NPTEL Online Certification Courses
Industrial Robotics: Theories for Implementation
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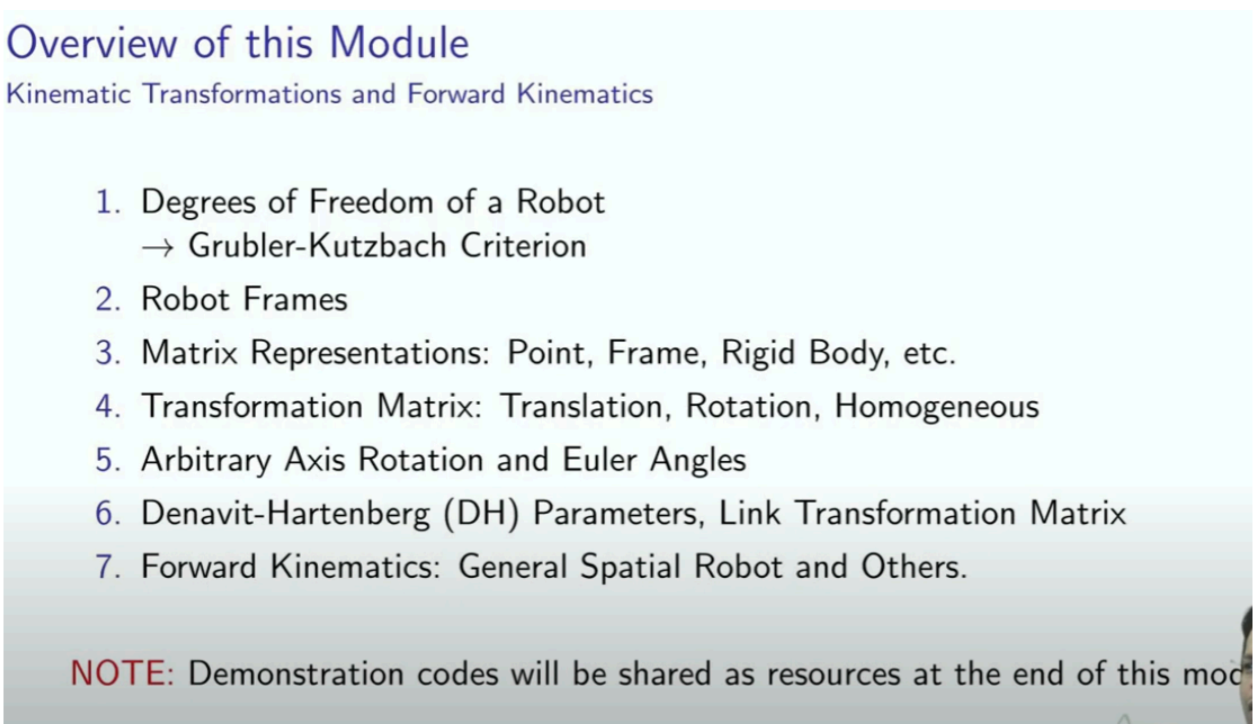
Week: 04
Lecture: 16

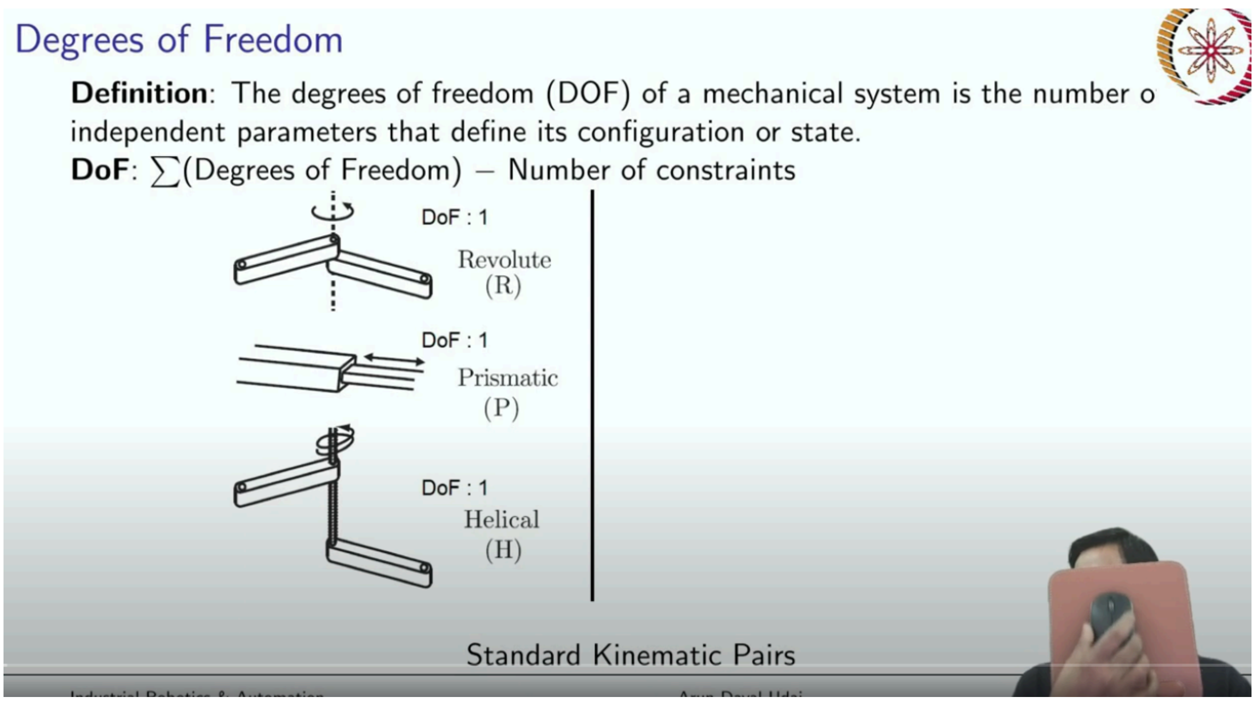
Degrees of Freedom and Kinematic Transformations: Translation

So far, we have understood robots, industrial robots, as the mechatronic machine that comprises electrical and electronic hardware, software and mechanical systems. So, in the previous module-your modules-you have already seen industrial robots' anatomy. You have understood its structure. We have seen how actuators are driving these mechanical links, huge mechanical links. Sensors are there. You understood sensor technology to some extent, and software is always a part of various systems which are there, ranging from control of the actuators to sensor data acquisition, processing, and calculating for kinematics, dynamics, trajectory, etc. From the module and on, you will understand robots as a mechanical system. Moving forward, today's module is module 4. We will be doing kinematic transformation and forward kinematics, that is module 4. In today's lecture, we will be dealing mostly with degrees of freedom and kinematic transformation, especially translation.



So, let us move ahead. So, this is the overview of this module: kinematic transformation and forward kinematics. So, we will be starting with the degrees of freedom of a robot. That is a very important parameter that you have seen earlier. We will discuss that. We will see how the Grubler-Kutzbach criterion helps us find out degrees of freedom. Robot frames, different robot frames which are there, we will understand. Matrix, representation of point frames, rigid body, etc. Transformation matrix. This is a special kind of transformation which is mostly used in robots to do transformation, mostly translation. The rotation and homogeneous transformation, we will understand better when we are actually doing it. So, arbitrary axis rotation Euler angles, we will understand. We will talk about that. Denneby-Hortonberg parameters, that is, to do forward kinematics of industrial robots. Forward kinematics we will be doing for various robots, ranging from one of two degrees of freedom, three degrees of freedom and finally, two industrial robots, generally special robots, which is six degrees of freedom robot. So, we will be looking at them and all the demonstration codes which will demonstrate all these principles which are taught in this module. I will be sharing those at the end of this module as a resource. So, let us move ahead.

We will be starting with degrees of freedom and kinematic transformation in this module, especially translation. We will be talking about.

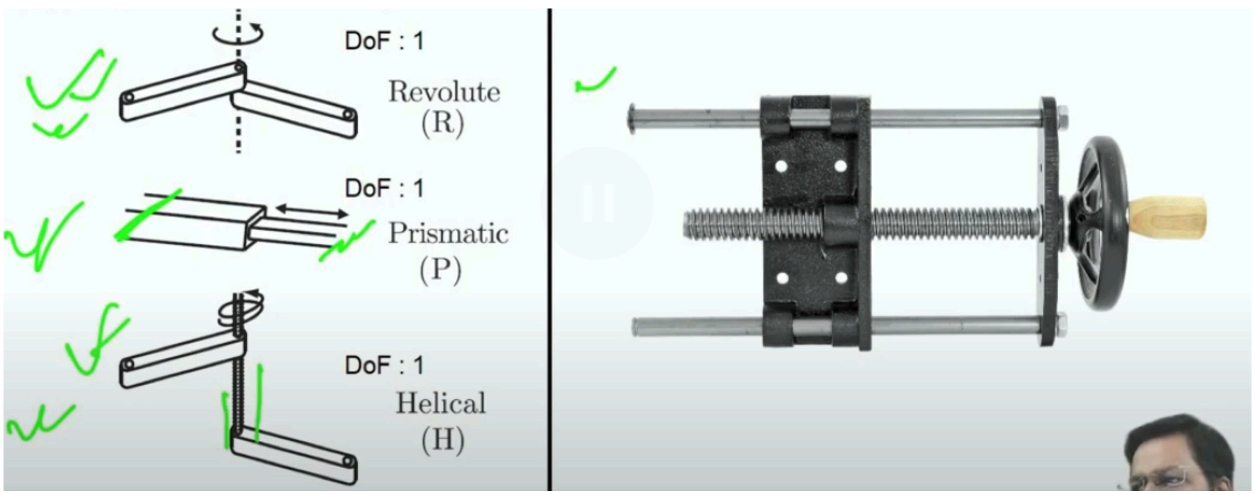


So, yes, degrees of freedom. So, that was a very important parameter that you saw when we were discussing different performance criteria and different classifications of industrial robots. So, how a robot is classified? It was also classified based on the degrees of freedom. What is it? So, that important parameter, let us formally define that and understand how it can be calculated.

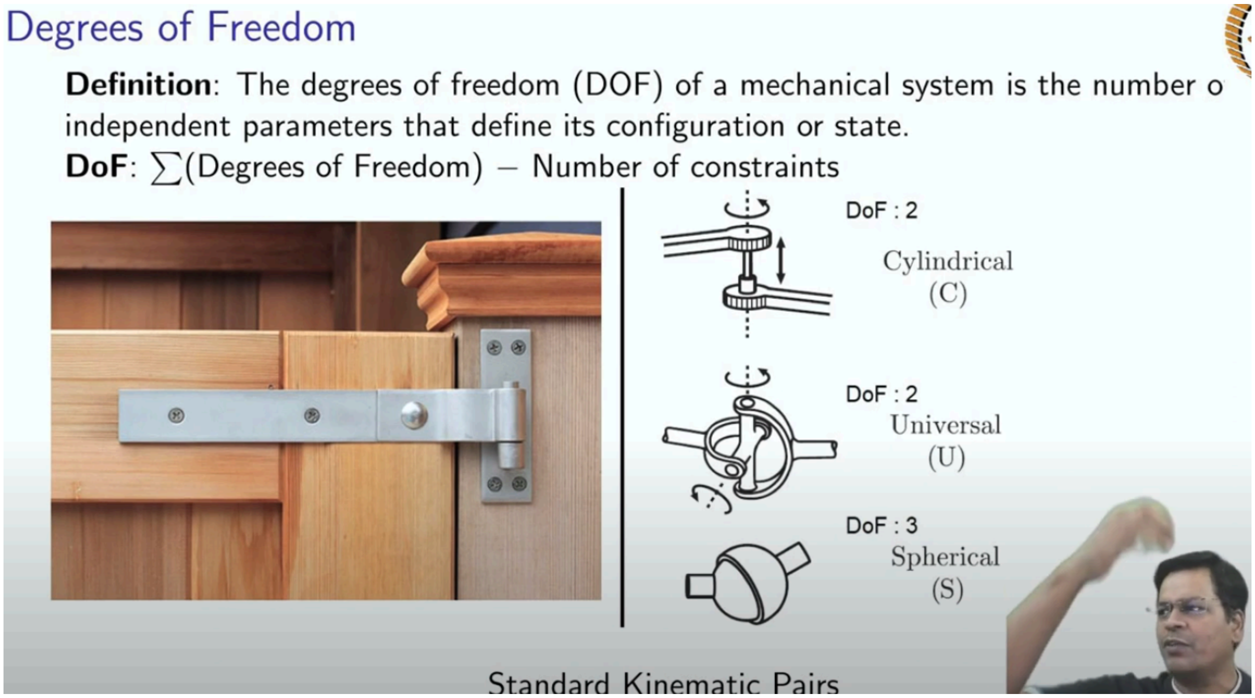
So, let us move ahead. So, yes, the degrees of freedom of a mechanical system, not just a robot. It is the number of independent parameters that define its configuration or state. It can be alternatively defined as the number of independent constraints that you can put to make a mechanical system fully constrained. So, you will understand it better by doing it. So, yes, let us first understand degrees of freedom as such. So, any special object, any special object, may it be a mouse or anything, in this space is free to move along the vertical direction, horizontal direction, along the x-axis and y-axis. Let us say this is x, this is y, so three translations it can do, and three independent rotations it can do as well. So, yes, a total of 6 independent degrees of freedom it has. So, in this space, as many constraints we put, the remaining degrees of freedom are the degrees of freedom of that object. So, if you keep on constraining your object from moving. So, let us say if it is a mouse pad and if it is rolling on this surface if you can see it clearly. So, what you see, it can move along this plane. So, it can move along x. it can move along y. if I say it should not come out of this plane, so it cannot move along z axis, that is coming out of this surface. So, what else it can do? It can rotate about its axis. So, all together, it can move along x, along y and rotation. So, two translations and one rotation it can do. So, you got it. So, why was that? Because it was constrained to move along the remaining three things. So, if I say it can move along x and move along y, it was unable to move along. Rotation degrees of freedom along y, rotation degrees of freedom along x. So, two were constrained. 2 were completely constrained, got it? And as I said, it should not leave the surface. That means it is constrained to move along a vertical direction perpendicular to the plane of this plane. So, it was constrained by 3, so the remaining was 3. So, six minus three constraints. So, the remaining was 3 degrees of freedom for this mouse if it is on this surface. So, got it. So, this is how it can be explained. The degrees of freedom of an object are always total degrees of freedom that maximum degrees of freedom, that is, six minus the number of independent constraints which are there.

So, let us discuss one by one. So, this is one degree of freedom system where you have a link connected with another link. like this, with a revolute joint. So, it has just one degree of freedom. Why It cannot travel like this (to and fro), it cannot dislocate like this. That means it is constrained to do all three translations. Now let us say because it is an axis, it cannot rotate like this. It cannot rotate like this. 2 rotations are also constrained. So, two rotations and three translations are constraints. So, the body which comes next, which initially has 6 degrees of freedom to 6 minus 5, that is 1 degree of freedom system, and this joint, you know, can be defined by defining that particular remaining 1 degree of freedom. If you can define the angle between the first link and the second link, which is there, you have fully defined this mechanical system. So, the number of independent parameters, as it is said in the definition, also defines the configuration or state of the mechanical system. So, that is what is degrees of freedom. So, two ways I have defined it. First, the number of independent definitions that are required and the total minus the number of constraints that you can put. So, either way, it is coming out to be 1. So, now let us see what could be an example of this. It can be simply your door hinge. Your door hinge is very much like this.

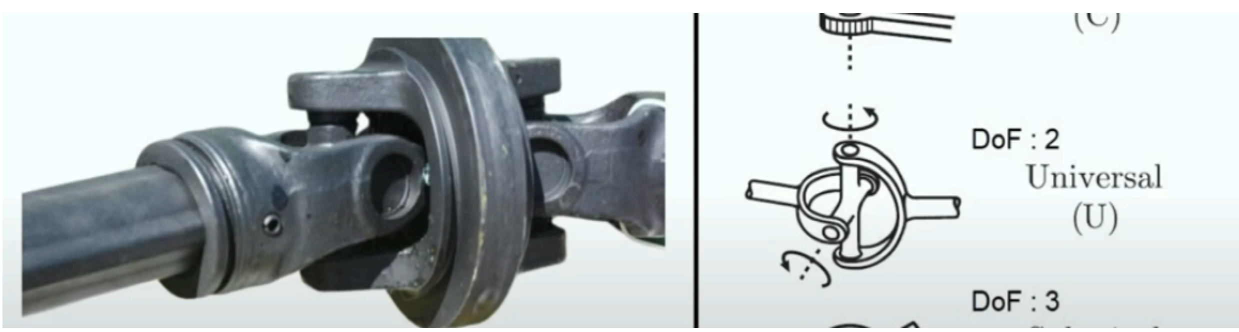
Another similar one is this one (Prismatic (P)). So, what is this? It is just like a syringe and a piston. So, it can be like this. It is just like a telescopic joint. It is commonly known as a prismatic joint. You see, normally, in earth-moving vehicles these types of actuators are there which actually move those joints of huge robots or maybe huge hydraulic systems or pneumatic systems or earth-moving vehicles. So, everywhere you see those. So, that has got 1 degree of freedom. Again, just by defining the displacement of this link with respect to this link. You have defined the system. So, it needs just one parameter to define this mechanical system completely. Alternatively, because it is a piston, which is one inside the other, it cannot rotate. It cannot rotate, which means two rotations are constrained. As it is a rectangular thing, it cannot rotate about its axis. Also, three rotations, all three rotations are now constrained. So, how many translations it can do? It can do just one translation and cannot do the remaining two translations. So, it is there are five constraints. So, six minus 5, it is just 1.



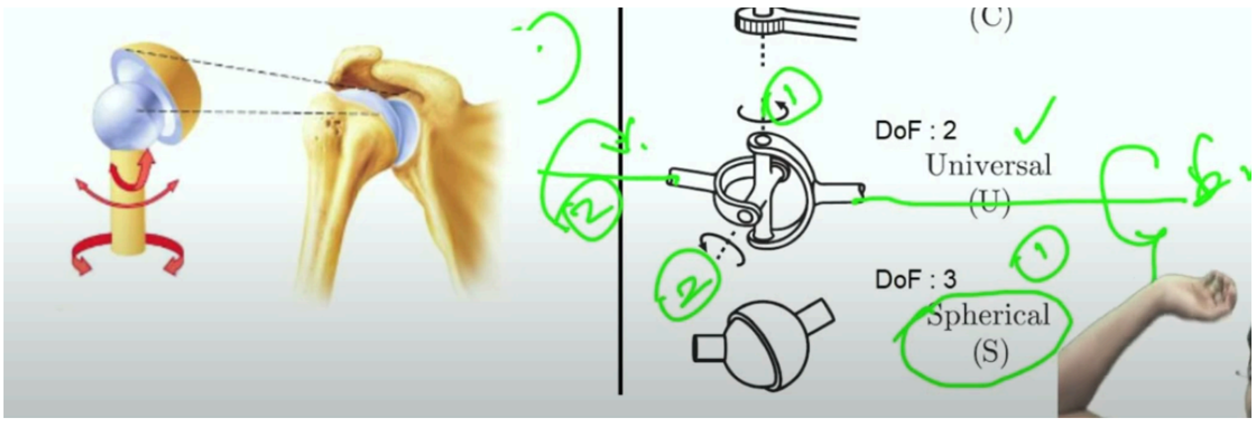
So, again, a similar one is Helical (H). This has got 1 degree of freedom. Although it appears if it rotates, it also translates. It is what It is: a link which has a thread over here and moving on this thread. So, you are unwinding it in a spiral way. So, if it rotates, it also translates. So, effectively, just one definition is enough to constrain this system fully. If you can constrain this, you can weld it, and you do not allow the second link to rotate with respect to the first one. You have finally defined it. So, the degree of freedom is 1.



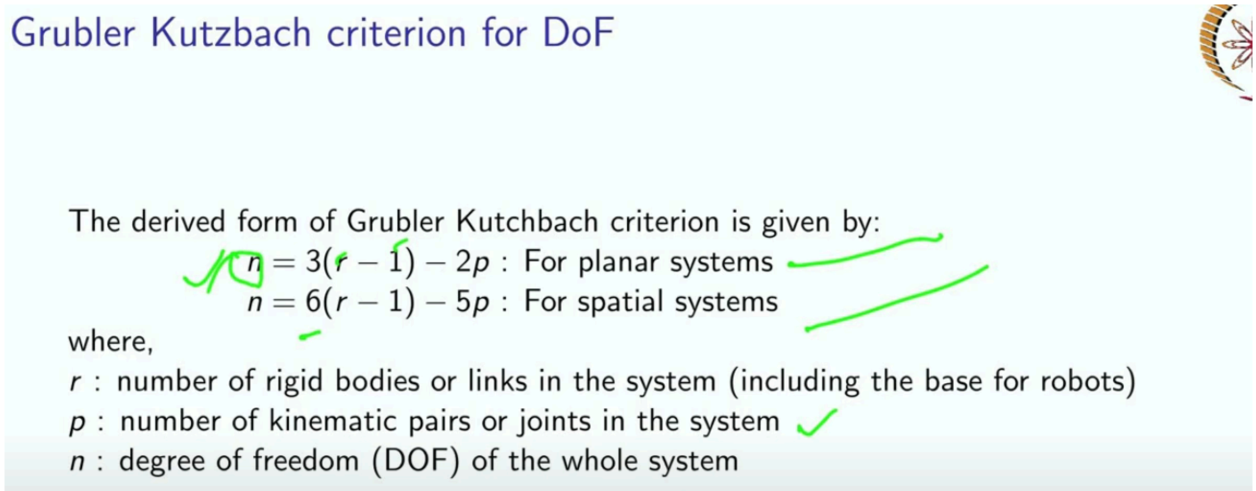
A similar one is another example. That is a cylindrical joint. So, this is similar to a hinge of a huge gate. Huge gates are put in like this. It is put from the top, and it can even be taken out like this: There are two hinges which are there, which normally support your huge gates so that it is easy to take it out. Just lift it and take it out while putting it back. You can do it. So, this is a cylindrical joint that allows rotation of the gate with respect to the wall on which it is mounted, and it also allows this vertical lift in order to take it out. So, that is maybe a cylindrical joint. That is a cylindrical joint, A universal joint. So, you see how many definitions you require for this. It is two only. So, if you are not allowing it to move vertically upward, you can constrain. You should also constrain the rotation degrees of freedom. Degree of freedom, rotation, degree of freedom. So, constraining that will fully constrain it. So, two definitions are required.



Again, a similar one is 2 degrees of freedom. That is a universal joint. This is used in moving vehicles. That is, this power is coming from your engine, and this goes to your differential, where two wheels are fitted. So, you have something in between so that it allows while rotating your wheel, transmitting the power from the engine to the differential drive, link one and link two bend in between. So, this axis has got two different axes of rotation. While it can rotate about the whole of the system can rotate Individually, one relative to another one, the previous one. There are 2 degrees of freedom. One is this one, and the other one is this one. So, two independent degrees of freedom. This type of joint is very common, even nowadays. In your screwdriver, you may have this attachment so that you can transfer the power. You can transfer the torque to your screw remotely, like this, even if your axis is not aligned with the screw. So, those attachments are in your screwdriver set also. So, you must have seen this. This is known as a universal joint. It has 2 degrees of freedom.



The spherical joint is known as a spherical joint. It has 3 degrees of freedom. It is a ball socket joint that is there in your shoulder also. It is there in your hip bone also. They are similar. So, how are they connected? You can move it like this. You can lift your arm, raise your arm. Like this, you can move it horizontally. So, two are there, you see, and this, your arm, can rotate about its axis as well. So, what do you see? It has 3 degrees of freedom. If you can constrain all 3 of them, you can fully define your joint. You can fully define your mechanical system, which is this joint. So, that is, three definitions are required. An example is your shoulder joint. So, these are a few examples how different joints are constructed and how degrees of freedom are assigned to these joints. And how to calculate joint. There are two ways, as I told you. So, that is all.



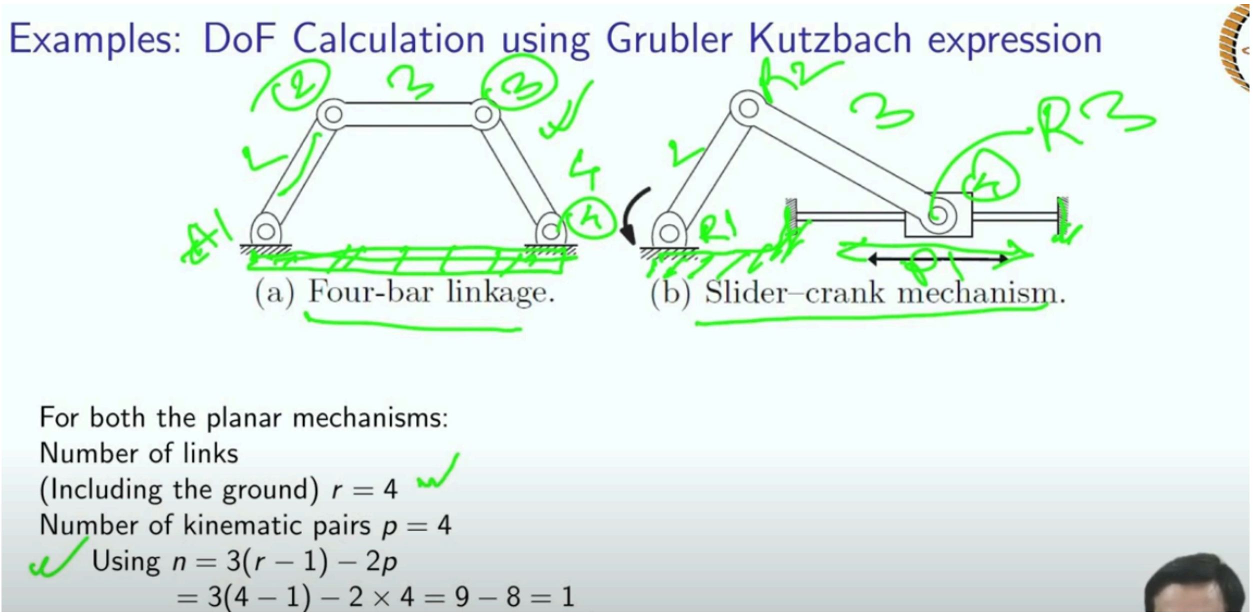
So, moving ahead to how to calculate it, Grubber Kutzbach's criterion for degrees of freedom is very popular. I wont to go very much deeper into it how it is derived for now. So, yes, you can take it for granted, at least for the planar system. It is given by. n is equal to 3 times r minus one minus 2p. So, as it is written here.

$$n = 3(r - 1) - 2p$$

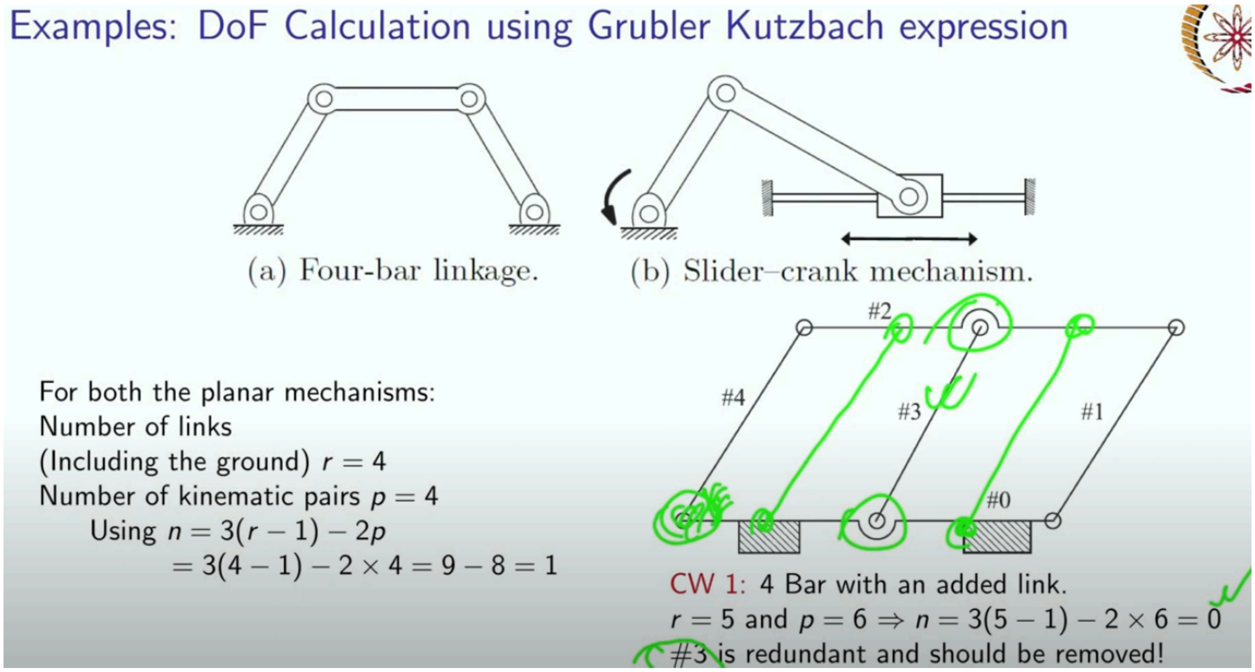
So, what are those r and p? r is a number of rigid bodies or links in this system for which you want to calculate the degree of freedom. n is the degrees of freedom, and p is the number of kinematic pairs. So, if you have any background in the theory of machines or maybe engineering mechanics, few chapters you must have gone through it. So, kinematic pairs, or joints in the system. They basically couple two independent mechanical bodies together in different ways, As you have seen earlier in these examples. So, those were all kinematic pairs. Let us use this and do some examples. You will come to know how it is used and mind it, and it is for spatial systems if it is not a planar system. It is a spatial system like a spherical joint. In that case, you have to use the formula which says six times of r minus one minus 5p.

$$n = 6(r - 1) - 5p$$

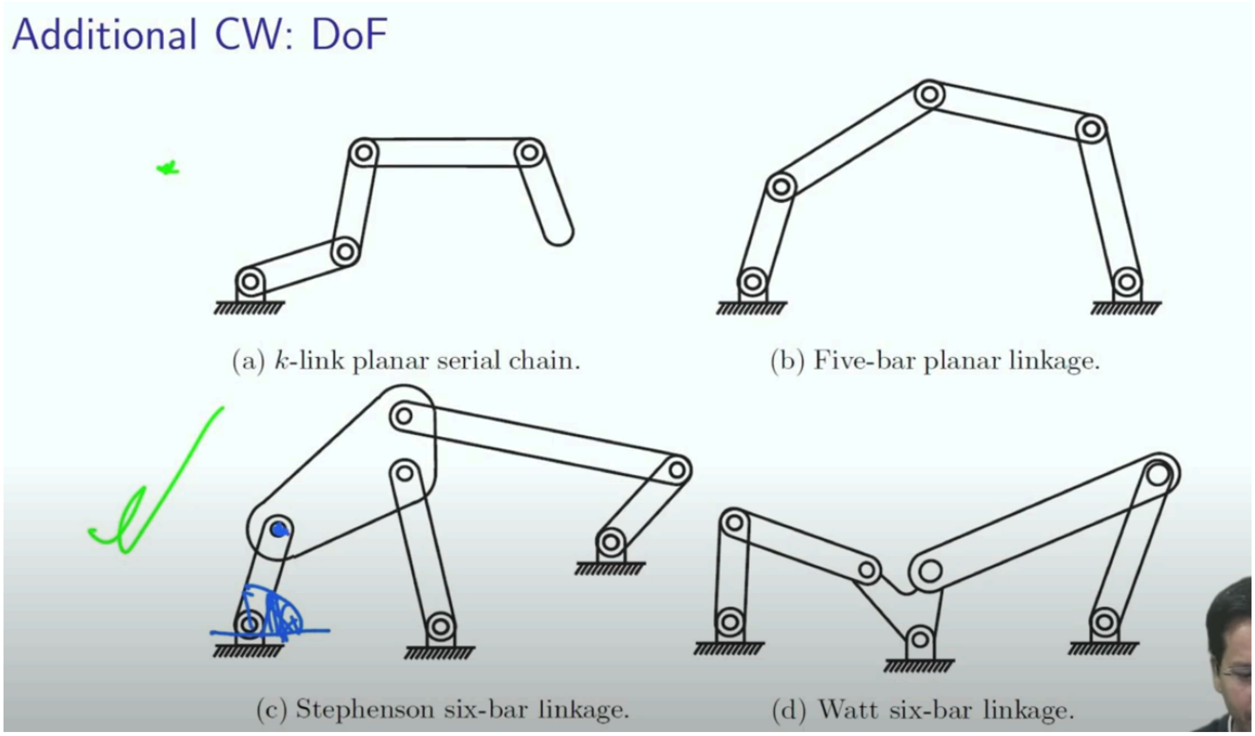
So, this is for a spatial system. There are some mechanisms which can go out of the plane also, so that is what. So, for those, you have to use the second one.



So yes, let us just do one example. That is a Four-bar mechanism, which is a very, very popular type of mechanism which is used mostly in mechanical systems. I will give you a few examples, also with some pictures here. So, these are a few. So, yes, for both the planar mechanism that you can see, the Four-bar mechanism, Slider Crank mechanism. So, yes, number of links, including the ground. So, the ground is this one. This also is a link. So, this is a mechanism consisting of 1 link, two links, three links and four links. So, a number of links are four kinematic pairs. You see one joint, a second joint, a third joint and a fourth joint. So, four links and four joints. Suppose you use the first formula for the planar mechanism. So, what do you see? 3 times of r minus 1 minus 2p. So, it is three times of 4 minus one minus two fu into 4. So, it is nine minus 8, and it gives you one how this one is made. Actually, This is your one link. This is your link, also. So, this is your first link. This is your first link. Second link: This is the third link. And you have another link which is not visible here. It is actually the fourth one, which is the slider, which is sliding on this ground surface, this surface. So, that is four links. What are the joints here? So, one of the joints is revolute joint, R1, revolute joint, R2, revolute joint which is here is R3, and one of the joints is P1. This is a prismatic joint which allows your link to slide on this rod. So, there are 4 links and 4 joints for this one also.



So yes, can we do some tricks over it? So there is a common query which comes along a 4 bar mechanism, quite a lot of time. So, can it be made like this? How many degrees of freedom which are there? So, you see, this also is a 4 bar mechanism with additional support which is here centre support. It is just a parallelogram with a centre support. So you can make it move like this. While this will go back and forth, it can go like this: So how many degrees of freedom, If you just count them? So, the number of links that come are 1, 2, 3, 4, 5. So, 0 to 5. 0 to 4, that is 5. And how many joints? You see 1 joint, 2 joint, 3, 4, 5, 6. So, effectively, this should come as 0 degrees of freedom. But you think, you see, if you constrain this one (one of the corners), if you weld this joint, the whole of the system is constrained. That means you just need one definition to define the whole of the system. At least 1 degree of freedom should have come. So, I hope you can see it clearly now. So, yes, this is what I am talking about. So, what is here is this number 3 link. This link with both these joints is redundant. They are redundant, and they should be removed before you start calculating degrees of freedom. So, you can have many such supports here. That does not mean all of them together should be included while calculating degrees of freedom. So, mind it, you have to remove any such support any duplicate links which are performing no function apart from some support and some different functions.



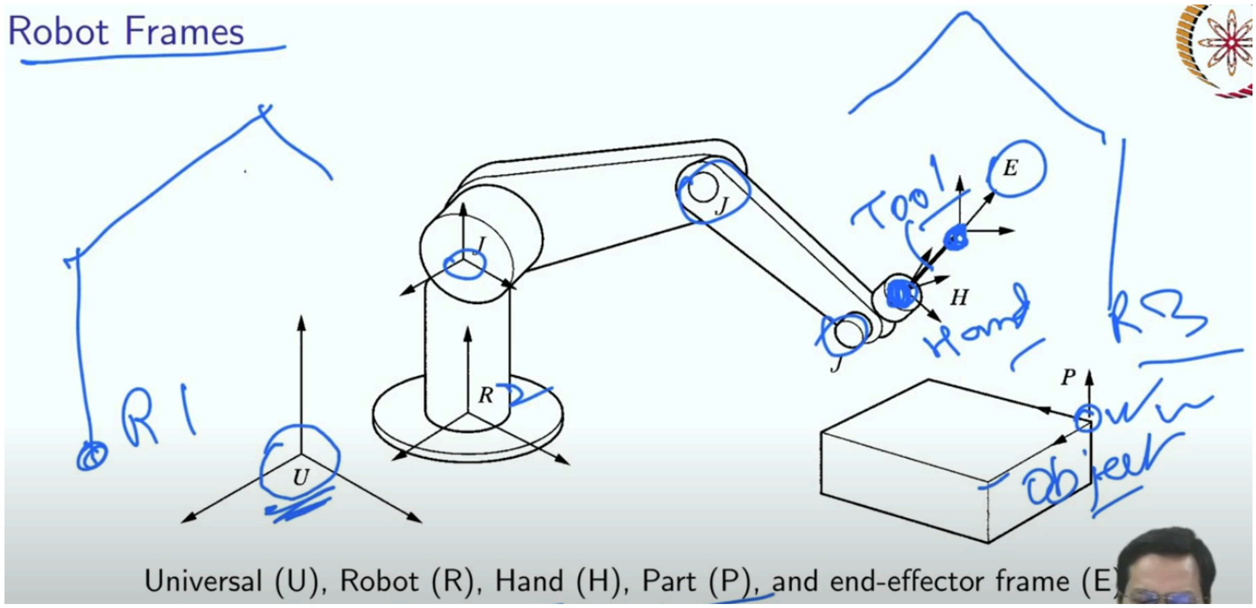
So, yes, now let us do some exercise further on this. So, yes, this is it. So, I will come to this (k-link planar serial chain (a))at the end. Let us start with this one (Stephenson six-bar linkage (c)). This is a good place to begin with. So, what do you see? There are many links, you see. So, the best place, the best way to find out degrees of freedom for this complex system is to try doing it otherwise, instead of using the Groppler-Krachbach criterion or using the number of definitions that you can define to define your system. Both ways will prove it to be a lengthier one. So, how I go for it is: you try to weld your system and try to constrain your system. The final constraint that you are able to put, and this system is unable to move, is the degrees of freedom.

So, let me just weld one of the joints. What is happening? At least, this one appears to be. This is like a four-bar mechanism. So, at least this one is a four-bar mechanism. So, this piece has got one degree of freedom. Okay, So let me freeze this first. So, what will I do? I will just freeze this one. So, I will freeze this one. I won't allow this to move now. So, what happens? So, this point is fixed now From the ground. This point is fixed. So, what does it become? It becomes: this distance is fixed because you cannot move this length. So, this was already fixed. This joint is fixed. So, it becomes a triangle. So, this distance is also fixed. Okay, This is also fixed. Okay. So, what happens Now? If this point is fixed? This point is fixed. This distance cannot be changed. Okay, And this distance cannot be changed. That means even this is fully constrained. That means this: any triangle, any triangle, is a constrained system. Okay, You cannot move it at all. Okay, So this is a triangle which is formed if you go this way. So, this is fully constrained. That means the whole of this is something like a single block which is attached to the ground. So, if I have just constrained one of them, the whole of this system is now fully constrained. So, if it is constrained, that means this point is frozen, this is frozen, Okay, So this solid structure which is here that you can see, is fully frozen. Okay, This point is also relatively fixed. That means this point is fixed. So, if this point is fixed, this was already fixed to the ground. That means these two points cannot move relative to each other. That means now, again, I can draw another triangle. So, that is this one. This is a triangle. So, this also is a fully constrained system. That means the whole of this system is fully constrained by freezing just one of them. So, this is all. So, that means it has one degree of freedom.

Now, let us go to another mechanism, this one (Watt six-bar linkage). So, what do you see here? So again, let us start constraining the system. This looks like a 1, 2, 3, 4. So, this is one degree of freedom. So, let me constrain one of them. I will freeze this one. If I freeze this one, this whole system is frozen. So, this was a 4 bar system. If one of the joints is frozen, the system is frozen. That means you are gone. Now, if this is fixed, that means this also is fixed. Okay, That means the remaining 4 bar mechanism, which is here, is also fixed. That means freezing just one of them. Freezing just one joint is freezing the whole of the system, fully defining the system. That means this, again, is a degree of freedom system.

Now, let us move ahead and do yet another one (Five-bar planar linkage). So, this is a case when I will go again by constraining the system. Let me just freeze one of them. I have frozen this one. So, this point is now frozen. This point is now fixed. Okay, This one is fixed. This one was already fixed, but it remains with this 4-bar mechanism. This is a 4-bar mechanism. So, what is it? This again has got one degree of freedom, one degree of freedom remaining. So, even after freezing one of them, you are remaining with one degree of freedom. So, if you freeze further, so that will constrain this remaining 4 bar mechanism. So, it is fully constrained. So, you need at least one or two joints to be frozen, fully defined, to define this mechanical structure fully. Okay, So this is having two degrees of freedom done. So this is how you have to move.

Now, the last one (k-link planar serial chain (a)), which is the simplest one, but I came here last. There is a reason for it. So, it is just like a serial chain system, just like your industrial robot. So, in the case of an industrial robot, if you freeze this one, this can move, this can move, this can move. If you freeze this one, still, this can move, and this can move. If you freeze this one, this can move. If you freeze this one, it is fully constrained. So, finally, you have to freeze as many joints as it has in order to define the system fully. This is a serial chain system. If it has n joints, it has n degrees of freedom. So, the number of axes matches with number of degrees of freedom, got it? So, this is your industrial robot, which is the simplest of all.

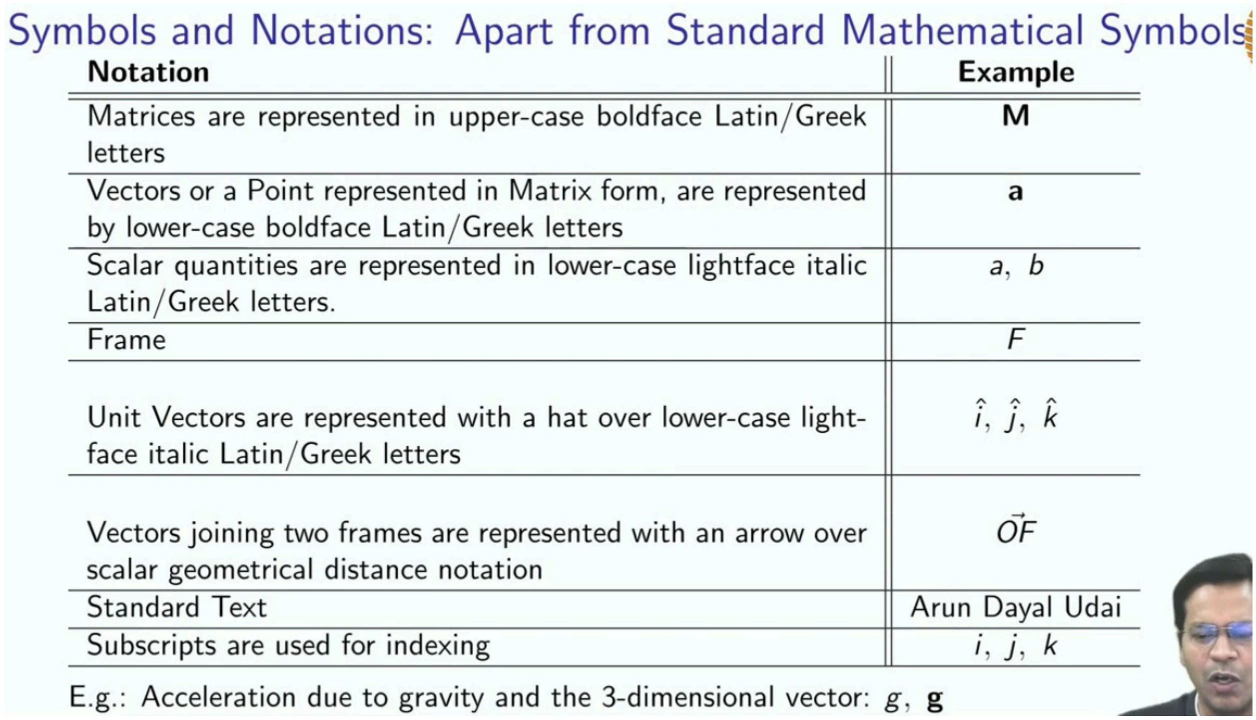


Now, let us start with robot frames. These are robot frames. So, how many frames are there in a robot and robot-associated system which is there in the workspace? So, let us start defining.

So, the first one is a universal frame. So, this is the frame in which you may have many other robots also. You may have a robot here, you may have a robot here. So, all the robots R1, R2, and R3 are defined with respect to the universal frame. So, this is the first frame which is there on your industrial shop floor with respect to that, all the robots are attached. So, that is the first frame which is there.

The next one is the object frame. The object that you want to handle, the object that you want to handle. This may be attached with a frame here or may be any convenient location on this object, so because it is a solid object, this frame, once it is fixed. It can be placed anywhere on this object. To fully define the object. Also, The position of the frame will define the object. The orientation of that frame will also define the object's orientation. So, that is the reason. So, placing one frame is enough. So, that is known as an object frame.

So, and then you have the robot's end-effector frame. So, this is the point which is to be taken to the object to do something. So, that is known as your end-effector frame, the robot's end-effector frame. This is your part frame. And then you have a hand frame. So, this is your hand frame. Why is this known as a hand frame, and how they are different from the end-effector frame. So, on top of your hand, you have various tools. That goes, you may be holding a pneumatic gun today to tighten a bolt. The next day, you may be holding just a grinding machine to do the grinding job. So, everything is associated fitted to the hands. That is also known as a robot's flange when it is delivered, and it has a specific dimension. We have talked about it already. So, yes, your end-effector. So, this is your tool. So, the robot's hand is attached to a tool, and the tool ends with a tip, which is known as your end-effector frame. Got it? And then you have the robot's frame. You know you have many other joints that may also have their frame. That is to define different links of the robot. With respect to the previous link. We will talk about this later when we will be going forward with kinematics. Let us move ahead.



Symbols and Notations: Apart from Standard Mathematical Symbols

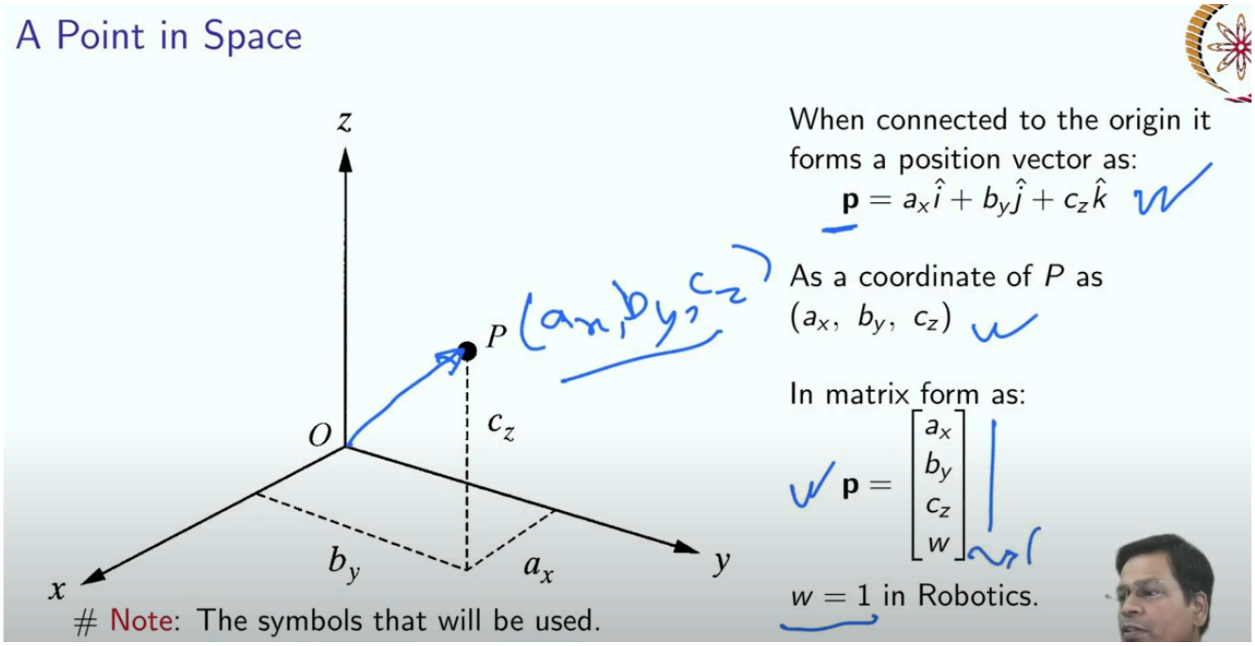
Notation	Example
Matrices are represented in upper-case boldface Latin/Greek letters	$\mathbf{M}$
Vectors or a Point represented in Matrix form, are represented by lower-case boldface Latin/Greek letters	$\mathbf{a}$
Scalar quantities are represented in lower-case lightface italic Latin/Greek letters.	a, b
Frame	F
Unit Vectors are represented with a hat over lower-case light-face italic Latin/Greek letters	$\hat{i}$, $\hat{j}$, $\hat{k}$
Vectors joining two frames are represented with an arrow over scalar geometrical distance notation	$\vec{OF}$
Standard Text	Arun Dayal Udai
Subscripts are used for indexing	i, j, k

E.g.: Acceleration due to gravity and the 3-dimensional vector: g, $\mathbf{g}$

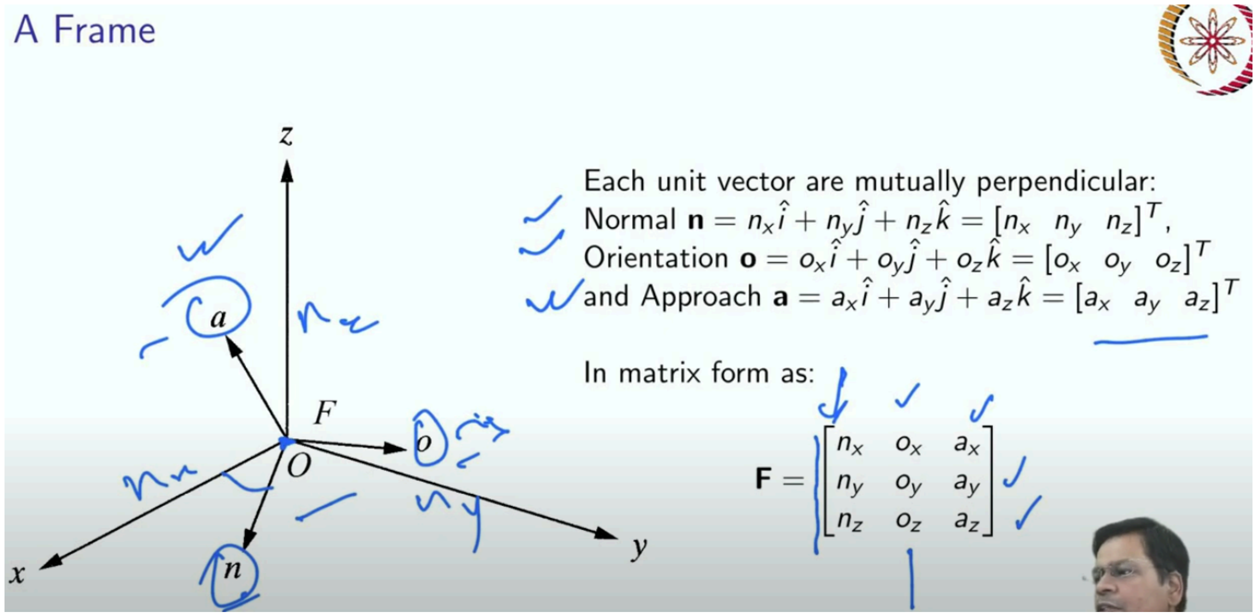
So, yes, the pose of a rigid body. To define the pose of a rigid body, what we have is a different matrix representation that we will be discussing now.

So, these are some of the symbols that you can quickly go through. You can have a snapshot of this so that you can easily follow my slides. I will be using this. So, all the matrices will be denoted with M. Why that? So, Because I have a smaller one to do something else. Let us see that one. So, vectors, all the vectors. So, I will be using straight letters, bold, small letters, straight, not italic and bold. Okay, so again, all the scalar quantities, italics, not bold. They are lowercase. Also, lightface it and italics.

Frames will be defined as italic capital letters. Unit vectors with a cap on top of it: Okay, unit vectors with a cap on top of it. So, i, j, and k may have caps, but sometimes I do. I can even use it like this. So, I can just make a bold I small letter to denote I as well. So, that is there. Vector joining two frames so O is a frame. F is another frame. Vector joining two frames can be denoted as with the arrow on top of it. That is good enough. Or maybe a capital bold OF should be good enough. So, both are equivalent. But this is the one which I will be doing if at all it is required. A standard text like this that I will be following. Subscripts that are used for indexing are like this (i,j,k). So, this is just an example of acceleration due to gravity, and a three-dimensional vector will be denoted like small g, italic, not bold g. The small letter is g, which is a three-dimensional vector because vectors are here. So, this is just a summary sheet that I will be following.

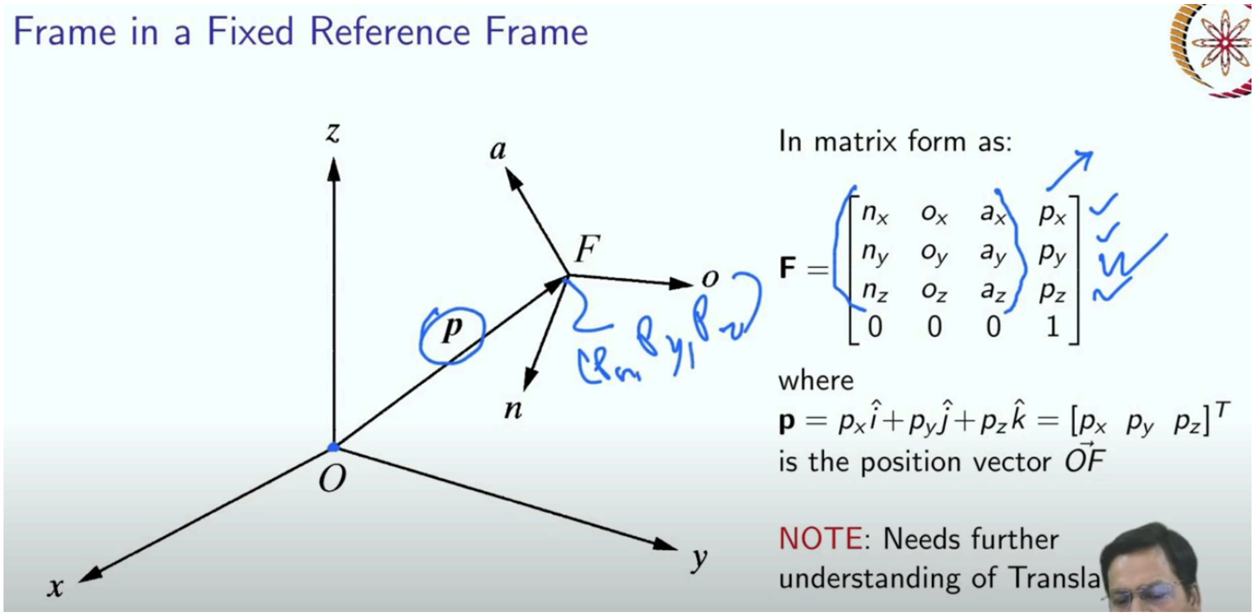


So, now let us move ahead. So, yes, how a point in a space can be defined. So, we are very much used to doing this. So, position vector p, mind I am using the same symbol. Just now, I have summarised you. Okay, so this is your vector. So, basically, it is this vector which is the position vector of a point, which is given by ax, by and cz. that is the coordinate of that, in matrix form. From now on, we will be using it like this. So, a vector is now can be written as a column matrix of this kind with four elements. In robotics, w is equal to 1 always. So, a point will be represented as ax by cz in the column. So, effectively, w becomes equal to 1. So, that is your point. So, a point in a space will be denoted like this.

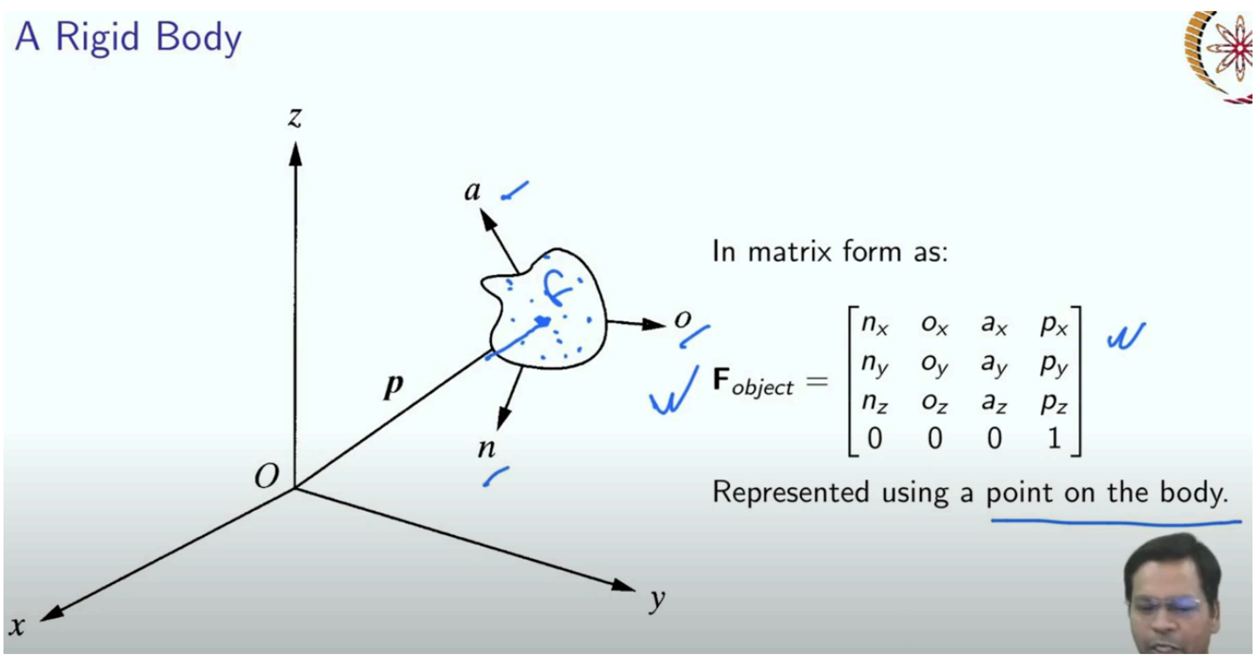


Now, a frame; so, how a frame is represented. So, the frame is a triad of three vectors which are intersecting together, and the point represents the location of the frame where it is located. And those three orthogonal vectors, let us say if it is normal (n), orthogonal (o) and approach (a) vectors, n o a, commonly known as. So, there are different other terminologies also which are used for this, but I will be taking it as n, o, a, which is fine, or UV is equally good. So, yes, I will be using this. So, n o a are three vectors: Projection of n along x is nx, n along Y is ny, and N along z is nz. So, that is arranged over here as nx, ny and nz.

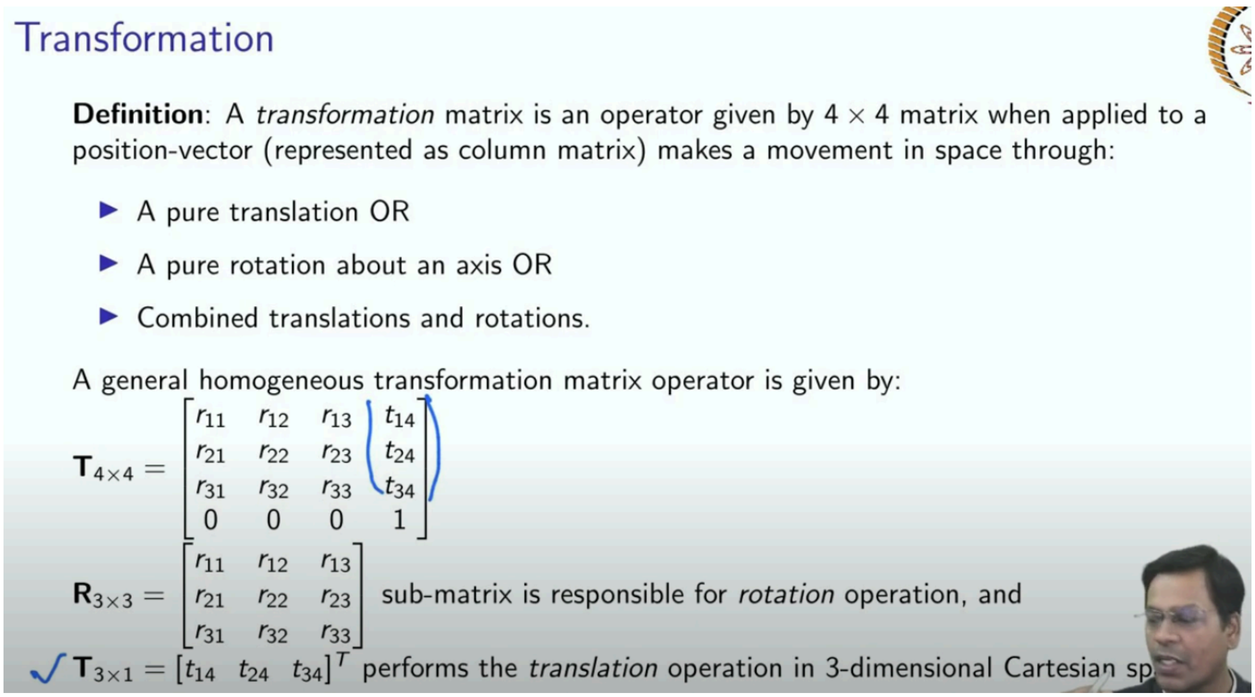
Similarly, O has three components: Ox, Oy, and Oz that is put here. a also has got three components, that is ax, ay and az that is another that is the third column. So, all three are orthogonal vectors. They are put in matrix form like this: Okay, so altogether, this is your matrix form for a frame to be represented. Normally, they are normal vector, orientation vector, orthogonal or orientation vector and approach vector. So, they are like this. So, the transpose of all of these are vectors. Each of them are mutually perpendicular to each other.



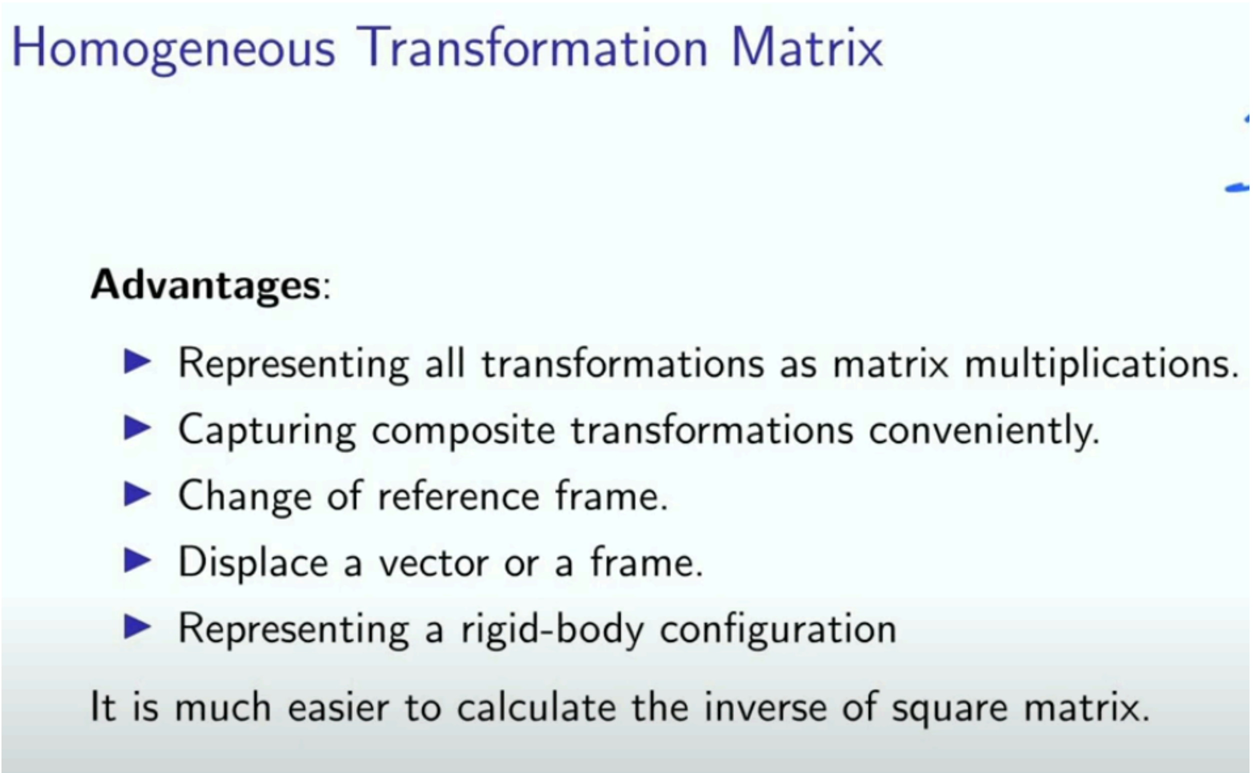
Now, frame in a fixed reference frame. So, this frame is now displaced from here, and it has come to a new location. Okay, so it is no more at the origin. So, there is a vector again with which your frame is dislocated from the origin. So, it does not just have orientation; and it also has a position dislocation. So, in order to accommodate all of those, your new frame notation will be like this. It has this, which actually tells the orientation of this frame, whereas the last column tells you the position vector: px, py, pz, which is the location of this. px, py and pz, that is the location of this frame with respect to your global frame.



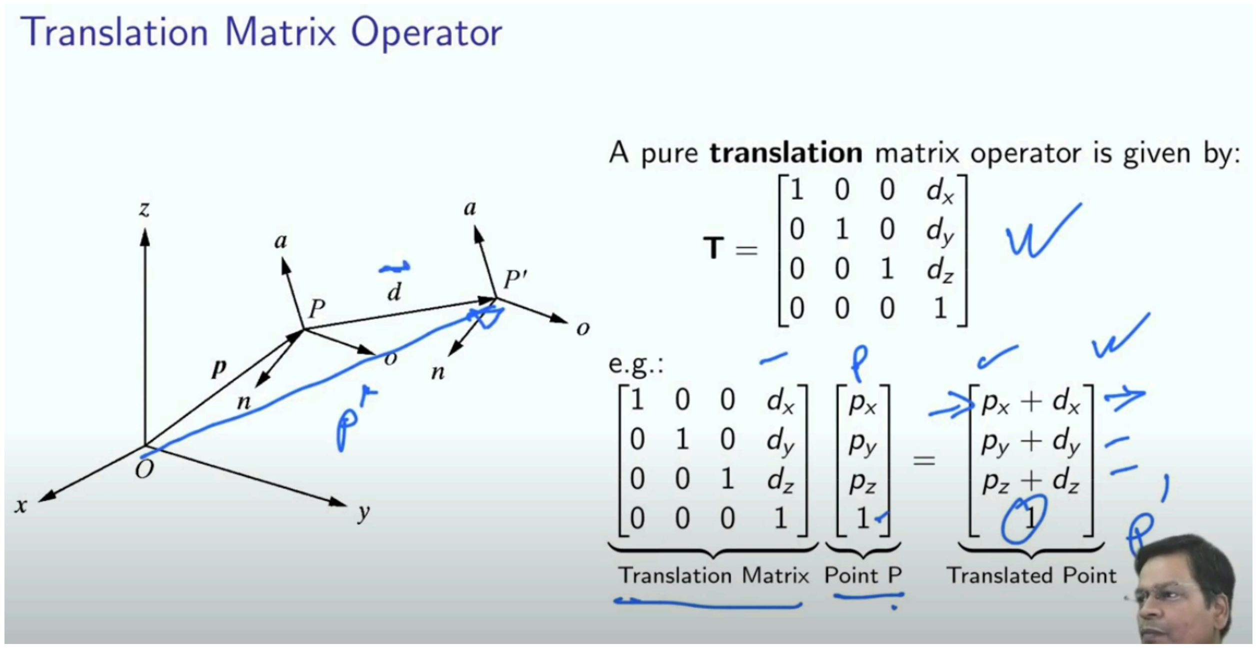
Now, let us say how a rigid body can be denoted. So, as I told you, if it is a rigid body, there is no relative displacement between any two points which is there on that body or within that body. So now, if it is fixed to this rigid body. So, the orientation and position of this frame will change, along with the orientation and position of the object. So, what happens? So, if you can define your frame's location. So, that will fully define the rest of the object. So, if it is a frame which is here, okay, so with which it is connected, so rest of the object points, all the points which are there on this object as because they cannot move relative to this F, this point, Okay, so whole of that object is fully constrained. Suppose your frame orientation and position are constrained. So, all this is defined, which means your frame is fully defined. It cannot rotate, it cannot translate, so your object also cannot rotate and translate, and it defines the object also. So, this is exactly similar to the frame definition. So, represented using point, any point on the body, and it can be any point on the body.



Now, what is transformation? Now, that we have already defined a different frame, different body, okay, how to define that? How to define a point? So, transformation matrices are operators. They are operators given by 4 cross 4 matrices. when applied to a position vector represented as a column matrix, as we have just seen. So, that makes a movement in space through a pure translation. If you apply a transformation matrix to a column vector, column vector or a matrix of points. So X, Y, Z and one will do pure translation. We will see by example also. So, it can do a pure rotation about the axis. It can combine translation as well as rotation using these 4 cross-4 matrices, and you can do combined translation as well as rotation. That transformation can be done. How We will see by demonstration. So, in general, that is known as a homogeneous transformation matrix operator, which is given as a transformation matrix. 4 cross 4 is the dimension of that matrix 4 cross 4. So, it contains these elements: R11, R12, R13, so like this, and then you have a column that comes at the end. So, for each one of them, what are they? So, R matrix 3 cross three sub-matrices, which is here. Sub-matrix is responsible for the rotation operation of that frame, and it has a last column, this one, that performs the translation operation of a three-dimensional Cartesian space vector.



So, let us see what advantages this can offer. It can represent all the transformations as a matrix multiplication if there is a matrix. The transformation matrix, which is applied on a point, is okay. It can transform this point into a new point. So, these are. This is the transformation operator applied on a point, sometimes applied on a frame also. So, what it can do? It can represent all the transformations as matrix operators because you know this can do rotation as well as translation. Okay, So it can capture the composite transformation. Conveniently. It can do composite transformation, that is, the mixture of different rotations, maybe translation in between, so you can have multiple transformations can be captured in a single matrix. So, it can be a point which is applied with the first transformation, the second one, and the third one. All these are equivalent to a single transformation of 4 cross, four matrices applied o.n. P gives you a new point. So, that is what. So, multiple operations can be conveniently packed in a single four cross-four transformation matrices. Okay, It can change the reference frame, as I told you. It can change the vector. This point with respect to the frame can be changed, or your frame itself can change the point remains there. So, yes, you can do multiple things. We will see by example as we move on. Represent the configuration of a rigid body. So, as you know, because rigid body is defined using a frame. The frame was nothing but rotation and translation, which was there. These three were zero, and this is one. So, this is what is your rigid body configuration? Okay, Exactly. It looks like your transformation matrix. So, this transformation matrix now represents the position and orientation of your rigid body. So, yes, it is much easier to calculate the inverse of a square because, you see, the transformation matrix is four cross 4. You can quickly find inverse transformation. We will see why it is needed, how it is done, and what are its limitations? Okay, by the inverse of a square matrix. How to go for it?



So, yes, the translation operator is using this matrix. How to do a translation operation? This is just one example. I will do So. yes, pure translation. As you know, this was orientation, So I have made it a diagonal element as 1. So, that it becomes a unit matrix and does not do any rotation operation. So, the only thing that remains here is this translation part. So, that is dx, dy and dz. So, this was your initial point at p. Finally, it reaches a new point, location, that is, p dash. They are. This point is to be displaced by a distance, which is d.

d has projection, dx, dy, dz along x, y and z direction. Okay, So, this is how I want it to be done. So, this is your transformation matrix that can do that. Let us see how it is. So, this is your initial point, which was p, and when you multiply the translation matrix, this one to this point, that means you are applying a transformation to this point. And what kind of transformation is this? This is a translation transformation matrix. Okay, it is pure translation applied on p. You got a translated point that is p dash. You can multiply this and check. How is it? 1 into px, so px, plus 0 into py, 0 into pz, plus 1 into dz, so it becomes px plus dx.

Got it?

Similarly, other rows are there, and finally, the last one will remain here as 1. So, yes, now this is your new position vector for this. Got it. So, this is your P-dash vector. So, it has changed from P to P dash. It is now translated by a vector d. So, this is how this operator works.

That is all for today. So, next class, we will be doing pure rotation matrix, arbitrary axis rotation and how to represent rotations using Euler angles. So, this is what we will be doing next class. That is all for today. Thanks a lot.