NPTEL Online Certification Courses
Industrial Robotics: Theories for Implementation
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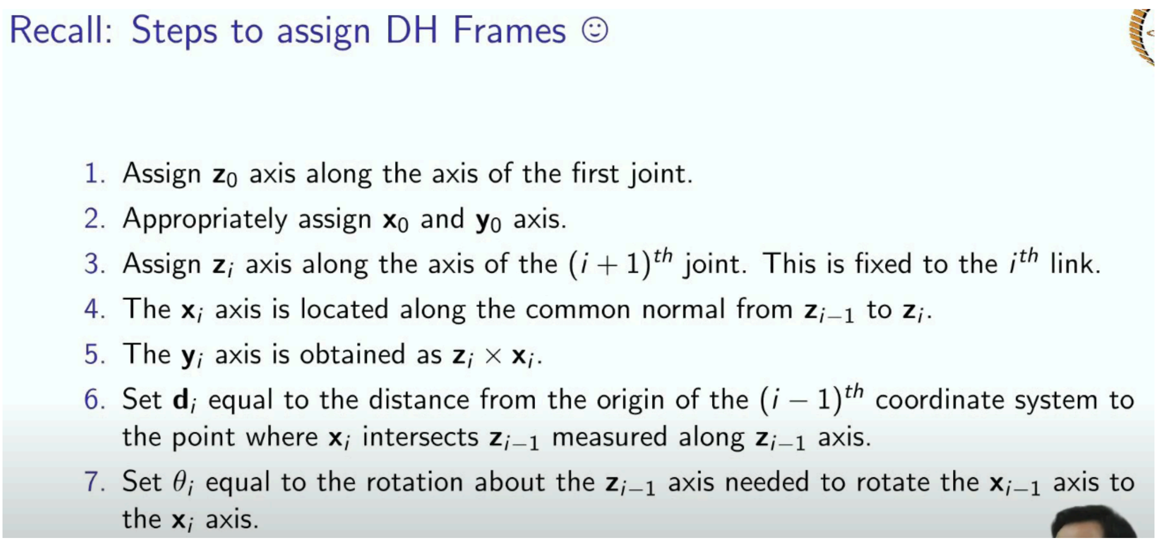
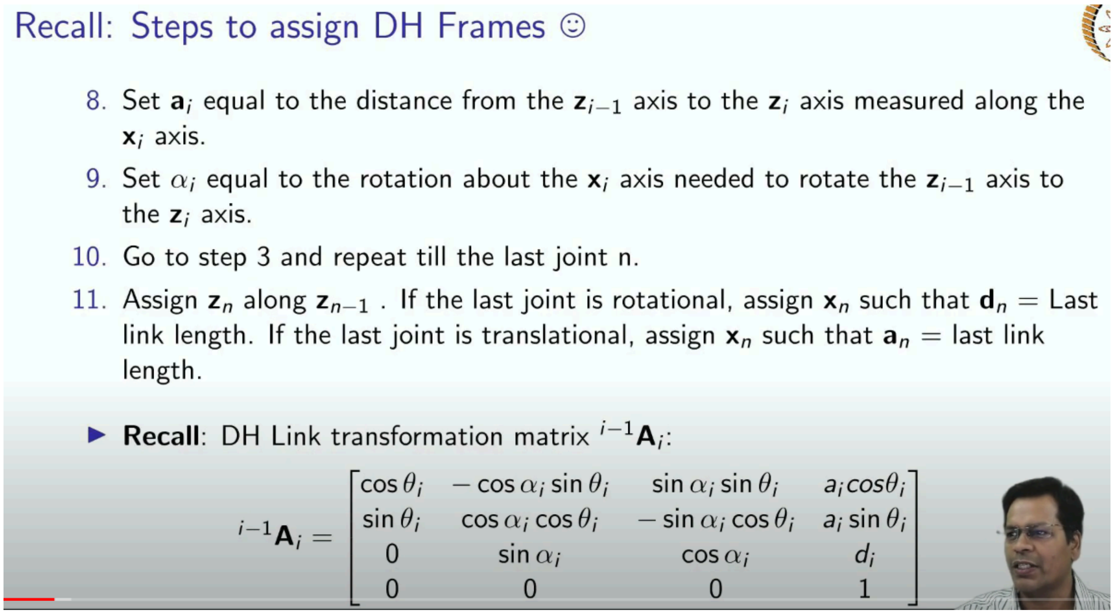
3 DoF Cylindrical Robot (Spatial), SphericalWrist, Cylindrical Robot with Wrist SCARA Robot

So, welcome back. In the last class, we understood what is the link, joint and DH parameters and we did a DH parameter table for at least one of the examples, that is, 2 degrees of freedom robot. So, in continuing further with that, let us do some more examples of special robots now.

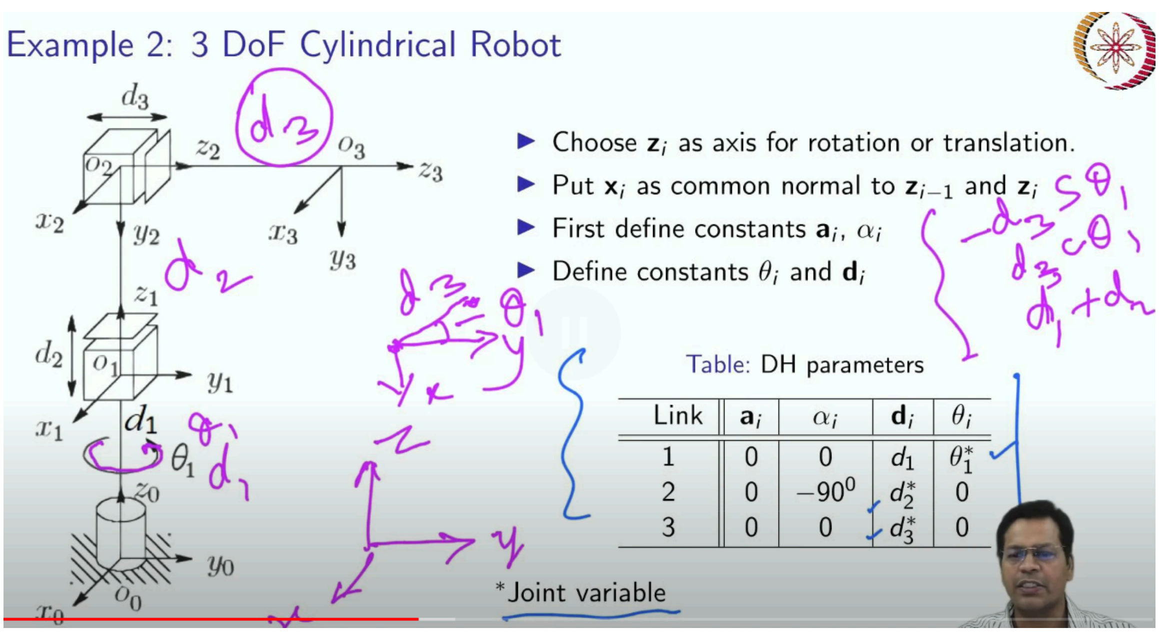


So, special robots are robots which can position themselves in space, Cartesian space in X, Z location anywhere in space. It may have some arrangement later on as an attachment like a spherical wrist or maybe just a plunger or something like a tool; some other tool may be a parallel robot mounted on top of a serial robot so that it can do further orientation manipulation.



So, let us begin. Let us just recall the steps to assign DH parameters. So, you see, you need not mug it up; you may just take a snapshot of this.

So, yes, you also should recall the DH link transformation matrix i minus1 Ai($^{i-1}$Ai). So, let us move ahead quickly to the example. So, here is one 3-degree-of-freedom cylindrical robot. So, let me just show you a small video to make you recall how it works. So, this is how it was. You see, the arm which is horizontal to the ground can extend its length, it can extend its length and it can rotate about a vertical axis.



Let us quickly do this one. So, this is your cylindrical robot kinematic representation. So, this is your first degree of freedom that was at the ground up. So, this is to rotate the whole of the two prismatic joints. One is here, and the other one can change the radius of this. This arm point can move backwards and forth, back and forth in this direction. And this can move like this up and down. So, there is some minimum height to which it can come, and there is some maximum height to which it can go. So, quickly we can do, we can place the Z axis along the motion axis. So, the first thing you should do is choose Zi as the axis for rotation or translation.

So, even if it is a rotation axis, we can quickly put the first axis like this. Here is your one. It is your first axis that becomes your Z0. And then you have another one, yet another one which is here, this is your Z1, this is your Z1 and then the third one is here which is like this. So, this is your Z2. So, from 0 to 1, 1 to 2 and 2 to 3, it requires a total of four frame placements, one which is permanently attached to the ground that is 0 0 0, X0, Y0, and Z0 that is not rotating that is the fixed frame about which first link rotates.

So, now, put other parameters as such, other parameters are we need to put Xi as common normal to Zi-1 and Zi. It is one of the steps of placing DH parameters. First, place the frames, and then we will assign the parameters. So, Xi is to be put. Xi is common normal to Zi-1 and Zi. Let us look at the first link, which rotates about the ground, so Z0 and Z1 are here.

Z0 and Z1 are here because there is no distance between them; both axes are along the same axis. So, you can draw a common normal anyway, so it is convenient to put the common normal in the original direction where your X0 is as over here. So, I will quickly put it as common normal like this: this is your X1 also, got it this is your X1. So this is Z0, this is Z1 common normal to that is X1. Now, let us put this is your Z2, this was your Z2, and this is your Z1, this was your Z1.

So Z1 and Z2 again common normal to them. It is X2, so this becomes your X2, got it and then this Z2 finally terminates over here aligning with Z3. Z3 is just a dislocation of the second frame to the new location, that is, the third frame, which is O3, which is your end effector frame as well, got it? So, now put the constants Ai and alpha i. What is Ai? Ai is the distance between two Z axes, that is, the link length, and alpha is nothing but a link twist. So, you see, there is no angle between Z0 and Z1. Both are aligned in a single axis, so there is no link twist, so Z and Z there is no angle between them that can be measured along the X axis, so alpha 1 becomes equal to 0. Then, talking about alpha 2, Z1 to Z2, you see there is some angle, so if this is your Z1, this is your Z1, this is your Z2. So, you have to rotate clockwise in order to align with Z2, so it is alpha two equal to minus 90 degrees.

Then is your alpha 3. Again, Z2 and Z3 are aligned together, so alpha 3 is equal to 0. Now, let us see what the link lengths are. Link lengths are the distance between Zi minus one and Zi measured along Xi. Let us see if it is there; Z0 and Z1 are aligned, and there is no distance between them. Z1 and Z2, again, there is no distance between them; they intersect at this point, and similarly, Z2 and Z3 are along the same line. So, again, A3 is also equal to 0, so A1, A2 and A3 are all equal to 0. So, you have assigned alpha and A. Now, moving ahead, so what is theta I and Di? So you see carefully here theta 1 is there because that is the joint variable for the first link, so the first link rotates with theta 1 with respect to the frame O0 that is attached to the ground, so theta 1 exists, whereas there is no theta 2, there is no theta three that is not there, that is not there, got it? Now, just see if ds are there, so d is nothing but joint offset, so that is the distance measured along the Z axis between X0, X1, X2, X3 like that, so d1 is there you see it is the distance measured from O0 to O1 that is frame 1, so this is your d1, so that exists that is having a length of d1. Now, what is d2? It is again the distance between measured along Z, so it is here, so d2 is here, but that is because it is a prismatic joint, so d2 is a joint variable, and d2 is a joint variable.

Example 2: 3 DoF Cylindrical Robot (Spatial)

$$ {}^{0}\mathbf{A}_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}\mathbf{A}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \because \sin(\alpha_2) = -1 $$

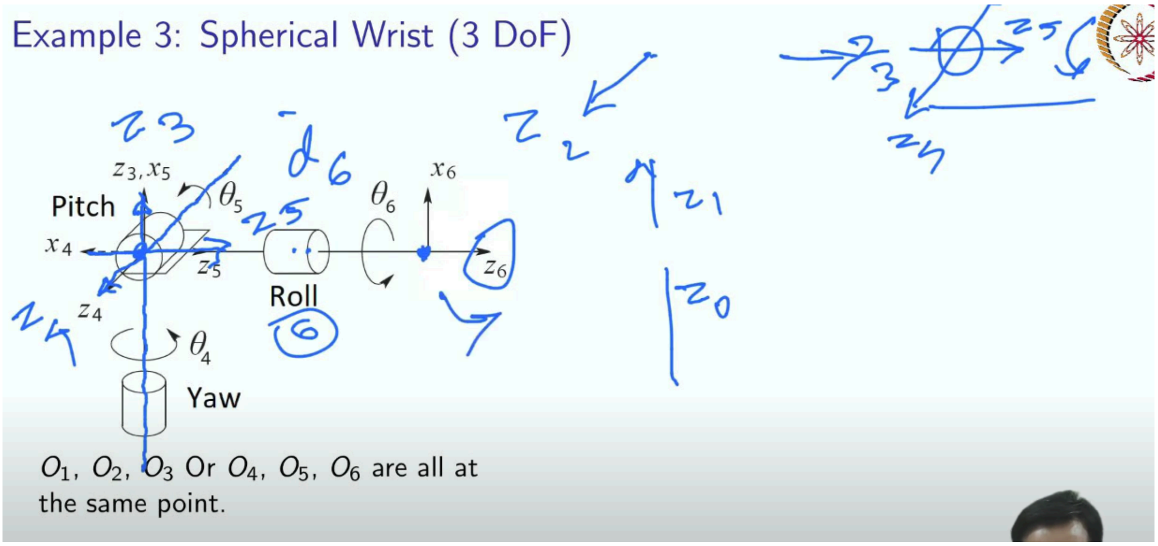
$$ {}^{2}\mathbf{A}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}\mathbf{T}_3 = {}^{0}\mathbf{A}_1\, {}^{1}\mathbf{A}_2\, {}^{2}\mathbf{A}_3 = \begin{bmatrix} C_1 & 0 & -S_1 & -d_3 S_1 \\ S_1 & 0 & C_1 & d_3 C_1 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Now, this one is your d3 that is measured along Z2 between two axes, X axis, so is d3. So, d3 is again it is a prismatic joint. So, it is a joint variable, got it? So, these are the parameters. Now, let us put all of them together in the table form. So, this is your DH parameter table. I have marked joint variables as theta 1, d2 and d3. If you remember your DH parameter link transformation matrix this one, you have to use this one every time. So yes, let us just quickly go to that page. If I can substitute my DH parameter values to the link transformation matrix, I will get ${}^{0}A1$, ${}^{1}A2$, ${}^{2}A3$, like this ${}^{0}T3 = {}^{0}A1\,{}^{1}A2\,{}^{2}A3$. You can try doing it yourself and concatenate all of them together, taking the product of them together, you get the forward kinematic transformation matrix four core four homogeneous transformation matrix like this, so this is it. So, this is your forward kinematic transformation matrix. It shows the end-effector position, and this shows your end-effector orientation. So, can we compare any of these so quickly at least this one you can understand if you can see your previous figure. So, this is your, how much was that? So, you remember it was d1 plus d2. Z is d1 plus d2, so, if this is d1, this is d2, so perpendicular distance along Z, this was your Y, this is your X, and this is your Z. So, total perpendicular distance is along Z is d1 plus d2. Coming to X and Y, can we do that as well. So, this is your d3, okay, and that rotates about the vertical axis by an angle theta 1, you see that is theta 1. So, how much will it be that? So, X, along X if you take the projection of d3, looking from the top, so this is your Y, this is your X. so your link d3 goes like this, this is your angle theta 1. so this point coordinates, X coordinate will be $-d3Sin\theta1$, okay. Now, how much will be your Y coordinate. It will be d3 cosine theta 1, got it? So, these are the three which we can quickly verify. It is not that trivial in many robots, but yes, at least for this one, you can quickly verify.

Now, can you do orientation verification also? So, what was that? So, it is a projection of the U, V, W frame over here it is U, and it is V and U, V, W frame along X, Y and Z. So, can you do that as well, so the projection of X along X, projection of X along Y, projection of X along Z. So, you can quickly understand if this rotates your, this axis, this axis X3 rotates by an angle theta 1. so you can quickly find out, so along X will be X3 cos theta 1. So, X3, if it is the unit vector, it is simply cos theta 1. Similarly, along Y, it will be X3 sin theta 1. Along Z, it will be 0. So, you see, this is your unit vector, column vector, the first column of the transformation matrix.

Similarly, you can work out for other matrices. Y-axis projection, you can quickly find out. So, this projection along X will be 0, along Y will be 0, but along Z, it will be because it is pointing downwards. It should be 0, 0 and minus 1. So, that is how it can be worked out. So, if you see it here, it is cos sin 0, 0, 0, minus one and similarly, this one, so these are the three projections you got. So, this is your orientation matrix, which basically is the orientation of the end effector frame, and this is the position of the end effector frame. So, this is how forward kinematics can be interpreted.



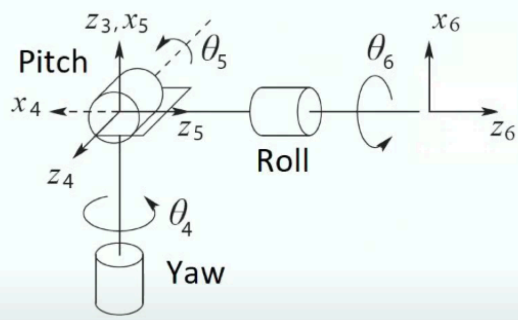
So, now let us do one more example that is very, very important because most of the industrial robots are fitted with this kind of joint. So, let us first make you understand what it looks like.

I will just switch the virtual robot that I have here, so this is it. So, you see, you can move it, so the first axis was something like the vertical axis. If you can see it here, this was your vertical axis, which is fitted here. So, the first axis can make it go like this, okay, and the second axis can make it do this, and the third axis can make it do like this. So, forget about the first three. What are the last three? Okay, can we do it as well? So yes, this is your fourth axis, which can rotate the whole of the wrist centre point like this. So, where is your wrist centre point? It is the point which is somewhere over here inside the centre line, okay, so it does it, so this is your axis 4, this is your axis five again that is also coming meeting it at the same wrist centre point and then the last axis. In order to show this, let me make it like this, okay? If you can see it clearly, you see the last axis is now rotating that the black dial one that is the tool post on which tools are mounted. Any robot gripper is mounted welding tool is mounted. So, that is the flange with some holes, standard holes so that you can mount it, so this is it. So, you see, the axes are all intersecting; axis four is intersecting at the wrist centre point somewhere over here, axis five is also crossing that one only along this and Axis 6 is also very, very similar, okay. So, let me just show you Axis 6 once again. So, this is your axis 6. So that axis is also if you protrude it backwards. So, that also intersects at the same location. So, hope you got it. So, let me just switch it off and come to.

Let us come to the kinematic picture of that. So, here it is. The first frame that is visible out here is a part of the last frame of most of the three degrees of freedom positioning robot. So, this is the orientation part of it, okay? So, this spherical wrist basically gives you the orientation. The first three give you the position. So, let us begin with placing the Z axis first. So, this one is part of the last frame of a three-degree freedom system positioning system. So, this is your last frame of that three-dip system, so this is your Z3, got it? So, that is coming to the wrist centre point. Now, let us put the other cross axis which is there, so this is the second one for this one, so this is your Z4. If you remember your cylindrical robot that we did just now. So, that had a final axis, so first one was like this Z0, then Z1, then you had Z2, and finally you had Z3 that was like this, got it. So, from there, I am establishing this spherical wrist. so this is your Z3, then your Z4, and finally you will have Z5. So, here goes your sixth link that finally ends here. So this is your fifth axis that rotates the sixth link and sixth link at its end. It has a sixth frame that is Z6. It is nothing but a dislocation of the axis, which is here, the axis which is here, the fifth axis. Exactly that comes here and there is a distance between these two. So, this is your spherical wrist centre, and finally, this is the distance D6 because it is travelling along the Z axis. So, it is D6 that finally ends here, so this is your last frame, that is the tool frame. Normally, they are fourth, fifth and sixth axes. So, you can put it like this, this and this or in order to do it work conveniently here, theta1, theta2 and theta3 are equally good. So, if I put it all over here, so, using this DH parameter, I can quickly get this table. This table got it. So, this is the same. So, there is no AI because all the link axes are intersecting.

All the joint axes intersect. If they are intersecting, there is no distance between them, so link lengths are 0 and also, D1 and D2 are 0 because the distance travelled along the Z axis is also 0. The only distance that you see here is the last one, which is D3, which is here, or you call it D6. They are equivalent to convenience. So, this is D3, so this is D3 that is actually collocating from this point to this point. So, it is just making some offset along that line. So, that is your final thing, and theta1, 2, and 3 are joint variables. If you can measure angle alpha, that is the twist angle, that is the angle rotated about the X axis from the previous Z to the new Z. So, that is there. So, if you do it, you will get minus 90, 90 and 0.

Example 3: Spherical Wrist (3 DoF)

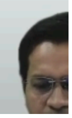


O_1, O_2, O_3 Or O_4, O_5, O_6 are all at the same point.

Table: DH parameters

Link	$\mathbf{a}_i$	α_i	$\mathbf{d}_i$	θ_i
$4 \rightarrow 1$	0	-90°	0	θ_1
$5 \rightarrow 2$	0	90°	0	θ_2
$6 \rightarrow 3$	0	0	d_3	θ_3

$$^0\mathbf{A}_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



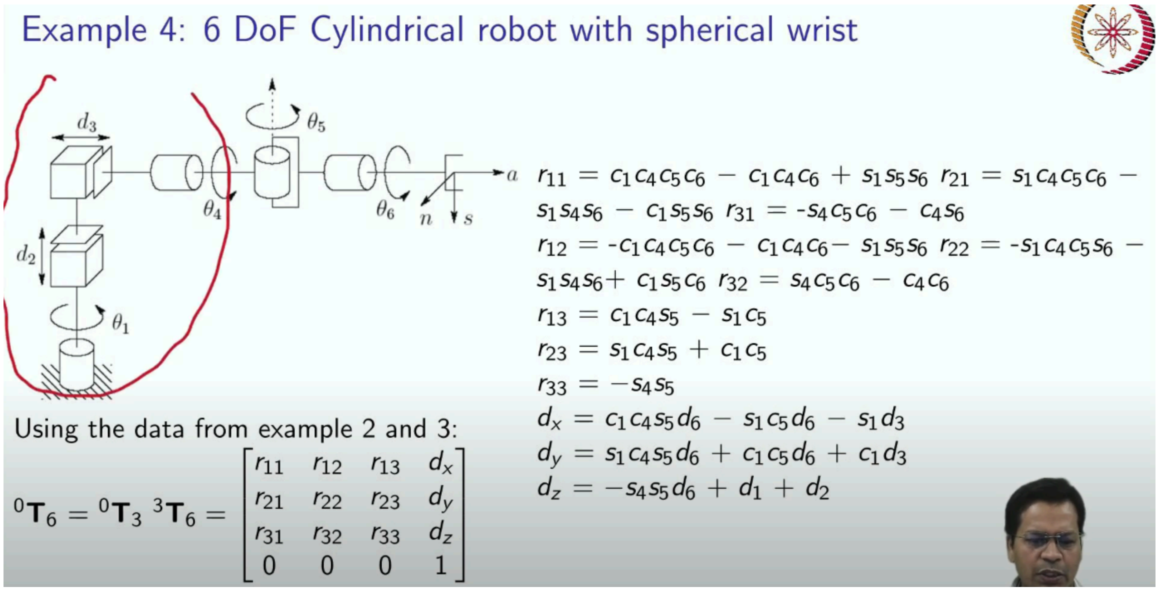
$$^1\mathbf{A}_2 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2\mathbf{A}_3 = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0\mathbf{T}_3 = {}^0\mathbf{A}_1 \, {}^1\mathbf{A}_2 \, {}^2\mathbf{A}_3$$

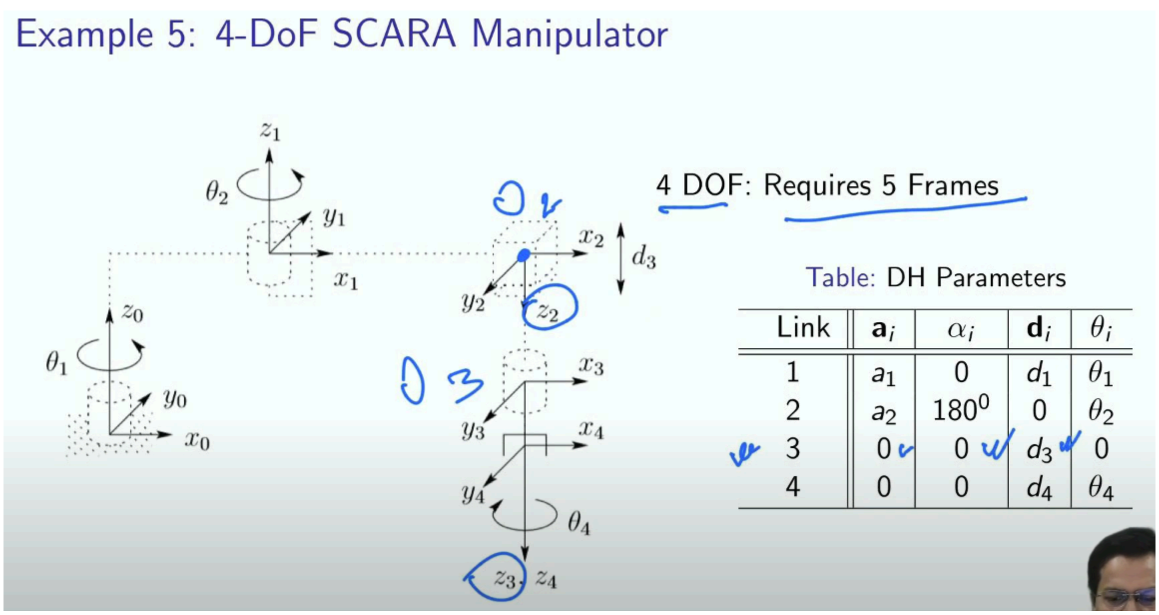
$$= \begin{bmatrix} c_1c_2c_3 - s_1s_3 & -c_1c_2s_3 - s_1c_3 & c_1s_2 & c_1s_2d_3 \\ s_1c_2c_3 + c_1c_3 & -s_1c_2s_3 + c_1c_3 & s_1s_2 & s_1s_2d_3 \\ -s_2c_3 & s_2c_3 & c_2 & c_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, this is how your DH parameter table can be worked out for this. So, the first transformation link transformation matrix goes like this. The second and third one are here. If you take the product of all of those, you get forward kinematic transformation for this spherical wrist, got it? So, this can be attached to the top of any robot. So, the first three axes will do their job, whereas these three will go as 4, 5 and 6 numbers on top of this. Instead of putting 1, 2, and 3 the way it is here, it is just 4, 5 and 6, and they are equivalent. So, 1, 2, and 3 for the first three joints of the robot. 4, 5, 6 for the spherical wrist.



So, now, can we do it here? It is what we did here. so immediately, if you can recall, this is your same old robot, the one which you have worked out just now. It was not included there. So, you ended maybe somewhere over here that was your last joint, last point, end affected point. Now, you have attached 4, 5 and 6, all the three intersecting axes. So, over here, it is shown like this. There may be an instance when you see it can be made exactly like this. So, this is all the axis is now. Axis 4 and 5 are collinear, got it. So, it becomes like this approximately, so you got it, so this is how it is drawn over here, so it is axis 4, this is axis 5, this is axis 6, got it. So, the spherical kinematic transformation matrix that you obtained in the previous slide and the forward kinematic transformation matrix that you obtained for the cylindrical robot earlier in example 2. You can take the product of both of them. So, effectively, you will get this. Finally, you will get this, and this is your thing. You can work out using any symbolic mathematics computation tool and you can arrive this way, got it.

So, now let us do one very popular robot that is known as the SCARA robot. So, let us just look at it. What does it look like, ok? This robot is known as the selective compliance assembly robot arm, and this is the FANUC one. Many Omron companies are making it, and many other companies also are making it. So I hope you got it. It has three axes which are there. The first one is the revolute axis. The first vertical one is the revolute axis. You still have another one that is the revolute axis, and the final prismatic one is there, making the whole of this go up and down, as well as it can orient about its axis. Now, it is working, and you got it.



Link	$\mathbf{a}_i$	α_i	$\mathbf{d}_i$	θ_i
1	a_1	0	d_1	θ_1
2	a_2	180^0	0	θ_2
3	0	0	d_3	0
4	0	0	d_4	θ_4

Now, let us again come back to the original kinematic picture of this. However, it looks quite boring here. You see, this is very, very important in order to put DH and do forward kinematics here. So, let us quickly do that. So you have the first axis, which is the vertical one that is Z0, which is shown here. The ground frame I have kept exactly like this is my forward. It is my other side of it: Y0, X0 and Z0. Above the first link is rotating, ok. So, this is the shape of your first link. If you can see this one, this is your first link. Here is your second joint, ok. So, this is your Z1, ok? So, the first link rotates about Z0, and you have a cylindrical joint, which is here. So, this is your Z0. It is your Z0, about which the whole of this is rotating, this yellow one that I have marked here, and Z1 moves along circularly.

So, this is your Z1, ok? Again, that is extended further, and you have a point here about which there was a prismatic joint that can go up and down. so this is your Z2, this is your Z2. I am placing all the Z's now, ok and this can go up and down to make a point. This end effector point has a frame, which is put here that is the fourth frame. Before that, there was an intermediate frame that was able to rotate the whole of this along the vertical axis. So, this is your cylindrical joint. So, this is your Z3, Z3. So, you got it, so you have Z3 and Z4 both aligned together, ok? So, it originates from here, this one originates from here Z3, O3 is here whereas O4 is here, this is your O0, and you have O1 here, O2 here got it, and then you have to find out all the d's that is the joint offsets, ok.

So, what are the offsets? So, 4 degrees of freedom require five frames. So, let me just quickly find out all the offsets here. Going from X0 to X1 along the Z axis will be your offsets. So, this becomes your D1. So, that is D1, ok and going from Z0 to Z1 along X1, so that is your A1, ok, and both the Z, there is no angle between them, so there is no twist, so it is 0. theta1 is the joint variable which is here, got it. Hope you have obtained it, hope you got it, ok? Now let us come to yet another one, that is the second link. You see quickly here. So, this is your A2, that is the distance between Z1 and Z2, got it, and both are facing opposite to each other. So, the angle between them is the link twist measured about X2. so it is 180 degrees, and no distance is travelled along the Z axis ok. So, this point and this point are moved along the X-axis but not along the Z-axis. So, that makes this link offset 0, and theta2 is the joint variable.

Now, let us see the third one. So, you see again, you have a link length that is equal to 0. Why? Again, there is no distance which is measured between Z2 and Z3, and both are aligned together. So, that makes it 0. Both are aligned, so the twist is also 0, and it is travelling from 2 to O3 by a distance d3, which is the extension of that prismatic link. So, this is the joint variable, and there is no rotation angle variable. Got it. So this is it. It is your third, and then the final one. Let us quickly do it again. So, now your fourth frame, which is here, is O4. ok this was your O3. so again, there is no distance measured along Z3 and Z4. Both are aligned together, and the link length is 0. There is no angle between them measured along the X-axis. So, there is no link twist, so it is 0. Again, there is some distance which is between them that is d4. So, that is basically the offset which is from O3 to O4 measured along Z that is there; and finally, this is a revolute joint. So, that is a variable. Theta4 is variable. So, this is how this parameter table is put, and if you can find out all the individual link transformation matrices, all 4, using the link transformation, homogeneous transformation matrix ${}^{i-1}A_i$ that we have just seen in the first recapitulation slide, I showed you so that.

Individual transformation matrices: SCARA

$$
{}^{0}\mathbf{A}_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}\mathbf{A}_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2 c_2 \\ s_2 & -c_2 & 0 & a_2 s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}\mathbf{A}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{3}\mathbf{A}_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}\mathbf{T}_4 = {}^{0}\mathbf{A}_1\,{}^{1}\mathbf{A}_2\,{}^{2}\mathbf{A}_3\,{}^{3}\mathbf{A}_4 = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 + c_{12}c_4 & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & -1 & d_1 - d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Orientation

Position

You can use the same thing here, and you can do it, and now you need to take the product in order to find out the forward kinematic homogeneous transformation matrix, ok? So, this is your thing.

It represents again your position, and this represents your orientation, three cross three (3x3) represents your orientation. At least a few of them you can quickly verify if you have obtained it correctly or not. so you see it is a1 c1, a2 c12. c12 means c cos of theta1 plus theta2, ok. I use this as a shortcut to write it here. So, in a compact notation, you can write it here. similarly c1 is cos of theta1, got it. so that is what is written here, and the same way I have put the sin is also. So, now, if you can see it from the top. Suppose you can see it from the top. So, this was the thing. So, this was your link length. this is your a1, this is your a2. The first one was able to rotate by an angle theta1. It was by an angle theta2. So, looking from the top, it is very much like a two-link robot as we have worked out earlier. So, it is theta1, this is theta2, this is a1, this is a2. So, whatever this point is, this point has xy coordinate. So, the same xy will be for the bottom one also because it is just travelling vertically up and down. So, whatever the xy which comes here same is the xy for this one also. So, the fourth point also got it. So, it is just like a 2-degree-of-freedom robot. Now, so xy will be a1 cos theta1 plus a2 cos theta1 plus theta2 $\{a1\cos\theta 1 + a2\cos(\theta 1+\theta 2)\}$. Similarly, y will be a1 sin theta1 plus a2 sin theta1 plus theta2. So, that is what is visible here.

Now, look at your vertical distance. So, vertically, you see you have travelled like this. So, you went up like d1, and then you came down by d3 and you came further down by d4, got it. So, total vertical travel will be d1 minus d3 minus d4. So, that is what is visible here. Got it, so at least the position thing you can quickly obtain. Ok, we can do this also. It shows the final z-axis. z4 is pointing downwards along the z-axis. So, that is what is here. So, you see, z4 is pointing downwards. So, it has components along x along y as 0. whereas the components along global z, z0, and z4 you see are oppositely directed. So, z4 has a minus one projection along z0. So, that is what is here. So, you see, you can verify other elements are because it was two DoF kind of thing. So, orientation is something like this, got it? So, this is how you can further cross-check it. So, this is your SCARA which is a very popular industrial robot which is used for PCB manufacturing.

So, that is all for this lecture. So, we will be doing further forward kinematics in the next class for the 6 degrees of freedom industrial robot arm. We will see various industrial arms also which are very much similar to the one which we will be dealing with, ok. so that is all for this lecture, thanks a lot.