NPTEL Online Certification Courses
Industrial Robotics: Theories for Implementation
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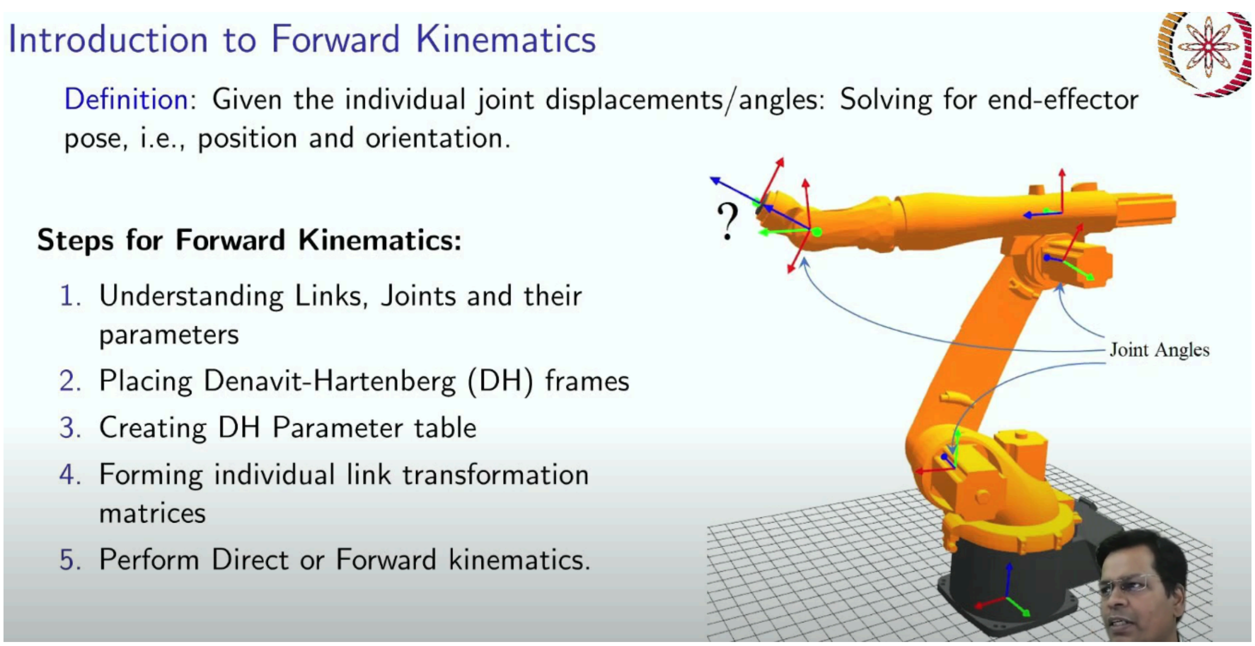
Lecture: 18

Link and Joint Parameters (DH Notations), 2 and 3 DoF Robots

Welcome back. In the last class we did degrees of freedom. We understood what it is and how the degrees of freedom of a serial robot or a general mechanism can be calculated. So, we also did transformation matrices, and we understood how a rotation matrix works and how a translation matrix works. So, moving further, now we have built up enough prerequisites to do forward kinematics of industrial serial robots. So, let us begin with that.

So, in today's class we will be doing kinematic transformations further, and we will be doing forward kinematics of industrial serial robots. So, we will start with link and joint parameters, that is, DH notation, which is commonly known, and we will go ahead with the DH parameter table build-up. We WilL start creating forward kinematic matrices, and we will do some example problems as well. Today, If time permits, we will do 2 and 3 degrees of freedom.

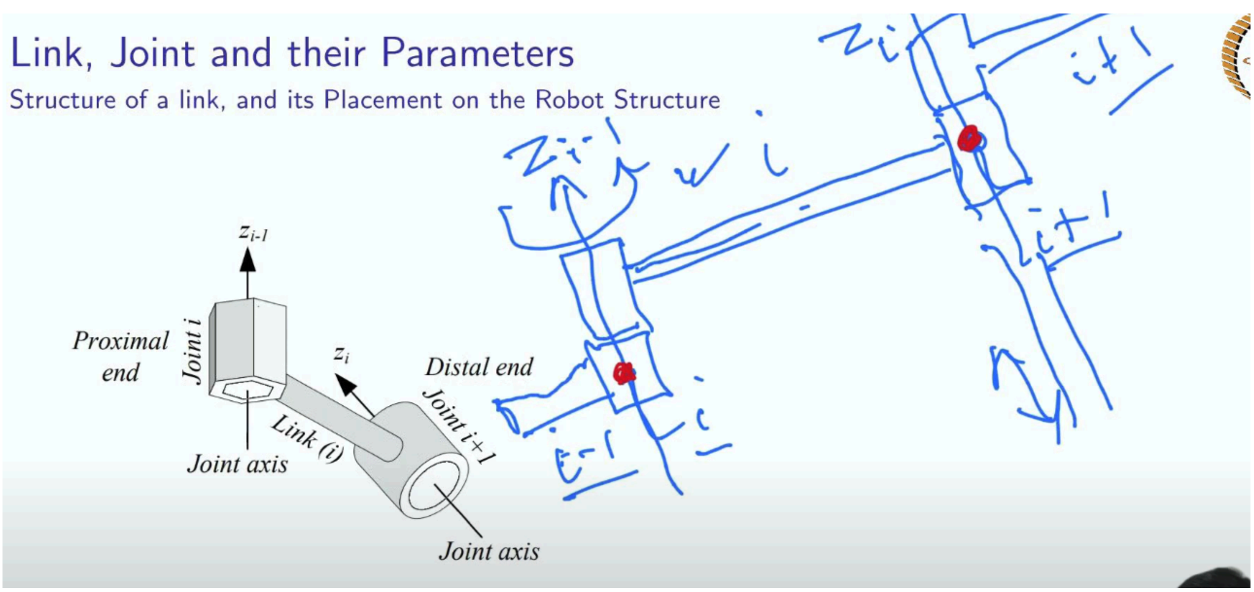


So, let us move ahead. So, what exactly is forward kinematics? Forward kinematics is something like you are given a robot, and you know its joint angles. So, you know its joint angles. So, you have a link, you have a joint, you have a link, you have a joint, and that serial finally terminates at a place. So, if at all you want to go somewhere, you have some joint angle which is to be put to the joints. So, provided you know your joint angles, where is your arm and how is it oriented? So, that is what is to be found out. So, that type of problem is known as a forward kinematics problem. So, it is defined as, given the individual joint, displacement, and oblique angles. Why displacement? Because the joint can be a rotary joint. It can be a prismatic joint also.

So, in the case of a prismatic joint, it is displacement. In the case of the angular joint, it is angled. So, solving for the end effector pose is forward kinematics. Pose means position and orientation. You are going there, and you are orienting there in a particular way. So, we will see when we will actually do it.

So, the steps for doing forward kinematics are as follows. So, you know, initially, we will understand the link joint and their parameters. We will try to parameterise the whole of your robot, link, your system so that it is easy to define and relate them using some notation. Those notations are popularly known as Denavit-Hartenberg notations or Denavit-Hartenberg parameters. So, we will also place DH frames on different joints to relate between those two joints. So, that is known as the DH frame.

We will create the DH parameter table and the Denavit-Hartenberg parameter table. We will see when it comes and forming individual link transformation matrices. Furthermore, we will form link transformation matrices for all the links which are there, which are connected, one after the other, and finally we will perform direct or forward kinematics. So, these are some of the steps which are necessary to go through before we actually do forward kinematics.

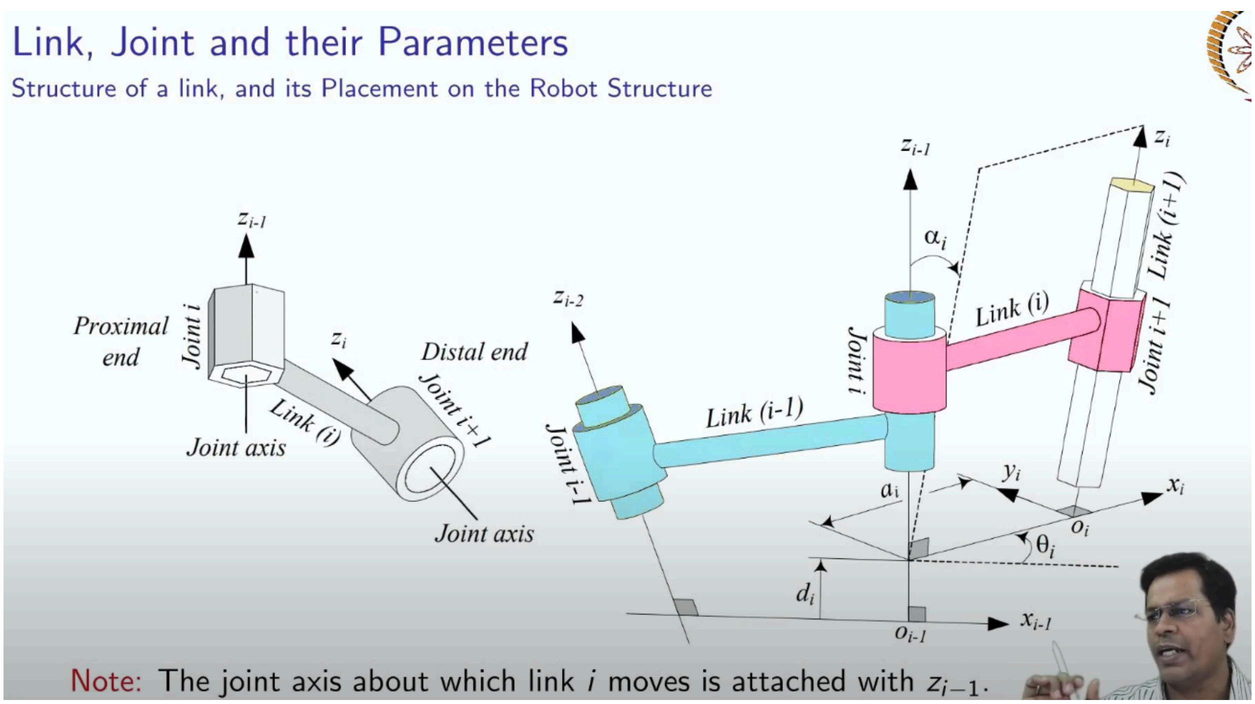


So, yes, link and joint parameters. So, the structure of the link normally has a link with two ends. So, one of the ends is connected with the previous link, which is there in the series. So, that is ending with the joint also. So, this is a previous link. This is a previous link: i minus 1st link. So, that has ended with the joint i. This joint i actually connect to the link. Finally, this link also terminates with a frame which is attached to this, which is called i plus 1th joint.

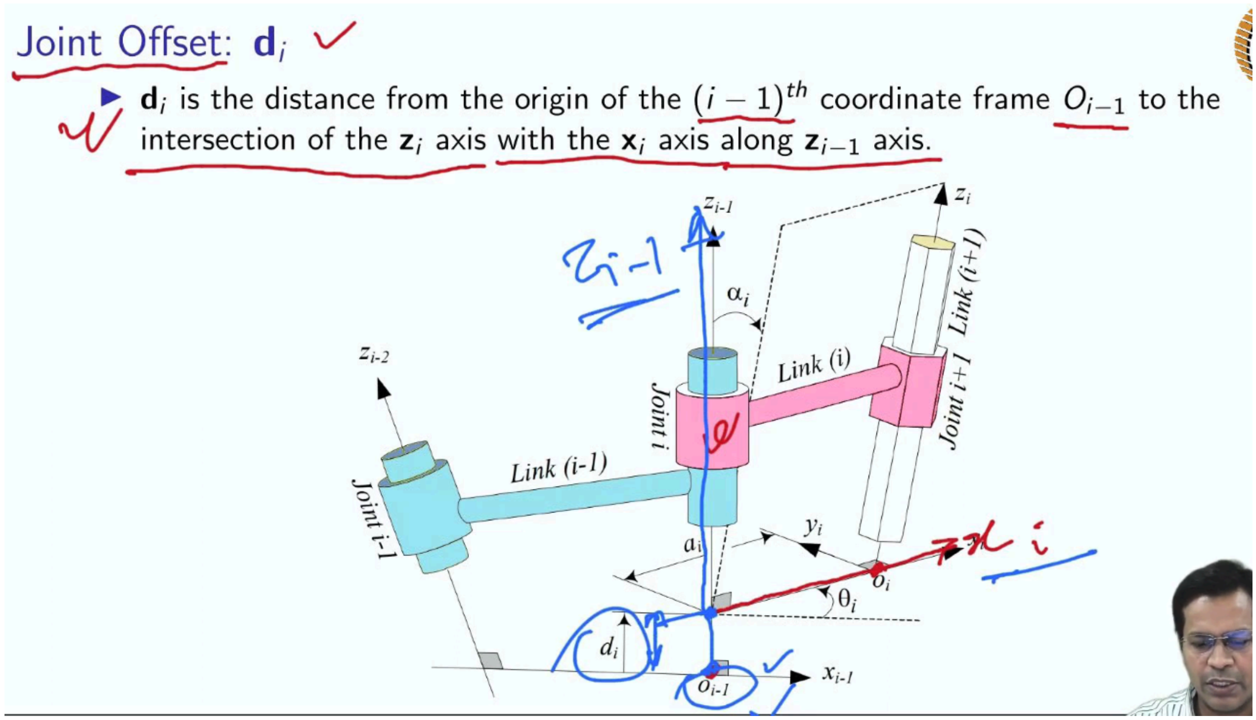
So, this joint will further connect, maybe some other link in the series so that it may go ahead like that. So, it starts with i minus one link. i minus one link end with the ith joint. On the ith joint, the ith link is placed. ith link ends with i plus 1th joint on which the i plus 1th link is fitted. So, these are the joints, these are the joints. This is one axis, this is the second axis and so on, so forth, So that keeps on going.

So, this may be a revolute kind of joint, or it can be a prismatic joint which can move in this direction. So, the next link which is connected to this may move like this. So, either way, this axis is all placed with the Z axis. So, they are all placed conveniently with the Z axis. So, this is Zi minus 1 (Z_{i-1}), that is, the joint axis, and this is link i, and then you have the distal end. That is Zi.

So, this is Zi. So, this is the part of a frame which is attached over here. So, this is the part of the frame which is attached to the link i. So, that is here. So, this is part of the frame, which is attached to the link i minus 1. So, this is the frame i minus 1, and this is frame i. So, this is all. Links are all made. So, this is the distal end, this is the proximal end, the nearer one, and this is a joint axis. These are joint axes.



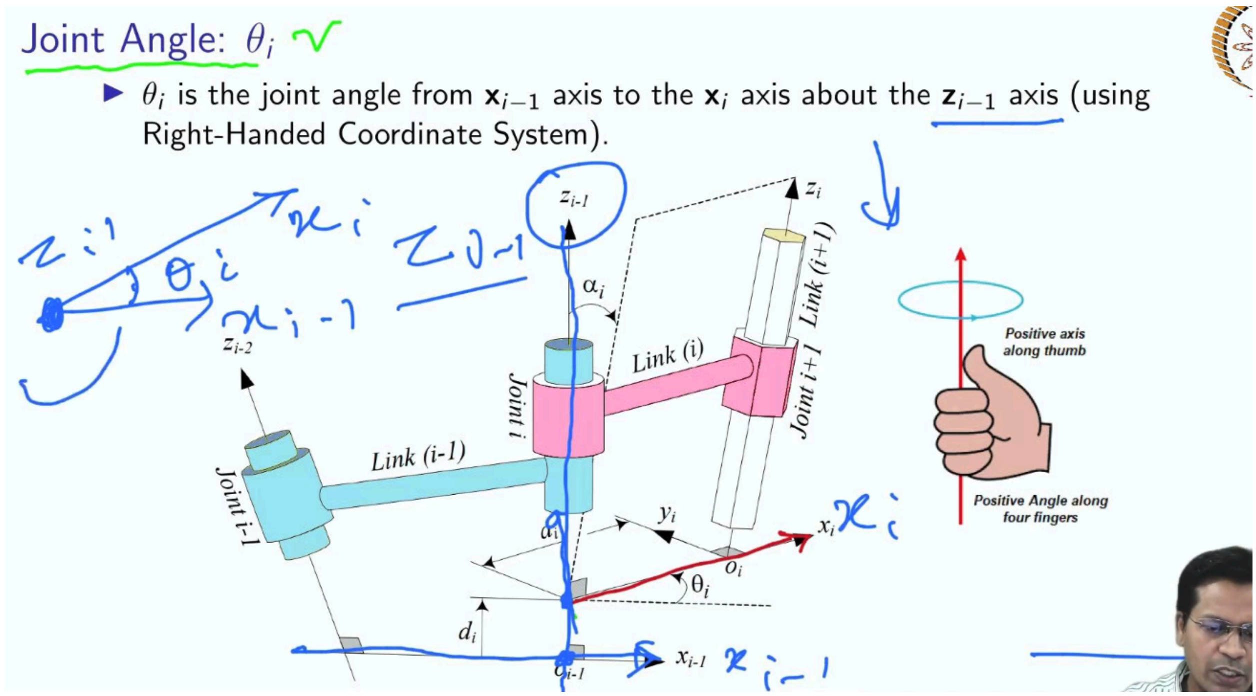
So, yes, this link now becomes part of a serial chain system. So, you should note that the joint axis about which link i move is attached with Zi minus 1. So, you see, Zi minus 1, as I have just told you. So, now let us formalise the whole of the link with different parameters which actually define a link.



So, starting with that. So, the first parameter is known as a Joint Offset (d_i). From now on, I will be denoting that using d_i. So, d_i is the distance from the origin of the I minus 1 (i - 1)th coordinate frame to coordinate frame OI minus 1(O_{i-1}) to the intersection of the Zi axis with the xi axis along the zi minus one axis (z_{i-1}). So, reading it may be quite difficult to understand. Let us look at the figure directly.

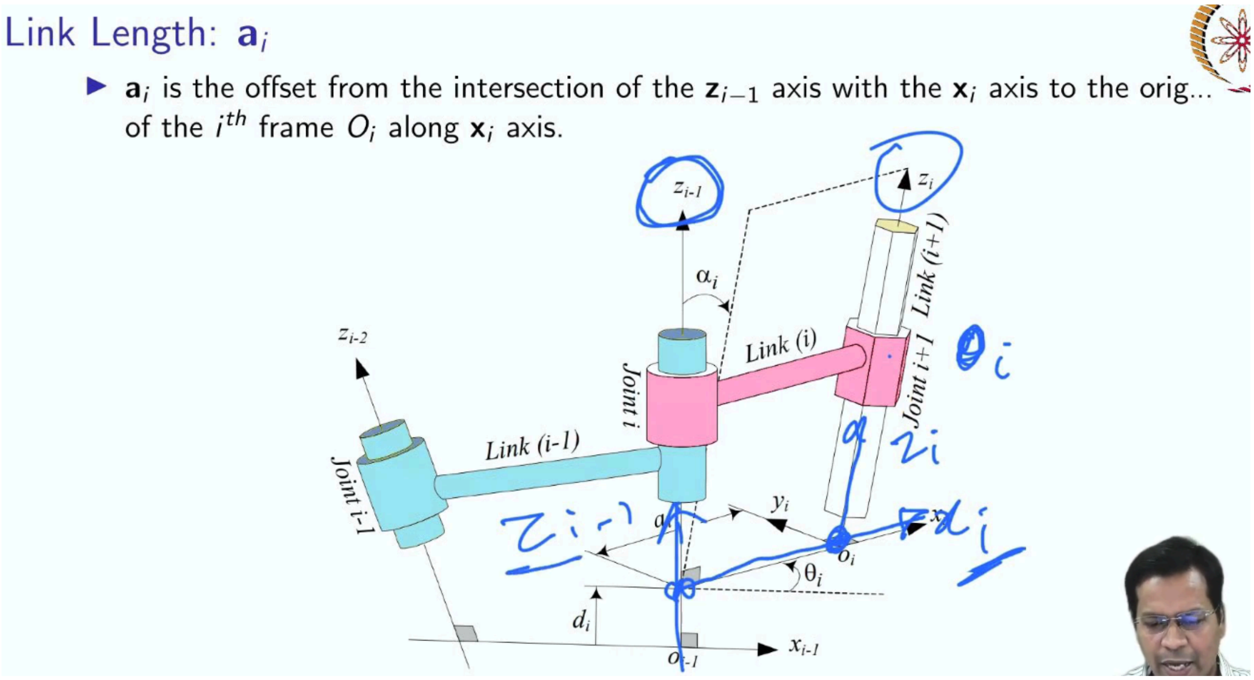
So, where is it actually located? So, let us say you have two links which are connected. See my hand. So, yes, this is one of the links. The next link comes here. So, they are dislocated by both the links. This link and this link. They are dislocated by some distance. So, the axis which was there was the Z axis, on which you have another link which is connected and that moves. So, distance moved along ZI minus 1 (z_{i-1}), because this is the link which ends with the frame Zi minus one (z_{i-1}), and you have another link that comes on top of it, and they both move relatively. So, the link i move with is relative to link i minus 1. So, that is the joint. So, this is the joint. So, there is some distance between those two links.

They are calculated as, so this is your frame, i minus 1, from here to this is a common axis which intersects both of them. So, this is your xi x-axis, which is fitted at the end of the link i. So, this is part of frame i, which is fitted at the end of the link i. So, this is there, and this intersects Zi minus 1 (z_{i-1}) over here. Over here, and this is your distance between Oi. This was the frame of the previous link that was ending here. So, this is your frame. So, from this frame to the point of intersection of xi and Zi minus 1. So, this is the offset which is there. So, this is the offset. So, you have two links which are connected like this. So, the offset between them is measured along Zi minus 1 (z_{i-1}), and it is the distance between Oi minus one (O_{i-1}) that is frame and the point of intersection of xi and Zi minus 1 (z_{i-1}). So, that is this location. These two are the points between which it is measured. So, moving at this is a Joint Offset.

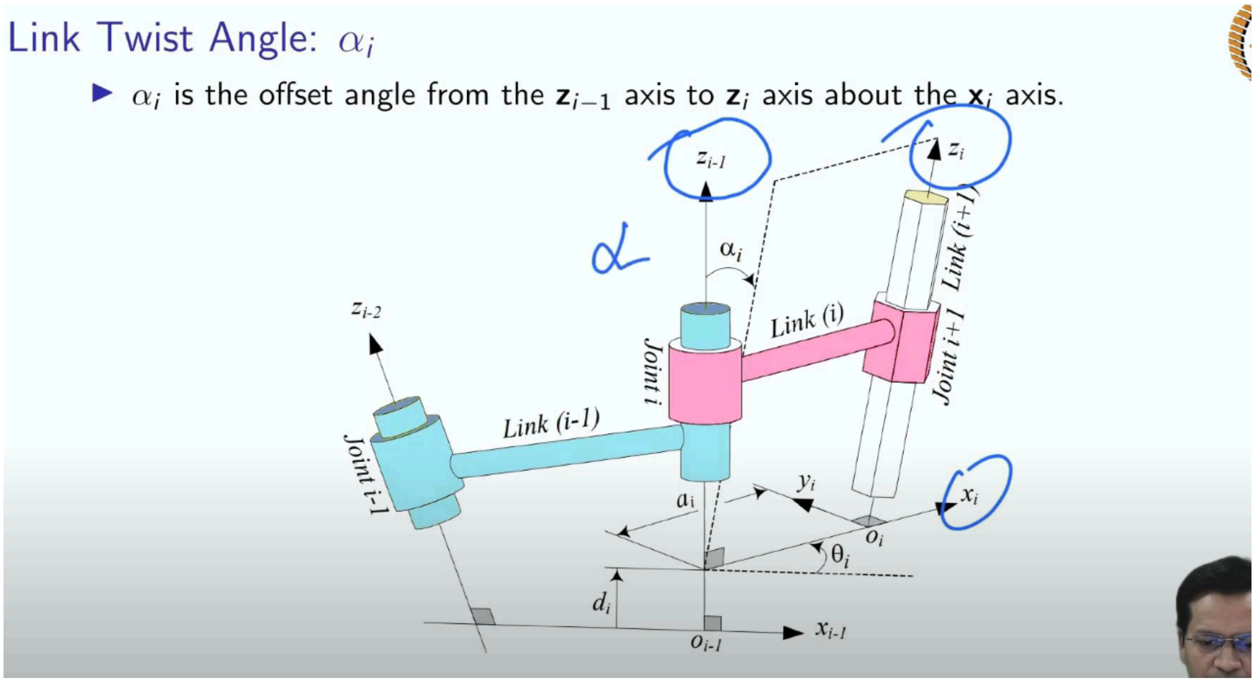


Now, the next important parameter is the Joint Angle (*ϴ*i). In the case of a revoluted joint, this becomes the joint variable. Also, This may be a variable because both the links move relative to each other, So there is some angle between those two links. So, this is the actual measure. So, where should it be measured, and how it should be measured. We are coming to this. So, this was my final point, where your x-axis was intersecting to z axis. So, this was the point. So, yes, this is your x_i axis, and the previous frame ended somewhere like this.

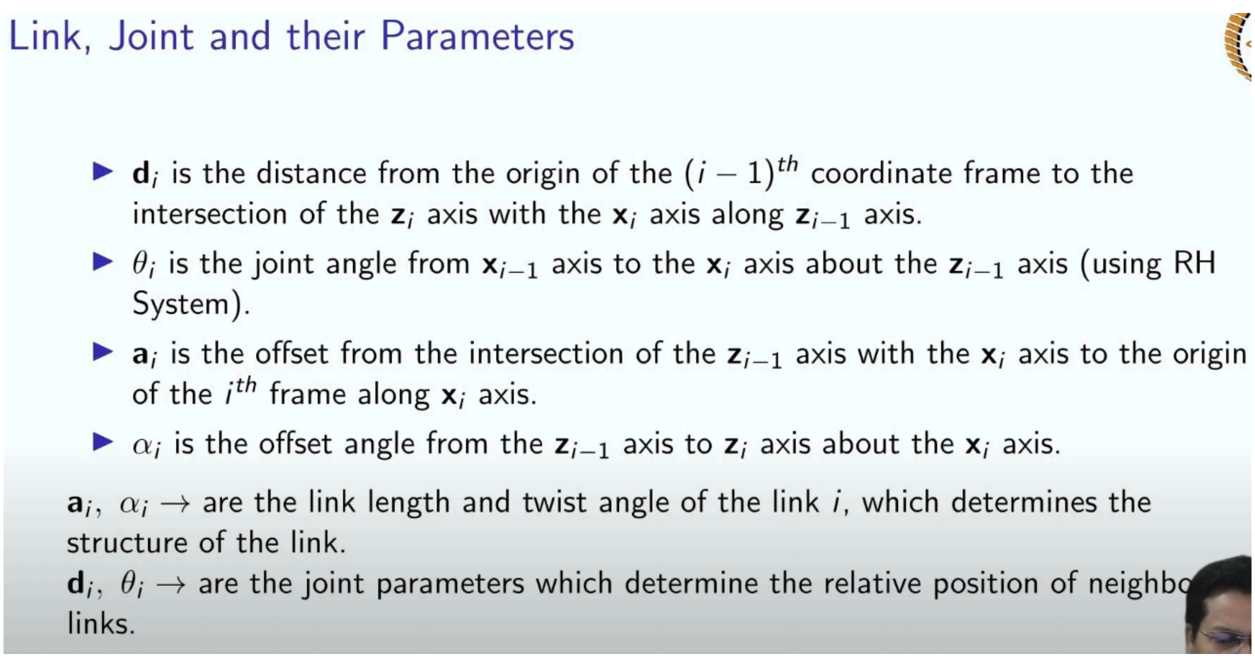
So, the previous frame ended here. So, this was xi minus 1(x_{i-1}). So, about this zi minus 1 (z_{i-1}), look carefully at the figure where I am drawing. So, this is your zi minus 1. So, this is the line. So, about this line? If you look from the top, if you see from the top, there is some angle between xi minus 1 and xi. So, this is your xi. So, the angle between xi and xi minus 1 is theta i (*ϴ*i), and this is your point through which zi minus 1 is passing through. So, you got it. So, this is the angle. So, theta i is the joint angle from the xi minus 1 axis to the xi axis, measured about zi minus 1 axis using a right-handed coordinate system. So, if it moves counterclockwise, looking from the top, it is positive. So, it is measured like this. Got it? So, this is your second parameter.



The fourth one is the Link Length (a_i). So, it is actually the distance between zi minus 1 and zi. zi is fitted at the end of this link i. It is part of the frame oi. So, zi is there, and xi is there. So, xi is also the part of the frame. So, xi is here, zi is here. So, this is it, and this is the point. And there is a similar point which existed here also. So, this was your, this was your, this is your ZI minus 1. So, this is your zi minus 1. So, distance is measured along the xi axis. The distance measured along the xi axis between the axis is zi minus 1, and zi is link length a_i. So, here is your link a_i. Got it? So, this is known as the link length.

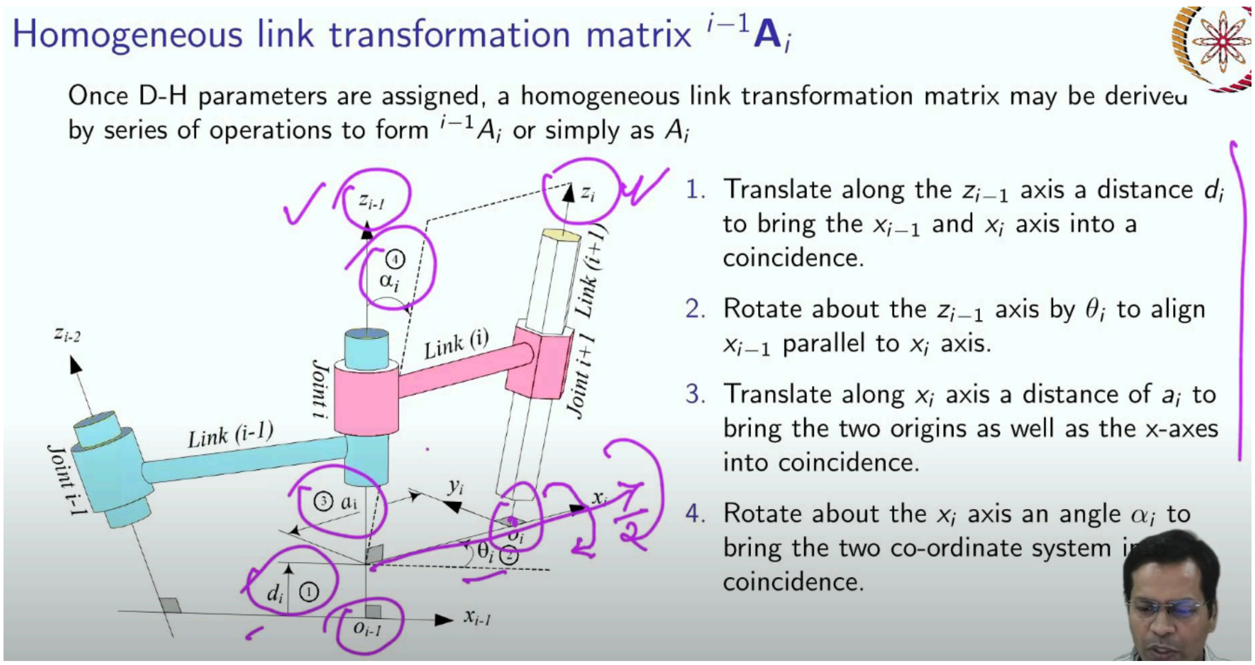


Now, let us move ahead. So, what is the twist of a link? As rightly said, the name actually speaks. So, this is the angle between zi minus 1 and zi. So, it is angled between zi minus 1 and zi measured along the xi axis. Counterclockwise is positive. It also follows the right-handed convention. So, yes, as per the thumb rule, counterclockwise is positive. If you go like this, This direction is your x-axis, So that is your angle. So, this is your alpha i. So, it is *zi* minus 1 and z_i axis, measured about the x_i axis. It is known as a Twist Angle ($\boldsymbol{\alpha}_i$).



So, four parameters are there. So, these were the four important parameters and this is what fully defines your link. So, the first two parameters, two parameters which I have noted here, ai and alpha i are link length and twist angle. So, once the link is made, once it is made, okay, it is maybe a twisted one. So, it is. Once it is twisted, it is forever twisted for that particular link structure.

So, this defines the structure of the link. Alpha is the twist, and length is the distance between the two axes which are there. One is about which the link itself is moving, and the other one is about the frame which is attached to the link itself, that is, the ith frame. So, these two parameters, d_i and theta i, are joint parameters which determine the relative position of neighbouring links. So, two of them were offset by d_i distance. In the case of prismatic, d_i is the joint variable. In the case of the revolute joint, theta i is the joint variable. Got it? So, these are the two parameters. Okay,



Now, let us form a Homogenous link transformation matrix ($^{i-1}A_i$). Now, we know the two frames which are there at the beginning of the link and the end of the link. Can I find out the relative transformation between these two frames? So, that is what is a homogenous link transformation matrix? Okay, Once the D-H parameters are assigned. A homogenous link transformation matrix may be derived by a series of operations, starting from Ai minus 1 Ai ($^{i-1}A_i$). It is known as Ai minus 1 Ai ($^{i-1}A_i$). Starting from i minus 1th (i-1)th frame to ith frame. So, that is it, Or we will be simply calling it Ai instead of calling it Ai minus 1 Ai.

So, let us see how we will go about it. So, yes, this is these are the four things that we did just now. So, the first thing that we did was move from Oi minus 1 to Oi. When we went from Oi minus 1 to Oi, the first thing that we did was move by a distance d_i, moved by a distance d_i along zi minus 1. Okay, Along zi minus 1. So, this is the first thing.

We did a translation about Z. We came to this point. Now the second transformation, Now the second one. What we did this time is we have already reached here. Now we have to. We are still

aligned on this. So, now we have rotated by an angle, theta i, and we came to a new direction that is given by x_i. Okay, This is there. So, this is your second one. So, you have moved the first one. You did was this one. The second one is moving from this to this by an angle theta i about zi minus 1. So, this is rotation about the Z axis by an angle theta i. you came like this.
Now, what to do? So this time I have to. I have already moved like this. I have already rotated like this. Now, I will translate from this point to this point by a distance a_i, that is, the linked length, so that I reach this point. So, this is a translation by a distance Ai along the X-axis. This is the translation of the X axis by a distance a_i. and you reach here. Are we done? No, We still have something remaining.

So, for both the zi and zi minus 1, the new frame is aligned. Still, I have to bring it to zi, actually. So, what now? I have to do it. This time, I will be rotating about the Xi axis by an angle alpha i and will bring zi minus 1 to zi. So, this is the final transformation. So, four transformations are there. First, displacement along zi minus 1 by a distance di. Next, rotation about zi minus 1 by theta i (**ϴ**i). Next, translation along the X-axis by a distance a_i. Next rotation about the xi axis, to finally align to zi and reach Oi. So, these are your four transformations. So, finally, in order to reach from here to here, you have to do these four transformations.

Corresponding transformations are ...

1. Translation along z_{i-1} axis:

$$\mathbf{T}_{z_{i-1},d_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Rotation about z_{i-1} axis by θ_i:

$$\mathbf{R}_{z_{i-1},\theta_i} = \begin{bmatrix} cos(\theta_i) & -sin(\theta_i) & 0 & 0 \\ sin(\theta_i) & cos(\theta_i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Translation along x_i axis:

$$\mathbf{T}_{x_i,a_i} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Rotation about x_i axis by α_i:

$$\mathbf{R}_{x_i,\alpha_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\alpha_i) & -sin(\alpha_i) & 0 \\ 0 & sin(\alpha_i) & cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, four consecutive transformations, which I have just covered, are these. This is translation along zi minus 1 axis by a distance, di. So, this will be your transformation matrix. Next will be rotation about zi minus 1 axis by an angle theta i. So, zi theta i. So, this is your rotation matrix. And the third one is translation along the xi axis by a distance, ai.
So, this is it. And final one is rotation about the Xi axis by an angle, a twist angle that is alpha i. So, this is rotation about the X-axis. Alpha i is here. So, these are your four transformations.

Now look carefully where none of them were global transformation. That means the first transformation was dislocated from the starting frame, and then it was a relative rotation, again relative, again relative. Finally, you reach the endpoint. Okay, That is Oi. Okay, So all were relative transformations. Only the first one was the principal axis transformation. You started from your principal axis over here. It is i minus 1.

Homogeneous link transformation matrix ${}^{i-1}\mathbf{A}_i$

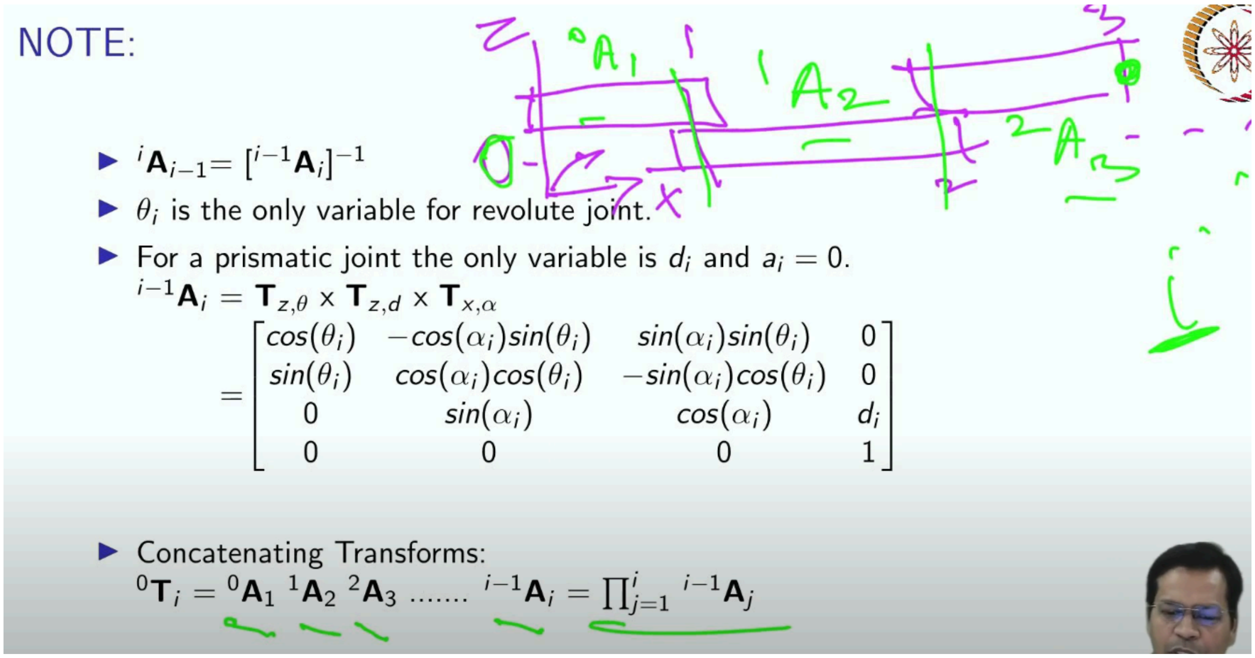
$$ {}^{i-1}\mathbf{A}_i = \underset{①}{\mathbf{T}_{z,d_i}} \times \underset{②}{\mathbf{R}_{z,\theta_i}} \times \underset{③}{\mathbf{T}_{x_i,a_i}} \times \underset{④}{\mathbf{R}_{x_i,\alpha}} $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta_i) & -sin(\theta_i) & 0 & 0 \\ sin(\theta_i) & cos(\theta_i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdots $$

$$ \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\alpha_i) & -sin(\alpha_i) & 0 \\ 0 & sin(\alpha_i) & cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{i-1}\mathbf{A}_i = \begin{bmatrix} cos(\theta_i) & -cos(\alpha_i)sin(\theta_i) & sin(\alpha_i)sin(\theta_i) & a_i cos(\theta_i) \\ sin(\theta_i) & cos(\alpha_i)cos(\theta_i) & -sin(\alpha_i)cos(\theta_i) & a_i sin(\theta_i) \\ 0 & sin(\alpha_i) & cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

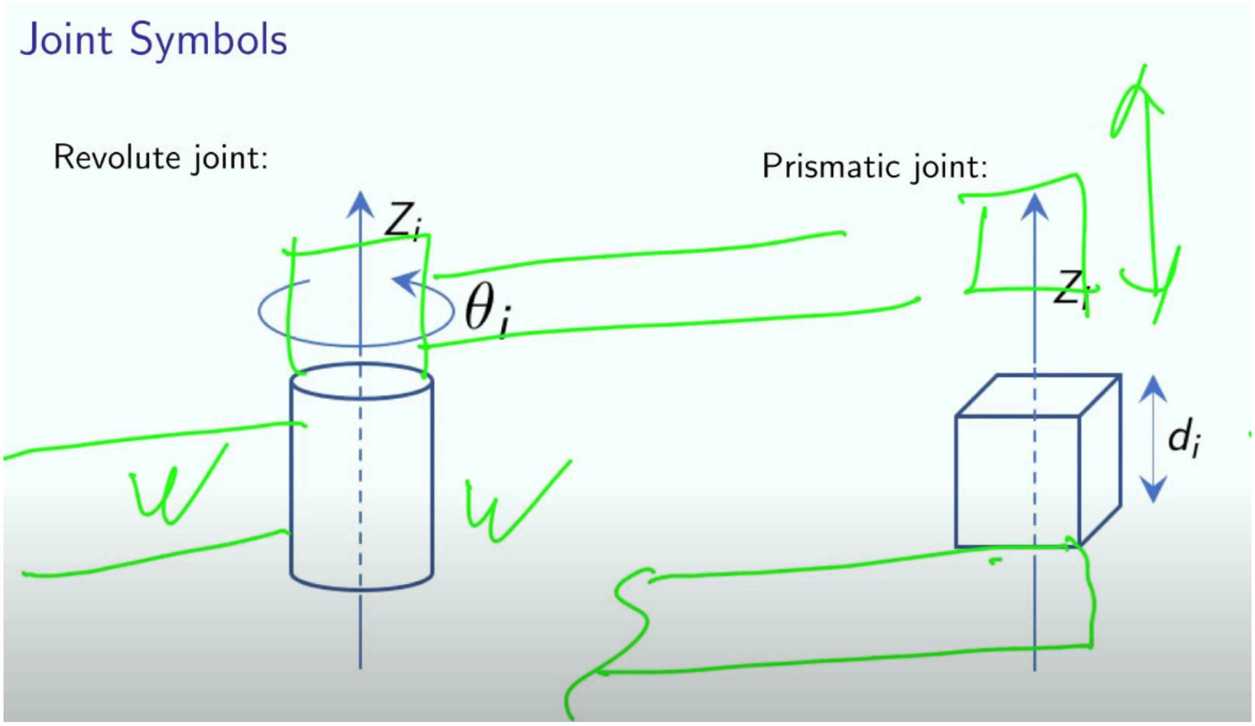
The next transformation was relative. So, that is why all four transformation matrices will be post multiplied. So, this is your first one (Tz, di), this is your second (R_z, θi), this is your third (T_{xi}, a_i), and this is your fourth (R_{xi}, α). If you put them all together, finally, you will get a link transformation matrix, a homogeneous matrix, which says i minus 1 to me. It will take you from location i minus 1 to me. So, this is your location, relative location of i, TH frame from i. and this is your orientation change. Okay, This, as usual, it, is 1. These are 0. So, this is your homogeneous transformation matrix. So, this is your link transformation matrix.



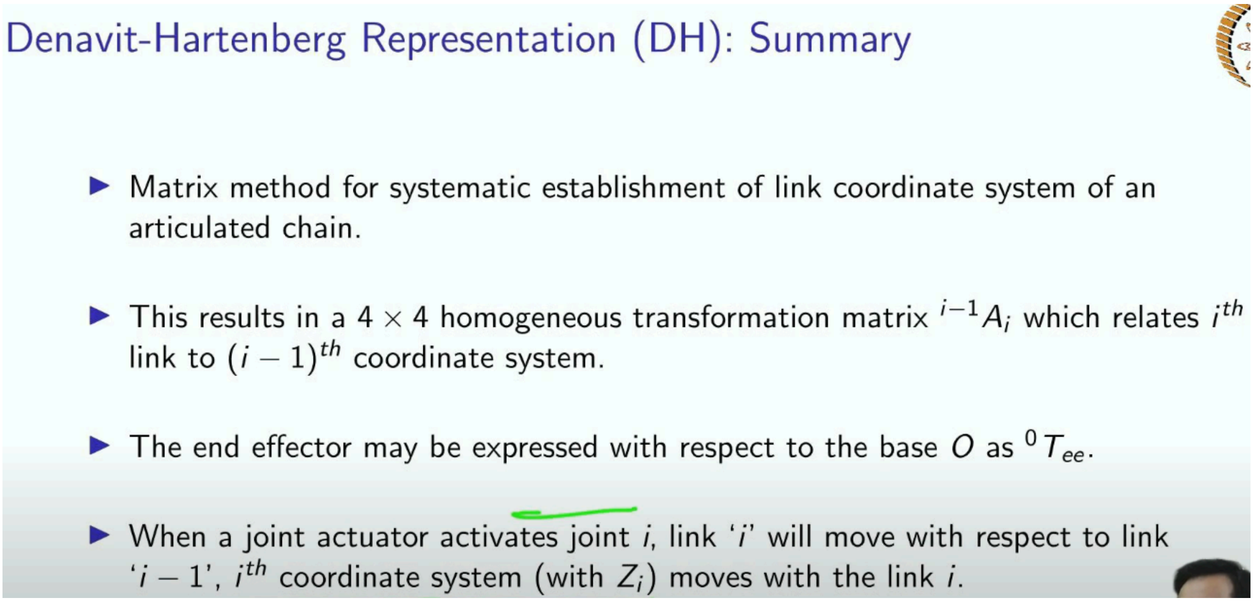
Now, we will move ahead. Please note there are a few things here. So, any matrix, transformation matrix, if it is i with respect to i minus 1, if you have to do inverse transformation. You have to reach; otherwise, that means this frame is expressed with respect to this frame, ith with respect to i minus 1. If I say I have to represent this frame with respect to this, this simply will become inverse.

So, this is it, Got it. Okay, So theta i is the only variable in the case of the revolute joint. Everything else is constant. di is not a joint variable in the case of the revolute joint. So, once two links are placed, they always have the same offset. Okay, So for prismatic joint, DI is variable. di is variable, whereas Ai becomes equal to 0. Link length becomes equal to 0, and you can only move along zi minus 1 axis by a distance di. So, this is there. Okay, So we can simply use the previous transformation matrix.
Just substitute ai is equal to 0, and you will get this. So, this is it. So, now, concatenating all the transforms. So, let us say you have the first link, you have a second link, you have a third link and so on and so forth. You have multiple links here, and this is started with the 0 frame, which is mounted on the ground. Okay, So this may be the z-axis, this is the y-axis. So, it can be like that. And there are other frames which are coming as 1, 2, 3, and so on and so forth. So, each link has its transformation matrix, from the starting to the ending frame.

Okay, From here to here. Okay, So that is it. So, if it is link 1, for link 1, it is 0 to 1. For link 2, it is, it takes you to 2, it starts from 1, for the third link, it takes you to 3, starts from 2, likewise. So, finally, if you want to reach here, you have to multiply all of them together, concatenating them, so 0, 1, 1, 2, 2, 3. Finally, you have to go till i minus 1 i. So, with respect to ground and ith the link frame, that is the frame which is finally attached to the ith link. So, if you want to reach there, you have to take the product of all of them. So, this is the product of all of them. So, this is it. So, this is how you reach the end point of a link. Here, in this case, it is the third link, i equal to 3. Got it, Okay?



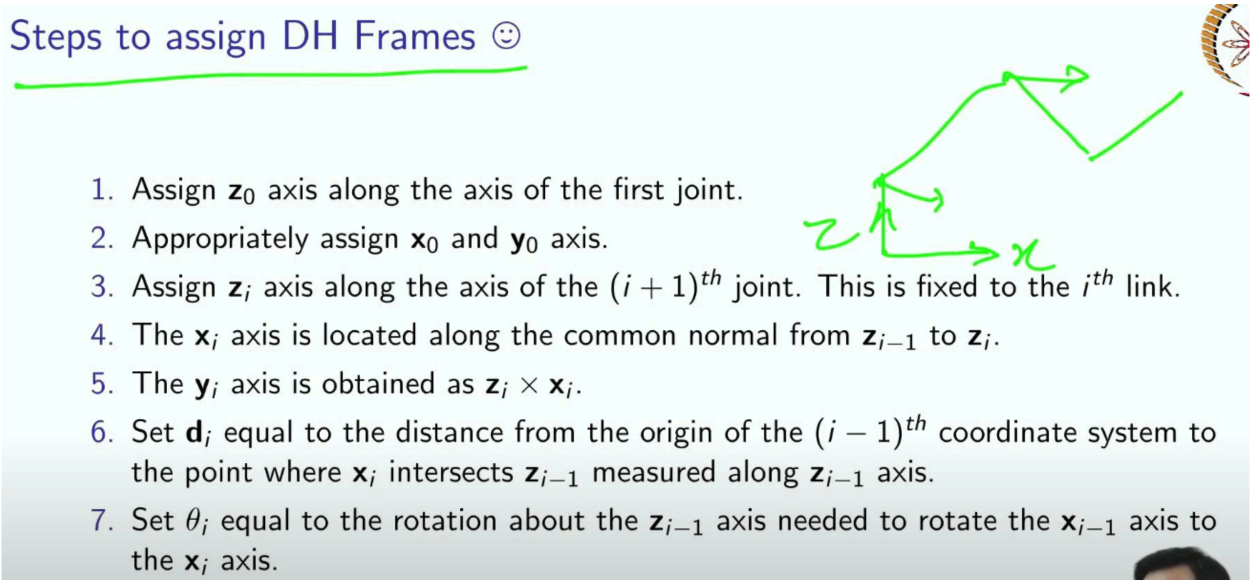
So, let us now formalise the way we are going to draw a robot. We cannot draw complicated shapes. So, rather, we will be representing them using a link, which is something like this. So, this is what this is a joint. If it is a revolute joint, we will represent them using a cylinder with a Z axis placed like this. Theta i is the joint angle, So you may have a link which is something like this. So, on top of this, you will have another link that goes here. So, this is your joint. And another one could be simply like this, in which you have a link fitted with a prismatic joint like this. So, you may have another body which has which can move, like this: Okay, So this is your prismatic joint, Got it? So, these are the two important joint symbols.



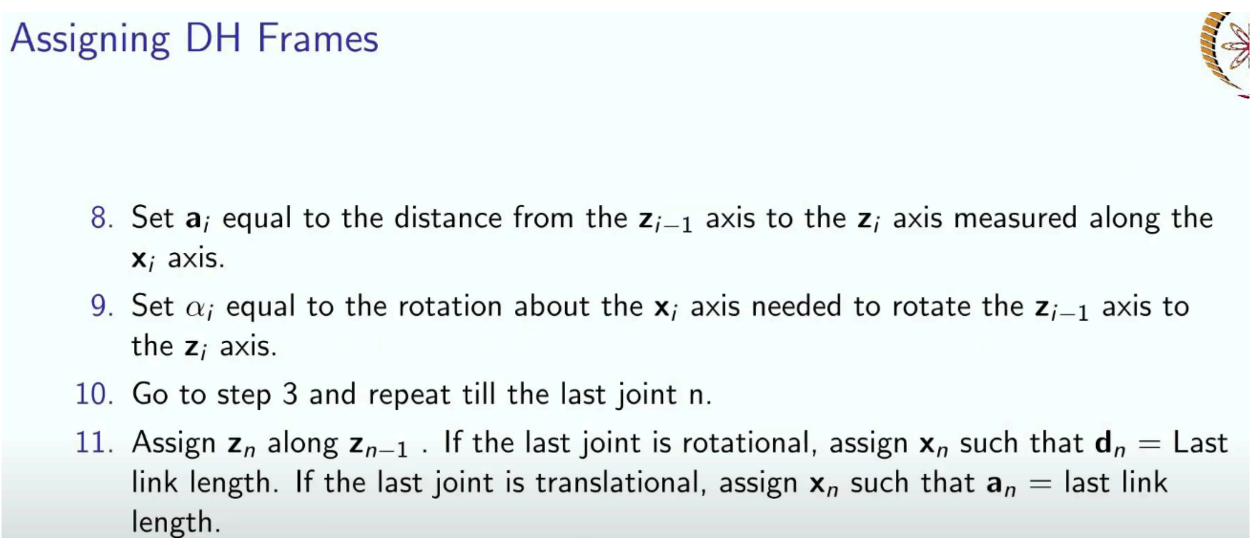
So, now, in summary, DH represents what it did. So, it is a Matrix method for the systematic establishment of a link coordinate system of an articulated chain. Articulated means it is a serial chain fitted one after the other links are fitted one after the other.

So, the next is this results in a 4 cross 4 homogeneous transformation matrices i minus 1Ai ($^{i-1}A_i$). That relates to the link to the i minus 1 coordinate system, and i can start from 1. Okay, So this value i can take 1 to the number of degrees of freedom. Okay, And the end effector may be expressed with respect to the base if you can do concatenation of all those matrices, of all the link matrices. So, ultimately, what you will get is $^0T_{ee}$, starting from base frame 0 to the end effector frame. Okay, So the final transformation matrix will have this having end effector position and end effector orientation: 0, 0, 0 and 1.

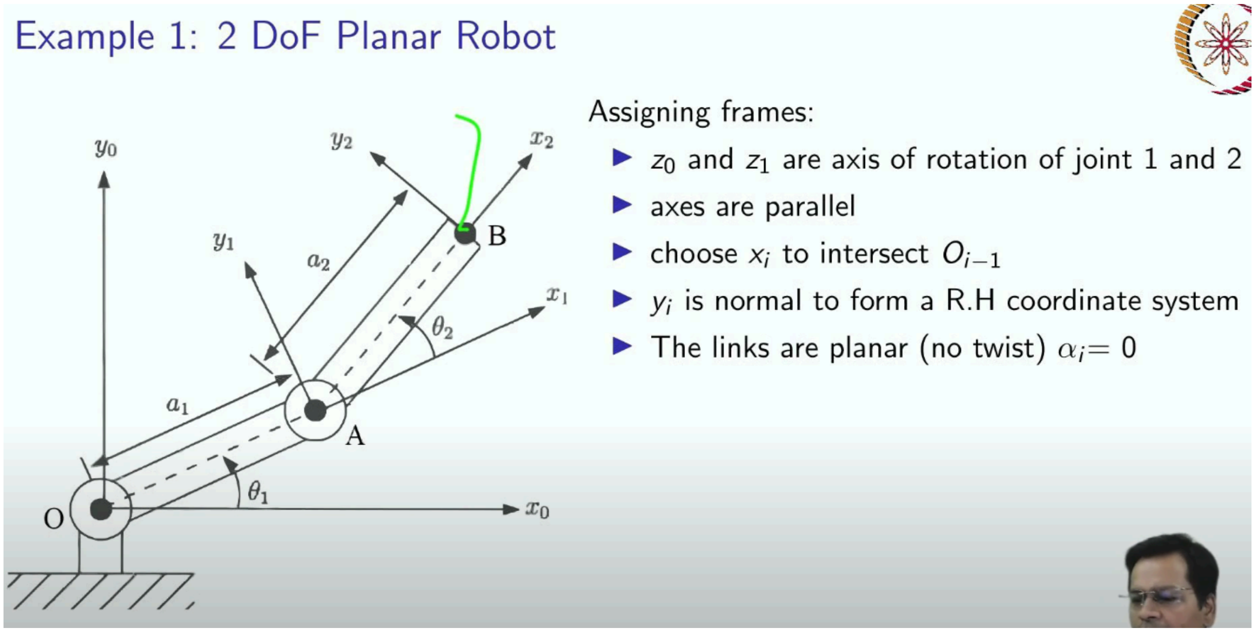
So, it is like this, and when the joint activates, the joint i activate. The link i will move with respect to the link i minus 1. Okay, and i^{th} the coordinate system, which is fitted at the end of link i, that will move along with link i.



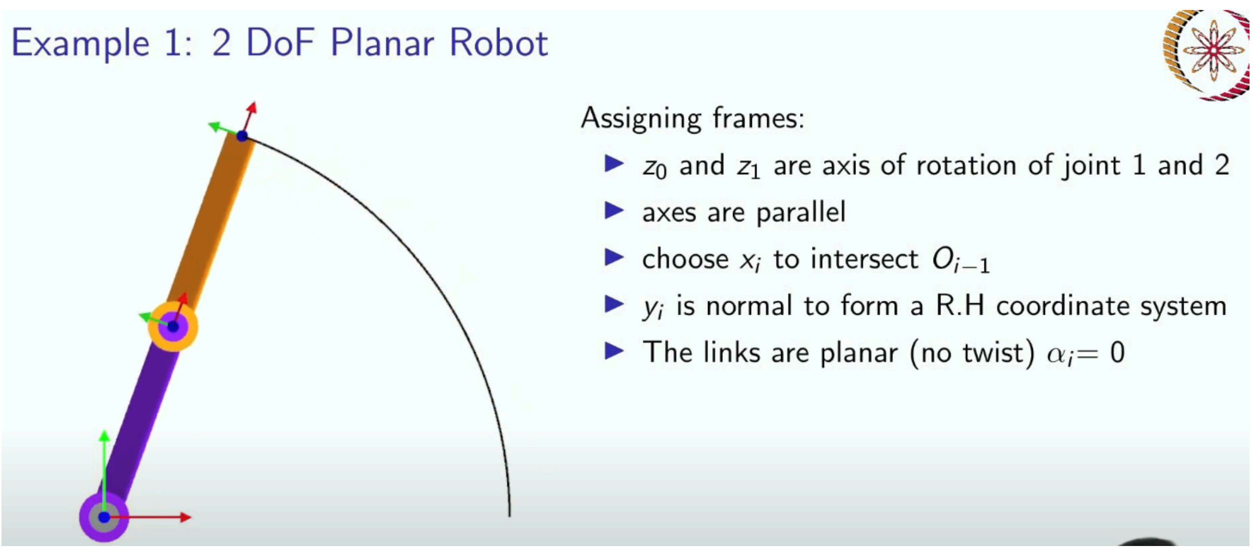
So, these are the steps to assign DH frames to a serial chain system if you are having a robot which is like this. So, how to assign coordinate frames? So, can this be X? This can be Z again. The next one will be Z, where all the X, Y, and Z will go. The only thing that you know now is the axis of rotation. They have a Z-axis. Okay, They have the Z axis. That is all. So, these are the steps. So, that is the reason it is smiling here because it is very difficult to mug it up. We will do it by practice. So, today, we will do one small example, and you will see it is not that tough to remember this also by practice. We will do 4-5 examples in this module and hope you will be very much familiar with this.



So, yes, these are a few more steps that here forgot. All these now need not mug up, need not repeat here.

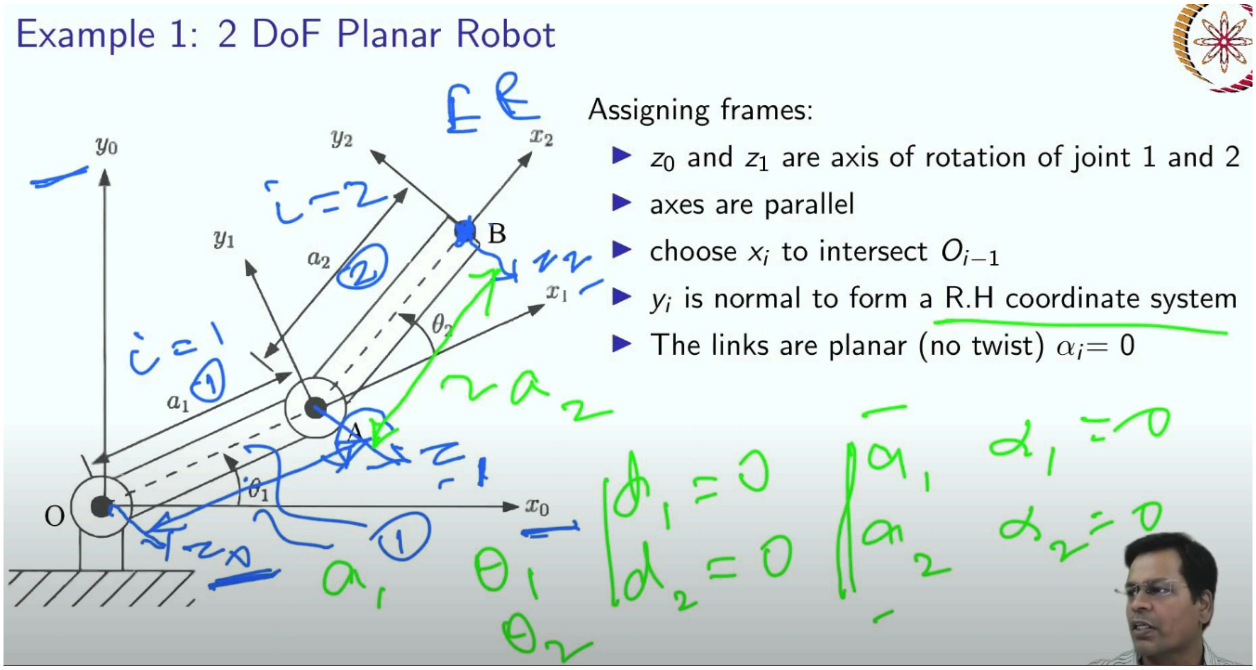


So, let us quickly begin with 2 degrees of freedom planar robot. So, what do I actually want to do here? I want to place the frame 0. I want to place every link with a frame, and I want to find out the transformation matrices, the link transformation matrices, that I can do, concatenation, and finally, I can reach till end effector.



Got it. So, let me see if I can explain you well using this figure. So, this is our robot. So, you see, it can move. So, let me just quickly move it so that you can understand how this link is connected, how the links are connected. So, you see, it can move like this. So, you have first linked with one colour fitted with the ground frame-red and green. It is showing Okay, And you can simply move it like this. So, two joint angles are there. You can just move one of them, Okay, Or you can move both of them. This is only moving the second link with respect to the first one. So, if I want, I can just move the first joint without moving the second one.

So, this is how it will move. So, I have just commanded it to go from 0 to 90 degrees, first joint. So, this is what the two degrees of freedom robot will look like.

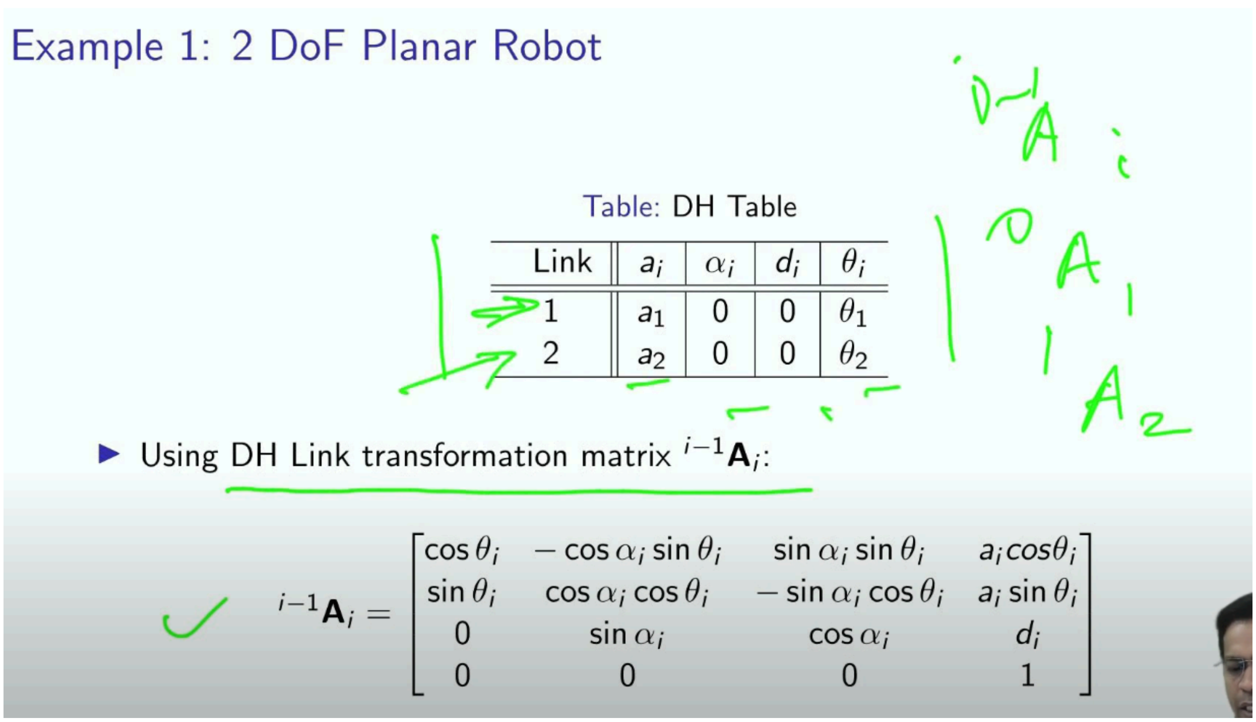


So, this is the figure which quickly represents the same structure. So, this is your joint. So, you know the first axis which was there. This is your first axis. So, I will use a convention from now and. So, I will use red and red colour for the X axis, green for the Y axis, and blue for representing the Z axis, So that will be convenient. So, yes, let us quickly go and put those frames here. So, yes, this is your Z axis, about which the first link is moving. It is out of this plane. So, in order to make it visible, I have just drawn it like this. So, about the Z axis, the Z0 axis is attached to the ground. Okay, That is the first frame that is always attached to the ground, about which the first link will be moving. So, this is your first frame named as 0,0. So, Z0. X0 and Y0 can be conveniently placed the way you want your robot's end effector, end effector to lie in whatever space. Okay, So accordingly, you can put X0 and Y0. So, if it is all positive, you want. So, this should be X0 here, Y0 here. So, it follows our right-handed coordinate system: X cross, Y should be Z. So, yes, so Z0, so theta 1 will be measured.

So, first link. So, this is your first link, Okay? That is running on. This is your i is equal to 1. It is running on 0, Z0 axis, and this itself is ending with a frame on it. So, that will be given as Z1. Why is Z1 like this? Because Z becomes the axis of rotation for the second link, which is here. This I is equal to 2. Okay, Finally, that also will be ending somewhere. So, this can be just a dislocation of this frame. You can put it here. So, this also, fortunately, I have put it like this. It can be placed any other way. So, this is your second frame, which is at the end of link 2. So, this is your link 1, this is your link 2. So, link 1 ends with Z1, and link 2 ends with Z2, and Z0 is the frame about which link 1 rotates. So, you got it. So, the distance between you see this link, this

system is completely planar. So, the distance between Z0 and Z1 is your link length. So, that is the link length, given as a1. Okay.
Similarly, the distance between Z1 and Z2 is your link length 2. So, this is a2. as it is a planar system. So, there is no dislocation along offset along the Z axis. Okay, So both the systems are exactly lying in the plane, not like this.

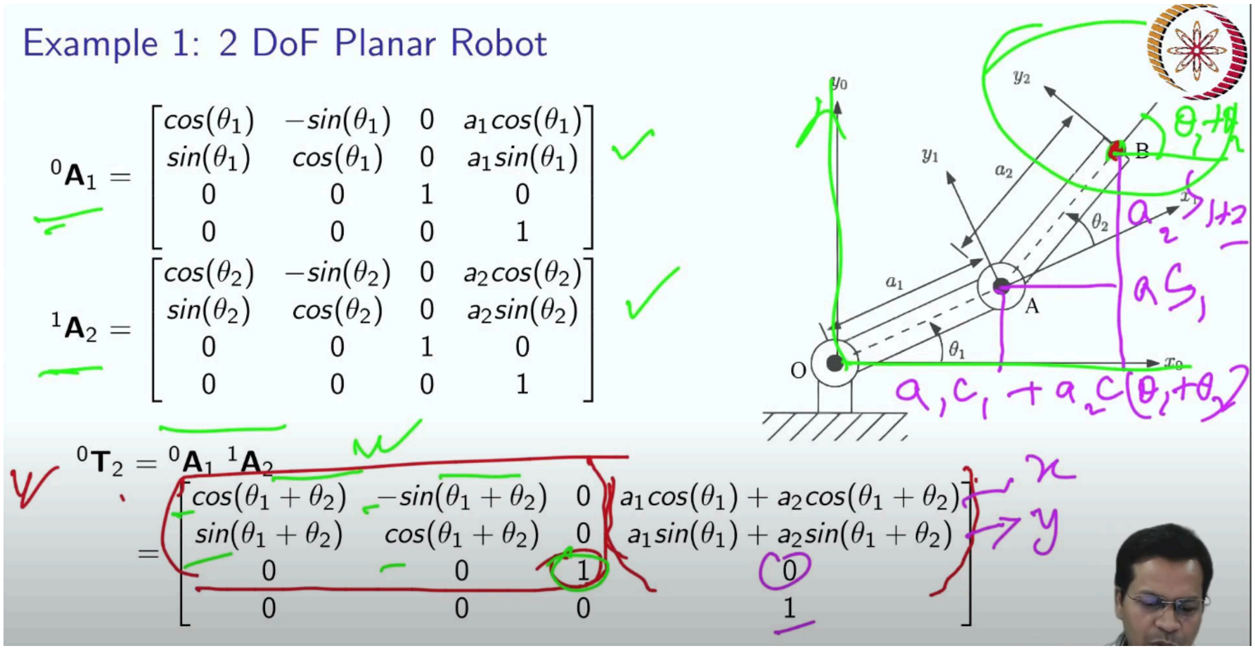
So, the d1 is equal to 0, and d2 is also equal to 0. The only thing that remains is a1 and a2. Those are the link lengths. There is no twist. You see, all the Z axes are parallel. So, you have a link with both the Z axis about which it is rotating, and this also has a link frame. Okay, So both are parallel to each other. So, there is no twist in between. So, there is no angle between Z1 and Z2, Z0 and Z1. So, alpha 1 and alpha 2 will also be equal to 0. So, d1, d2 is 0, alpha 1, alpha 2 is also 0, a1 and a2 are link lengths, and what remains are joint variables. Theta 1 and theta 2 are joint variables. Okay, That is the only variable. So, the same thing is written here. This is how we can put the frames. Angles will be measured in the right-handed coordinate system. Okay, So that is all.



So, now I can quickly put my DH table with the parameters that I have just said. So, for link 1, ai. this is your link length, it i.S a1. Okay, Similarly for link 2, link length is a2, alpha i for both, because there was no twist, and there is no angle between any of the two consecutive Z axis.

So, alpha 1 and alpha 2 are equal to 0. Similarly, this link is completely planar. So, d1 and d2 are also equal to 0. These are joint variables. Okay, These are joint variables. Okay. So, now this is your DH parameter table. Okay, We have reached here. So, DH parameter table.

I have just transferred it here. We already know the link transformation matrix, So it starts from ai minus 1 to me. Okay, So if I put, I am equal to 1, so that is your first one, that is with respect to 0, and I will put all the values which are here, Similarly for the second one, which is with respect to the first one. So, I will put this one, and I will use all these values to obtain that in this matrix. So, this is the one ($^{i-1}A_i$) which we just now derived link homogeneous transformation matrix. I will substitute these values here,



and I will get exactly to these two transformation matrices. Got it? So, these are the two. So, quickly I have just put them here. So, these are the two and taking the product of them will take me to the end effector point. So, this is your end effector point. So, if I have taken product of this, finally I have got what? 0 to 2.

$$^0T_2 = {}^0A_1\,{}^1A_2$$

So, that tells me the end effector position. One can quickly at least derive this one. So, what is that? You just see it here. So, it is a1 cos theta 1, and this distance is a2 cos theta 1 plus theta 2. So, the sum of them is your x coordinate. So, this is here. Similarly, y can be obtained as a1 cos theta sin theta 1 and a2 sin theta 1 plus theta 2. Got it? So that is here. This is y and z because it is planar. So, z is always equal to 0 and this. So, got it. And you can quickly verify even the rotation matrix, at least in this case. So, this is your rotation matrix. You see, this is our rotation matrix about the z-axis by an angle theta1 plus theta2. So, end effector frame. You see, its combined angle is theta1 plus theta2. So, this frame end effector frame is now rotated with respect to the ground frame that is x0, y0 by an angle, theta 1 plus theta 2. Got it.

So, that is quickly here. Cos, theta minus sin, sin and cos. So, this can give you the orientation. So, position and orientation. You got it quickly.

Thanks a lot for this lecture and the next class. We will be moving ahead with three degrees of freedom, a Spatial robot. We are done with planar. Now, we will learn what a wrist is and how that can be represented using the DH parameter. We will attach this wrist on top of the cylindrical robot only, and we will make a robot with six degrees of freedom, the cylindrical robot with a wrist. And we will do one of the industrial robots, the SCARA Robot, if we get time. So, that is all for today. Thanks a lot.