NPTEL Online Certification Courses
Industrial Robotics: Theories for Implementation
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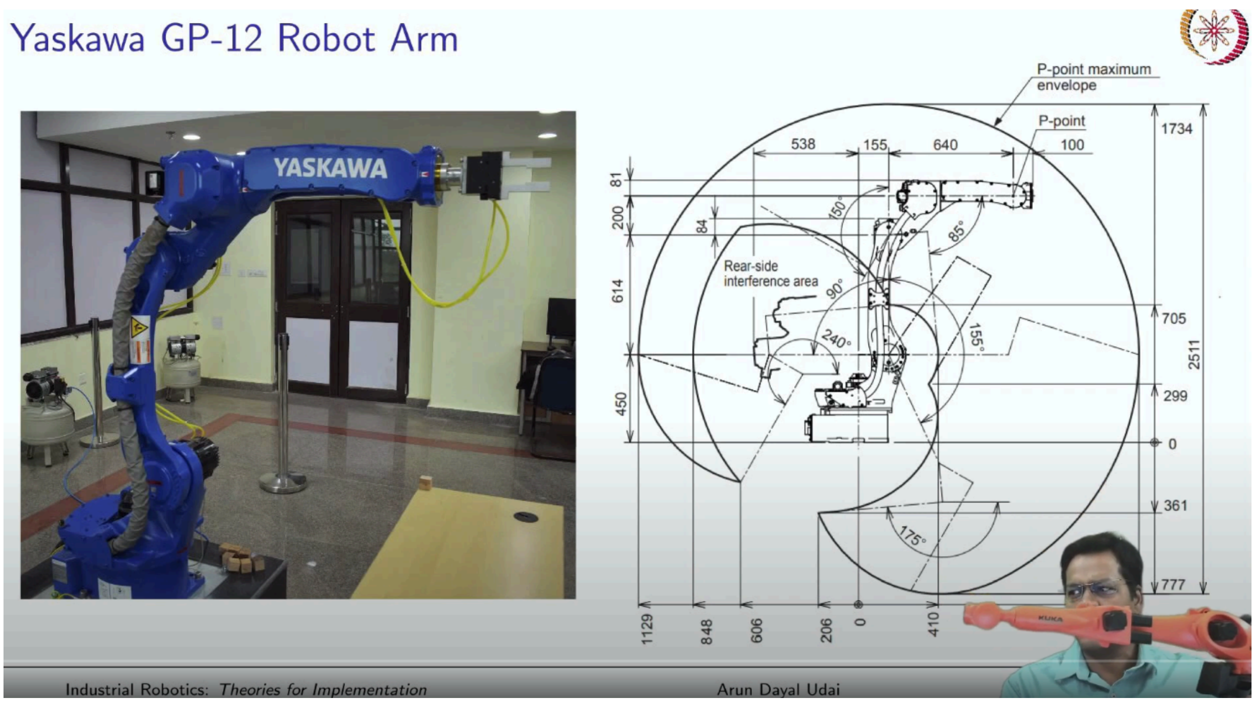
Lecture: 23

Inverse Kinematics of a 6-DoF Industrial Robot

Welcome back. In the last class, we discussed mostly planar manipulators. We did inverse kinematics of 2R and 3R planar manipulators. We also did SCARA, which is a three-dimensional robot spatial manipulator. So, today, we will discuss what a 6-degree-of-freedom industrial robot is. So, let us start.

Mostly, all the industrial robots have 6 degrees of freedom, and they have almost similar kinds of architecture. We will discuss what are the different types of manipulators that you might see when you actually start working on any of these. So, we will do the Yaskawa GP-12 robot now. So, let us begin.

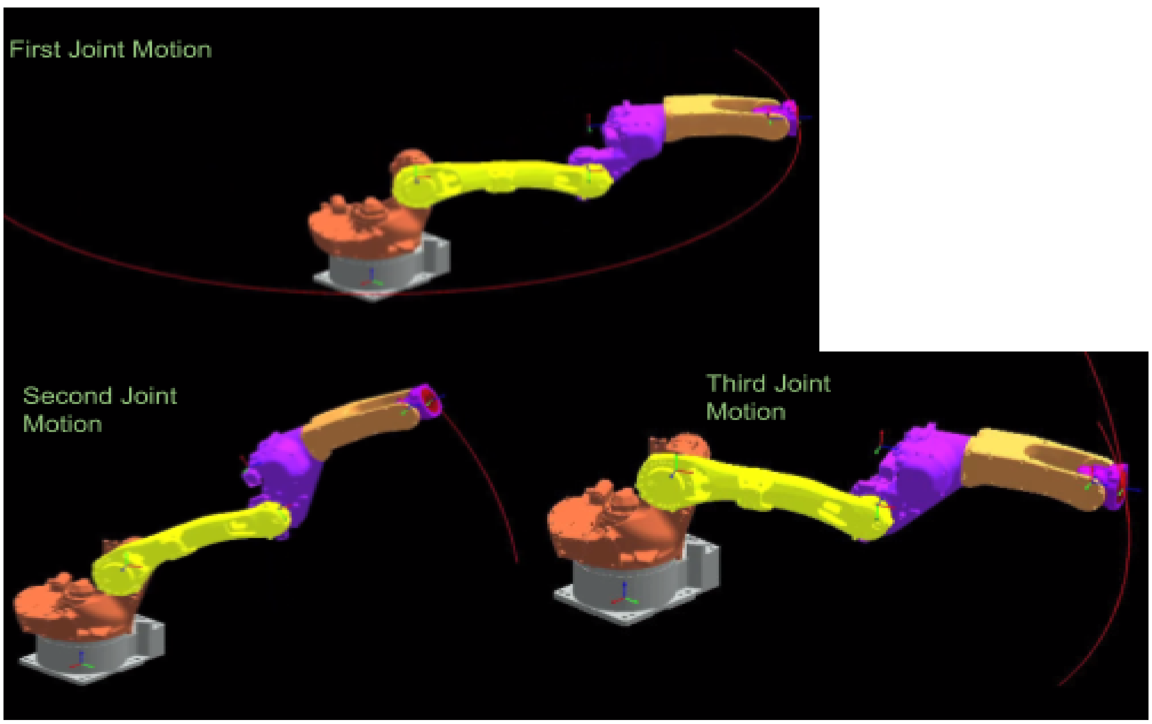


So, you see, this robot is of the kind when you see you have one of the links which is not straightforward. What do I mean? I will show you here. Normally, you see this robot. This is one of the robots for which you can arrange it like this. The whole of the link can go straight like this.

Can you see it? You see this joint and this joint, and all the joints which come next can be arranged in a single straight line. So, when it goes like this, it can go perfectly straight parallel to the ground, and it can make this kind of motion. But this is not the case with quite a lot of robots, and your link structure goes a little differently the way it is there with the Yaskawa GP-12 robot. This is quite a common architecture robot where maybe you have actuators arranged a little differently, but the shape of the link is almost the same with some dislocation out here. You see, you have one of the joints which are here, another joint goes here, and finally, it comes to the center point.

So effectively, the shape of the link is something like this. So, you see a small bit of displacement over here, and also, you have a link length that comes here. So, this is what is the shape of that link. So, if you talk till the center point, till the center point, till here, you have a first link that is a little twisted kind of thing. It is twisted by 90 degrees.

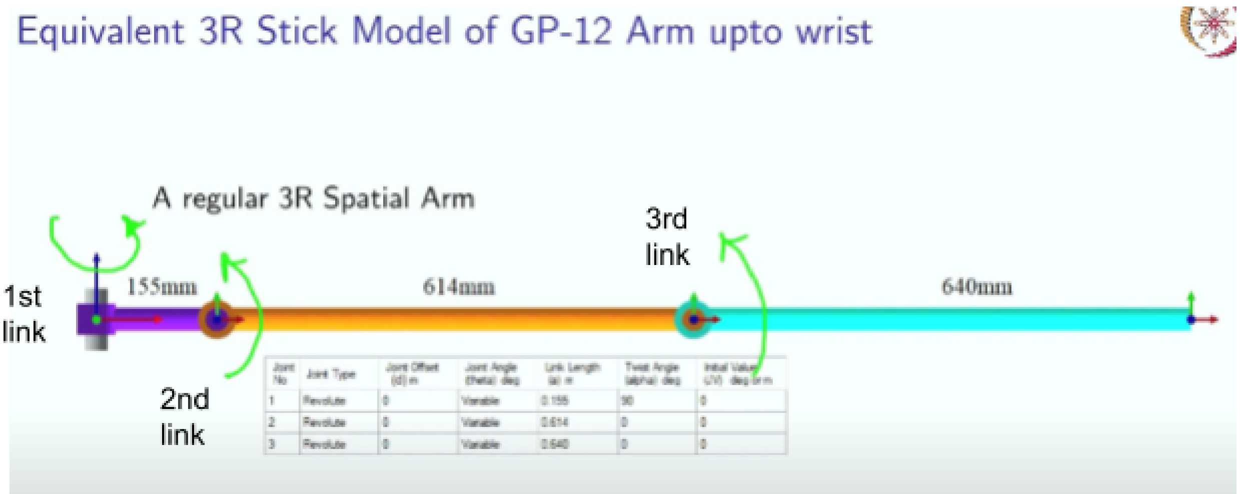
So, you have the first axis that is like this, the second axis over here, the third axis which is over here.



So, the shape of the first link is something like this. So, you see, the architecture goes something like this. So, what do you see? The first joint is fine. It can make a motion, which is something like this. And the second joint, if that moves, it moves from here. The third joint can go something like this.

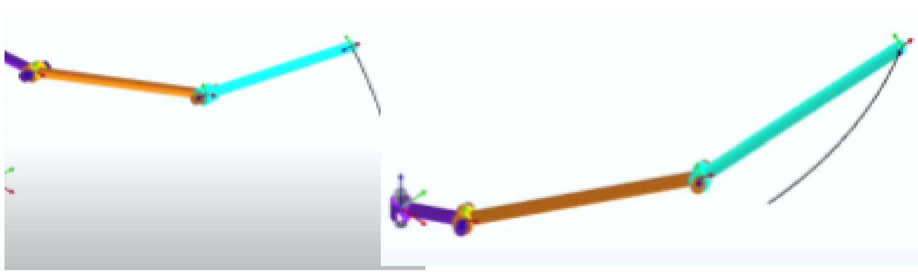
So you see, it is very very much similar to a 3R spatial arm with first-degree, first, second and third links arranged like this. But the shape here is a little different. If you see this one very closely, the shape here you see you have an axis which is over here. So, you have an axis that is here and this rotates like this. This can rotate like this.

So overall, it intersects somewhere over here. This is the point where it intersects. So, the effective shape that goes to this is something like this. From here to here, and it is like this till the center point. So, this is what I meant.



So now, let us come back to our original slide. So, this is what I meant. So, normally, your 3-degree-of-freedom spatial robot was something like this. So, the first link had a motion like this. The second one can go like this.

The third one can take it further like this. So, that first vertical axis can rotate the whole of the plane containing the 2R manipulator along.



Let me just show you the kind of motion it can make. So, you see the robot, if it moves, it can move like this. If it doesn't rotate its first joint, so rest of the 2 joints can make it move in a plane.

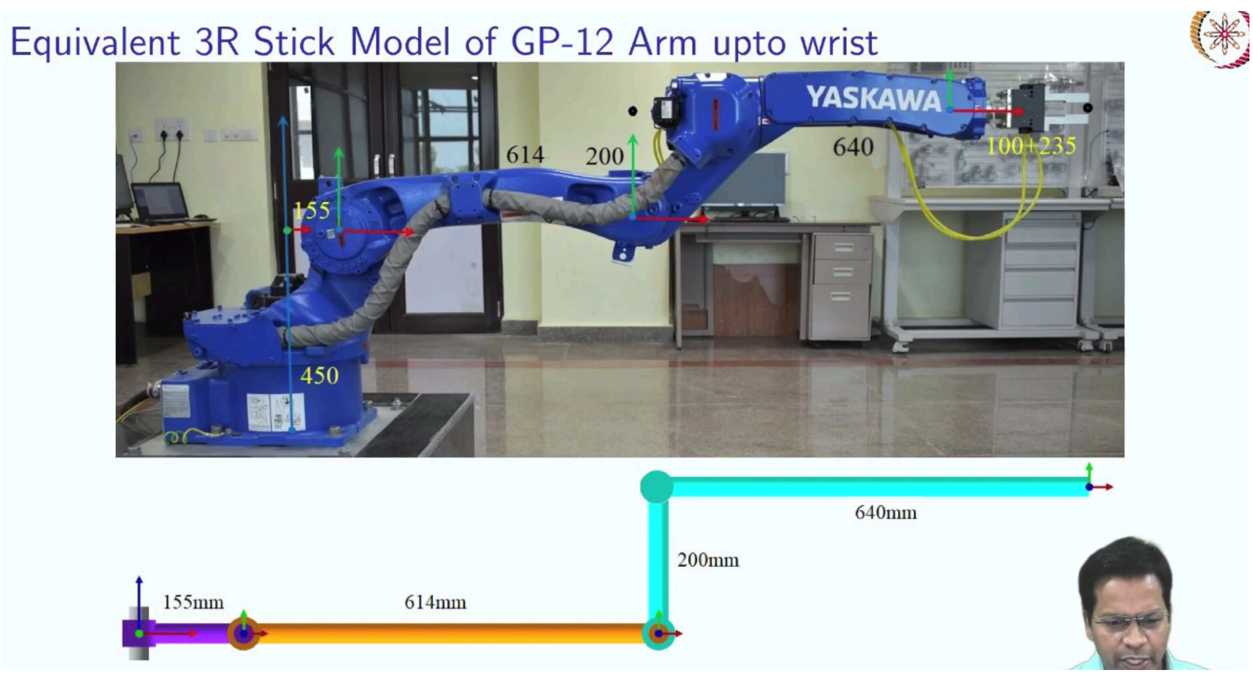
Let us say it is something like this. So, it becomes very easy to visualise this. So, let us say it is in this configuration. The second and third links have already attained some angle, and this is there in a plane. Now if I move it, it can make motion something like this.

To make things very very simple what we can do, we can make this frame the ground frame situated over here. So, I can quickly make the first offset like this. This robot is now like this. So, now, if I say to move it can move like this. So, this is the spatial 3R manipulator that you have seen earlier also.

But you have some offset, which is over here, that was the link length. So, moving ahead further, this was your standard 3R manipulator. So, this is how we have reached here. This is the effective DH parameter for that. So, this was vertical theta1, this is theta2, this was theta3.

This is the standard manipulator. But if you look carefully at your robot, it has the first link, which is here. The shape of that is something like this. The second link is from here till here. Whereas the third link has got a very different shape.

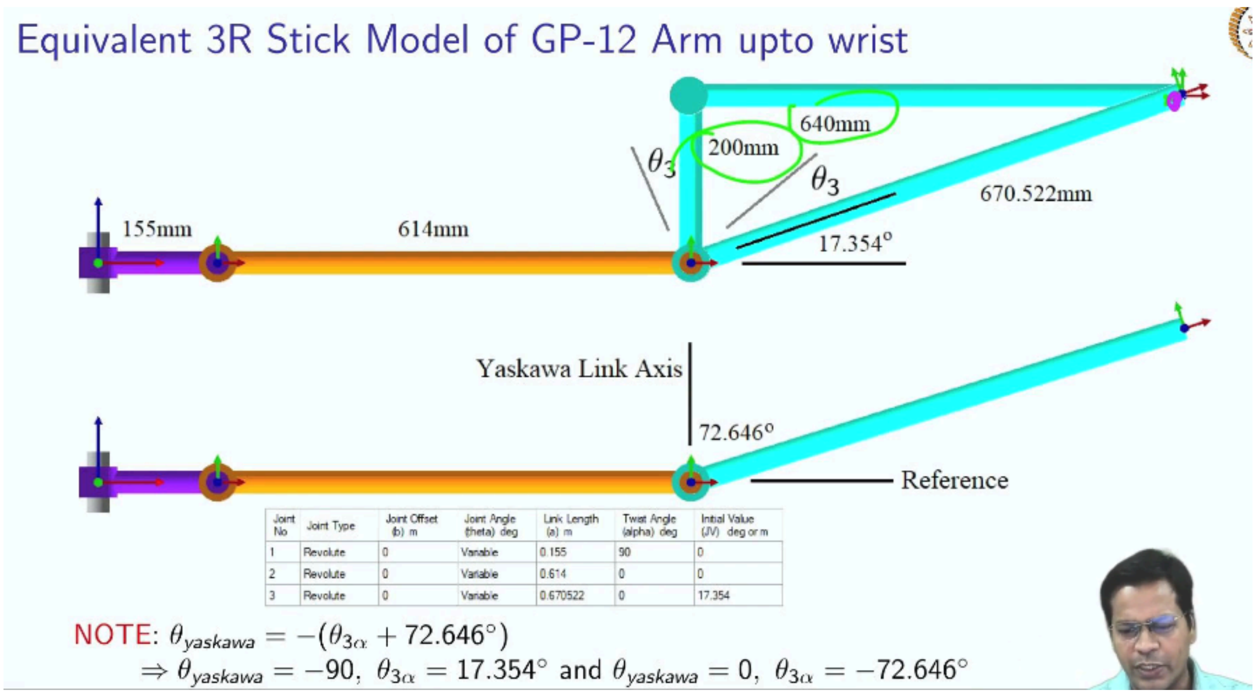
It is something like this till the centerpoint. So, it has to contain that dimension, which makes it something like this. So, this is what I am trying to make. So, this was your standard 3R manipulator. But I don't want to continue with this. I want to make it very similar to the first three links of my Yastava GP-12 robot arm. So, this is how it is.



So, now I have made it in a way that it can take up this dimension that is over here that is 155mm. This dimension is like this that is 614mm, and there are 2 dimensions, which are here that is 200, so it comes here and 640 mm over here. Now, it is this. Now, it has reached the end effector. For 3R, it is the end effector, but actually, after that, it also continues further till the tip after this is the spherical wrist. So, for 3R, this is the end effector; otherwise, it is the wrist. So, now I can do inverse kinematics using our standard 3R spatial robotic arm.

So let us do that. So, now I will try to consider that on this. So, why you cannot take this in a standard DH parameter is that in the DH parameter, you can have displacement along the X axis. So, this is your X-axis, and the distance between two Zs, 614mm becomes the length. Then you have another Z, which is situated over here. You can move in this direction that is from here till here, which is by 640 mm, but you cannot move along the Y axis.

So, where is your Y-axis? This is your Y-axis. You cannot make it move like that. You cannot have a DH parameter that can take care of this. So, what we can do here we can replace this the way it is shown in the next figure.



Joint No	Joint Type	Joint Offset (b) m	Joint Angle (theta) deg	Link Length (a) m	Twist Angle (alpha) deg	Initial Value (JV) deg or m
1	Revolute	0	Variable	0.155	90	0
2	Revolute	0	Variable	0.614	0	0
3	Revolute	0	Variable	0.670522	0	17.354

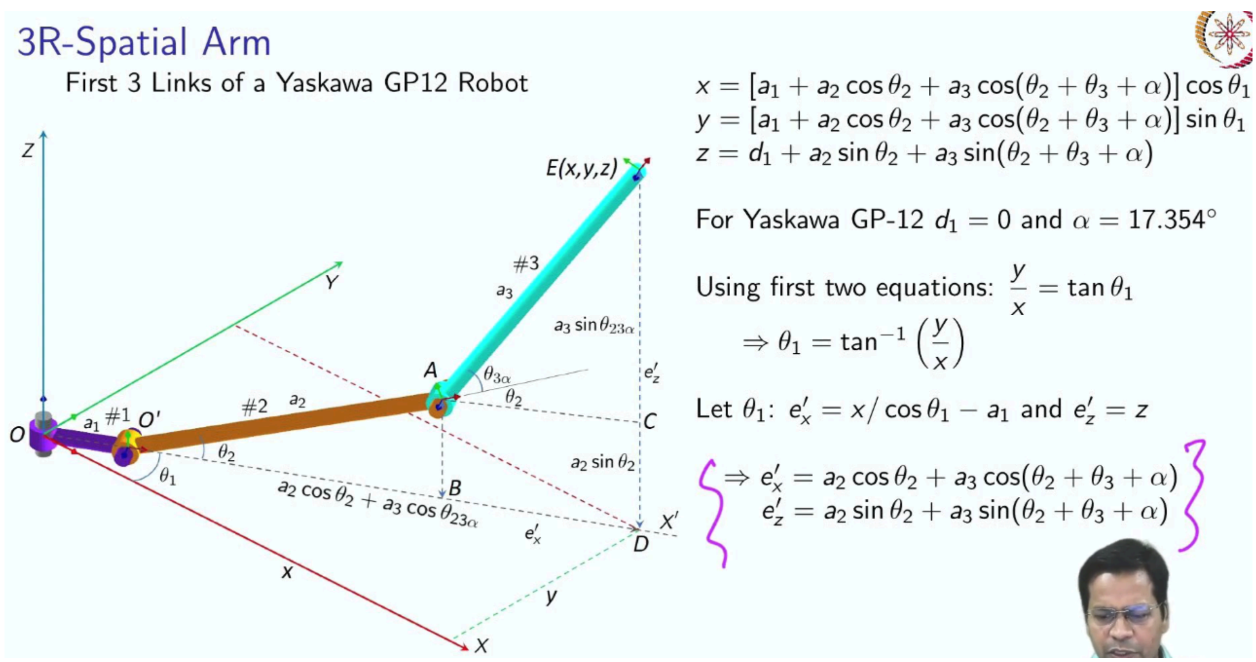
So, instead of making this length like 200 and 640, I can create an effective distance length which can take me to the same point it is the end effector.

I am only talking about the end effector position, not the orientation. Orientation definitely will change, but yes if I talk about the end effector position so, you reach the same place. Now, my link is something like this. So, it takes me from this to this with a new length that is squaring and adding. Taking the root, I bring it to 670. 522mm.

So, this is my new link that moves between two Z axes. One of the Z axes is here, and the other one is here, so it is moving along the X axis like this. So, the new length is 670mm, and there is no 200mm and 600mm distance. There is just one link length that is 670mm and it can be put in a DH parameter. With the initial joint angle, this time, if at all, you have a joint angle that measures from this horizontal reference. So, it has already moved by an angle of 17. 354. So, I call it alpha. So, this is theta three by default. We used to call it theta3. Now, we will call it theta3 alpha, which means theta3 plus alpha. So, when theta3 is 0 degrees, it already has an angle that is 17.354 degrees. That is simple trigonometry I can find it out. So, in the case of Yaskawa it measures theta3 like this from this vertical to this got it.

So if at all it is moving like this. This is theta3. If I have to make it move this way from here, this is our reference frame, so this is your new theta3. So, effective theta three, if it moves by theta3, will be theta3 plus this alpha. So, we call it theta3 alpha. So, now the new angle of this link, the third link, moves by an angle theta3 the total angle with standard ds is theta3 plus alpha got it.

This can be taken care of by having a ds parameter which is like this, and with respect to Yaskawa. It is something like this. So, if you measure it with this reference frame, it is like this. So, when Yaskawa shows 0 degrees, that means this link gets aligned to this, so our actual link new link has already rotated by minus 72.646 degrees. So, our new robot is like this with a link length 670.522mm got it. It has already moved by an angle of 17.354 degrees, okay. So, that is my new robot. Rest everything remains the same. So, now I will solve the inverse kinematics problem.



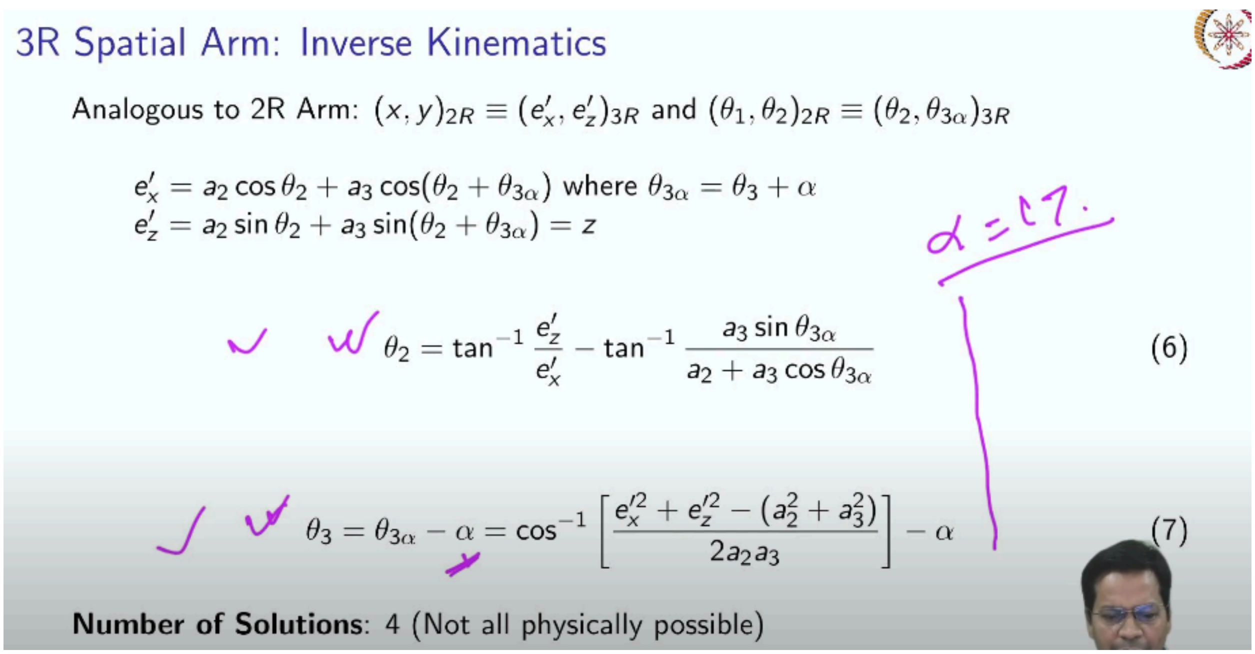
This is my robot. I want to solve inverse kinematics for this. So, x y, z can be straight away obtained from the forward kinematics equation only thing that has changed is the angle that is theta3 becomes theta3 plus alpha. As you know this link has already rotated by some angle that is alpha. This one got it. So, that angle is considered here. Rest everything is same this time. You also have a1 which is a1 as compared to the previous 3R spatial manipulator that we have done.

Now, I will solve this. So, for GP-12, d1 is equal to 0; where is your d1 taking it to this? This was your d1. This becomes equal to 0 and I placed my first frame over here that is attached to the ground.

So, using the first two equations, I can quickly obtain theta1. So, theta1 is this done, and ex' is x by cos theta1 minus a1. ez' is equal to z minus d1. So, this is your eZ'. Got it. Now, I will move ahead. I have created these two equations.

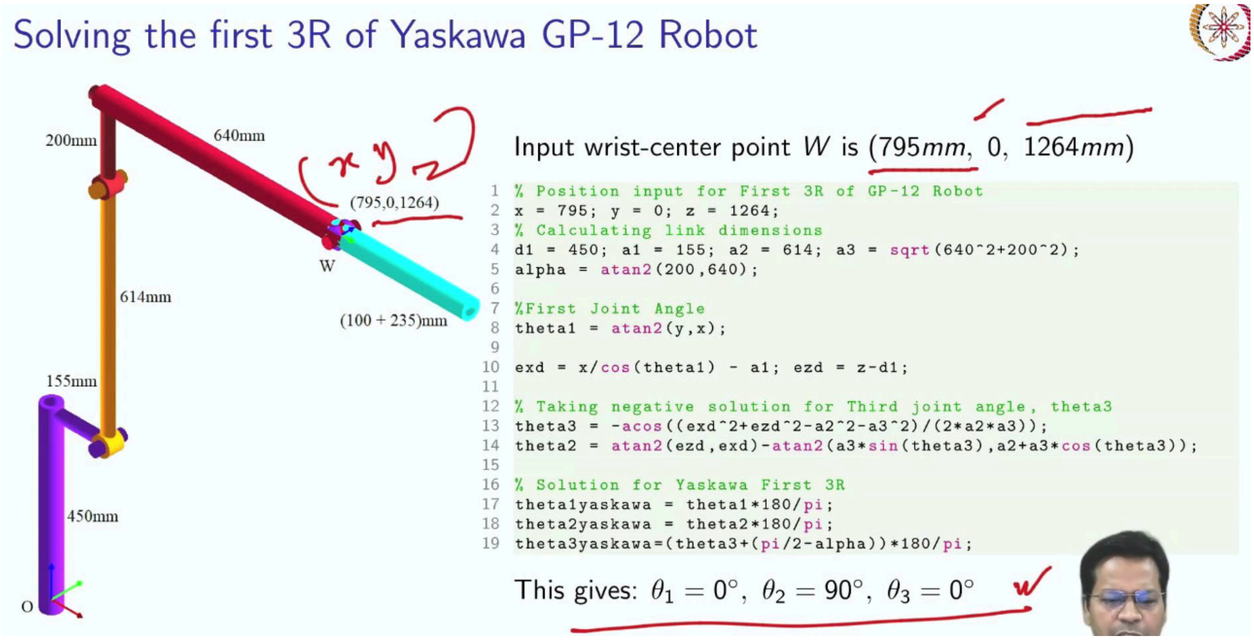
$$e'_x = a_2 \cos\theta_2 + a_3 \cos(\theta_2 + \theta_3 + \alpha)$$
$$e'_z = a_2 \sin\theta_2 + a_3 \sin(\theta_2 + \theta_3 + \alpha)$$

This is very much like a 2R manipulator planar manipulator. So, you can solve it. You can directly see the 2R manipulator subset, which is here and on that plane, which is a plane of X' and Z'. Got it. X' and Z'. Got it.



You can solve it and you can quickly obtain your this where three alpha you know it is theta3 plus alpha. Got it, so exactly copying your 2R manipulator solution takes you to theta3 and theta2, which can be easily solved. theta1 was solved here theta1 was here. So, now you have theta2 and 3. So, this is solved exactly like your 3R spatial manipulator with a little bit of changes to that so that you can consider the free angle of 17 points. Whatever angle you have that is for an alpha, that angle is already considered here, and the rest of everything remains the same. So, now you can obtain the angles for the Yaskawa robot.

So this is your Yaskawa robot. Now, if you know the risk centerpoint location, you can calculate all the angles theta1, theta2 and theta3. Got it. So, you have done inverse kinematics up till this point. Got it. So, moving ahead, let me take the whole robot now.

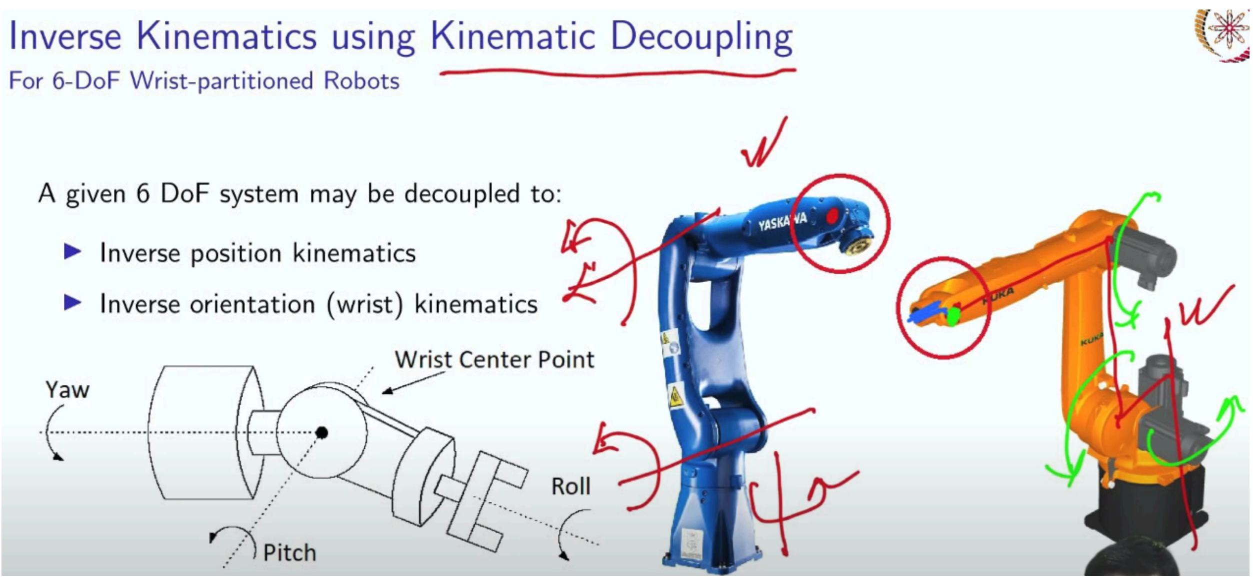


This is a sample set for which I have solved it for this configuration where you have X equal to 795mm for this, Y is equal to 0 and 1264 mm for this along Z. So, this is the coordinate which is given for the wrist center point. I have just verified that using this MATLAB code which is here, gives me this.

This gives: $\theta_1 = 0^\circ,\ \theta_2 = 90^\circ,\ \theta_3 = 0^\circ$

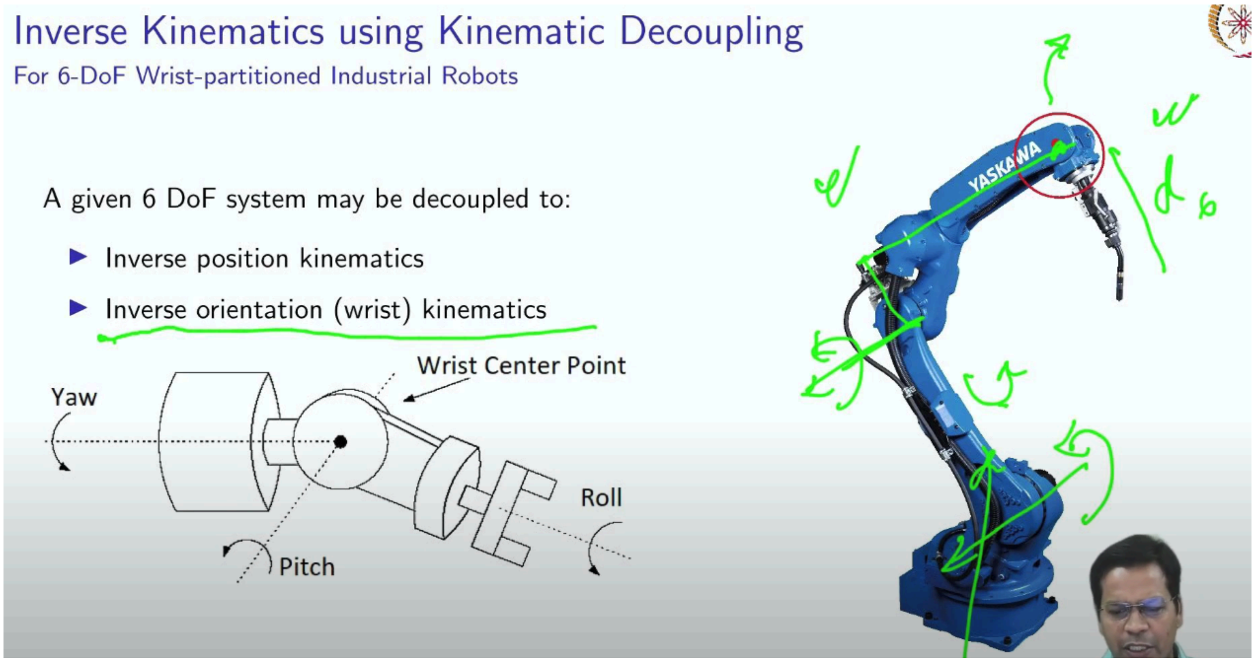
That is the correct solution for this which is quickly visible also. So, this is how this 3R for Yaskawa GP 12 here is solved. So, this was the positioning part of my Yaskawa GP-12 robot.

Now, I will do the wrist partitioning method and segregate this robot into two halves. The first part will be till here is the positioning part next part is this after the rest is an orienting robot.



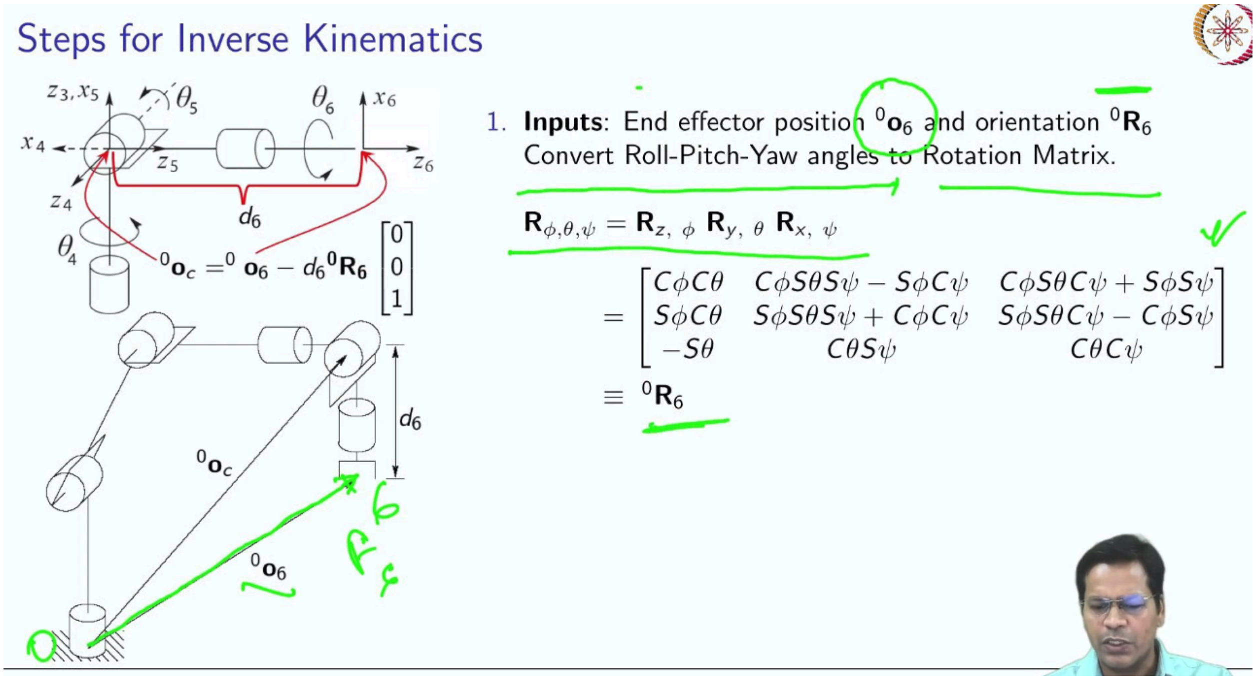
So, I will just change it. So, normally, kinematic decoupling can be done even with any standard industrial robot. So you see, it has the first three lengths, which are like a standard spatial 3R manipulator. This was the vertical axis, which can go like this, then you have one axis that can make it move like this, another axis which is here that can make it move like this, and finally, you reach till the centerpoint. Same for this is the KUKA robot, and this is the Yaskawa robot. Again, you reach here, that is the centerpoint. You have the first vertical axis with some offset here, and then you have two distances, this one and this one. So, you have the first axis that can move like this second axis, which can make it move like about this and the third axis, which is like this.

After that, you reach the centerpoint. So, positioning the workspace is very, very clear. Now, let us look at the orienting link. So, the last link, which is hereafter that, is this one for this, and here you see this one. So, that is the orienting link. So, after the rest, you have another three intersecting axes that make it a spherical rest that is here.



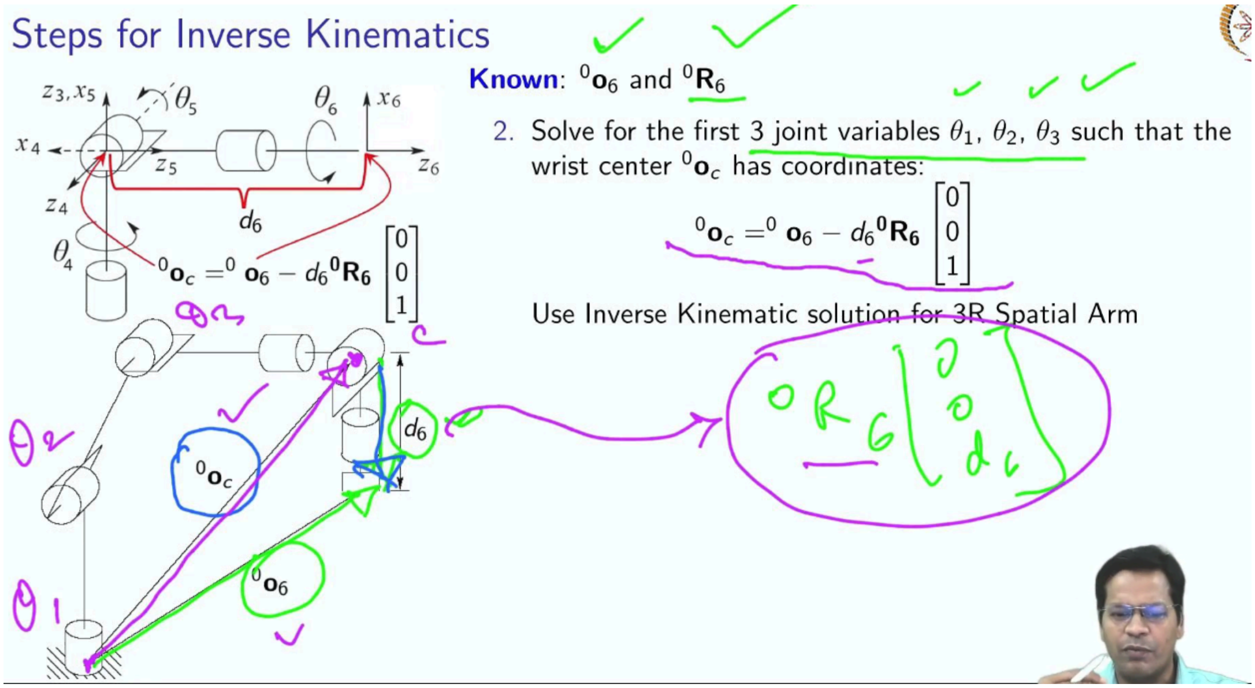
So, this is your actual Yaskawa GP-12 robot in which you have seen your third link, which is shaped like this, and you know you have already solved for that you have one axis which is like this. You have another axis which is making it like this. Your first axis, which was vertical, you know, so that used to rotate it like this. So, this is how it is till here it is 3R spatial manipulator, the modified one that we have discussed today. So, this is how you reach this, and after that, you have the orienting link. So, this is the link d6 that we initially had when we discussed this parameter of any six degrees of freedom spatial robot. So this is the distance.

So, you have inverse orientation wrist kinematics that is accounted over here. Positioning kinematics is accounted till the first three links. So, this is how it is decoupled.



Let us work it out. So, yes, the first step that you know for this kind of robot is you know the end effector position. The end effector position is the position vector in this one ($^0O_6$). I will just draw it over here. This is the location that you already know, which means you know this vector from frame 0 to frame 6 given by code that is the position vector here. So, this is well known. You also know the orientation of the end effector, which means you know the frame, which is attached to the sixth link.

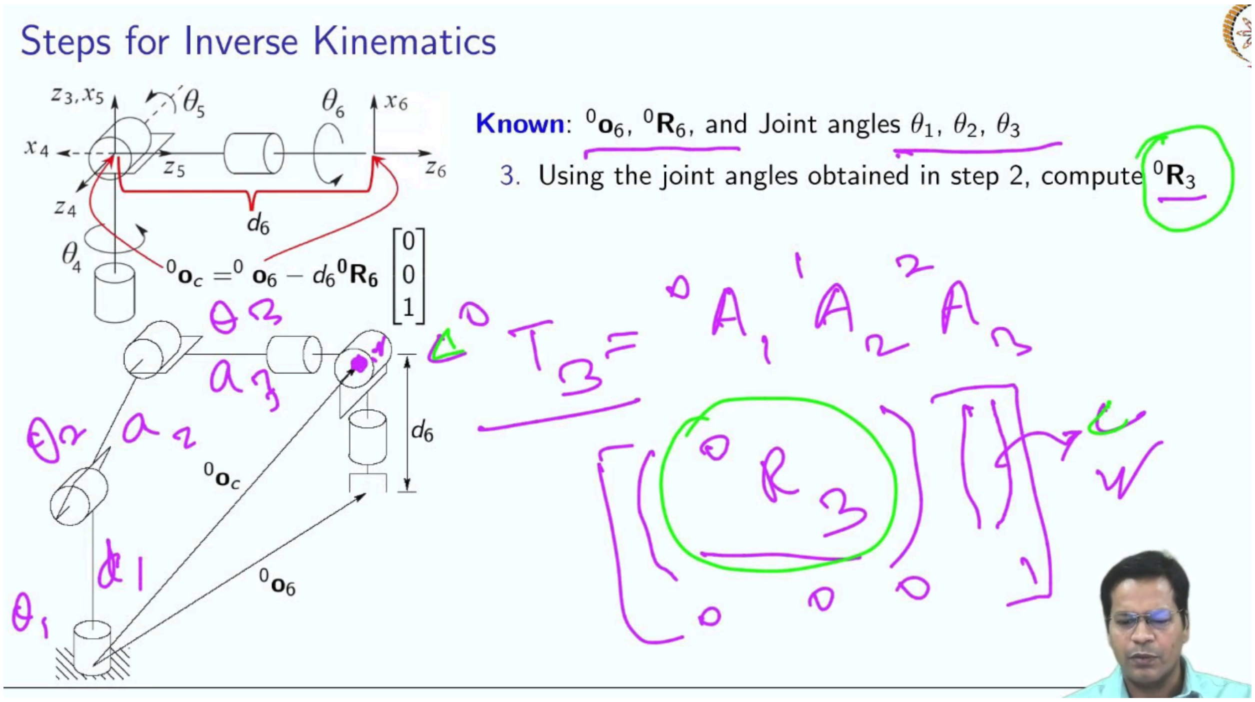
So, you know the orientation matrix for that because you know the roll pitch yaw of your last link, which means you know the orientation of the last link. How do you know? Convert the roll pitch yaw angles. Normally, your industrial robots will tell you to roll pitch and yaw angle with respect to the base, that is, with respect to zero. So, you have to convert that to the rotation matrix. You can quickly do it using the RPY transformation matrix that we have studied. So, you can use this, and you can get to this. So, this is how you can obtain rotation matrix 0 or 6. You already know this. You have inputs that are given here, that is, OR6, and the roll pitch yaw angle was given you have converted it to a rotation matrix. So, these are the two pieces of information that you have acquired from the given one.



Now, moving ahead, you have this. You have this now. You have converted to the RPY2 rotation matrix. You have this. Now you have to solve for the first three joint variables, theta1, theta2 and theta3. You already know this. This vector is known. This distance was known. This distance, which is from the wrist Centrepoint to this distance, is well known. Through your robot data sheet you can just take out that distance that is d6. As per DH it comes out to be T6. You already know the orientation of your last frame.

That is, 0R6 is known. You have to multiply the vector 0 0 d6 and what do you get? You get this vector. This vector is obtained. Got it? So, once you obtain this vector, you can quickly obtain the wrist centerpoint vector. That is the vector which starts from the origin and till here. This is your wrist centerpoint. Got it? That is this C point. So, this is easily obtainable using a vector triangle. So, instead of writing 0 0 d6 you can take out the D6 and write like 0 0 1.

Your matrix goes here. That is okay. So, this is your vector which is here. Got it? So this vector and you already have this vector so that you can calculate this. So, this is your first step when you have calculated the wrist centerpoint. So, this one. So, once you have obtained this point, you can calculate the rest of the three joint angles. The first three joint angles, theta1, theta2 and theta3, using the techniques that we have learnt today, is the modified 3R spatial manipulator. Earlier, we did it for a 3R regular manipulator. So, both are applicable to different kinds of robots that we have seen in our first slides. So, this is all.

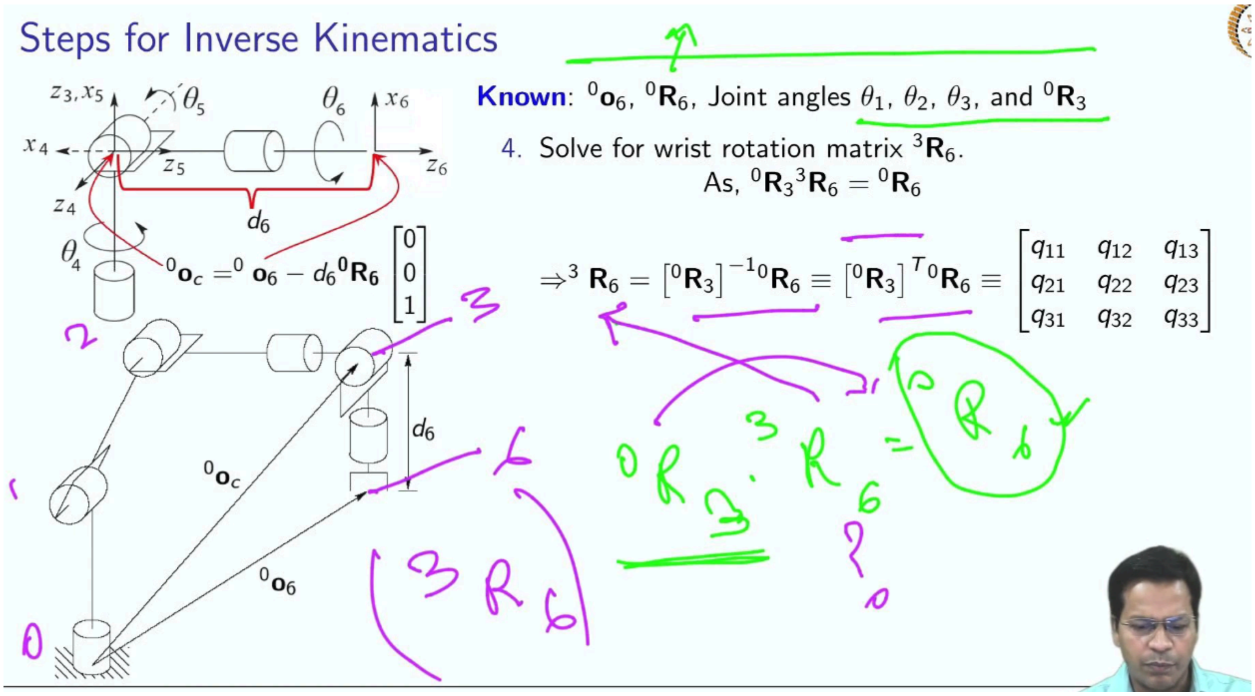


So, now you already know these and the first 3 joint angles. So, if the first 3 joint angles are known, you know theta1, you know theta2, you know theta3, and you already know the information that was d1, a2 and a3. Those were the distances. So, you can quickly do forward kinematics and you can obtain this frame. This frame means you can do forward kinematics by multiplying $^{0}A_1$, $^{1}A_2$, $^{2}A_3$. These are the link transformation matrix, and finally, what you get is $^{0}T_3$. So, what is this?

$$^{0}T_{3} =- {}^{0}A_{1}\ {}^{1}A_{2}\ {}^{2}A_{3}$$

This has the position of this frame that goes here and this is your rotation. So, this is how you can obtain 0R3. So, this part is 0R3.

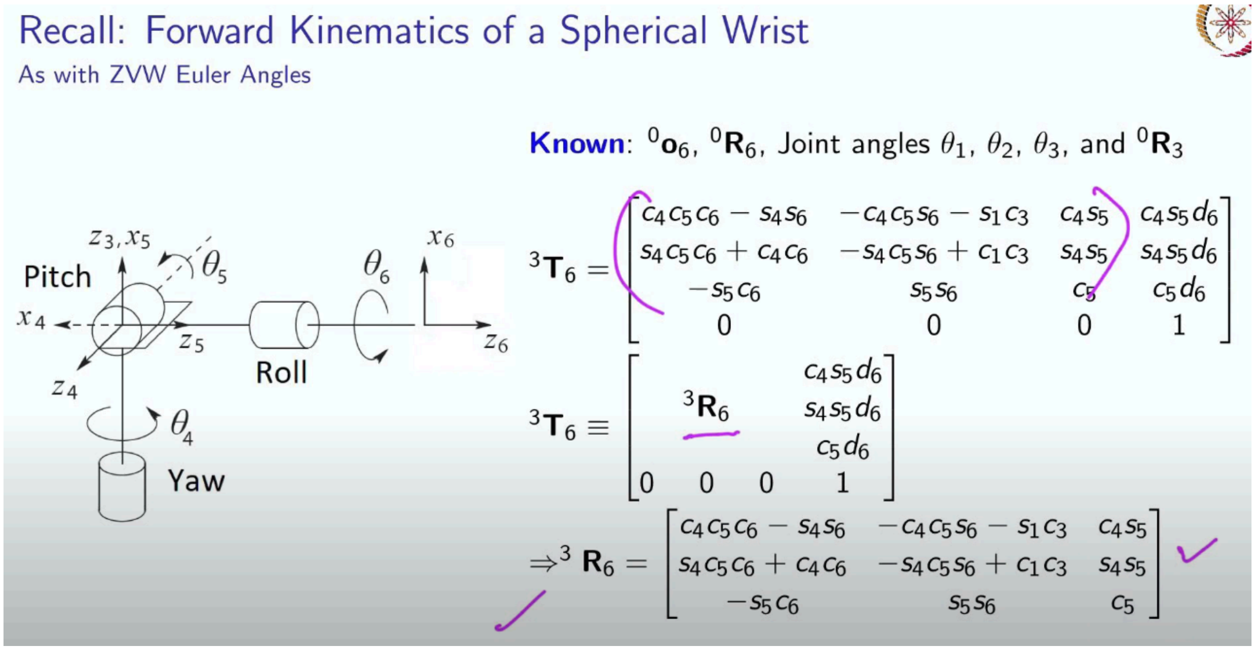
This is the position that is already you know it is this position. So, once you know your joint angles theta1, theta2 theta3, you can calculate the rotation matrix for this. This is your transformation matrix homogeneous transformation matrix that has parts of this point C. This was your C, and you also have this part that shows the 0R3, which is very much required for the rest of the inverse kinematics problem that we will be doing. So, we have obtained 0R3 using the first three joint angles using forward kinematics. This is step 3.



So, now, step 4. So, we already have all these values. So, 0R3, when taken as a product for 3R6, will give you 0R6. 0R6 is already known. You already know the end effector orientation with respect to the base that we took quite earlier because that is the input from the roll pitch yaw angle you converted this directly.

So, this is known. This is just now we came to know using the first three joint angles. So, I have just taken the inverse of that to find out this one. So, this is found out using this relation. So, you just take the inverse of this and take it to the other side of this so you can take this. So, the inverse of the rotation matrix can be quickly obtained by taking the transpose, you know.

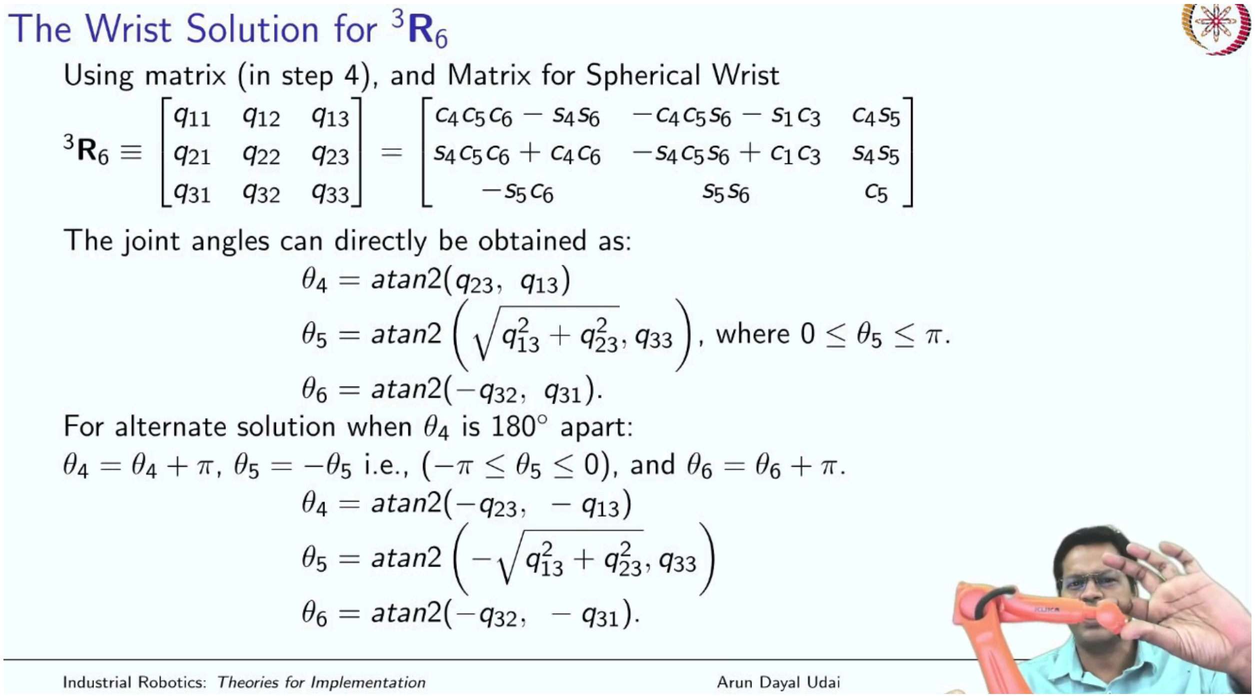
So, it is this. So, now what I have obtained is 3R6, which is the transformation matrix from this to the end effector. Not the complete transformation matrix. So, we already know that because we know this x, but still, what I need now is only this that is 3R6, which is the rotation matrix from frame 3 to frame 6. This was your 0, this is 1, this is 2, this is 3. So, from 3 to 6, you know it is here. So, this is your orientation matrix that is what you know with respect to the third frame.



Recall the forward kinematics of a spherical wrist that we have done using also using ZVW Euler angles. You have yaw angle, you have pitch angle, you have roll angle. This can be arranged in a straight line. So, if you know the last 3 joint angles, theta4, theta5 and theta6, you know that it is a spherical wrist.

So, you can directly write that transformation matrix like this, and including the distance d6, it becomes this. This (${}^3T_6$) is your Euler angle transformation for the same, and you can obtain that using the ZVW Euler angles that we have studied very much in the beginning. In terms of variables it has theta4, it has theta5, it has theta6.

This is your rotation matrix that is from 3R6. So, that is here. So, I can write it like this. This part as a sub matrix is here. The remaining is like this and can be extracted. This is exactly this centermatrix 3 x 3 that is ${}^3R_6$. If you want to see it clearly, I will just vanish. So, this is it.



As we have discussed in step 4, we obtained 3R6 with some values. Now, we have, in terms of variables, used a matrix for a spherical wrist. Just the previous slide, we saw this. Earlier, we have already obtained this. Now, equating both of these, you can extract the information.

The first one, you see, theta4, can be obtained from q23 and q13. See the last column. This one, this two. So, if you divide these two, you can quickly get this by this is equal to tan theta4 is equal to whatever are the values which are here. You can take an arc tangent solution, and you can obtain theta4.

Similarly, theta5 can be obtained if you can see it clearly, it is q13 and q23 and q33. q33 is this one, this and this squaring, adding and taking the root. What you can extract is sin theta5. Sin theta5 can be directly extracted. So, cos square theta6 plus sin square theta6 will go to 1, and finally, you will be left with sin square theta5 taking root. It is a sin theta5. So, sin theta5 comes here, and cos theta5 comes here. And that is what will give you theta5. So, that is how you obtain theta5.

Now, the angle theta6 is obtained using q32 and q31. Again, you have two similar values that are to be used. q32 and q31. You just see where it is q32 and q31. So, these are the two. Just use them. If you divide this by this, what will you get? Sin theta6 by cos theta6. So, it is tan theta6, R tangent of that will give you this. But this has some limitations. It can vary only theta5 from 0 to pi.

The alternate solution is theta4. When theta4 is 180 degrees apart that means you have an alternate solution. How is it possible? Let me just show you how to use this model. What you see here is the first solution that can take you. Let us see this point. The same thing can be obtained if you rotate theta4 180 degrees apart and bring it down like this, and you reach the same place. Got it? So, if it is at this orientation, the same orientation can be obtained by rotating theta4 by 180 degrees and bringing it theta5 like this, and you reach the same orientation.

So, that is what it is meant here. So, you reach till this. So, this is the alternate solution for that. That will give you theta6. So, you have obtained theta4, theta5 and theta6. theta1, 2 and 3 were obtained earlier using inverse kinematics of the 3R spatial manipulator subsystem, which was there once you knew the spherical wrist centerpoint.

So, theta1, theta2, theta3 was obtained theta4, theta5, and theta6 are obtained here. Got it? So, you see it has two solutions at least. So, overall, this type of robot has got 8 solutions, at least for this structure that I have. So, that's all for this lecture. So, in the next lecture we will see robot Jacobian that will relate the end effector rates to the joint rates. We will talk about motion, and we will try to find out the acceleration, and we will try to find out the velocities provided I know the joint rates.

We will discuss something which is known as singularity. We will discuss Jacobian, which is the rate relationship. That's all for today. See you then. Bye bye.