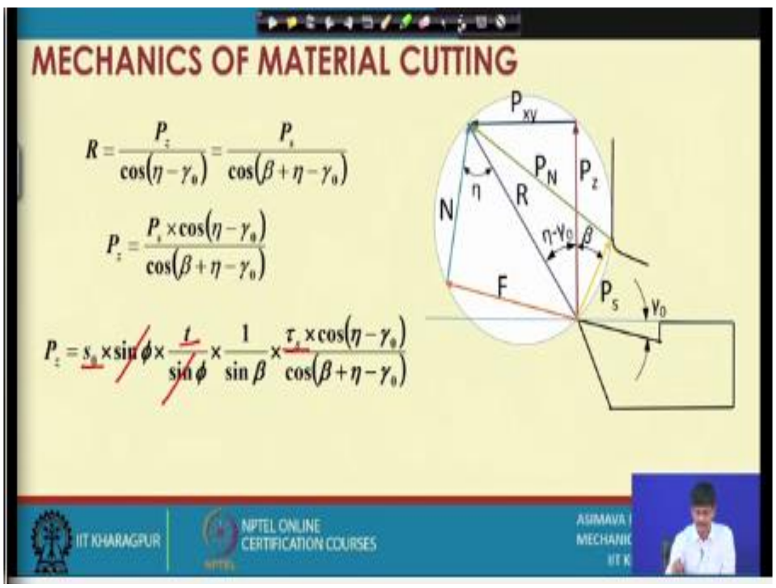
Metal Cutting and Machine Tools
Prof. Asimava Roy Choudhury
Department of Mechanical Engineering
Indian Institute of Technology-Kharagpur

Lecture-09
Measurement of Cutting Forces

Welcome viewers to the 9th lecture of the course metal cutting and machine tools. So, in this lecture we are going to cover some discussion on measurement of cutting forces. But previous to that whatever leftover portion was there for calculation of cutting forces, we will quickly have a look at that and then continue with today's discussion.

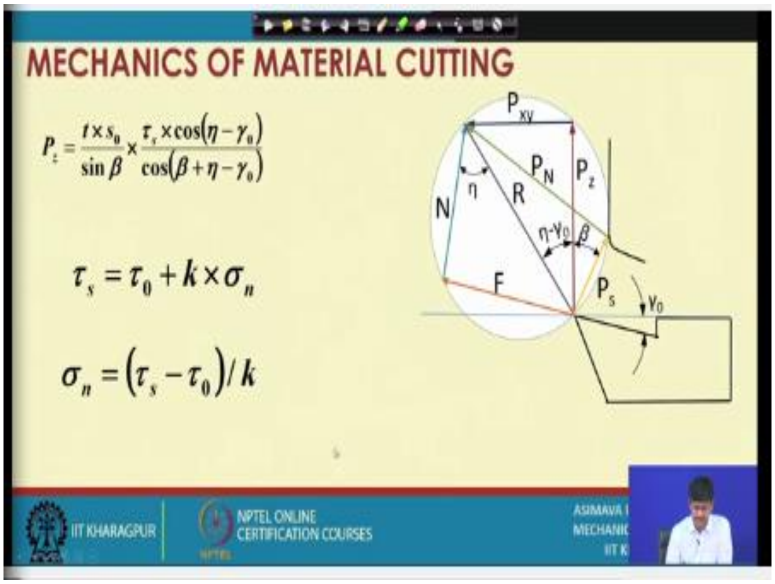
(Refer Slide Time: 00:49)



So, to start with, in the last discussion that we were having we derived the relation between the main cutting force P_z and the shear component of the resultant force P_S along the shear plane which is equal to P_S. And therefore P_z was found to be P_S into cos $(\eta - \gamma_o)$ divided by cos $(\beta + \eta - \gamma_o)$, we had proceeded up till this. And then we replace the value of P_S by the dynamic shear strength of the material and the area on which it is acting.

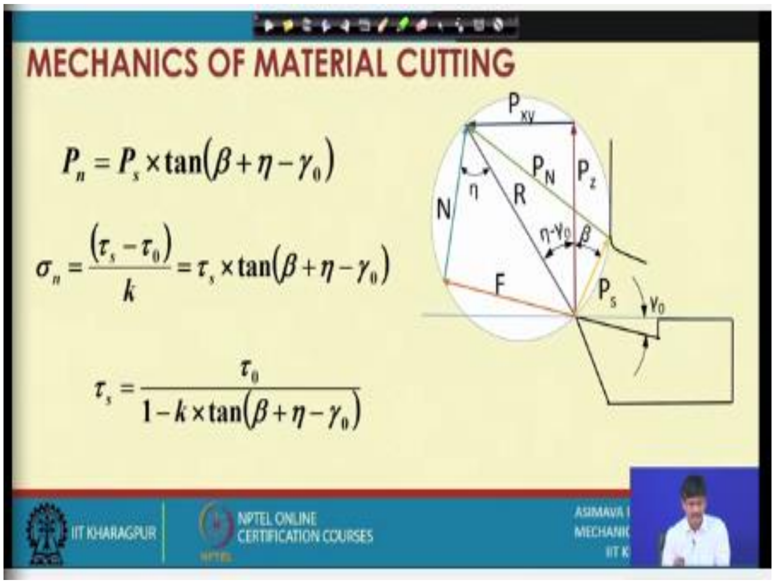
The projected area we had found was equal to t/sin φ into S_0 sin φ, so that sin φ cancels out, let me just do it here itself. That is this we can cancel out, so that we have $S_0 * t * \tau_S$ into all these trigonometric functions sin β cos $(\beta + \eta - \gamma_o)$ and cos $(\eta - \gamma_o)$. The trouble is we cannot handle all these angles because while we are knowing the value of γ_o, γ_o is known from the tool angles orthogonal rake. We do not have a concrete idea about what is the shear angle and what is η? So, let us quickly go through the possible ways in which we can find it out.

(Refer Slide Time: 02:31)



So, here this τ_S which is not known to us up till now, this τ_S is actually taken to be assumed to have a relation with a constant value τ_S plus it is a proportional part with the normal stress. How do you find out normal stress? We know the normal force on the shear plane that divided by the area of the shear plane that will give us the normal stress. So, that way if we divide sorry just let me go on to the next slide**.** So, just shifting the terms and getting an expression of σ_n in terms of τ.

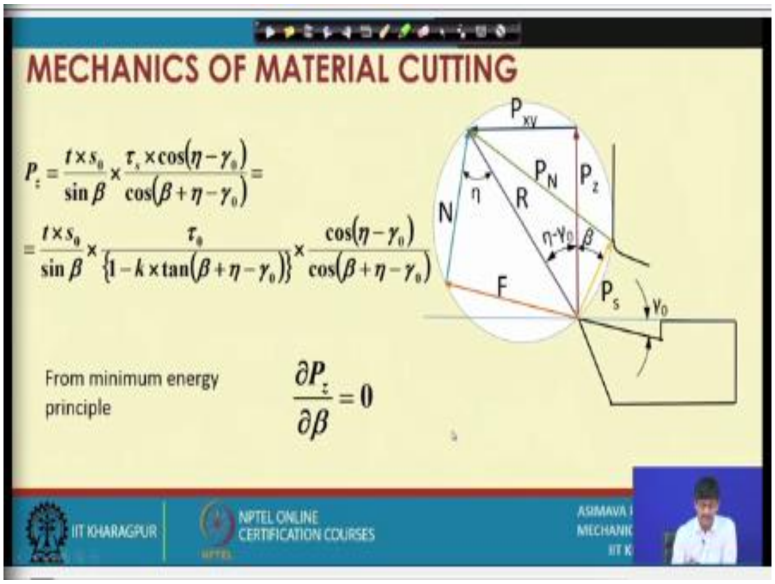
(Refer Slide Time: 03:26)



So, we also have a relation between the normal force, normal to the shear plane and the force tangential to the shear plane, we have a tangential relationship, why? You know P_n is here, P_s is here and this is the angle $\beta + \eta - \gamma_o$, so we have a tangential relation between these 2. So, if we divide it by the shear plane area, in that case we have σ_n replacing P_n by area of $A_s = \sigma_n$ and that is equal to P_s gets replaced by τ_s in the process and tan remains as it is.

If we rearrange the coefficients and find out the value of τ_s, because τ_s is having one component from here and another component from here. So, if we collect coefficients and express τ_s as a function of τ_o, we get a relation of this type. So, this is the constant of proportionality between the normal stress and the shear stress on the shear plane.

(Refer Slide Time: 04:45)

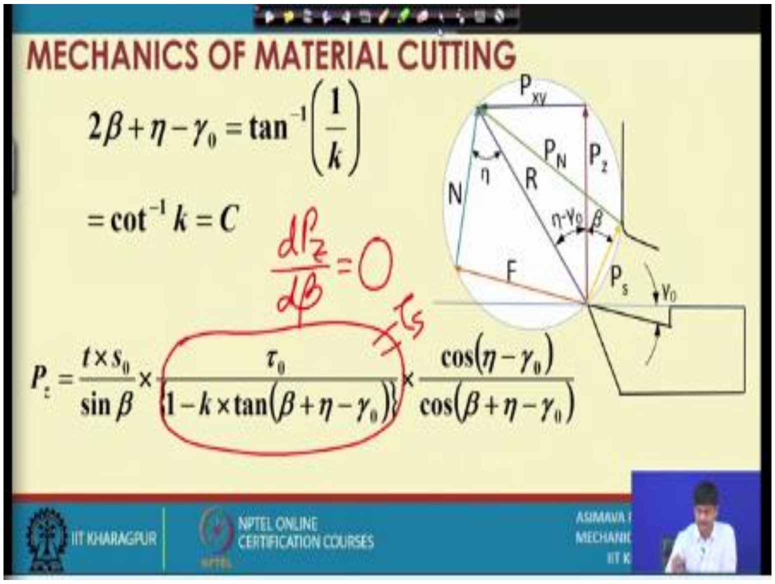


So, if we do that we can replace τ_s in terms of τ_o the constant portion of the shear stress and this term appears at the bottom, this added the term in the denominator appears. So, that the expression of P_z is t *S_0 into these things were previously there cos $(\eta - \gamma_o)$ sin β cos $(\beta + \eta - \gamma_o)$, additionally this term comes in. Now what is exactly the value of P_z? For this we use the minimum energy principle, that is in order for any process to be carried out it will always follow the natural process of expending minimum energy to achieve that.

So, if shear plane is creating the shear between lamina of the uncut chip, it is getting sheared off, it will happen in such a way that minimum energy is expended. Otherwise why should it go for any plane in which more energy is spent? It will go for that plane in which minimum energy is there, that will happen first. So, from the minimum energy principle the least amount of P_z will define this particular plane.

The least value of P_z occurs when this minimum energy is expended because energy is equal to P_z into V_c. So, if we differentiate with P_z with respect to β, we will be able to solve for that β for which P_z is minimum.

(Refer Slide Time: 06:37)



So, following that principle, if we differentiate I have not included the whole mathematical derivation here, if you are interested I will upload it in the form of a separate file. But believe me if you try yourselves, I am sure you will be able to do it. That means this particular expression I am sure you will be able to derive it yourselves very easily. That is if you try deriving this particular.

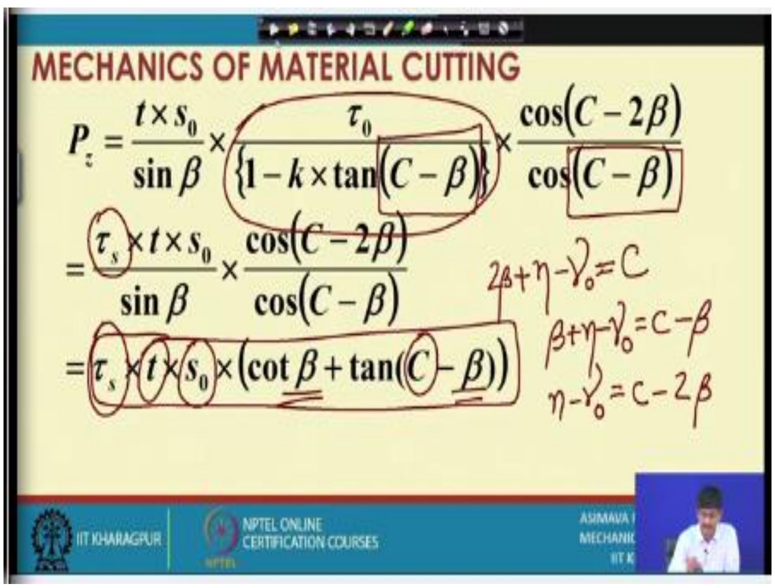
So, here as you can well understand all terms which are not containing β they will be treated as constants, so you might be having only 2 terms, one is $\sin \beta$ and another is $\cos (\beta + \eta - \gamma_o)$. And of course another term that is $(1 - k) * \tan (\beta + \eta - \gamma_o)$ in the denominator to be derived. So, I am sure you will be able to do it yourselves, so I am not including the formal discussion here.

So, that finally gives us a relation $(2\beta + \eta - \gamma_o) = \tan^{-1} (1/k)$, now what does this exactly mean? It means that the angles are going to have a relation in such a case, which case? For minimum energy to be expended to achieve chip formation, the angles are going to have this sort of a relationship. Is this the final thing in metal cutting? No, in fact 10 to 11 different types of theories are there what can be the relation between these basic angles.

And we are discussing just one of them; this is called Merchant's second solution following the minimum energy principle. So, that is equal to $\tan^{-1} (1/k)$ where that is equal to $\cot^{-1} k$ which is equated to a constant C which is called the machining constant in some literature. So, that ultimately we can replace the relation between these angles in this expression.

Now if you notice this term in the middle it is equal to τ_s. And if we keep it that way we do not have to bother about the changes that will be happening due to incorporation of this relation here. So, we can remove it from consideration by just keeping the original value of τ_s here. So, let us replace these values and see what happens?

(Refer Slide Time: 09:57)



Yes, in that case we find that the expression comes out to be all these things are untouched. This one expression since $2\beta + \eta - \gamma_o$ is equal to so what is it called? C. Therefore, we can write $\beta + \eta - \gamma_o = C - \beta$, and therefore we have replaced those values here. We have replaced that one here $\beta + \eta - \gamma_o$ has been replaced here and it has been replaced here**.**

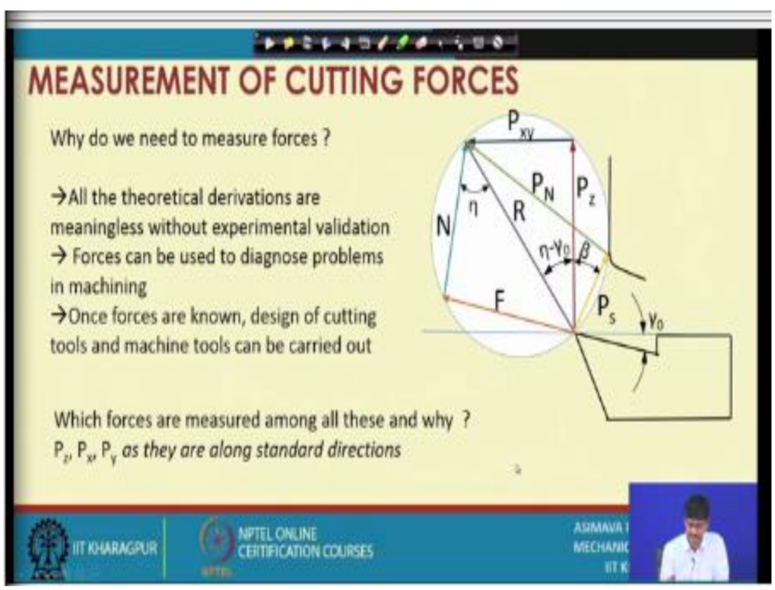
And therefore after replacement still we are knowing that this thing is equal to τ_s and we have straight replaced that formatively large expression here. So, that we have leftover sin β here, cos (C – β) from the other side of the denominator coming here. And of course previously we had cos $(\eta - \gamma_o)$ here which gets replaced by cos $(\eta - \gamma_o)$ is naturally equal to C - 2β. That we have replaced here and therefore we get this particular expression.

If you expand this expression, we will find that it will become equal to cot β + tan (C – β). And therefore we can say that this expression will give us an estimation of the main cutting force P_z, if we are knowing t, t we are knowing of course it is the depth of cut. If we are knowing feed, feed we are knowing definitely in millimetres per revolution it will be set by us only.

If we know the dynamic shear strength of the material, yes that will be known and of course if this machining constant is found out by us and of course as I said β. Now if you remember β, η and ζ they were related together, so that we can directly replace the value of β here. So, I leave this to you find out the value of tan β which we had found out to be equal to cos γ_o - divided by ζ - sin γ_o.

If you put it here we can replace β as well, so all the terms being known, we can find out the value of P_z. If P_z is known we can find the value of P_{xy}, and also we can find out the other values of like friction normal force F and N etcetera everything can be known. So, we will have in the next lecture some discussion of some numerical problems in which we can find out the force components of turning I mean force components during orthogonal turning. Now let us move on to the next subject of discussion.

(Refer Slide Time: 14:08)



The thing that we had proceeded to discuss today that is the measurement of cutting forces. Why do cutting forces have to be at all measured, is there any need? First of all, from common sense approach we can say that if we are measuring the cutting forces and rather unless we are measuring cutting forces, how do we know all these discussions that we are having about calculation of cutting forces to be correct?

So, in order to validate our calculations on cutting forces we need to carry out experiments to find out whether we are really getting those forces as per calculations or not? But you might ask me, what is the whole point of calculating and measuring cutting forces? Suppose we know nothing about them, what is wrong? First of all, if we do not know cutting forces, we cannot estimate what should be the dimensions of the machine tool on which this cutting is going to take place.

For example, suppose I am feeding a tool in the longitudinal direction with the help of a lead screw. So, the cutting tool is moving and it is experiencing feed forces axially to the feed screw which is making the carriage move. So, unless I know what sort of forces I am going to experience, how can I design the dimensions of this feed screw? So, for design calculations and other activities cutting forces will have to be known.

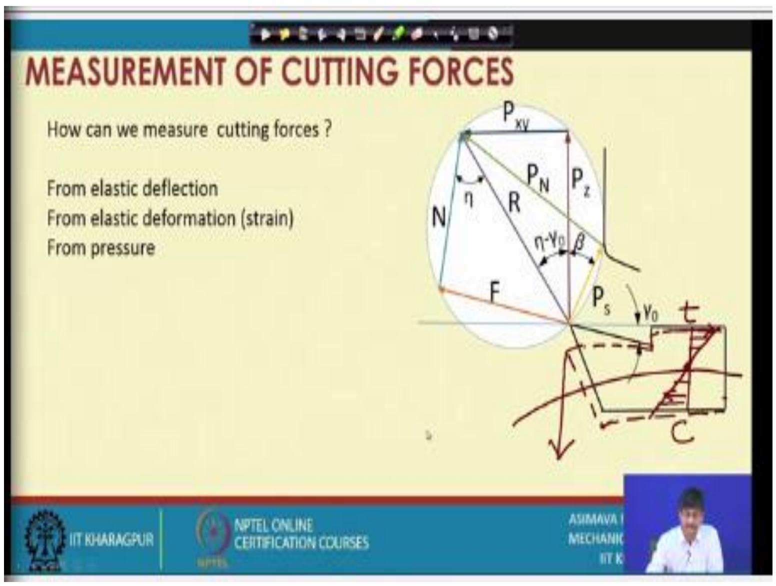
You might still argue that the machine tool was already there on which you are using the cutting tool. But it is the everlasting question about which came before hen, did the egg come first or the chicken come first? So, that way if you want to design machine tools, you have to know the cutting forces, you have to carry out the experiments. The experiments have to be carried out on a machine tool only, so machine tool must be present.

But in order to make the machine tool present design has to be done, I mean a measurement of the cutting forces have to be run in order to design it etcetera, that is absolutely essential. That is knowing the forces through experiments in order to use that data for machine design. So many other reasons are there, I am not touching all of them because these things I can give you references from which you can pick up this information.

So, what do we exactly measure? Do we measure for example P_s? Somebody is interested to measure P_s, so that you can directly put it into the calculation of that Merchant’s second solution, etcetera. Or P_n, generally we are interested to measure or rather we are capable of measuring forces in standard unchanging directions. And that immediately makes it clear that we measure it along P_x, P_y and P_z.

Why do not you measure it along say the shear plane? That is because the shear plane does not have the same orientation in every case, so that is out of question. So, standard directions are chosen for the measurement of cutting forces like P_z, P_x and P_y. So next.

(Refer Slide Time: 17:53)



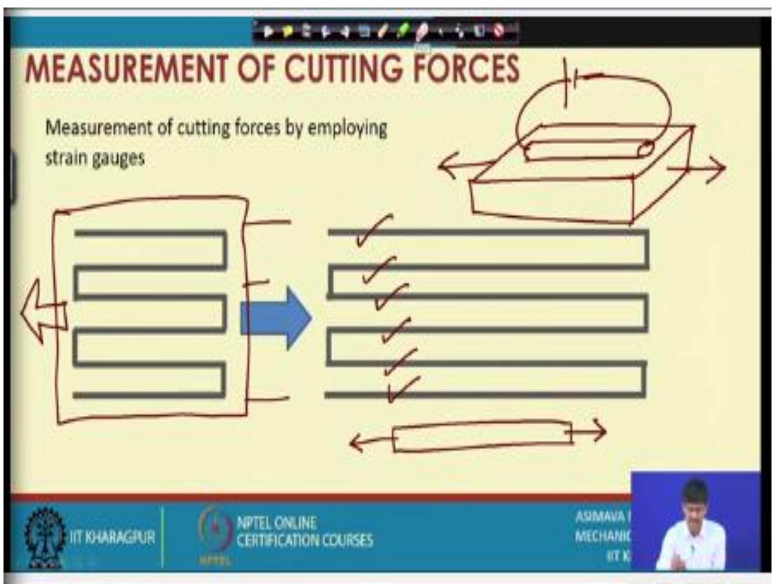
How can we measure cutting forces? Cutting forces can be measured from elastic deflection, now what do we mean by that? We mean that you know just a moment if under the action of cutting forces this tool deflects; why should it deflect? Because the force is in the other direction, this force mind you is on the chip, the cutting tool experiences this force because of which this is the force on the cutting tool.

It deflects like a cantilever beam, my drawing is not very perfect because it should have been here, let me make that correction, this line is not supposed to be here, that is right. So, it deflects as a cantilever beam, and from the deflections we can find out the forces if they are within proportional limit. So, elastic deflections can be measured and from that force can be estimated.

We can have elastic deformation that is strain, what do we mean by this? I mean that you know, if there is a cantilever beam and if this be the neutral axis, then we can estimate the strain here to be of tensile nature, bending of beams. And the strain on the bottom side is going to be compressive. And at the middle in the neutral axis there is no stress at all, so that we can expect something of this type to happen, stresses are like this.

And I am just giving some arrows here to make you understand. So, it is reversed, on the top it is tensile, on the bottom it is compressive. So, from these strains we can estimate what is the force within elastic limit. So, we can also find out from the pressure, so next let us see what do we have after this?

(Refer Slide Time: 21:01)



So, first of all if we want to measure strains, we can take the help of something called the strain gauge, how does the strain gauge work? The strain gauge it is like I will try to give you the idea in simple words. Suppose I have here a simple say a rod of whatever cross-section. So, this one I start pulling if I pull it, it will undergo a strain. Now in the same manner suppose I have a body, on top of which I have put a sort of conductor it is absolutely cemented onto the body.

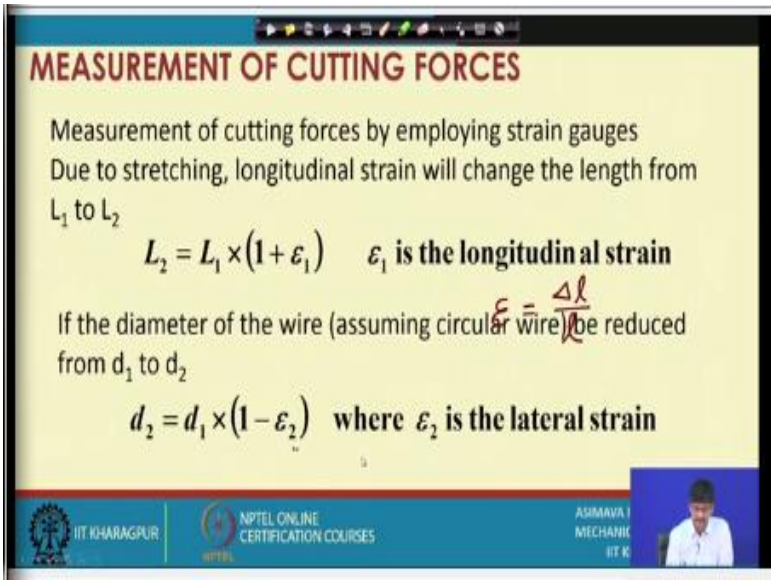
If I pull the body, the body is going to undergo longitudinal strain and this body will also undergo the same amount of I mean whatever its length qualifies to, it will undergo strain. There will be no relative motion between the top surface and this particular element. So, it undergoes longitudinal strain together with that it will undergo some change in the lateral direction also. But what do we achieve from that?

We want to say if it is undergoing a change in length, if it is say increasing in length due to tensile forces, and if its diameter is decreasing. If I measure its resistance or say I measure the current flowing through it, I will be definitely able to register a change. So, from this change I can estimate what is going to be what is the strain that is what. I will be estimating what is the strain from the resistance change, if they are proportional my work will be easier, I mean it would be possible to extrapolate also.

But the problem is how much should be this length, you will say 1 meter I will say that is impossible. On the cutting tool, when the cutting tool is mounted on the lathe, there is hardly any elbow space. You have to do things within a very small space, but small spaces will mean that this length will be limited and within this limited length you can hardly expect some appreciable change in resistance to take place due to strain.

So, what we do is within a very small space, say this particular space is restricted within a few square millimetres, maybe this side is say 4 millimetres that size is 6 millimeters like that. Within that we enclose a large number of zigzag conductors, some sort of pattern is chosen, so that the lengths are of large quantity. What about this side? This side hardly there is any length, almost negligible. So, all these lengths add up, this is the thing when the thing is stretched on 2 sides all these lengths get stretched so that within a confined zone of area I will be having considerable change in the resistance due to considerable amount of strain getting registered. So, let us now see what happens after this.

(Refer Slide Time: 25:27)

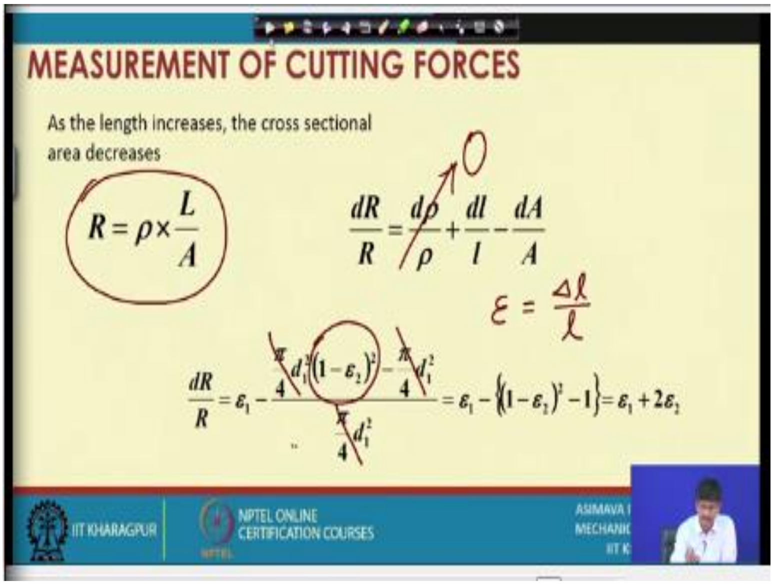


So, if we employ such strain gauges due to stretching longitudinal strain will change the length from L_1 to L_2. So, how do we estimate L_2? L_2 will be equal to L_1 multiplied by taking into consideration that ε_1 is the longitudinal strain. So, strain is defined as $\Delta l/l$, where l is the original length and Δl is the small increment in length. So, if we multiply it by the original length L_1, we will get the corresponding change in length.

Therefore, $L_2 = L_1 * (1 + \varepsilon_1)$, where ε_1 is the longitudinal strain that means in the direction of the stretch. We are considering for the moment, that it is tensile stresses which have arisen in the work piece or the part under of our interest, due to the forces which are applied. Tensile forces giving rise to tensile stresses, so this tensile strain. However, if we assume we can generalize it also if we assume that say the cross section is defined as a circular cross section of those strain gauge wires. In that case that will undergo generally a compression.

But it is this compression is definitely not going to be following constancy of volume rather from Poisson's ratio we will get a relation between this lateral strain and longitudinal strain. Lateral strain by longitudinal strain is equal to Poisson's ratio. But anyway the new diameter will be decreased by original diameter multiplied by $1 - \varepsilon_2$, so that is the lateral strain.

(Refer Slide Time: 27:53)



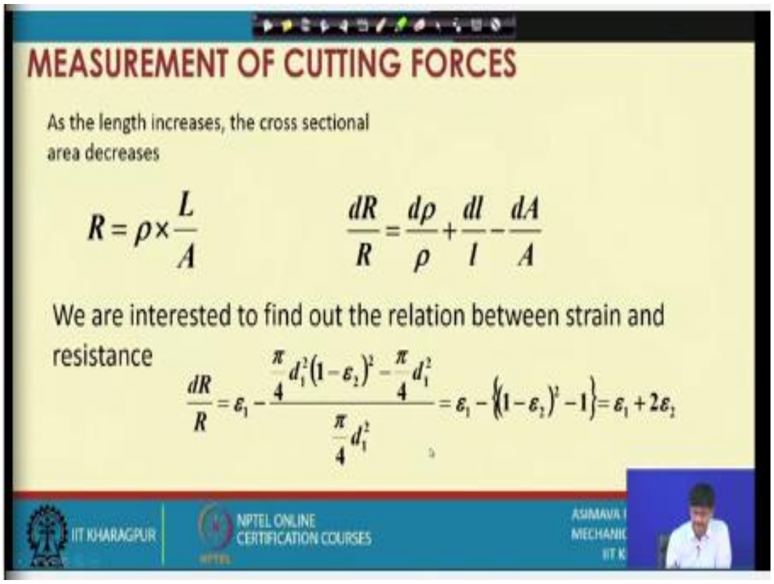
Therefore, if we have the resistance to be defined as ρ * L/A, we can differentiate this and get this particular relation between the differentials. Where do we what is the origin of this one? This is simply resistance is equal to resistivity multiplied by length divided by cross sectional area. Resistivity for the time being we will consider to be constant, it is not affected by any of these physical phenomena which are occurring like stretching etcetera.

So, in that case if we differentiate this we will get dR/R = dρ/ρ + dl/l **+** dA/A because it is in the denominator. This thing we will assign a value of 0, now itself there is no change in resistivity. Hence in the leftover terms what we notice is that dl/l is nothing but the strain and dA/A we painstakingly put in all the terms, so that we get π/4 into changed value of diameter square – π/4 * d_1^2 original diameter.

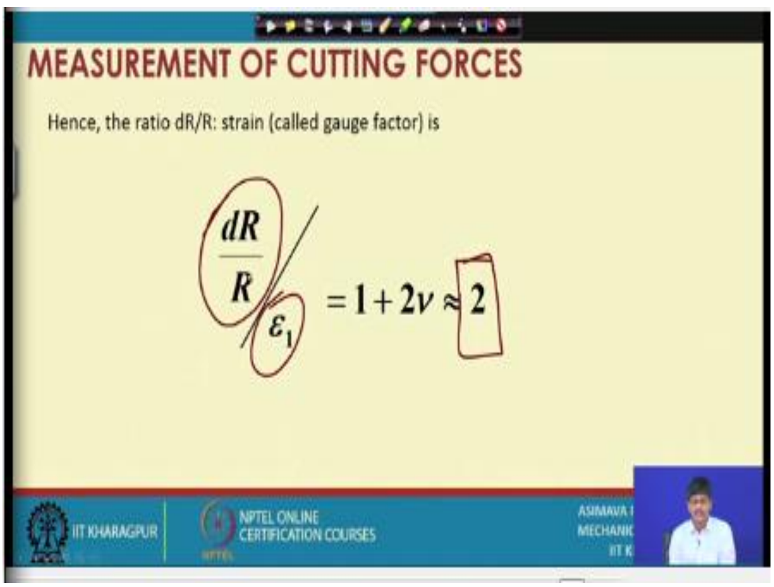
So, change in area divided by original area equal to ε_1 and naturally as d_1^2 is there, I mean d_1^2 d_2^2 is equal to this thing we get $(1 – \varepsilon_2)^2$. And from here we get 1, this thing cancels out, this thing will cancel, so this is simply this term - d_1^2, what do you call it simply this term – 1, so that is what has come. So, if we increase it binomially and not even binomially, one if we say that this is $(A – B)^2$, the square of that strain term we neglect.

So, that when we consider all these signs out comes $\varepsilon_1 + 2\varepsilon_2$. And hence if we divided by ε through the expression, we will get.

(Refer Slide Time: 30:46)



(Refer Slide Time: 30:48)



Just a moment, dividing the equation throughout by ε_1 we get dR/R by strain, that means change in resistance per unit strain is equal to 1 + twice Poisson's ratio. Because we will get ε_2 by ε_1 here, and that can be assumed to be equal to 2, which means that change in resistance per unit resistance divided by strain is actually a constant, these 2 things are proportional.

So, this brings us to the end of the 9th lecture. In the 10th lecture I want to exclusively discuss I mean numerical problems, and in the 11th lecture I will be completing some part of the calculation on measurement of cutting forces which I could not cover in this half hour lecture.

But that part will not be included in the second week's syllabus. So, whatever I discuss on measurement of cutting forces in the 11th lecture will be treated as syllabus under the 3rd week, so thank you very much.