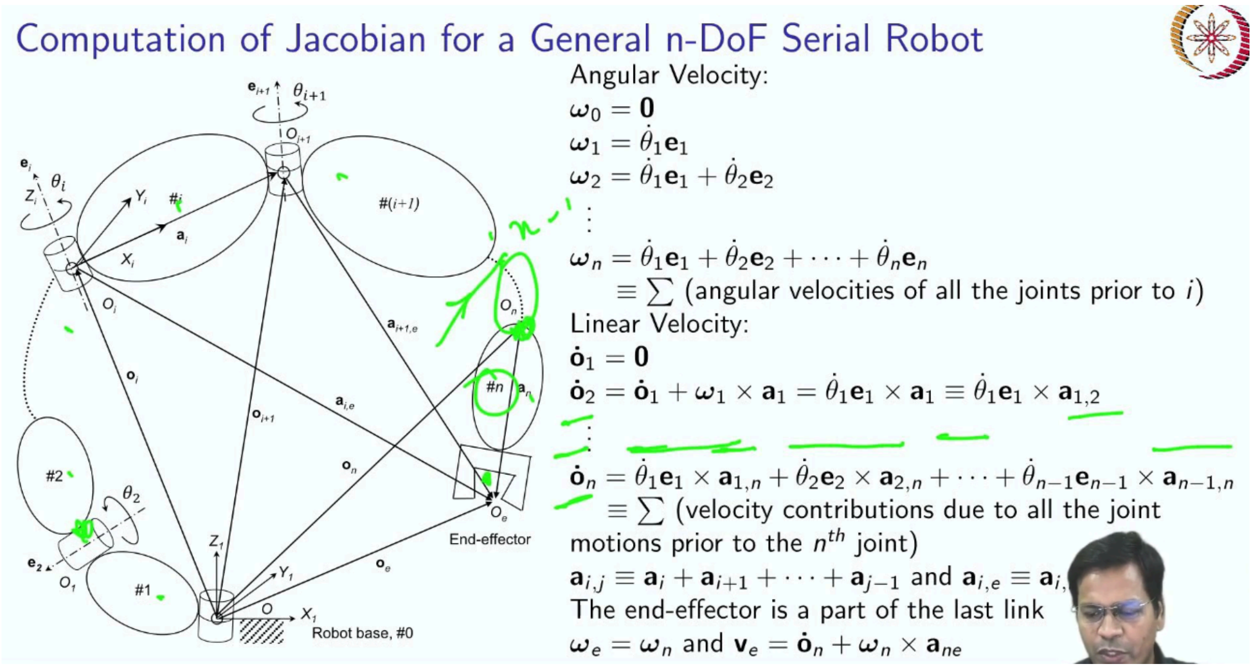
Now, what will be your o2 dot? Where is your o2? O2 is one dot plus omega1 a1. So, if this is your link, length, a1. So, there must be a vector which connects A1. This is you're a1 omega1 is the angular velocity of this, so this gets added on. So, this is what. This is the tangential velocity vector. So, this is your velocity of o1. So, that is 0. You know it is attached to the ground, so there is no velocity over here. So, whatever this is, you can quickly write it as this. So, it becomes simply theta1 dot e1 cross a1. So, this is equivalent to theta1 dot e1 cross a1, 2.

$$\dot{\mathbf{o}}_2 = \dot{\mathbf{o}}_1 + \boldsymbol{\omega}_1 \times \mathbf{a}_1 = \theta_1 \mathbf{e}_1 \times \mathbf{a}_1 \equiv \dot{\theta}_1 \mathbf{e}_1 \times \mathbf{a}_{1,2}$$

So, we will call it a1. So, a1 is the distance it starts from 1 and goes till 2. So, if you take it all together, it becomes very much like this before we come to this point.



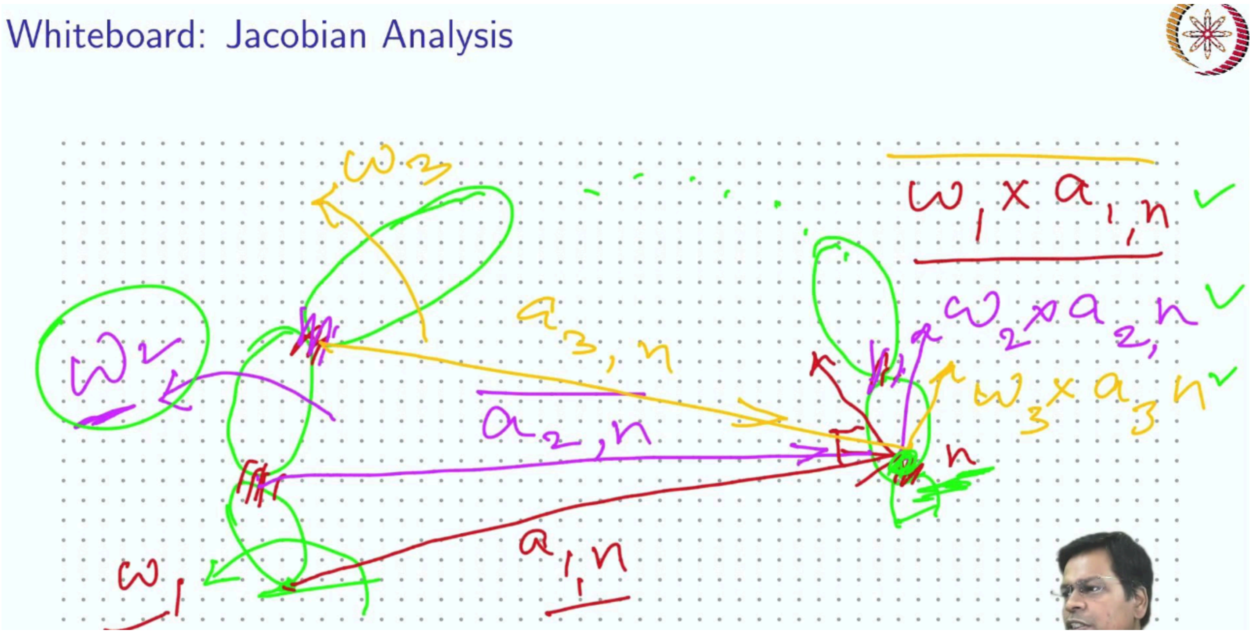
Let us discuss it differently. For any joint, let us say, if it is 3, the o3 dot will be the o2 dot plus omega2 a2. So, let us say this is your third link. This is your o3 vector. So, this is your o2 vector. And then you have a vector which is a2. So, this is your second link. At the end, you have o3. Before that, you have o2. So, velocity over here will be simply an o3 dot that is the sum of the o2 dot velocity over here plus whatever is the resultant of angular velocity that comes to this location. So, what is that? If this is omega2, this is an a2 vector. So, the cross-product of those 2 vectors will be reflected over here as a tangential component at location 3. This is the normal component at location 3, but because you know any normal component will be absent because this link length is not changing with time, So you have only this component, the tangential component, and you have already given the base of this link to a velocity of v2. So, that is o2 dot. So, it is the sum of these 2 vectors. Is the resultant velocity at this point? Got it? Now, can I put it like this? So, what is this o2 dot? O2 dot can simply be written as this from your earlier derivation that you have obtained. So, the o2 dot was because of the rotation of the link 1. So, you had e1, you have theta1 dot, that is the angular velocity. eI is the vector along the z1, which is the first axis on which link 1 is placed. So, this was there. This we have already derived. Now, can I replace this omega2 with these 2? You know, the angular velocity, for the second one will be the sum of the angular velocity of the first plus whatever is the joint velocity which is here. So, the sum of all the joint angular velocities put together till the second will come here. So, this is your omega2 cross, whatever the length of this link. So, this is your a2, this is youra2, got it? So that comes here. So, now I will simply rearrange that. I have put together theta1 dot e1, that is there here, that is here together and cross product with a1 plus a2. That goes to this bracket and the last term which is remaining is this: that comes here. But it is simply rearranging that. So, now you got what. So, what is this? theta1 dot e1? That is, as it is, a1 plus a2. What is that? So, this is your a1 vector, this is your a2 vector. So, the sum of this basically came here, the sum of those two. So, it is what it is a1 to 3. So, from here, this is 2, this is 3. So, this is the final vector, that is obtained. So, a1,3, it takes you from 1 to 3. So, that is the vector which is here. This remains as it is. I have simply put this as a2, as 2 to 3. It takes you from 2 to 3 points. So, this is it. This is nothing. But a2 got it. So, this is your O3 dot.



Now, let us come back to our original slide. So, what you see here, O2 was arranged similarly. O3 we have just derived. So, what is effectively On? If you take it further till n, you will see it is theta1 dot e1 cross 1 to n. So, 1, 2, 3, 4 till n, we have reached till here. So, this is your vector. So, this is your first term. The second term is from here till the nth frame. If it is taken till n, it has to be till n, 2 to n.

$$\dot{\mathbf{o}}_n = \dot{\theta}_1 \mathbf{e}_1 \times \mathbf{a}_{1,n} + \dot{\theta}_2 \mathbf{e}_2 \times \mathbf{a}_{2,n} + \cdots + \dot{\theta}_{n-1} \mathbf{e}_{n-1} \times \mathbf{a}_{n-1,n}$$
$$\equiv \sum \text{(velocity contributions due to all the joint}$$

Similarly, 3 to n, 4 to n. Finally, you will get n minus 1 to n. So, you have this vector. You must be having a link which is prior to this. That becomes your n minus 1 frame to n. This is the frame. So, that also will come. So, when you have to consider till n, you will see this particular equation.

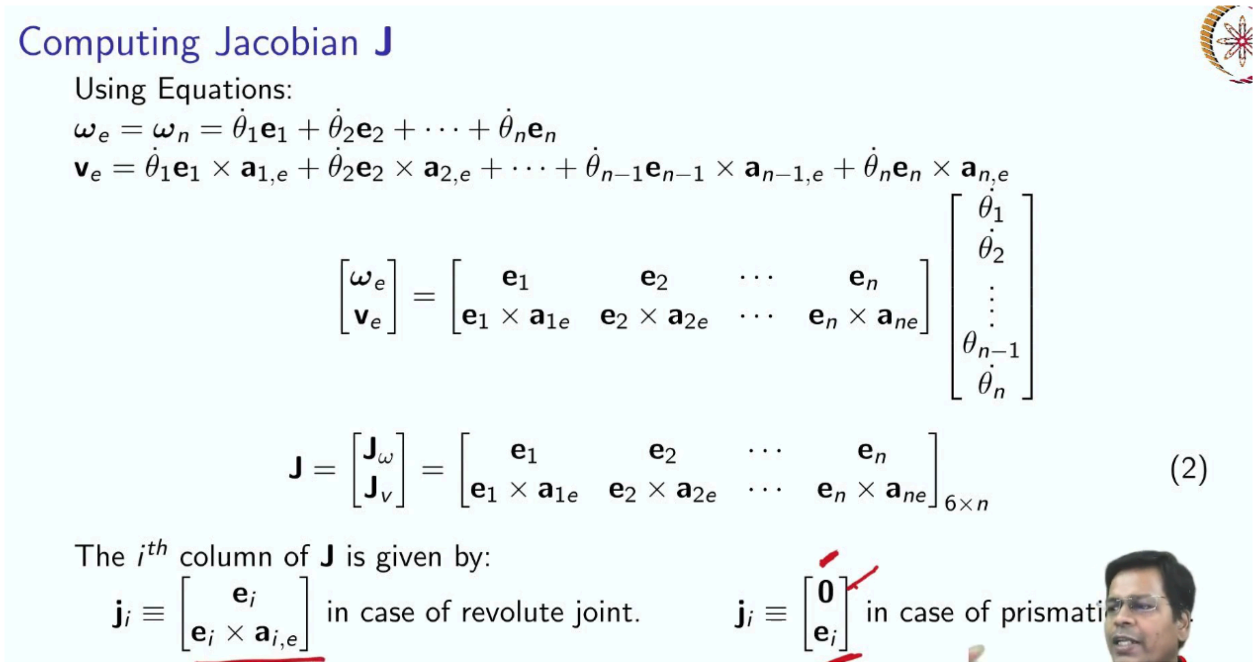


So, what effectively do you understand here? So, I will just draw it once again. I will make you understand now. So, let us say you have a robot, which is a serial chain robot, which is connected like this. So, you have n number of links which are there. There is n minus 1. Finally, n even you have an n vector, which is here. This is attached to a frame. So, this has some velocity. Let us say you have frozen all these. You have frozen all these. Note all the joints you have frozen. So, what will you see? This? If this is your angular velocity, 1. So, you have a distance which is like this a1n. If this is your nth frame, so 1 to n. So, this is the distance. So, you will see a velocity arising due to omega1 will be omega1 cross a1n. What is a1n? So, that is the velocity component due to this angular velocity. Now, let us do the second one. Now, I will take this motion: omega2. Again, you consider all these as present ones. So, omega2, because it is rotating, from a distance from here till n, will be a2n. So, this is the vector of the nth frame from the second, and this is your angular velocity. So, the component due to this will be omega2 cross a2n. So, you have two velocities. First, something like this, you will see, that is normal to this, and the next one is normal to this. The second one, and then you may be having yet another one. Let me do it once more. So, this link is also moving Omega 3. You have a third velocity vector, which is a3n. The third component will be omega3 cross a3n, and that comes here again, perpendicular to that. So, whatever is the nth frame velocity, you see it has components of all these components. So, all the omegas have a contribution to the velocity which is seen here.

So, this is what is reflected here. This is your first term, this is your second term, this is your third term, and so forth, till the nth term, n minus 1st term. But all these will have an effect in the end. So, what is this? This is just the distance, what we have just understood. Now, it connects i to the jth frame. So, these are all the links long which are added together to reach here. So, this is there. So, this was. This is explained here. Omega e is equal to omega n. You know, the last frame, which is fitted at the end effector frame, Omega n, is fitted at the nth link. So, omega n is equal to omega e, But in this case linear velocity will be velocity over here plus omega n, which is here omega n, times of a_{ne}. So, n to e you have a vector an e.

$$\boldsymbol{\omega}_e = \boldsymbol{\omega}_n \text{ and } \mathbf{v}_e = \dot{\mathbf{o}}_n + \boldsymbol{\omega}_n \times \mathbf{a}_{ne}$$

So, this vector comes here. So, that will give you the resultant velocity, Just like the previous one. This is your velocity.



So, now both the equations, the angular velocity equation and the end effector velocity component, can be written like this, As we have just derived. So, the angular velocity at the end will be the sum of angular velocities. The velocity of the end effector will be theta1 dot e1 times a1e. Similarly, a2e, an1e, a_{ne}. So, all the contributions will be added together till the end effector. You reach till this.

So, these are the two equations arranged together you can write it as omega, that is, the angular velocity and the linear velocity, like this in the matrix form, And the top one will have these are eI vector, e1, e2, e3. What are these? Basically, These are the z-axis that we normally place at the joint, Got it? So, these are nothing but this axis. And what are these? These are cross products that are this one, this one, this one, all taken, extracted out. Finally, you have a joint angular velocity vector, which is here. So, this is the final derivation that you were expecting. So, this relates the end effector rates to the joint rates. So, this is your Jacobian.

$$\begin{bmatrix} \boldsymbol{\omega}_e \\ \mathbf{v}_e \end{bmatrix} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_n \\ \mathbf{e}_1 \times \mathbf{a}_{1e} & \mathbf{e}_2 \times \mathbf{a}_{2e} & \cdots & \mathbf{e}_n \times \mathbf{a}_{ne} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_{n-1} \\ \dot{\theta}_n \end{bmatrix}$$

So, your Jacobian has two components. Now, First is your angular velocity components. These and the bottom one, this one will be known as the linear velocity component. So, you have j omega and Jv. So, this is basically of the dimension 6 cross n. Why, you know, This is what these are: all vectors of 3x1. So, if it is e1, it can be 0, 0, 1, or maybe some other three terms like that. So, this is 3 and similar to Y, this is 3, column of 3. So, 3, 3, it is 6 and n dimension robot, so it is 1, 2, till n. n degrees of freedom robot, so it is till n. So, this is the dimension of Jacobian. So, in the case of the revolute joint, it is with the term. Look like this eI, eI, cross aie.

$$\mathbf{j}_i \equiv \begin{bmatrix} \mathbf{e}_i \\ \mathbf{e}_i \times \mathbf{a}_{i,e} \end{bmatrix} \text{ in case of revolute joint.}$$

So, with term will look like this: any particular one, you can find it out. And in the case of the revolute joint, you see there is no angular velocity component that will go. So, that is making this term 0. And you know, velocity was only. It is not the tangential level, it was along the direction of the axis. So, it makes this like this in the case of the prismatic joint, Got it? So, this is the general expression of Jacobian that you need to remember. We will use this expression in the next class and move ahead further.

So, we will do a Jacobian of 2R manipulator 2 degrees of freedom, planar manipulator, exactly the one that we did, simply by taking a differential. We will use this particular relation, and we will try to find out the Jacobian. We will see why. Jacobian inverse is required and how to go for it. We will understand what Singularity is, and we will see acceleration analysis also. That is all for today. Thanks a lot.