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Industrial Robotics: Theories for Implementation
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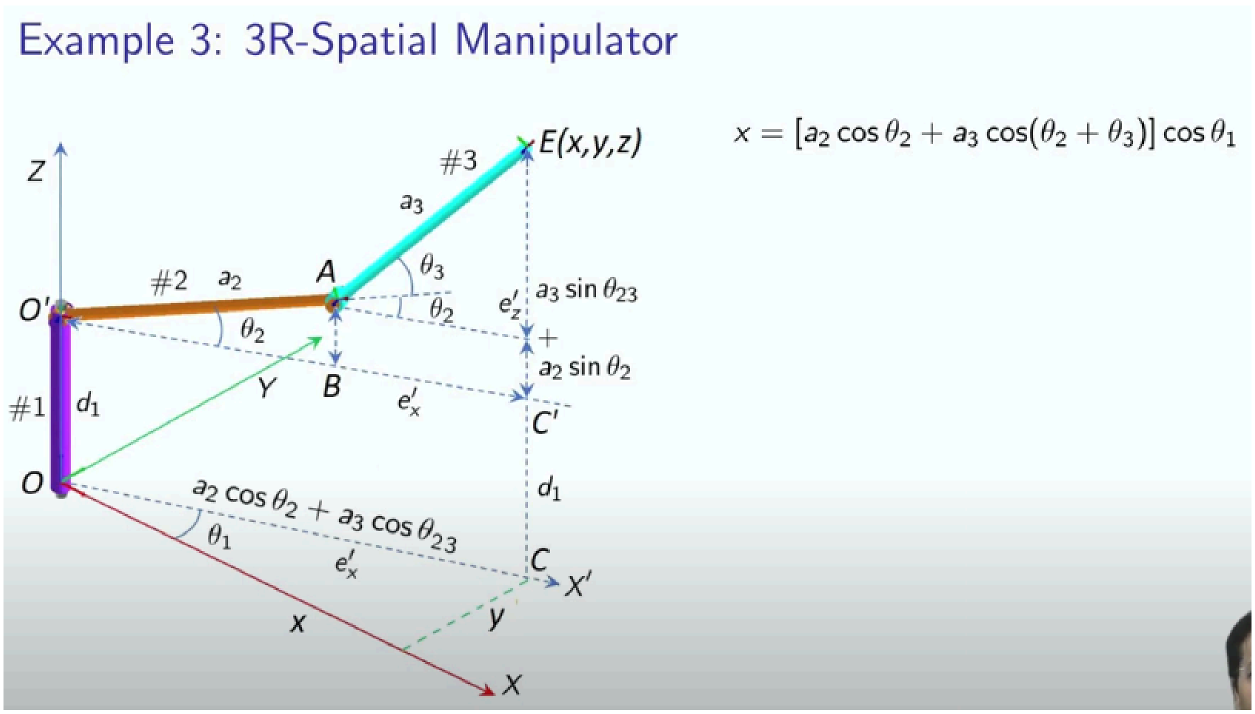
Lecture: 22

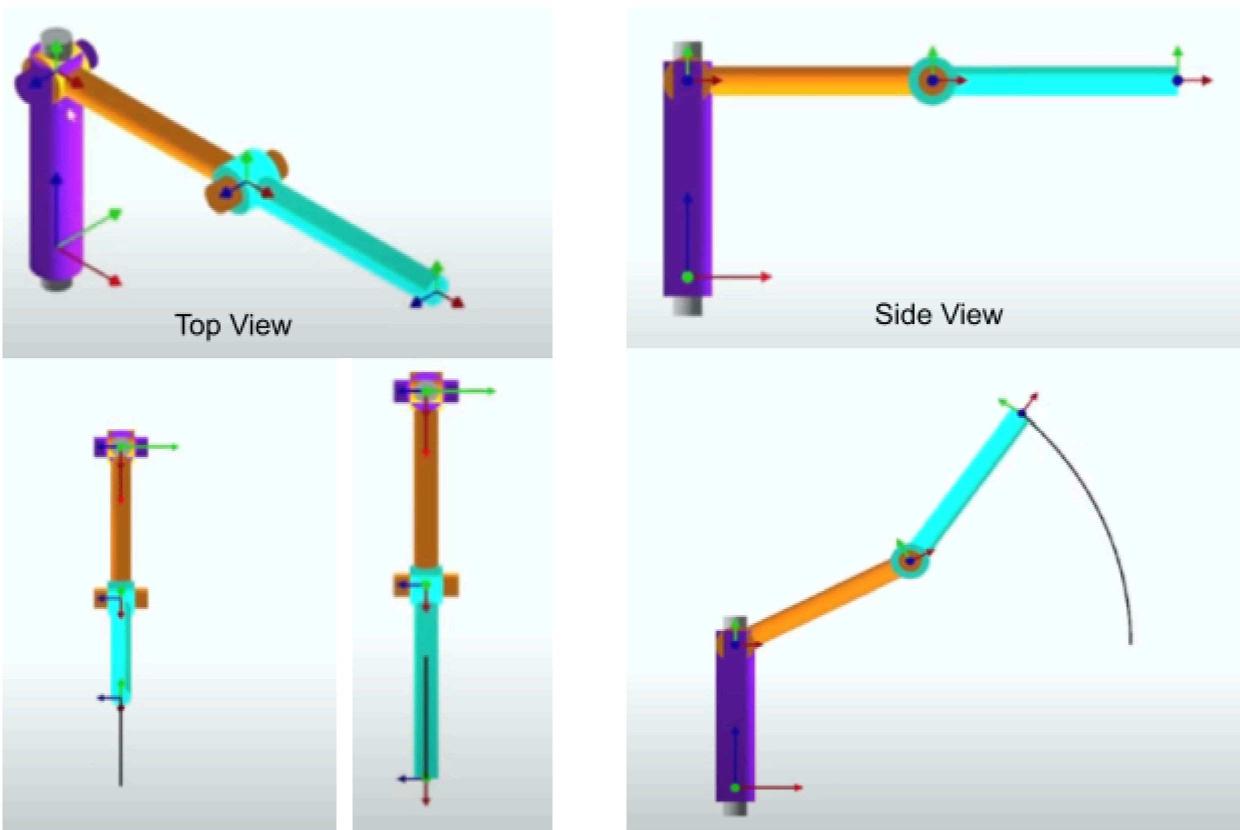
Spatial Robots - 3R, Cylindrical (RPP), 4-DoF SCARA Robot



Welcome back. So, I hope you are following the inverse kinematics that I did in my last class. I covered two degrees of freedom and three degrees of freedom robots. When I showed you how two degrees of freedom can be a subset of higher degrees of freedom robots as well. We saw two degrees of freedom solutions, multiple solutions, also even three degrees of freedom, multiple solutions, which is nothing but an extension of two degrees of freedom plus one degree of freedom system. So, moving ahead further, today we will be doing Spatial robots. Earlier, we did two degrees of freedom and three degrees of freedom, planar robots. So, today, we will be doing a robot that can achieve the position in a three-dimensional workspace.

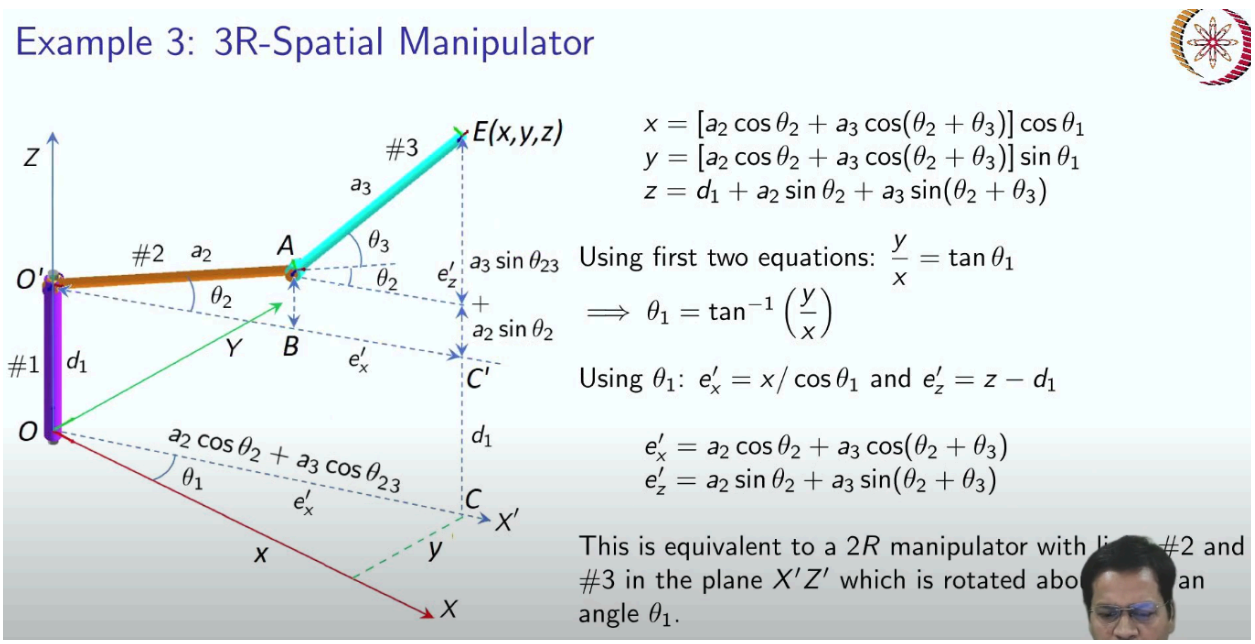
Along with that, we will also be doing one standard industrial robot, that is, the SCARA robot today and in between, we will also do three degrees of freedom cylindrical robots. So, these all robots are a few of the selected robots, which are very, very important because at least for two degrees of freedom robots, which is, you know, can be a subset of higher degrees of freedom robots. So, the solution gets very, very easy then and Spatial three R (3R) robots. That is, the first three links of standard industrial robots are mostly Spatial three robots.

So, moving ahead further. So, we will start with 3R-Spatial manipulators first. So, this is a Spatial manipulator to make you understand better. Let me just switch on some virtual environment that I have out of robot analyser software, which is a very popular software that was developed at IIT Delhi by Professor SK Saha and their team.



So, let me just use it to make you understand it better. 3R Spatial robots, basically, look like this. It has three joints. The first joint, as you can see, is vertically like this, where the first link is moving along the vertical axis, that is Z0 axis, which is shown here vertically, This one, and the second axis, which is parallel to the ground. It goes like this, which is shown here, and the third one is like this: I have taken two links. Both are of equal length. So, if you look from the top, it will look something like this. So, this is how it is. And if you look from the side so that it can go something like this, You can see how it moves. You see, it goes like this. It is exactly like a two-degree-of-freedom robot. So, this can move like this. It can come back to its original position like this. Got it. This time, if I look from the top, I hardly see there is any motion at all. The two links are coming exactly in the plane. Got the structure of this now. So, if I take it back, it goes like this. Now, without moving or making the second link and third link, exactly at the end location which was there. Now, let me move the first joint. You see how it moves. So, I am now moving the first joint to 45 degrees. So, it goes from 0 to 45 degrees. Got it? So, it is a complete plane on which the two degrees of freedom robot lies. You see that plane is moving. That plane is rotating. That plane carries two links, the first link and the second link, which was a subset of our subset. So, it is effectively two and three links, the second and the third link over here. So, if you look again in a view which is like this, you see how it moves: The plane completely, which carries the 2R subset, rotates about a vertical axis. That is the first degree of freedom.

Along with that, it rotates. The first joint makes the whole plane rotate about the vertical axis. So, this is how it is. So, let me just come back to the isometric view, which shows everything together. So, yes, now I will move the system, the whole of the system, from the first two links. So, this is how it is. You see, it moves The first two links. The first link moves from 0 to 45 degrees, whereas the second and the third link go from 0 to 30 degrees for each of them. This time, when you look from the top, it is something like this. The plane is rotating, and on the plane other two movements are there. That is the second and the third link, Got it? I hope you have understood well how this system is, So I will just switch off the virtual thing and come back to this kinematic picture.



So, forward kinematics you know you can do it straight away, using projections also, or using DH parameters. Both are equally good. So, you see X, X. Where is your X? So this is your X, This is your X, and Y is perpendicular to X and Z. So, you have Y here, and this is your Z. X,Y and Z axis. So, you see, this contains the 2R subsets. So, where is it? This is the link2 and this is the link3. That is completely contained in a plane and the plane rotates about this vertical axis, This Z axis, the whole of this rotates, Got it? So that is theta1, which is making it move. And this is your plane. So, this (OCE) is your plane. So, now let us analyse what could be the components of 2R subsets along different axes.

So, now, what is X? You just can obtain it directly from here. So, it says a2 cos, theta2. Where is that? This is your a2 cos theta2 is this, and a3 cos theta2 plus theta t. So, this is theta2 plus theta3. So, the cosine of that is this, and the same is this. So, total distance. So, this is your ex-dash. ex dash is here, and cosine of that, so this also is ex dash. So, the cosine of that with theta1 is actually your X. So, this is your X, Got it? So this is your X.

$$x = [a_2 \cos\theta_2 + a_3 \cos(\theta_2 + \theta_3)] \cos\theta_1$$

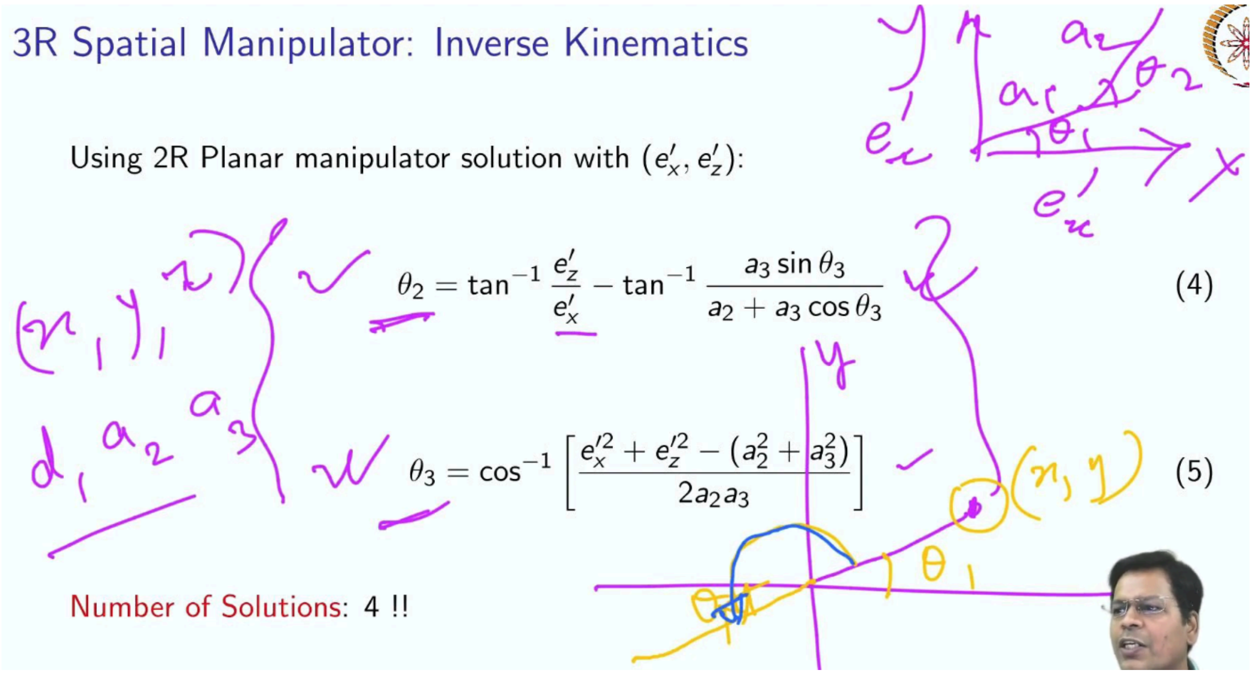
Now, let us see how Y can be done. So, Y is what It is. Again, Y, you see, it is a2 cosine theta2. So, you see a2 cosine theta2. This, again, is a3 cosine theta2 plus theta3. This is theta2 plus theta3, this one and finally this one. So, this is it. So, this is your ex dash sine theta1. So, if this is Ex dash, this is theta1, so this becomes your sine theta1. So, effectively, it is Y. So, here is your Y.

$$y = [a_2 \cos\theta_2 + a_3 \cos(\theta_2 + \theta_3)] \sin\theta_1$$

Now, let us look at how Z is taken. So, Z is simply d1. So, this is your d1. This distance is always there, So it brings it here. Now, how much is the remaining? So, a2 sine theta2. So, this is a2, this is theta2, a2 sine theta2. Then a3 sine theta2 plus theta3, so this is theta2 plus theta3. This is your a3. So, this is the remaining distance. So, d1 plus this, plus this; so this is your Z.

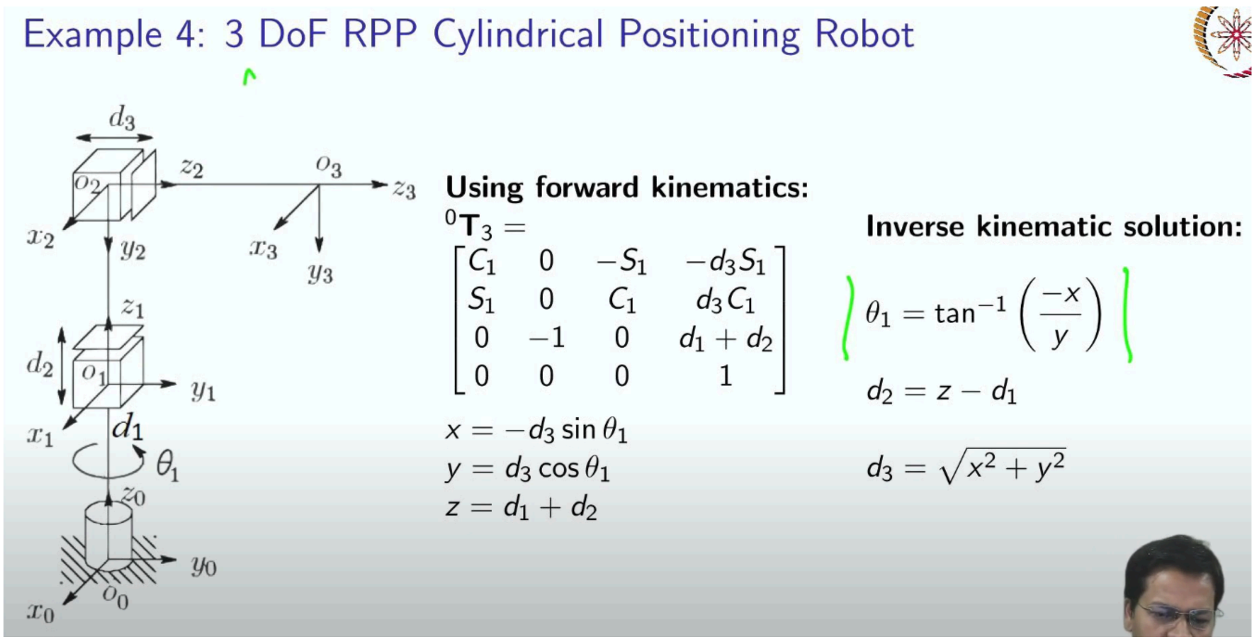
$$z = d_1 + a_2 \sin\theta_2 + a_3 \sin(\theta_2 + \theta_3)$$

So, this 3 is quite okay. You can directly obtain this by projection. Using these parameters also, you will come to this. So, now let us do inverse kinematics. So, using this I need to solve for theta1, theta2 and theta3. What is given is X, Y and Z. So, using the first two equations, if you just divide them, so Y by X, this quickly gives you tan theta1. So, theta1 can be directly obtained using the tan inverse of Y by X. You can see from here also: this is theta1, so if it is Y by X, so it is directly tan inverse of Y by X should be your theta1. So, you have obtained theta1. Now you know theta1 is theta1. So, once theta1 is obtained, you can obtain ex'. So, what is my ex'. ex' is simply X by cosine theta1, this is your ex', which is marked in this also. So, this is your ex'. So, this is it Ex' Again, ez'. So, that is ez' As I am marking. Just keep following. So, this is your ez', So this is your Z'. So that is equal to Z minus d1, both are known. Z is known, d1 is the distance from here to here- O to O', so this is known. X is known. theta1 is known from here. So, Ex' and EZ' are known. So, now you just look at carefully these are the two equations in which two unknowns are there: theta2 and theta3. And you already know ex' and ez', ex' and ez'. So, can you recall your two-degree-of-freedom system solution so effectively? This was your X, this was your X. Let us say: this is your X'. So, that is your new X axis and this is your Z' that is also equal to Z because both are aligned together. So, this is your 2R subset, and this is your 2R subset. So, now what we know is ex' and ez', two are known. This and these are known. So, you can quickly find out theta2 and theta3 using your 2R subset solution. So, you have already solved this kind of equation earlier in the 2R system. So, you can quickly make it like this.



So, this will give you theta2 and theta3. You just have to replace X with ex', Y with it was a planar system. So, we took this as X, this as Y. This was your system. This was a1, this was a2, theta1 and theta2. So, I am just replacing theta1 with theta2 and theta2 with theta3. So that is it. theta2 and theta3 is obtained. So, instead of a1 and a2, it is a2 and a3, got it. It is a2 and a3, got it. So, exactly this you can obtain. And instead of X, you have ex'. Instead of Y, you have ez' got it. So, this quickly, you can obtain theta2 and theta3. So, now you have obtained theta1, which was here, and theta2 and theta3 got it. So, all the angles are obtained. What was known was X, Y and Z, so that was input to the system. What was known was d1, a2 and a3. So, all the kinematic parameters- d1, a2 and a3 were known. So, all the parameters were known. So, now, if I tell you it has four solutions, how?

You can quickly see the previous slide, and you can quickly see this. So, the first solution, the one which is visible here, shows an elbow-down solution for the 2R sub-system. So, this is your elbow down. So, another one to reach the same place would be something like this: this becomes your new theta2, and this is your new theta3, so you know, by the earlier method. So, at least these two are easily visible. So, what are the other one? You can just see from here that theta1 is either both Y and X positive, both Y and X positive or both negative. Still, you will get theta1. So, what does it effectively signify if you look from the top? So, these were your four quadrants. Here is your robot when it is X and Y, both positive. So, this was the end effector position that gave you theta1, so the end effector position can also be obtained. Similar to theta1, you will see when you have the robot which is facing something like this. So, this also will give you the same tangent solution. So, X is negative, Y is negative. You will obtain the same theta1, Which means your robot is now completely rotated by 180 degrees, and you are coming back. You are coming back and you are obtaining going to the same X- Y position. Even with X- Y, both positive, theta1 can be theta1 plus 180 degrees. That is till here you can reach. So, this is where you are now, and now you are gone back. So, this is like you can do it, something like this: you are holding, like this, elbow down, you can rotate 180 degrees opposite and come back like this, and you can come back to the same location. So, that is what. So, if I can show you this robot so that it can go here this way, or you can rotate it, this robot now can obtain a similar solution going like this. Got it? So that is how it can be obtained. So, theta1 is theta1 plus 180 degrees and again, you have an elbow-up and elbow-down solution. So, 2 for the front. So, solution 2 from behind can do elbow and elbow down. Not all of them are achievable using any standard industrial robot because you have joint limits, you have structure of the robot does not allow you to do so. So, that is quite possible, but mathematically, it has four solutions. Got it.



Now, let us move to our old friend. So, this is a 3-degree-of-freedom cylindrical positioning robot. It was revolute, prismatic and prismatic. First was theta1, the next variable was d2, next variable was d3. So, all together, you see the forward kinematics we got. This can be written as you just take this portion out that says this is x, this is y, this is z. Got it. So, exactly that is.

x = -d3S1

y= d3C1

z = d1 + d2

$$x = -d_3 \sin\theta_1$$
$$y = d_3 \cos\theta_1$$
$$z = d_1 + d_2$$

Now, what is given is for an inverse kinematics problem, x, y and z are known. You have to find what is the joint variable, what are the joint variables. So, the joint variable was theta1, and this one was revolute, so this is unknown. The next joint variable is d2 because this was a prismatic joint. This was a revolute joint. For the next prismatic joint, you have d3, which is to be found out. So, all these are unknowns. So, these are known. So, you just have to solve it. So, let us see how to do it. So, the inverse kinematics solution here could be theta1 is simply, if you see it carefully, it is minus x by y. Just look at this carefully.

$$\theta_1 = \tan^{-1}\left(\frac{-x}{y}\right)$$

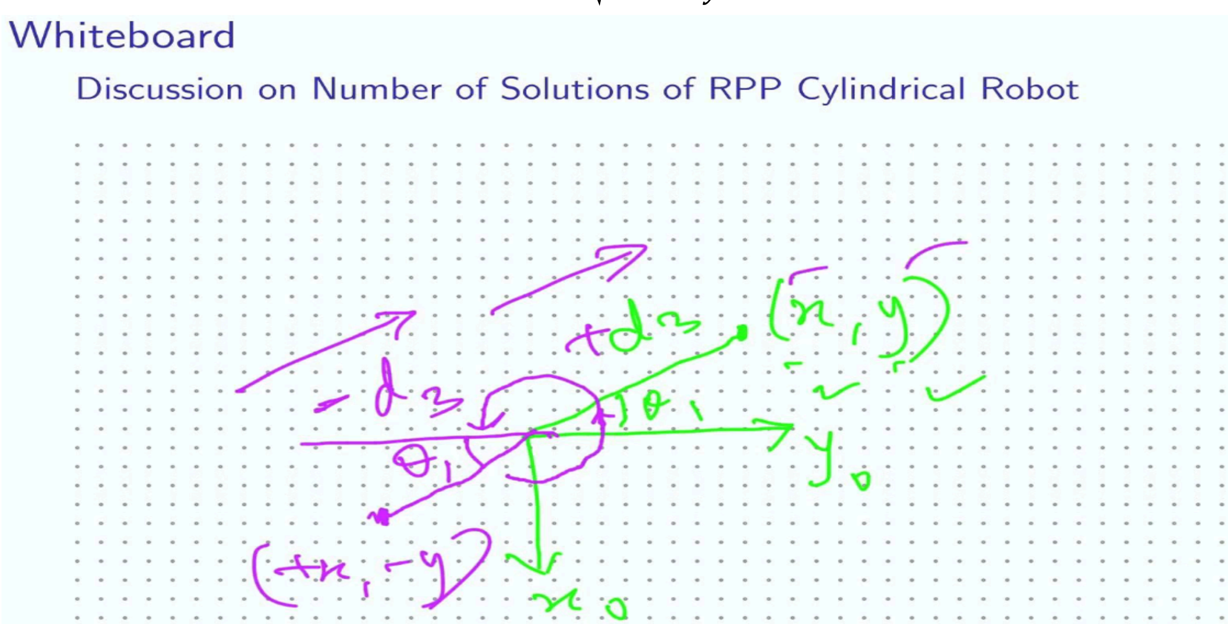
You just have to divide them. So, if you divide this by this, you will get a minus of tan theta is equal to x by y. So, tan inverse x by y minus x by y will give you theta1, so this is your joint angle, theta1, which can be obtained by just having x and y. it doesn't require z at all.

Now, how z can be utilised? So, you just have to use this equation now. So, d2 is z minus d1, so what is that? You know d1 is a constant. This is the offset from here to here. So, that was the offset. So, d1 was known. Z is because it is an inverse kinematics problem. Z is given, so both are known. So, you can quickly obtain d2, got it? So, d2 is obtained.

$$d2 = z - d1$$

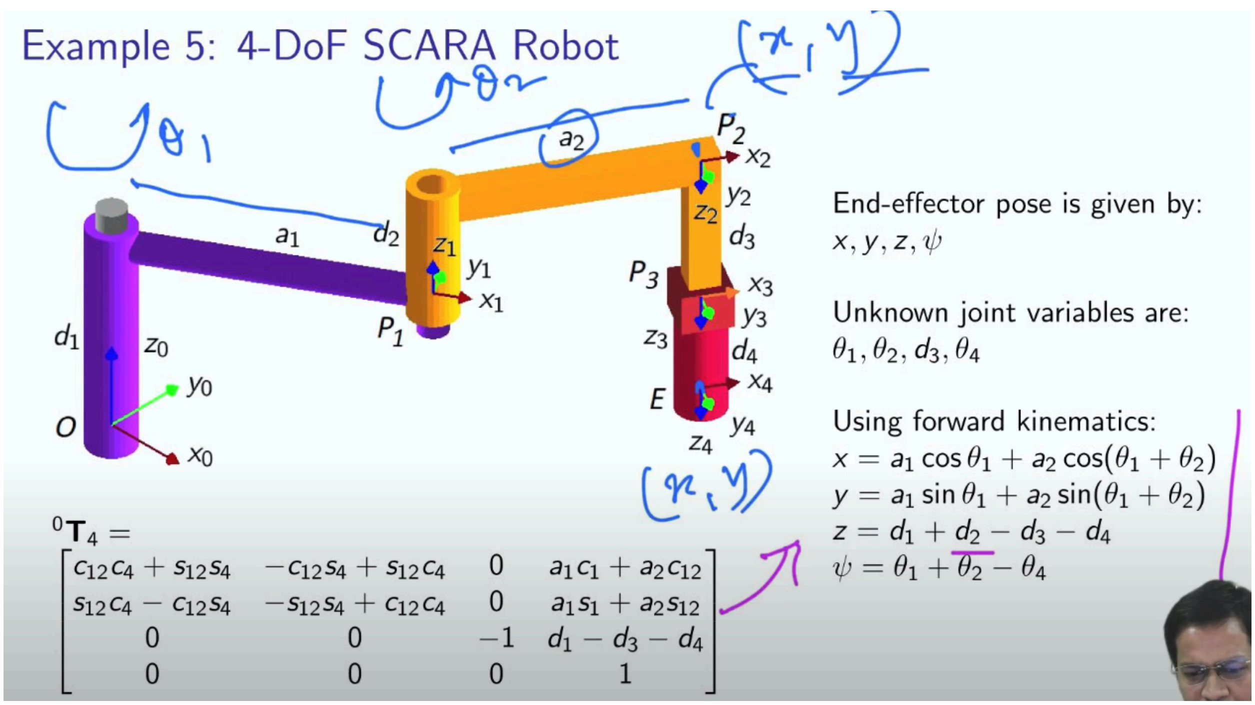
Now, d3 is just square and add take the root. so you can quickly obtain d3, so d3 is like this.

$$d3 = \sqrt{x^2 + y^2}$$



Now, let us analyse its multiple solutions on the whiteboard. So, you see, you also have a similar solution that we obtained earlier. Let me draw a whole of this system looking from the top, how it should be.

So, this is your x0, this is your y0, and your link is somewhere over here. If it is positive, theta1 goes like this. This and this is your distance, d3 that makes an angle, theta1, from the vertical. So, this is your d3, got it? So, you are here with coordinates x and y, got it? So, how many ways can I reach XY, so theta1 from here? You know it can be either minus x plus y or plus x minus y- both are equally good and take you to the same theta1 angle. So, physically, what does it mean? What is this? This is minus x plus y. x is negative, y is positive. So, you can also be in this quadrant. So, here you have plus x and minus y, plus x and minus y. You will again get the same angle. So, this also is theta1, this is an alternative solution for theta1, but this is taking you here. So, in order to go to the same place, instead of coming to d3 as positive, like this, this should take you d3 as negative, got it? So, this time, instead of coming out this way, you have to go back in this direction, got it? So, d3 is negative in that case. Got it? So your robot has turned, not just till here, it has gone till here. Got it? So, this time, instead of moving forward, you have to go backwards and achieve the same x, y. So, this is how you see you have how many solutions two solutions which are quickly visible here. Also, you can see that d3 is the square root of x square plus y square. It says plus or minus, is it not? So, the plus 1 is considered when this is the case. Exactly the 1, which is here when you have negative x. positive y minus is considered when you have a negative of y and a positive of x. Got it? So, these are the two solutions. So, you have to consider these while you are programming for the inverse kinematics of a cylindrical positioning robot. Got it? So this is it.

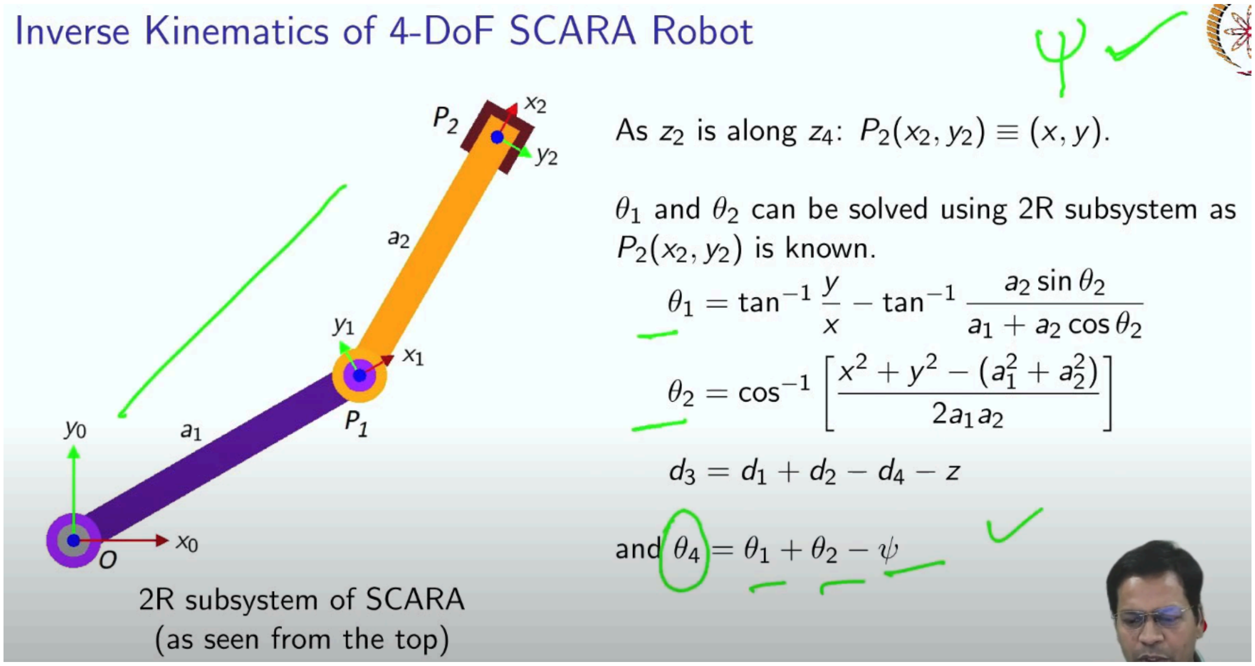


Now, let us come to one very important industrial robot that is known as the SCARA robot. It is a 4-degree-of-freedom robot. You have already seen that. So, this was the forward kinematic solution that we obtained earlier. There is some addition here. This distance (d2), we took it as 0 over there, so you don't see any d2 over here. Otherwise, you will also see this. You can quickly arrive at it like this. So, if this is your z, that was your z direction. So, what do you see? It is d1 plus d2 minus d3 minus d4, is it not? So, minus d3, so in addition to d1 minus d3 minus d4, z will be d1 minus d3 minus d4 plus d2, got it.

Others will be the same. So, the end effector pose is given by this. That is the x, y, z position of the end effector, which is here, and its orientation also, and total orientation. You know, it is the sum of all the angles that your robot has rotated. Your end effector has rotated along the vertical axis only because this is the robot which doesn't have any roll and pitch angle, and it can only rotate above the vertical axis. So, it makes things quite simple. So, if this is your theta1, this is your theta2, and you also have theta4, this was your prismatic, so theta3 was varying. So, the sum of all these angles is actually the total angle, psi, which is obtained, got it? So, unknown variables are theta1, theta2, d3 and theta4, d3 is this distance. This is a prismatic joint so that it can travel up and down. So, that is what. So, all these are unknown. These are known. Forward kinematic equation from here you can directly get to here.

$$x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$$
$$y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)$$
$$z = d_1 + d_2 - d_3 - d_4$$
$$\psi = \theta_1 + \theta_2 - \theta_4$$

You just see this change. Rest assured, everything is the same as we have obtained earlier. We have to use it. So, your 2R friend is again here. So, you see, this is your first link. This is your second link, is it not so? This is a1, which can rotate about the vertical axis by an angle, theta1. This is a2 which can rotate about the vertical angle, that is theta2, very nice till this point. And this point both lies one over the other. Okay, so whatever is XY for this end effector, the same will be the x, y for this point, is it not? If you know XY, which you know already, you can quickly calculate theta1 and theta2.



So, that is what we will be using to solve the SCARA as well. So, I will look at it from the top. It is a 2R sub system which is out of SCARA, as seen from the top. So, you see, it looks like this. So, whatever is my x, y that directly can be written as the position for P2, this is your a1, this is your a2, this is your theta1, again, you have a relative angle theta2, x, y is given. You can quickly find out theta1 and theta2, this is there. So, theta1 and theta2 can be found out from our earlier 2R solution that we did. Okay, so I have just copied it from there. So, this is it. This, you know, has got two solutions. Okay, it can be like this. It can be like this: this is your theta1, or this is your theta1, so again, this is your theta2, or this can be your theta2. Got it? So these were the two solutions. So, we will come back to these multiple solutions, but for now, you know you have already obtained theta1 and theta2, so d3, what is d3, d3 was also unknown, so you go back to your SCARA robot and just check what d3 was.

So, this is what you want to find out what all things which are known are d1, vertical distance, and d2, which are the structure of the link1 and link2. link1 and link2 structure, part of that, okay, and d3 is variable. D4 is again the offset from. This to this. So, that is again a constant. So, d1, d2, d4 are constants, whereas d3 is variable. Z is something which is known. Z is known, got it? So, let me come back to this. z is known. d1, d2, d4, you know, you know it. So, yes, d3 can be evaluated directly. So, there is just one unknown in the vertical direction. So, straight away you can obtain that. So, three of them, you have obtained the final one. So, now you already know theta1, you already know theta2, got it. End effector orientation was given. Psi angle, that is, the total rotation of the end effector, is also known. So, this is known. So, you can quickly obtain theta4, that is the vertical angle of rotation for your which link, this link, this angle, is theta4, so you have calculated theta1, you have calculated theta2, total angle: psi is given. So, from here. So, that is what is here, got it? So, yes, solving these systems is very, very trivial. Now, now that you know how to use your 2R subset everywhere, even your 3R subset, you can't use it. The one which we took today is a 3R spatial system. That is a subset which is there in most industrial robots. The first three links will be arranged like that.

So, yes, that is all for today. For the next lecture, we will be doing inverse kinematics of 6 degrees of freedom industrial robots. So, that is all. Thanks for today, bye-bye.