NPTEL Online Certification Courses
Industrial Robotics: Theories for Implementation
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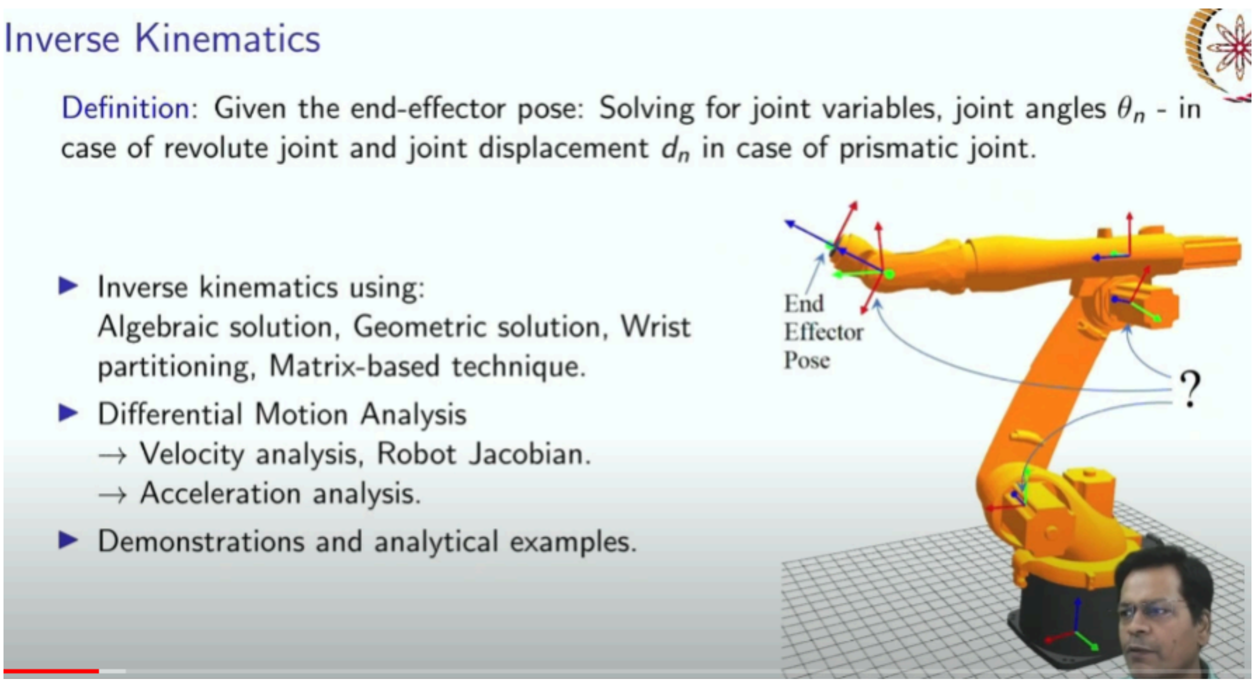
Lecture: 21

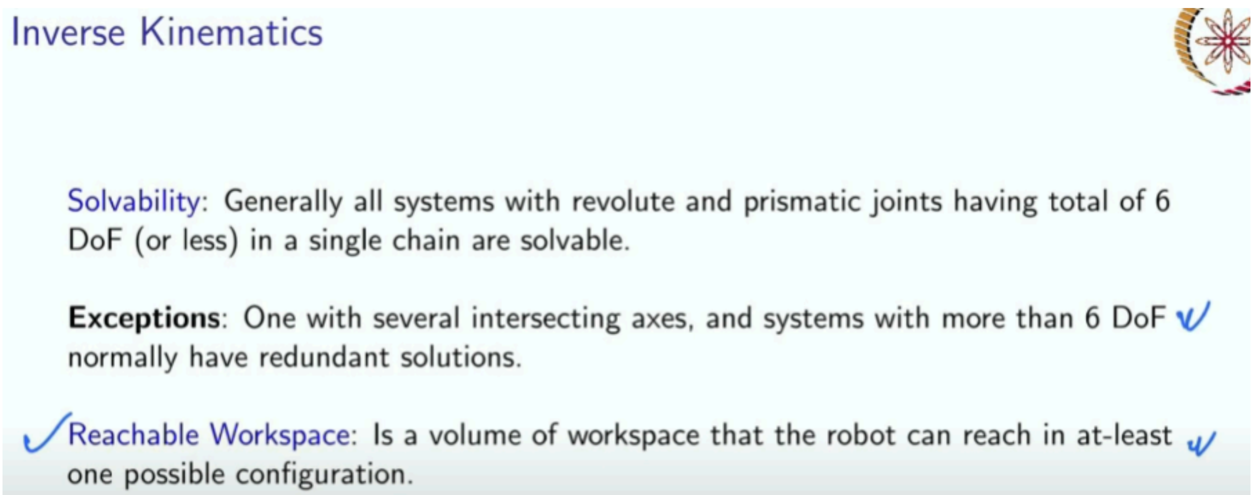
Inverse Kinematics: 2 and 3 DOF Planar Manipulator

Welcome everyone. So, in the last module, we covered forward kinematics problems. So, that is solving for end-effector pose, that is, position and orientation, given the joint angles. So, in this module we will be doing an inverse problem of that. Given the end-effector pose, we have to find out the joint angles. So, that is the quick definition of inverse kinematics. We will see as it comes, and we will also be doing differential motion analysis for robots, that is, to obtain velocity, acceleration and all the differential relations between the joint and the end-effector. We will try to do it in this module.



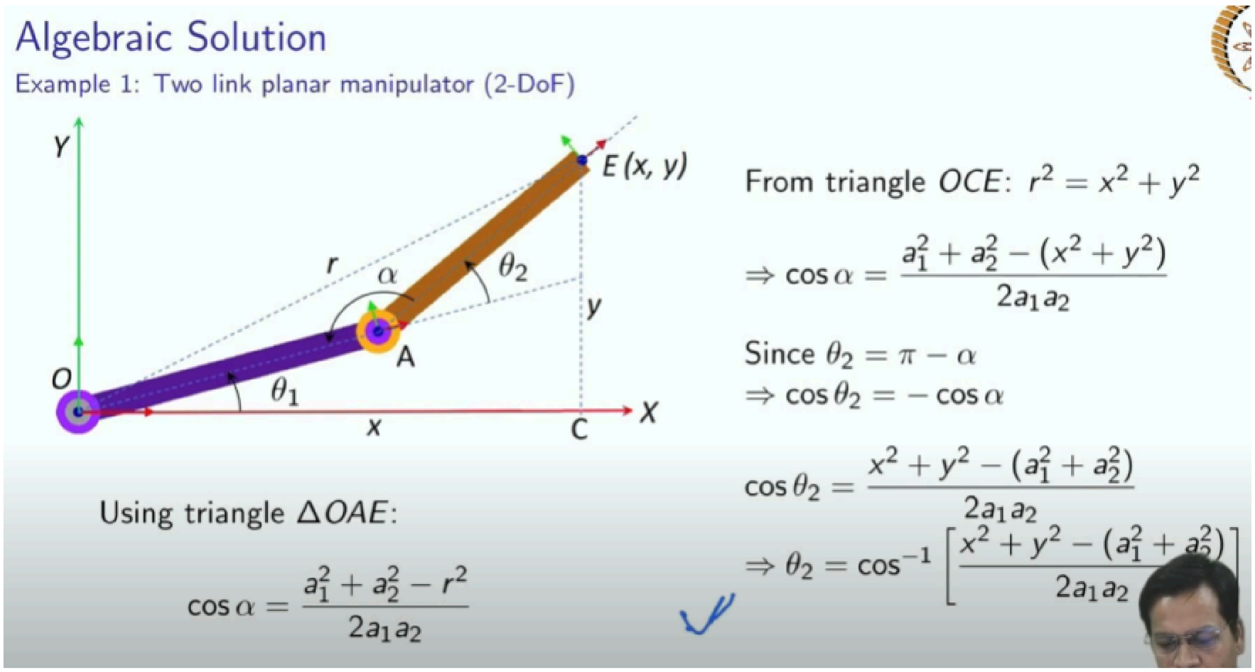
So, let us move ahead by a simple definition of this, that is, given the end-effector pose, solving for joint variables, that is, joint angles. In the case of a revolute joint, it is joint angles, theta ($\boldsymbol{\theta}_n$). In the case of prismatic joint, it is the displacement d_n. So, it looks like this End-effector pose. That is the position of this frame, the frame which is attached to the last link where you attach your tool that is the frame. So, it is here that the position of that will be given and the orientation of that will be given. That means the x, y and z axes which are attached to that frame.

The direction of that may be given. So, that is your input to this algorithm. That is the inverse kinematics algorithm. So, that will be given, and you have to find out the joint angles, All the joints that actually take your robot to that particular position and orientation. So, that is your inverse kinematics problem. So, in this module as a whole, we will be doing inverse kinematics using the algebraic solution, algebraic, trigonometric solution, and geometric solution for quick results from a few of the planar robots, which you can obtain risk participating method that is quite commonly used to solve inverse kinematics problem of six degrees of freedom, industrial robots. So, we will be doing that. And matrix based technique And differential motion analysis we will be doing.Velocity analysis. We will be trying to find out the relation between end effector rates and joint rates. That is related to using Jacobian. So, we will be doing that. Analysis for acceleration: We will be doing some demonstration for inverse kinematics problems. How to solve for different types of robotic systems, planar systems, two degrees of freedom systems, four degrees of freedom systems, and six degrees of freedom systems will be checked using some analytical examples.



So, let us begin. So, before we actually start, let me just familiarise myself with some of the important terms which we will be using quite often. Whether a robot is solvable or it is not solvable that we need to check. How do we get to know generally, that all the systems with revolute or prismatic joints have a total of six degrees of freedom in the serial chain? Serial chains are link, joint, link, joint, link, joint. So, that is how it terminates at the tool flange, So it is attached to the base. So, normally, for this kind of robot. This robot is mostly solvable using standard algebraic techniques until and unless there are a few exemptions, which are there, like you may have one with several intersecting axes, So that may not be solvable using standard techniques. So, there are many other techniques to solve them. It is not that it is not solvable. Systems with more than six degrees of freedom, like seven-degrees-of-freedom robots. The way our arm is like, It has seven degrees of freedom. Starting from here it is three degrees of freedom: one degree of freedom and three. So, you have infinite solutions to go to any of the positions. In any of the positions, you can go there and obtain the same orientation using an infinite number of ways. So, that is a degree of freedom system. So, those are a few exemptions that you cannot solve using a standard algebraic technique.

So, a reachable workspace. You also know this using your old definition that we discussed in the first module, where we discussed the industrial robot technical specification sheet. So, what did it say? It said it is the volume of workspace that a robot can reach in at least one of the possible configurations. So, at least in one of the possible configurations it should touch that point. That endpoint may be: maybe the boundary point, maybe the outer boundary or the inner boundary. So, not all solutions may be achievable over there, but at least one of them is definitely doable. So, these are a few definitions that we will be using while we are doing inverse kinematics problems.



So, let us begin with an algebraic solution for two degrees of freedom, a planar manipulator. So, you have also seen this when we did the forward kinematics for this. If you remember, when we arrived at, x is equal to x and y, given a1 and a2. Those were the link lengths. So, it was x is equal to a1 cos theta1 plus a2 cos theta1 plus theta2.

$$x = a_1\cos\Theta_1 + a_2 \cos(\Theta_1 + \Theta_2)$$

Similarly, you obtained y is equal to a1 sin theta1 plus a2, sin theta1 plus theta2.

$$y = a_1\sin\Theta_1 + a_2 \sin(\Theta_1 + \Theta_2)$$

This is what you obtained. That means, given joint angles, you can obtain x and y. Now, you have to do the inverse problem for this. So, you will be given x and y. That is the end effector position. This is a positioning robot. You cannot obtain orientation with this by controlling the input parameter. Those are the joint angles. So, yes, x and y will be given, and you have to solve for joint angles, that is, the joint parameters theta1 and theta2.

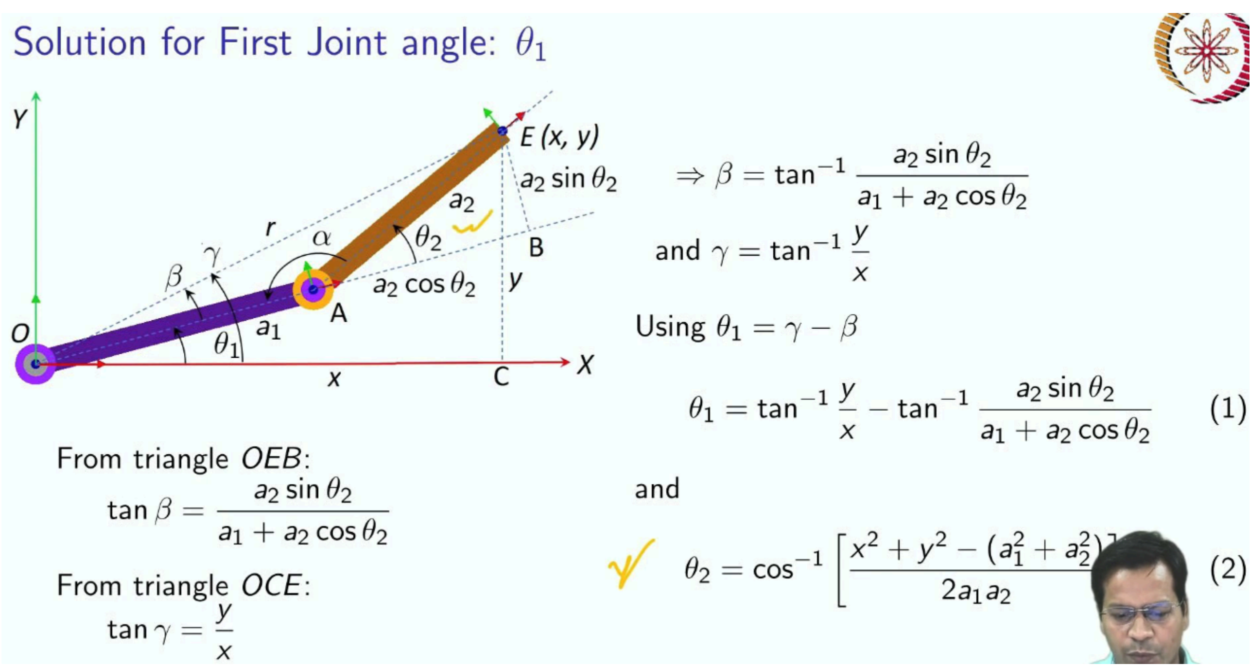
So, let us begin with this. So, using triangles OAE. △OAE, you can quickly write using cosine formulas from trigonometry, you know. So, that is cos alpha. Alpha is angle A, which is here. So, that is that may be written as a1 square plus a2 square minus r square. r is here, that is the length OE, that line which connects O and E, minus r square by 2, a1, a2.

$$\text{Cos}\alpha = \frac{a_1^2 + a_2^2 - r2}{2a_1a_2}$$

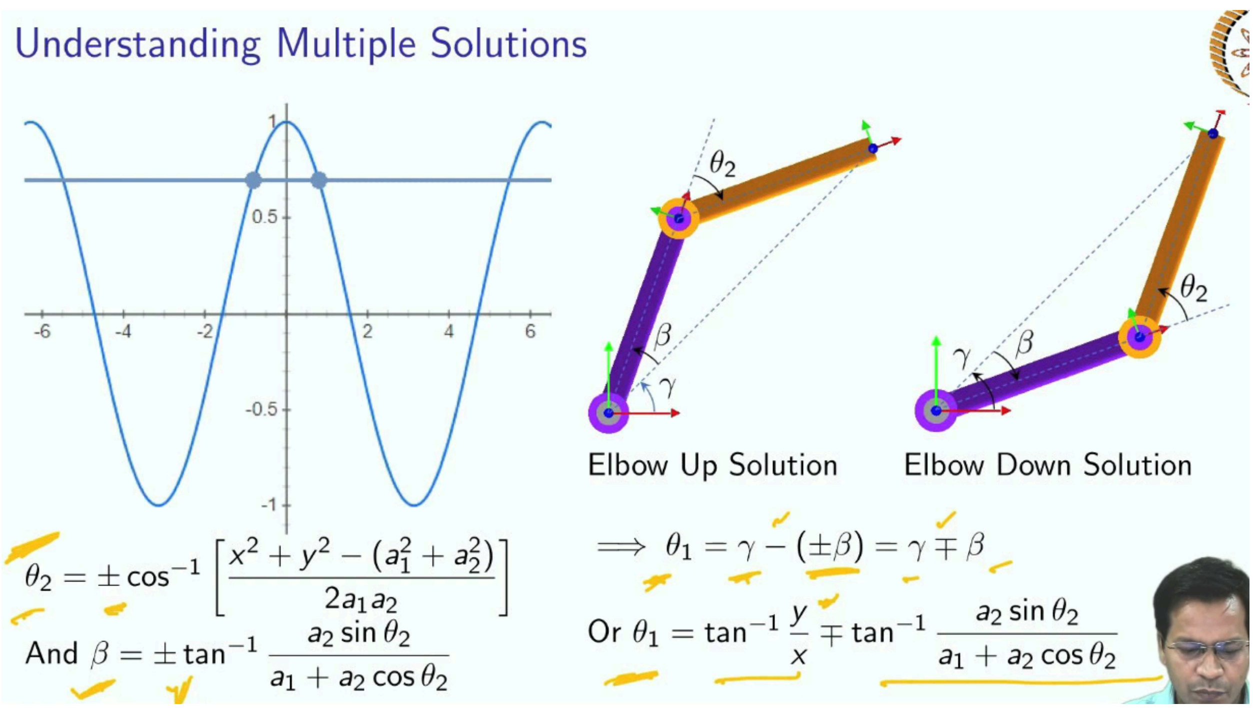
So, this is a quick formula that we can write. So, now, using triangle OCE. △OCE, which is here, using Pythagoras theorem, you know, r square is equal to x square plus y square. Done, this is quite trivial. So, quickly, I will replace r from here. So I can write cosine alpha is equal to.A1 square plus a2 square minus x square plus y square by 2, a1 a2.

$$\text{Cos}\alpha = \frac{a_1^2 + a_2^2 - (x2 + y2)}{2a_1a_2}$$

So, you know all these parameters. You know already a1, a2. Those are the link lengths. You begin the inverse kinematics problem here. X and y are also known so that you can obtain alpha quickly. So, alpha is obtained. You see, theta2 is related to alpha. You can write it as theta2 is equal to pi minus alpha. So, the cosine of theta2 may be written as a minus of cosine alpha. So, now the cosine of theta2 becomes x square plus y square minus a1 square plus a2 square by 2, a1, a2 got it. So, theta2 is equal to this. So, this is a quick output that you can obtain, given all the parameters.



I have to obtain the solution for the joint angle theta1. So, now that I have already obtained theta2, what I need to know is theta1. That is the first joint angle. Okay, so you see, using triangle OEB, this is a reconstruction that I have done. OEB: B is a point which is on the extension of the link length from A to B. So, just an extension which goes like this: and there is a perpendicular d.Rop from the end effector position on this line that meets at B. So, this is your 90 degree. So, tan beta, beta is this angle which is here, the angle which is here. So, the tan of beta may be written as a2 sine theta2. You see where it is. This is the distance EB. This is the distance EB that is here divided by a1 plus a2 cos theta2. So, that is this distance. It is this plus this. So, that is the distance divided by OB. got it? So, tan beta is equal to this. Okay. Now, the second one is from Triangle ONE. You see ONE, it is O, this is seen, this is E. So, that is again a right angle triangle. So, tan gamma, gamma is this time it is this angle. So, the tangent of gamma is equal to Y by X. So, this is Y, and this is X. You can quickly write it like this. Okay, so now you know beta, you also know gamma. You can quickly obtain what you can obtain: theta1, which is here. So, beta and gamma are like this. So, theta1 is equal to gamma minus beta. So, this is your gamma. This is your beta. So, theta1 is equal to gamma minus beta. So, you can quickly write these two values here. So, it goes like this: theta1 is equal to tan inverse Y by X minus. This got it. So, you have obtained theta1. So, this is what we have obtained earlier. So, this is theta2. Okay, so at least this says that the solution is done. Let us analyse multiple solutions to this.

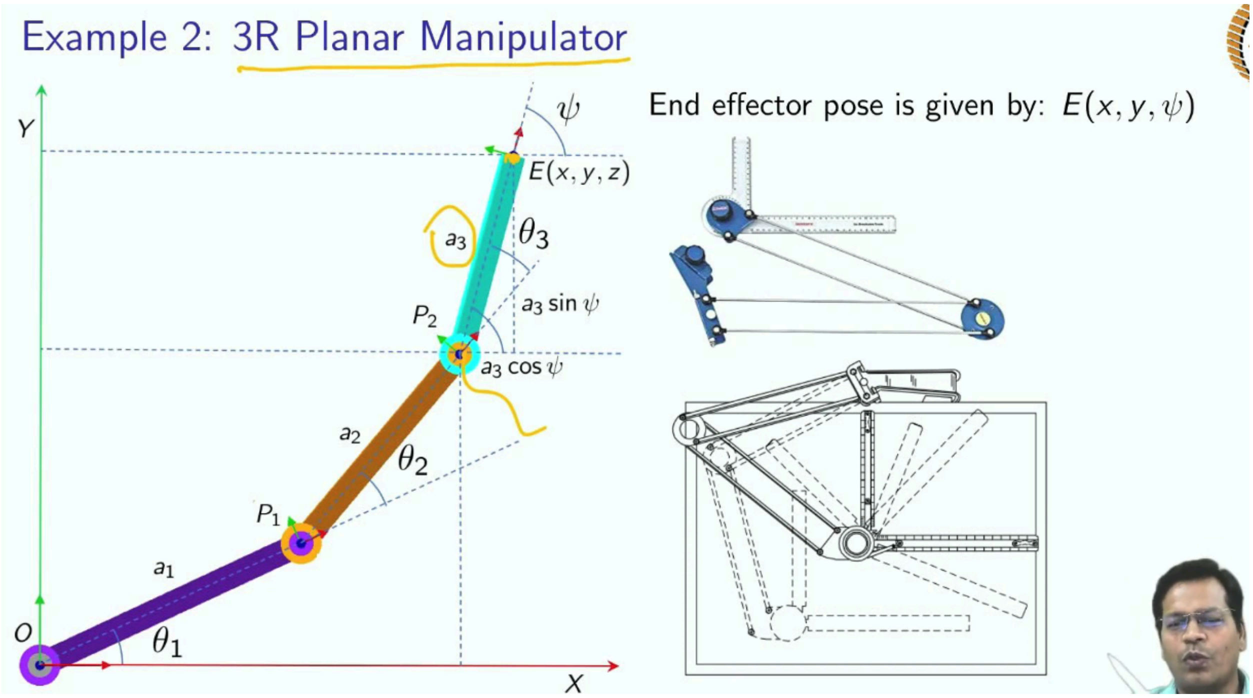


Now, looking ahead, so this theta2 which is here, theta2, if it can be plotted, it can, it will do something like this. So, you see, for a given value over here, you have two joint angles, which are here, so both are having the same value of cosine. So, theta2 can be written as, and because you know the cosine of plus or minus, theta2 will give you the same value. Got it, one which is here. So, once the value is given, theta2 can be plus or minus of whatever we have obtained.

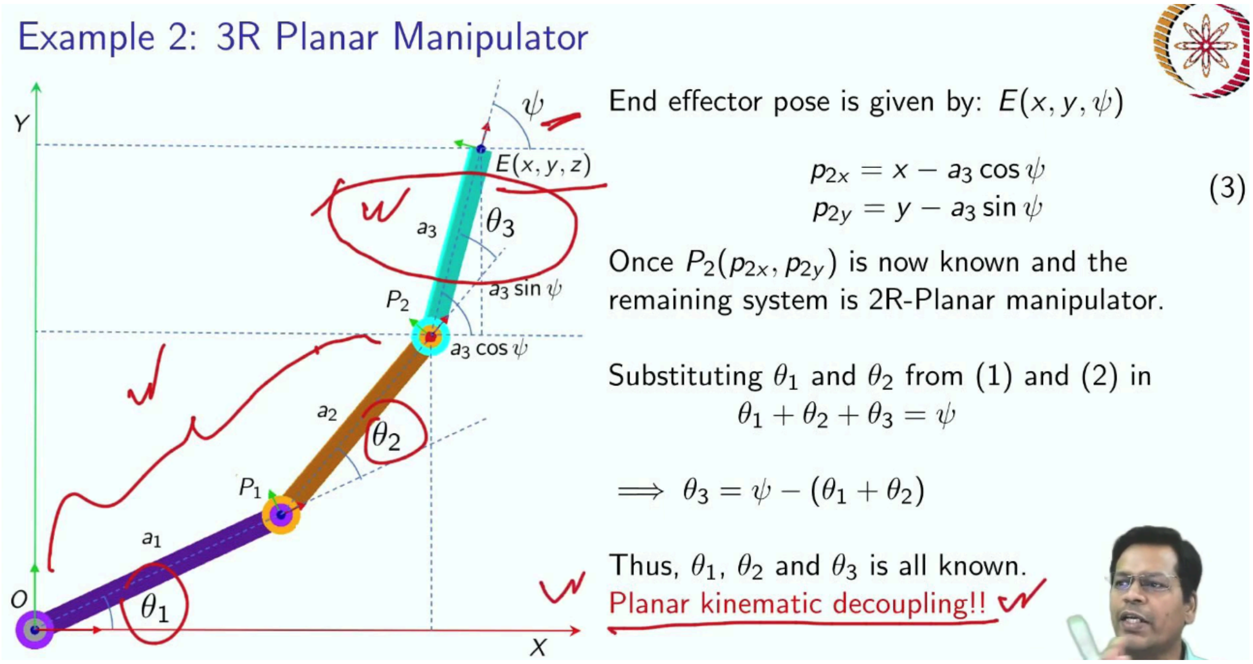
Okay, so now you see there are two solutions, how I will tell you. So, if you write beta as plus or minus this theta2, you know it is already plus or minus, same for beta2 also. So, that can be written over here. So, theta1 becomes gamma minus, plus or minus beta. So, in case it is a plus. So, because it is minus, it becomes gamma minus or plus beta. So, if the beta is positive, theta1 will use the upper one that is positive.

$$\Theta 1 = \gamma - (\pm\beta) = \gamma \mp \beta$$

So, effectively, this gives you theta1 as theta1, as tan inverse y by x minus plus. So, if beta, if beta is positive, beta is positive, you have to use negative from here. So, effectively, these are the two solutions. So, if this is theta2, theta1 will be the upper one if this is theta2, the bottom one. So, you have to follow the bottom one. So, what does it effectively mean? What is the physical significance of that? That means you have from the mean line, if you can see, this was your gamma. So, where is your gamma? If you remember, I have plotted it here, and also-this was your gamma, gamma minus and beta. What does it mean? It is actually beta.
Was this one? So one of them will take you to here- and theta2. You see, theta2 is plus or minus, so it can either be like this, traveling like this, or it can go like this with the same value. So, the value remains the same, but the direction is either positive or negative. So, this is the positive one, this is the negative one, okay, and this one is gamma plus beta. So, when this is plus, this is minus, you see. So, similarly for here, gamma minus beta. So, when this is minus, beta is minus, this is plus, you see. So, this is what gives you the complete solution. What does this physically mean? You see, it is an elbow up, okay, or elbow down solution. The one which we normally do is an elbow-down solution. We cannot do it otherwise. You know, we have our joint limits, and so is for industrial robots also. So, if at all it is analytically possible, that doesn't mean that your robot can also attain all those angles. They are normally limited by the structure of the robot okay, and the joint limits also. So, this is it. So, at least for now, take the analytical solution for granted. So, these are the two possible solutions which are there. So, remember this we will be using quite often for various other robots which are, which we will be doing as an example.



So, let us begin with the 3R planer manipulator. This time it is 3R. So, instead of just ending over here, this time, we are having one more link of length a3, and it is now ending here. What is an additional advantage? Is there any mechanism that you can remember which is of a similar kind? Yes, if you remember, you had an engineering drafter, the drawing tool that you used in probably the first year of engineering. So, that is very much similar to this. So, it had to link with a set square at the end which you can further rotate by a certain angle. So, using the first two link links, you can first two joints and the link combination. You can take that to any place on the drawing board, and the last thing you can do is turn to set the angle of the set square. So, that was to orient that. Okay. So, two links were quite good enough to take it to any place, whereas the third one can orient the end effector also.



So, total orientation is given by this angle. So, that is the combination of theta1, theta2 and theta3 because all were relative angles. So, in this case, what will be the input? So, you will be given the end effector pose that says XY also. XY is the position and orientation which is attained by the end effector. So, that is the frame orientation which is here. So, if this is your frame, okay, the orientation of this frame with respect to the ground may be given. So, this is the angle psi. So, that is what will be given. Now, I have to find out the joint angles: theta1, theta2 and theta3. So, can you see the existing solution that we just now did, that is, two degrees of two-degree-of-freedom robot? So, your two degree of freedom robot lies here. You see, it is here. So, you already, provided you know the solution, XY, which is the position over here. You already know the solution for theta1 and theta2 using the point XY, which is the point P2, got it? So, this is for P2(x,y). If you know this location, you can quickly get this. Can we know this? So, we will be using this solution subset, and we will try to get to this point, provided this information is given. So, that should be my approach. So, let us see how I am beginning. So, this point, P2x and P2y, X and Y coordinate of this point may be given as, if you can draw the triangle which is here. So, how much is this length? This is a3. a3 is the length of this, and psi is the total angle. This, okay, so that is this one only, which is already known. So, a3 sine psi. So, this distance is a3 sine psi.

Similarly, the distance which is here, which is here, is already a3 cosine of psi. So, if you can deduct these two distances from the end effector pose, you can obtain this. So, that is what is done over here. So, P2x is equal to X minus a3 cosine psi. Similarly, P2Y is equal to Y minus a3 sine psi done.

$$p2x = X - a_3\cos\Psi$$
$$p2y = Y - a_3\sin\Psi$$

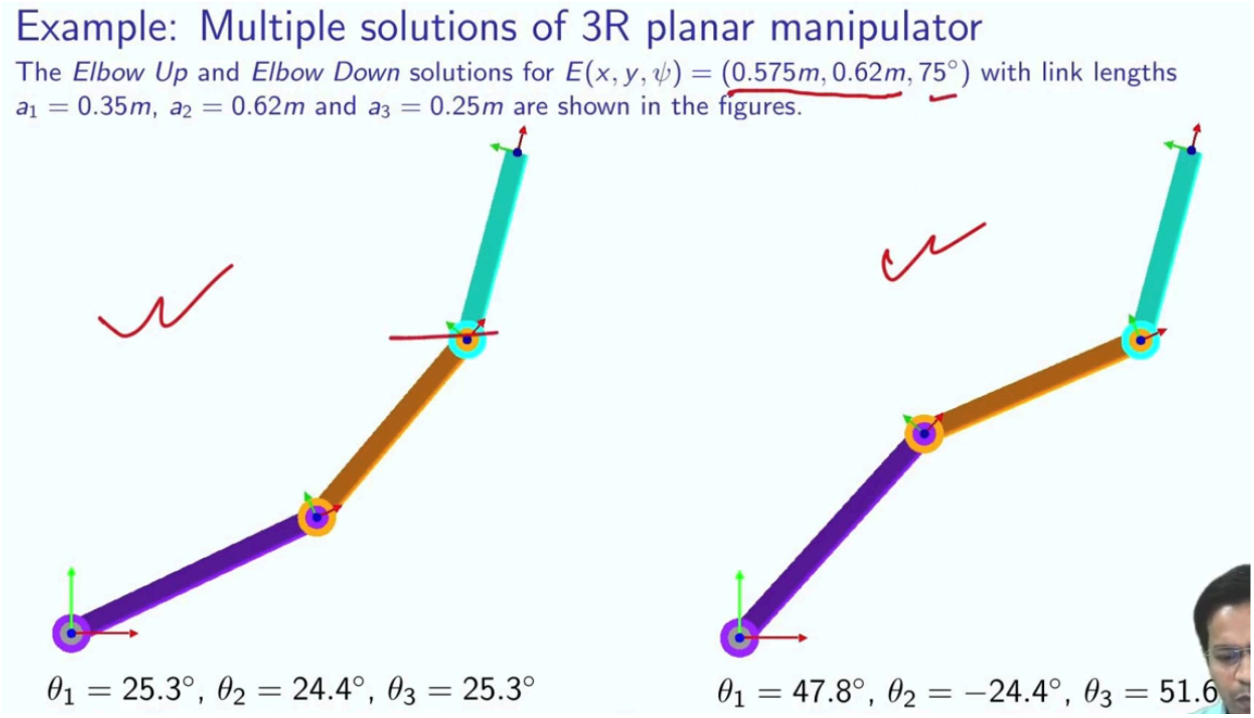
So, now you know this. So, once you know this, you use your existing 2R planar manipulator solution that we just now did, and you can find out theta1 and theta2. theta1 and theta2 are known using a 2R planar solution.

We already know this so we can solve this. So, once theta1 and theta2 are known, you know this. The orientation which is given is the psi angle sum of theta1, theta2, and theta3 got it. So, now you can write theta3 as psi minus theta1 plus theta2. Then thus, all the angles that are theta1, theta2 and theta3 are known. Okay, so is this the solution? Again, you already know at least this part of the subset has two solutions. One was this, other was this: so this was elbow down solution, and this is elbow up solution. So, up solution and down solution. Two solutions are there. So, can you visualise this? Any other solution apart from this at least we have? We have frozen our last link because you know the last link can only make an angle psi. So, the last link is frozen because that constraint is fixing it, the position is fixing the tip, and the angle is fixing this tip as well. So, both ends are clearly defined. So, you cannot move this link once x, y and psi is given. So, this point is totally fixed. So, for at least this point remaining, two links can have two solutions. So, intuitively, you can say it has two solutions again. Okay, so this is this.

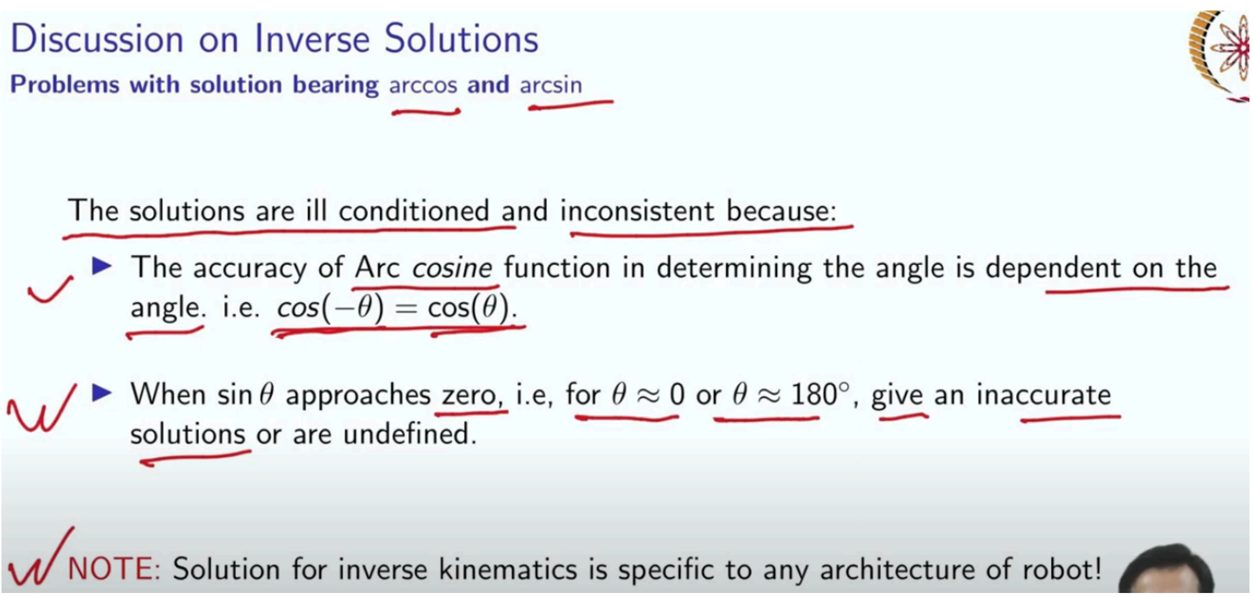
$$\theta_1 + \theta_2 + \theta_3 = \psi$$

$$\theta_3 = \psi - (\theta_1 + \theta_2)$$

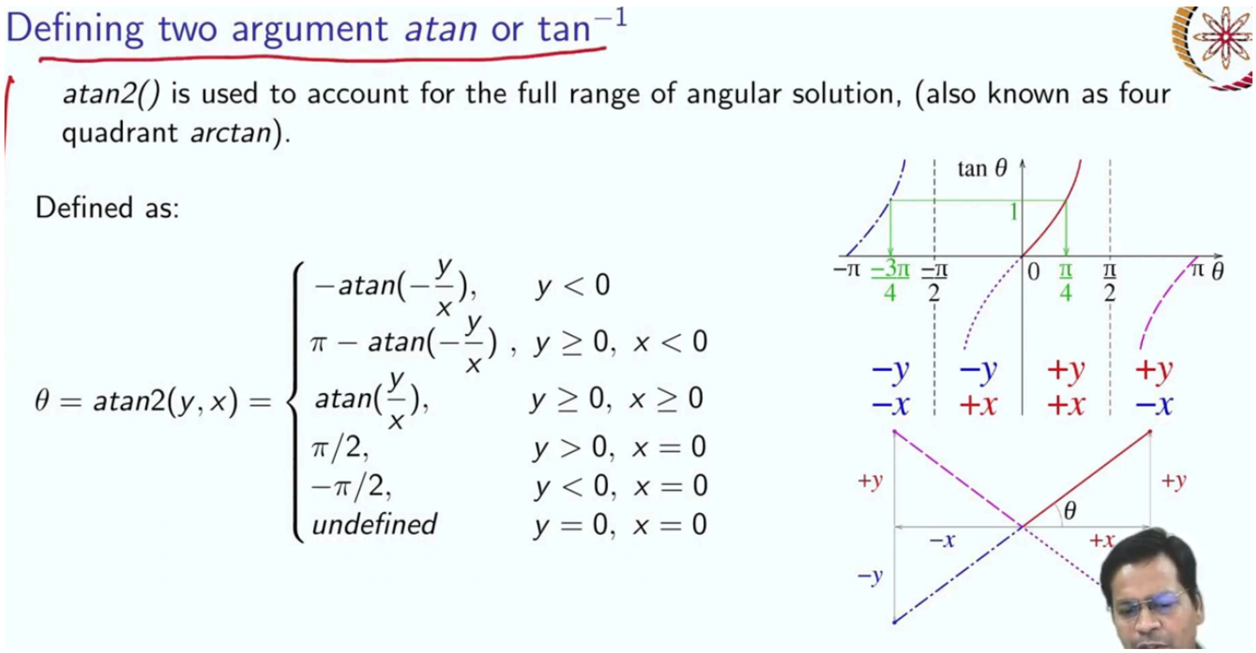
I call it planar decoupling. Why, you know it is now decoupled. The problem itself of the inverse kinematics problem of position and orientation, solving for joint angles, is now decoupled. So, you have to solve using position and using the rest of the links to solve for theta1 and theta2, whereas this was solved by using the orientation angle which was given. So, orientation and positioning robots are separate. So, this part of the system could give you orientation, and this part can give you a position. So, you have just now decoupled the problem into two different problems. You solve for angle, and then you solve for position. Effectively, you solve for all the angles. This I am telling you because later on we'll be using the same principle to solve for any six degrees of freedom solution where the same approach will be used to solve for industrial robots using kinematic decoupling.



So yes, moving ahead in the same line. So, these are the two solutions that I just now discussed. This is just an example. I have taken some inputs. This (0.575m, 0.62m, 75°) is the end affected position. This is the angle, which was 75 degrees. For this, this position was given as this, and I could get this many angles using the same set of algebraic solutions that I have obtained. Got it. So, these are two different solutions.



So, now, let us move ahead. So, there are a few problems with the solution that we have obtained for two degrees of freedom and three degrees of freedom system. So, both used arccos and arcsin. These solutions are ill-conditioned and inconsistent because you know the accuracy of the arccosine function in determining the angle is dependent on the angle. Also, the cosine of minus theta is equal to the cosine of theta. So, both are the same. So, there is some ambiguity. You have to take care while you do programming for this. Also, for sine, as sine theta approaches zero, theta approaches zero or 180 degrees. It gives an inaccurate solution, and sometimes, they are undefined if it comes in a denominator. So, as theta tends to zero, you know it. It is very, very difficult to obtain those solutions. So, the solution for inverse kinematics is specific to any architecture because, as you have seen just now, each structure of the robot has its own solution. There is no generic solution like a serial chain in forward kinematics that we did earlier in the previous module. Okay, so you had a specific solution. Any general robot can be solved using that, but there is no general solution for inverse kinematics, unfortunately. So, this is there.



So, in order to get rid of these types of problems, sine and cosine function, try to get as much as possible tangent solution, or you can. You know you can easily convert sine a to tangent, cosine to tangent one angle solution using trigonometrical functions. So, if you know the tangent solution, you can quickly obtain it like this. So, this is the special algorithm which uses what is known as four-quadrant arcs tangent solution is defined as. This has the possibility to take care of all the positives and negatives. Let us say if it is a tan y by x. It will give you the same value for positive-positive of y and x and negative-negative for y and x. So, you know the problem now. So, this is the solution for this.

$$\theta = \text{atan}\left(\frac{\pm y}{\pm x}\right)$$

So, it defines atan2. It is known as atan2. At least in Matlab it is commonly used also. So, it says it is a function of y and x, whereas when y is negative, you have to use this.

$$-atan\left(-\frac{y}{x}\right), \qquad y < 0$$

When y is greater than zero and x is negative, you have to use this.

$$\pi - atan(-\frac{y}{x}) \ , \ y \geq 0, \ x < 0$$

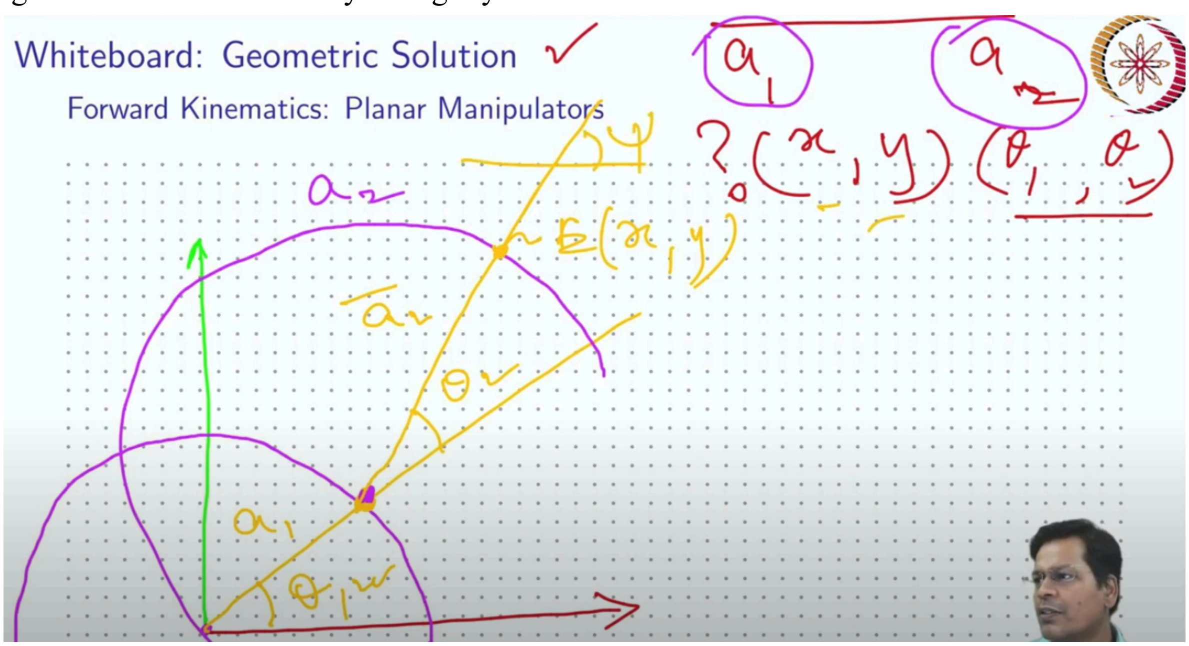
So, actually, it takes care of all the quadrants, okay? So, if this is y, this is x. So, where you are lying, you know it physically, if x and y both are positive, you can quickly use the way it is this one.

$$atan(\frac{y}{x}), \qquad y \geq 0, \ x \geq 0$$

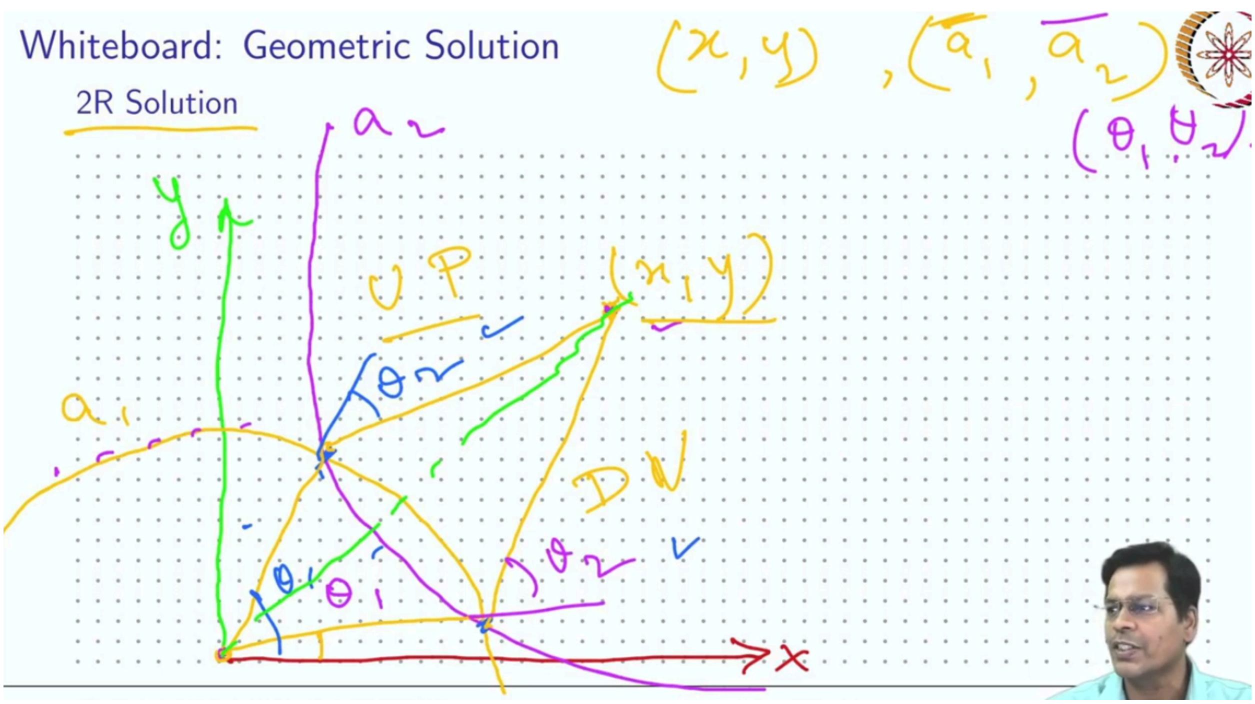
So, x is positive, y is positive, and you can directly use the way it is given. So, that is where your solution should be. But if y is negative, you know. If y is negative at the same one, you know your angle will be like this. So, that is the reason: if y is less than zero, you have to use this one.

$$-atan(-\frac{y}{x}), \qquad y < 0$$

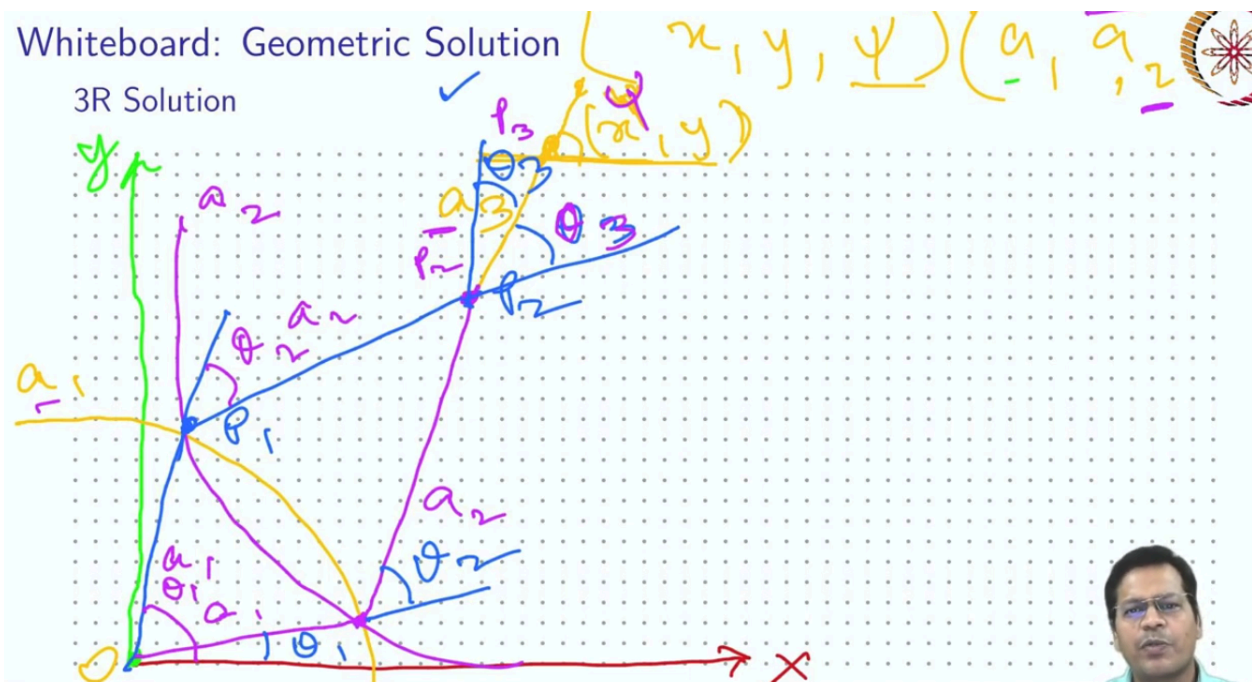
And if there is also the possibility that in the case of a tangent solution when x tends to zero, the angle tends to your value, tends to get infinity. So, in that case angle will be given as pi by two, you know? So if y is greater than zero and x is equal to zero, in that case, you have to use this, and if y is less than zero, x is equal to zero, it becomes minus 90 degrees. Both are equal to zero. That is not possible. So, it is undefined, got it? So, it takes care of the complete range of angles from minus pi to plus pi and for all the quadrants. So this is why it is very, very useful. Try to get the tangent solution, or try to convert the cosine ones to a tangent one, and then you can use this. Okay, so this is to be taken care of when you solve the inverse. This will give you a complete range of solutions without any ambiguity.



So, let us do it. Let us first understand what is the geometric solution before we actually do it. So, how can you draw a two-link using? You can do forward kinematics of a two-link system using a geometric approach. So, what you know is link length a1 and a2. What I want to know is the position x, y. Position x, y for the given theta1 and theta2. Those are the joint angles. So, you are given this, you are given this. You have to find out x, y, how you can do it. You have a coordinate system now. So, this is your x. okay, this is your y. now, I'll be drawing the robot. Okay, so here goes your robot. So, you just have to understand. This is your a1, so this is always constant. So, you can quickly draw a circle with radius a1. I don't know how good I am, but yes, it looks okay. So, this is your a1. Okay, so this is your a1. A1 can go anywhere, but you can always draw a straight line, a straight line that is at an angle theta1. So, wherever it intersects, that means this is the length, a1, this is the angle, theta1. So, this should be the tip of your first link. Got it from here. Your second link should originate with an angle, theta2. So, now you understand what I am going to do. So, this is your second point, from where you can draw another circle of how much radius, this one a2 radius. So, now I'll also draw a circle with radius a2, a2, and again I'll draw, a straight line at an angle, theta2 originating from here. So, this time, I'll again draw a line at an angle, theta2, from here. So, this is a2 wherever it intersects. This becomes your a2, and this becomes your end effector point XY, got it? So, this is how you can do forward kinematics using a geometric approach. So, you see, the same you can do for any number of serial chain systems. If it is planar, you are given a1, a2, a3, theta1, theta2, theta3. You can quickly obtain the end effector pose position, and also write XY. You can quickly obtain. Just by measuring the angle, you can obtain the orientation of the end effector frame. So, this is how geometric approaches are to be done, at least for planar.



Let us quickly do an inverse kinematics solution for 2R that is, two revolute, the same planar system which is given. So, what you will be given is x,y. What additionally you will be given is link lengths, that is, a1 and a2. Let us see if we can solve for theta1 and theta2. So, this is to be solved, so let us do it. So, again. I'll draw the horizontal x-axis and I'll also draw the y-axis, which is here. I am given the point in its workspace, which is x, y. So, this is already given. x, y is given. It is within the workspace of the robot, mind it. Now, can I get the complete configuration of my system? So again, I'll do the same thing. I already know the link length: a1. So, it should start from this point. So, I'll just draw a circle of radius a1. I am done. I know my second link is going to end over here. The second link will end over here. Okay, so I also know its length. So, you know what I should do now. I'll just draw a circle of radius a2, radius a2. So, this is my second link to possible solutions. Second link possible configuration: it should lie. The first end will lie over here, whereas the second lane end will lie somewhere on this circle. Similarly, for the first link, the first end will lie here, whereas the second one will lie anywhere on this circle, wherever it is intersecting, you see. So, it is intersecting here, and it is an intersection here. So, these are the possible intersecting points of both the links. So, you see, you can draw your link one like this and link two like this. So, this is one possible solution. Another one is this one. So, this is your elbow-down solution. This is an elbow up solution for the given XY position you have obtained. So, this is your theta one. This is your theta two. Similarly, you have another solution, also here. So, this is again your theta one, and this time, this is your theta two. Got it? So these are so theta two. Theta two should be the same in magnitude, okay, whereas this is theta one, theta one, if at all. You are interested in finding out the gamma, which was the intermediate solution. It should be somewhere over here. Got it? So, now, you can visualise what we have done, okay, so this is the approach for solving any planar system. Revolute, at least prismatic, can also be done using geometrical drawing. Got it? So this is it.

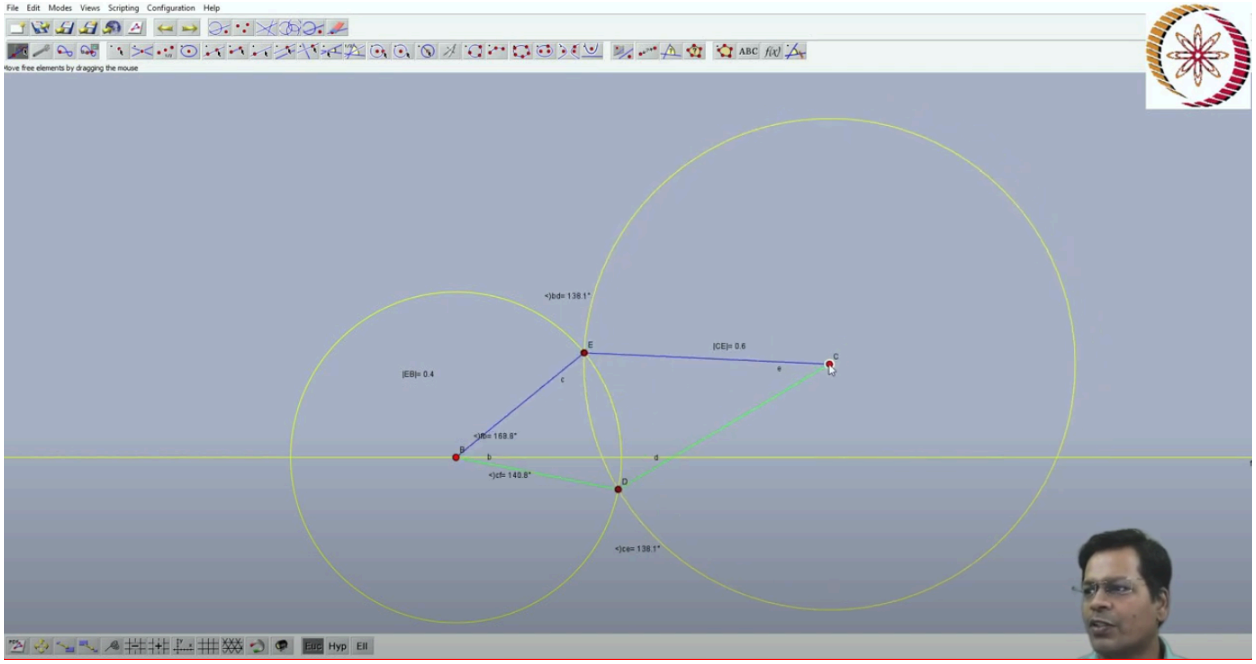


Now, can I do it for 3R also? Yes, what? All things which will be given? So again, you will be given x, y and the angle, total angle psi that is the end effector pose. So, this will be given length will be given a1, a2 and a3, got it. So, all these will be given. Can I do inverse kinematics? Now, let me again start doing it in the same way. I'll start with the x-axis, I'll draw my y-axis, also got it. So, start from something that you know. Again, I'll do the same thing. I know my link length, a1, and the place where it starts, so I'll start from here. I'll draw a circle of radius: a1, done from here. Again, I know the last link point. So, x, y I already. I also know its slope. So, can I draw the slope quickly? So I also know its slope, and it's so from this point, the end effector point has to make an angle psi here and draw it backwards, take it to this point by a distance a3. So, you know this last link length. That is a3. You also know from the ground it is making an angle psi, got it? So, let me rewrite it once again. So, this is your angle, which is known from the ground. This distance is known. So, you have already reached the point p2. Okay, this was yo. You're p3, which is end effector 0. P1 is somewhere on this circle which is here. Now, let me do this. A2, so a2 starts from here. The second link starts from here. Length is given by a2. So now, it should lie somewhere on the circle again. So, let me draw this circle once again. It should be something like this: so this is a circle with a radius equal to a2, starting from the p2 point now, if we connect all the points. So, this is one solution, got it? This is one solution. This is your p1. This is your p2, this is your p3, and similarly, you have yet another solution that is given by this one. This is the point of intersection. This is your second link. Okay, so now all the link lengths I can write. So, this is your a1, a1. This is a2, this is a2. So, what are the joint angles?

You can quickly measure it. Measured it from here. So, this is your theta1. This becomes your theta2. Okay, theta3 is: this is your theta3, got it? So you can just measure it. You can just measure it.

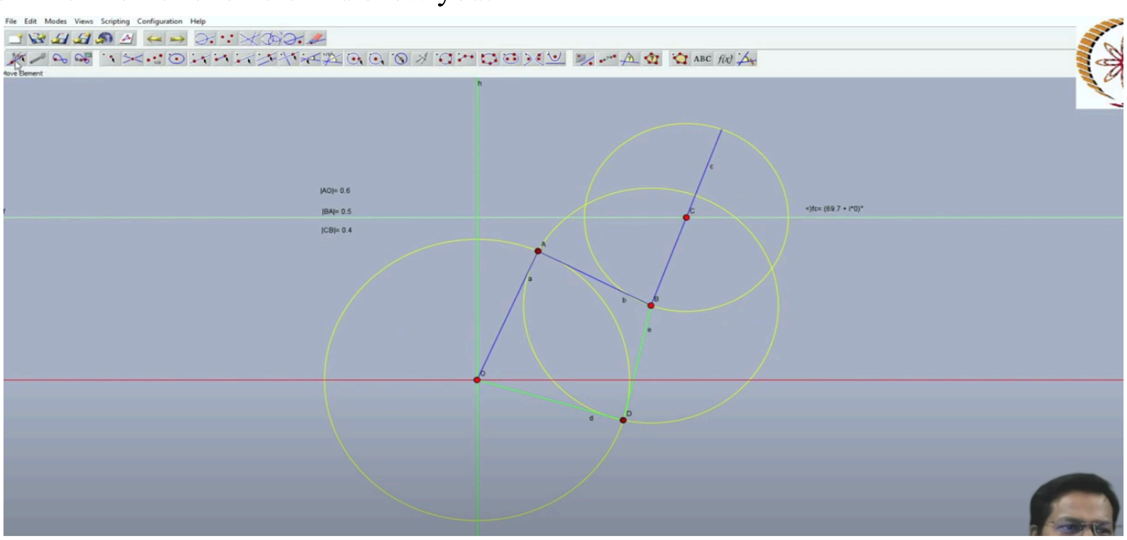
Similarly, the second solution is somewhere over here. This is your theta1. This becomes your theta2, and yes, I have drawn actually going from this way. This should be your theta3. So, this is your theta3, going from this way, from this side, this is your got it. So, overall, this is your possible solution. So, you see, you are done with a 3R solution using a geometric approach. You can use any geometric tool like Genera or maybe Cinderella or this type of freeware piece of software, which is downloadable from the internet, and you can try doing this very elegantly. So, sometimes you can use AutoCAD or any CAD software also to do this so that you can precisely measure the angles and do this kind of thing without getting into the analytical solution.

But that doesn't work much with the three degrees of freedom system. Using Cinderella, I'll just demonstrate how this is done. I have done it for you.



So, this is a tooling system which I have done. So, this is the solution. Same geometrical approach. I have used it. I have taken the link length as 0. 6 mm. 0.6 units, not mm, not centimeter. So, 0.6 units for the link, length 2 and similarly 0.4 for the first link, and I could get this solution. So, you, it allows you dragging, like this. So, if it disappears, that means it goes beyond the boundary, and it is not solvable. So, if you go beyond that, it is not solving.

You see, there are two solutions. One is elbow down, shown by the green one and elbow up is the. So, there is also an inner boundary. After that you cannot get the solution. You see, you are unable to get the solution. Got it. So, you can dynamically display the angles. So, I have done so. I also have another one here and show you.



It is the three degrees of freedom solution here. You see, here I have how I have done what I have done, got it. There are three links. There are three links, you see. So, this is your endpoint, okay, your first link that goes from this joint to this joint, okay, and then from here to here. That is the second link. And the third link is from here to here, same approach. I did it geometrically, using Cinderella. So, this is the point from where I draw this one, this circle. This is the point from where I have drawn this circle, and I have drawn this line, which is at the angle which is given, that is, the psi angle. So, I reached here because I knew wherever it would intersect this circle, that should be my third link. So, this is the point. That is the endpoint. From here, I have drawn this. So, that goes till here. There is the form here. I did this circle that is of second link length. Those were intersecting at this point and this point. So, I made a whole of the robot. Now, I can drag it and see all the possible solutions, and it can vanish like this. So, these are the two possible solutions and, mind it, I can also change the orientation, which is like this. End effector orientation can be changed, maintaining the position that is given by this point, that is, at c. I can change the orientation and check what could be my solution for the rest of the robot. Got it. So, this is how you can do it geometrically. Very elegantly, it will show you got it.

So, that's all for today. So, in the next class, we will be doing special robots. We'll start with a 3R spatial robot. We'll be doing 3 degrees of freedom, a cylindrical robot that we have done earlier forward kinematics for that, and we'll move ahead to 4 degrees of freedom SCARA. We have done forward kinematics for that as well. That's all. Thanks a lot.