

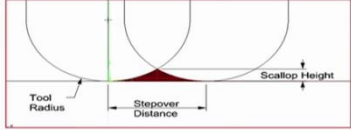
Cutter Path Generation (Concluding Part) and Current Status- CNC Machining and Related Processes

Welcome viewers to the 19th lecture on the open online course “Computer numerical control of machine tools and processes”. So today we will have the concluding lecture on cutter path generation for machining free form surfaces and in the remaining time we will discuss some of the current trends in this particular area.

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Isoscallop Approach

- Scallop height is kept constant which leads to uniform surface roughness
- Requires less machining time
- secondary cutter paths are derived from master cutter path
- Redundant machining is minimized



To start with, we have already finished the discussion on Isoparametric machining and isoplanar machining, we have discussed in details, forward step calculation for Isoplanar machining and today we will be taking up the last part Isoscallop machining, which produces equal height of roughness scallop all through the surface. So scallop height is kept constant, which leads to uniform surface roughness, this requires less machining time because there is no redundant machining, secondary cutter paths are derived from master cutter path and the master cutter path is generally taken to be one of the edges of the surface. So this shows two positions of the tool along two different I mean two adjacent cutter paths and this is the material which is leftover uncut and this height is called as scallop height.

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Surface derivatives and curvature

$$\begin{aligned} L &= \mathbf{R}^{uu} \cdot \mathbf{n} & E &= \mathbf{R}^u \cdot \mathbf{R}^u \\ M &= \mathbf{R}^{uv} \cdot \mathbf{n} & F &= \mathbf{R}^u \cdot \mathbf{R}^w \\ N &= \mathbf{R}^{ww} \cdot \mathbf{n} & G &= \mathbf{R}^w \cdot \mathbf{R}^w \end{aligned} \quad \dots (1)$$

$k = \text{local curvature along cutter path } R(t) = \text{normal curvature } k_n$

$$k = \frac{L \left(\frac{du}{dt} \right)^2 + 2M \left(\frac{du}{dt} \right) \left(\frac{dw}{dt} \right) + N \left(\frac{dw}{dt} \right)^2}{E \left(\frac{du}{dt} \right)^2 + 2F \left(\frac{du}{dt} \right) \left(\frac{dw}{dt} \right) + G \left(\frac{dw}{dt} \right)^2} \quad \dots (2)$$

$$I = E \left(\frac{du}{dt} \right)^2 + 2F \left(\frac{du}{dt} \right) \left(\frac{dw}{dt} \right) + G \left(\frac{dw}{dt} \right)^2 \quad \dots (3)$$

So this is just a reminder, we have gone through these expression before, these are the derivatives for example the derivative of the surface in the U direction dot product with itself gives a particular value which is equated to E that is it is given a name E, F is the dot product of $\mathbf{R}^u \cdot \mathbf{R}^w$, G and the dot product of $\mathbf{R}^w \cdot \mathbf{R}^w$, et cetera, these things we have discussed last time. This is the expression of the curvature which is taken to be 1 by R in this case and it contains all those constants which have been defined previously like L and N and E, F, G. Together with that we have a du/dt and dw/dt multiplier, which can also be simplified to du/dw and this varies with the direction and which we are considering the curvature.

For a free form surface which is curved, the curvature might be different in different directions, while L, M, N, E, F, G, they are invariant with the direction du/dt and dw/dt or du by dw , this varies with the direction and this is the 1st fundamental form of the surface, we will come across this very very soon.

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Forward step Calculation

$$\Delta t = \sqrt{\frac{8e - 4e^2}{kI}}$$

$e = \text{specified tolerance}$

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In isoscallop machining in this particular publication, the forward step was found out without iterations, we had iterations in case of iso-planar machining, find out the forward step, but here the forward step was found out without iterations and it was expressed as a parametric increment along the cutter path which is expressed as a function of a third parameter t on the surface $R \cup W$, so what do we have here? We have here this particular cutter path, along the cutter path this tangent has been taken at this initial point, our target is to find out the incremental increase which will allow us to reach this point designated by parameter t_2 , from parameter value t_1 .

So mind you, this is the parameter value while the curve position vector is $R(t_1)$ and $R(t_2)$ at these points respectively, this R is not the radius, this R however is the radius, this one, 1 by curvature I am sorry, this might be slightly confusing be alert about this particular difference. Now, whenever we are considering the tangent okay 1st of all let me state that here it was stated that Δt that particular parametric increment along the cutter path expressed as $R(t)$ that particular parametric increment which will give us acceptable forward step is expressed by

this expression, $\Delta t = \sqrt{\frac{8e - 4e^2}{kI}}$.

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$$R^t = \frac{\partial R}{\partial u} \times \frac{du}{dt} + \frac{\partial R}{\partial w} \times \frac{dw}{dt}$$

$$R^t \cdot R^t = \left(\frac{\partial R}{\partial u} \times \frac{du}{dt} + \frac{\partial R}{\partial w} \times \frac{dw}{dt} \right) \cdot \left(\frac{\partial R}{\partial u} \times \frac{du}{dt} + \frac{\partial R}{\partial w} \times \frac{dw}{dt} \right)$$

$$= \left(R^u \times \frac{du}{dt} + R^w \times \frac{dw}{dt} \right) \cdot \left(R^u \times \frac{du}{dt} + R^w \times \frac{dw}{dt} \right)$$

$$= E \left(\frac{du}{dt} \right)^2 + 2F \left(\frac{du}{dt} \right) \left(\frac{dw}{dt} \right) + G \left(\frac{dw}{dt} \right)^2 = I$$

$$|R^t| = \sqrt{R^t \cdot R^t} = \sqrt{I}$$

$$R^2 - (R - e)^2 = \left(\frac{|R^t| \Delta t}{2} \right)^2 \quad \Delta t = \sqrt{\frac{8e - 4e^2}{I}}$$

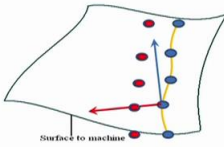
So what do we have here? R^t that means the tangent along the cutter path can be expressed in a chain form, this is the complete differential so this is basically dr/dt expressed in the chain form because R is basically a function of u and w and $R^t \cdot R^t$ we take the dot product of the tangent to the cutter path with itself, we have these two dot producted and ultimately it gives rise to this expression how do we have this, $\frac{\partial R}{\partial u}$ is simply expressed as R^u , $\frac{\partial R}{\partial w}$ as R^w and then we multiply this $R^u \cdot R^u$ will give us e as defined before. So this way, this term will simplify to this term it is extremely simple, straightforward, you can try it out yourself and this is nothing but the 1st fundamental form that we had previously defined and therefore we equate this to I .

So that R^t this is the dot product, so it basically it involves the square of the magnitude therefore, the absolute value magnitude of R^t is nothing but root over 1st fundamental form and after getting an expression of this R^t , what we do is we bring in Pythagoras theorem where $R^2 - (R - e)^2$ must be equal to this side square, this is expressed as $\left(\frac{|R^t| \Delta t}{2} \right)^2$. And we express this particular distance as nothing but $R^t \Delta t$ therefore, half of it is equal to $R^t \times \Delta t/2$ okay, because R^t is basically $dr/dt \times \Delta t$ will give us this complete distance assuming that this particular angle is so small that it can be well considered to be in the direction of the tangent.


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General equation of a tangent $P(t)$ on a surface $P(u,v)$ is

$$\frac{dP}{dt} = \frac{dP(u,v)}{dt} = \frac{\partial P}{\partial u} \cdot \frac{du}{dt} + \frac{\partial P}{\partial v} \cdot \frac{dv}{dt} = P^u \cdot \dot{u} + P^v \cdot \dot{v}$$


If there are two tangents at right angles to each other

$$\left(R^u \frac{du}{dt} + R^v \frac{dv}{dt} \right) * \left(p^u \frac{du^*}{dt} + p^v \frac{dv^*}{dt} \right) = 0$$


From this one we can easily simplify, I am leaving this to you we can easily simplify. That Δt comes out to be this one, so we are avoiding iteration and getting an expression of Δt increment of parameter along the cutter path in order to give acceptable forward step. So after this we come to sidestep, how is sidestep taken in case of iso-scallop? One method could be, once we see one existing cutter path moving this way and there are forward steps approximated or points calculated as per forward step, their locations are already determined that means these blue small blue circles are the positions at which if the tool is placed, it will give us acceptable forward steps that part is done.

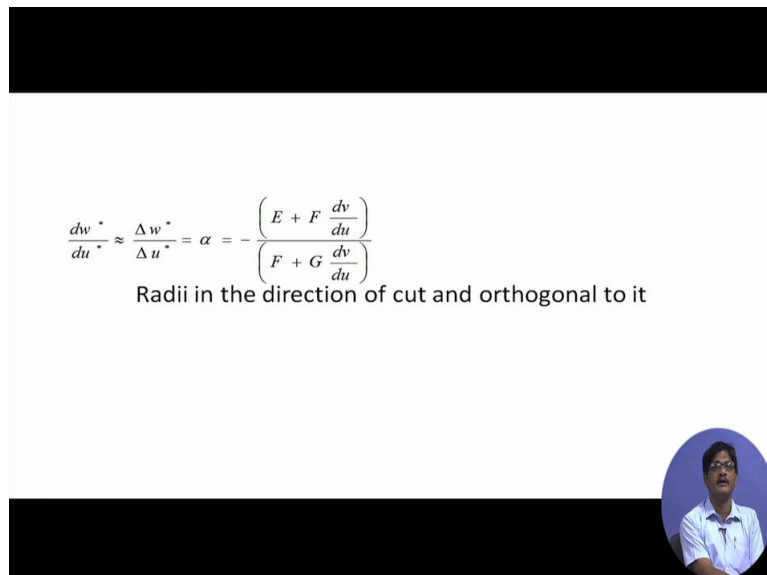
Then we are trying to find the positions of the cutter path, cutter contact points on the potential cutter path on by the side of it, which has not all found out up till now, we are trying to calculate the positions on this particular cutter path which will give us acceptable sidestep. For this what we do is, we say that okay this particular step that we have to take is definitely going to be defined by a vector, we simply find out its direction and magnitude which is all there for a vector. So first what we say is, let us 1st find out in which direction we should consider the sidestep at this particular location of the cutter position on the first cutter path.

So, on the 1st cutter path if we look at it at this position, the tangent to this cutter path which is basically once again that R^t that gives us this particular tangent gives us the direction of the cutter along this cutter path at this point. We will take sidestep at a direction perpendicular to it, so we will say that tangential direction on the surface at this point making 90 degrees with the cutter tangent okay that particular tangential direction will mark the direction of the sidestep. So what we say is, I am sorry suddenly R have been replaced by p by mistake and I

mean w has been replaced by v , please understand that this basically represents nothing else but the tangent.

That means please read it as $\frac{\partial R}{\partial u} \times \frac{du}{dt} + \frac{\partial R}{\partial w} \times \frac{dw}{dt}$ nothing else. This expression of general tangent we already we have come across, so what we are trying to establish here is that there are basically two tangents that we are considering and they are dot producted to give us a 0, which means they are neutrally orthogonal that means they are perpendicular to each other. One tangent is, already the existing tangent in the cutter path direction and the other tangent we do not know about it, we are going to solve for it. So this is the relation that defines the orthogonality of cutter path and sidesteps direction okay. Here it is interesting to note that this expression represents the tangent along the cutter path and R^u and R^w , they are figuring here.

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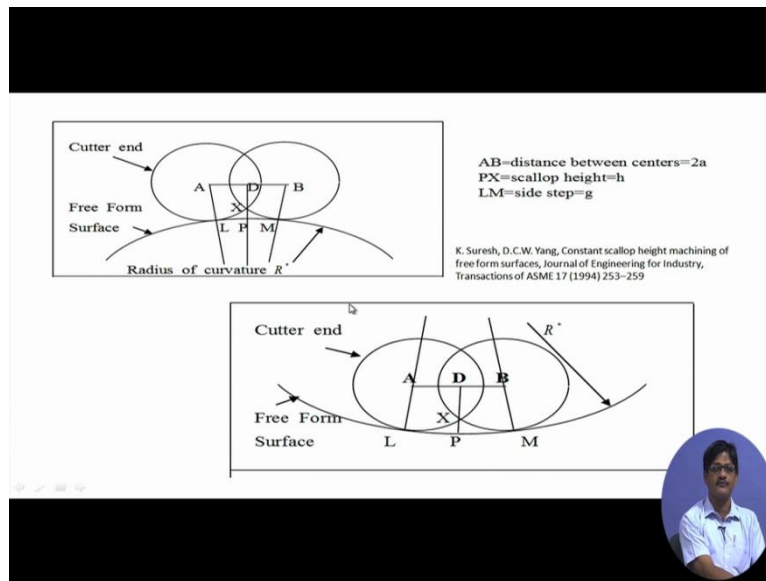


$$\frac{dw^*}{du^*} \approx \frac{\Delta w^*}{\Delta u^*} = \alpha = - \frac{\left(E + F \frac{dv}{du} \right)}{\left(F + G \frac{dv}{du} \right)}$$

Radii in the direction of cut and orthogonal to it

And in this one also R^u this should be R^u and R^w , they are also figuring here, why? Because R^u and R^w they are invariant, they do not depend upon the direction, du/dt dw/dt , et cetera these will be dependent upon the direction in which the tangent is being considered. So from the previous expression the dot product, we will ultimately have dw/du we cancel out the dt terms, dw/du giving direction of the tangent in the sidestep direction to be this one, so it is related to 1st of all the surface constants and it is also related to the direction of the cutter path, dv/du should be actually dw/du this is related to the cutter the cutter path direction at that particular point.

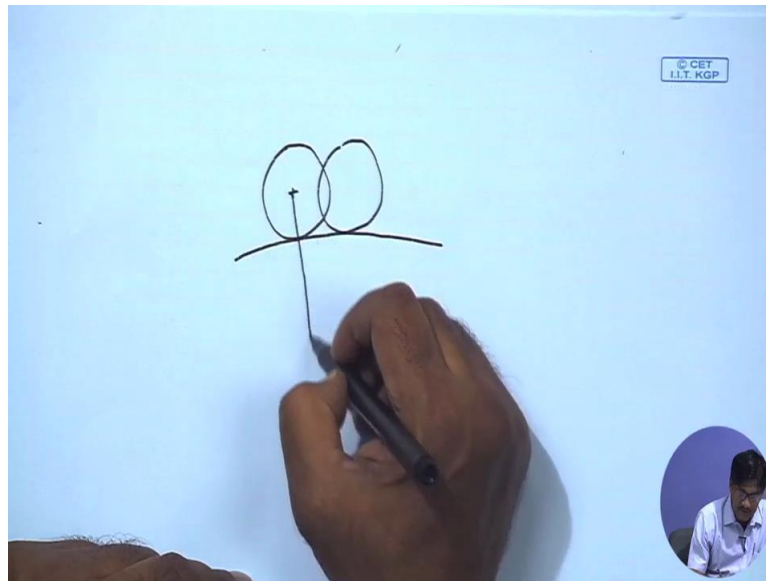
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So after having established this one that is we now know the direction in which the cutter sidestep has to be taken, we are trying to establish the magnitude how much should be the cutter cutter sidestep magnitude. For that we take the help of simple geometry and we say that if the surface be convex, there will be 2 positions of the cutter on adjacent cutter paths okay, these are side-by-side on separate cutter paths and this is the small amount of material left behind uncut and PX determines denotes the height of the scallop okay it is written here, PX = scallop height, AB = distance between centres obviously these are the centres of the two positions of the cutter round nodes and we are calling it twice A and the sidestep however is the distance between the 2 cutter contact points at L and M and therefore, LM denotes the side steps.

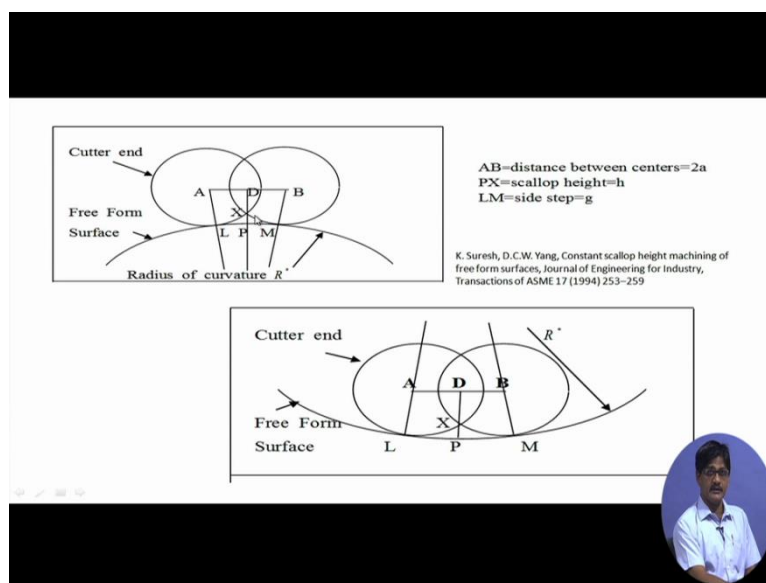
So what we are going to establish from here is that since these will be ultimately meeting at the centre of curvature of the surface therefore, we have similar triangles in which LM divided by AB will be equal to radius of curvature of the of the surface divided by radius of curvature of the cutter plus radius of curvature of the surface, this we have written down. Later on or we can quickly write it down here on the on paper.

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That is if we have 2 positions of the cutter here and if this be the surface, this be the first line I mean the normals containing the radius of curvatures, so we are basically saying this is if this be the sidestep and if this be the centre distance and if this be big R radius of curvature of the surface orthogonal to the cutter path and this is the radius of curvature of the cutter, we can say $R + r$, centre distance divided by sidestep must be equal to $R + r$ divided by R this is what we are establishing that means we are establishing a relationship between the sidestep and the centre distance, why are we doing this, because ultimately we have to establish how much is this particular magnitude.

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This magnitude once we establish, we will say that we have got the complete information about the sidestep vector, its direction is already established. If I know this particular magnitude, I can say that yes I know how much distance I have to shift in order to get the sidestep which will give me a definite scallop height, this is the scallop height okay. So coming back to the figure, this from this particular figure it is possible to calculate the scallop height by a few Pythagorean relationships let's see that.

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Convex curvature: O is center of curvature of surface

$$OD = \sqrt{OB^2 - BD^2} = \sqrt{(r + \frac{R}{2})^2 - a^2}$$

$$DX = \sqrt{BX^2 - BD^2} = \sqrt{r^2 - a^2}$$

$$OX = R^* + h = OD - DX$$

$$h = \sqrt{(r + R)^2 - a^2} - \sqrt{r^2 - a^2} - R^*$$

$$a = \sqrt{r^2 - \left(\sqrt{(R^* + r)^2 - a^2} - R^* - h \right)^2}$$

$$SS = \frac{2a}{\left(1 + \frac{r}{R} \right)}$$

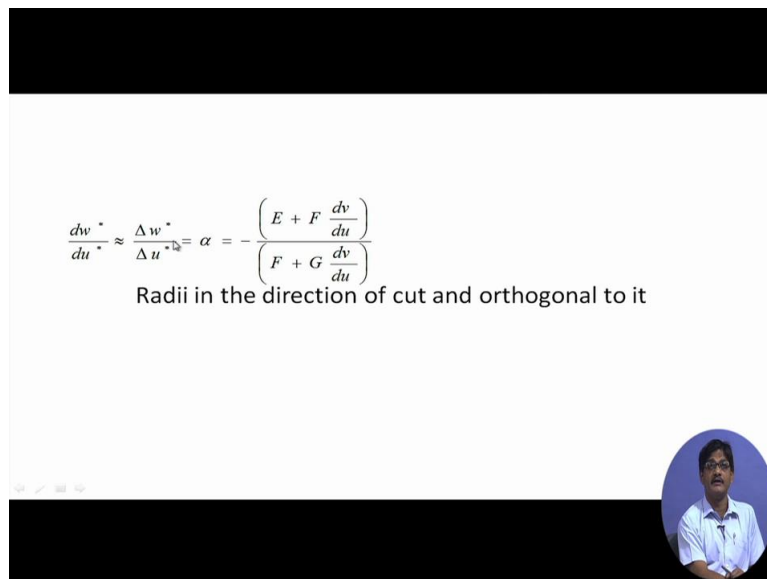
If we have this calculation carried out, I am not going through the calculation but I will just point out one or two of them. What we can say is that here there will be one right angle triangle XDB so that we can say XB^2 , which is nothing but the radius of the cutter, radius of the cutter Square must be equal to $XD^2 + DB^2$, DB happens to be half the centre distance so that we can induct centre distance into the calculation with cutter radius and this particular distance XD. XD will also be related the scallop height and radius of curvature of the surface in a direction perpendicular to the cutter path.

So this way these relations are drawn up, it is taken from this particular reference and I am not going through them in detail but you can definitely follow it and this is the relation which is coming from the geometric similarity triangles, triangular similarity relationship which we visited just now and from there from there this A value can be replaced here in terms of small r, big R that means radius of curvature of the surface and A okay. So this is this relation is containing a inside so that it can be iteratively solved or separation of variables can also be done so that it can be solved completely in terms of this particular parameter h, big R, small r.

One replace this, we get a get an expression of sidestep which comes out to be a function of these 3 things; small r radius of the cutter, radius of curvature of the surface and scallop height. So if we specify a definite scallop height for a particular tool and surface combination, we can find out the sidestep magnitude at any point on the surface. Once we establish that, last of all we introduce one expression that is the sidestep the current sidestep position being P and the final sidestep position I mean the current tool position being P, the sidestep position of the tool being P star, their different can be expressed as a tailor series expansion involving only the first derivative as $P^u \bullet \Delta u^* + P^w \bullet \Delta w^*$.

So in this tailor series expansion ultimately we will get the solution of Δu and Δw giving us the parametric increments from the present position of the cutter to the sidestep position of the cutter, which will yield the recommended value of surface roughness. This is one equation and the other equation if you kindly remember the equation.

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$$\frac{dw^*}{du^*} \approx \frac{\Delta w^*}{\Delta u^*} = \alpha = - \frac{\left(E + F \frac{dv}{du} \right)}{\left(F + G \frac{dv}{du} \right)}$$

Radii in the direction of cut and orthogonal to it

This was the one expression that we got for Δw and Δu , this is one equation and other equation we visited just now this one okay.


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$$(P^* - P) = SS, \quad (P^* - P) = P^u \Delta u^* + P^v \Delta v^*$$

$$\Delta u^* = \frac{\pm SS \left(F + G \frac{dw}{du} \right)}{\sqrt{EG - F^2} \sqrt{E + 2F \frac{dw}{du} + G \left(\frac{dw}{du} \right)^2}}$$

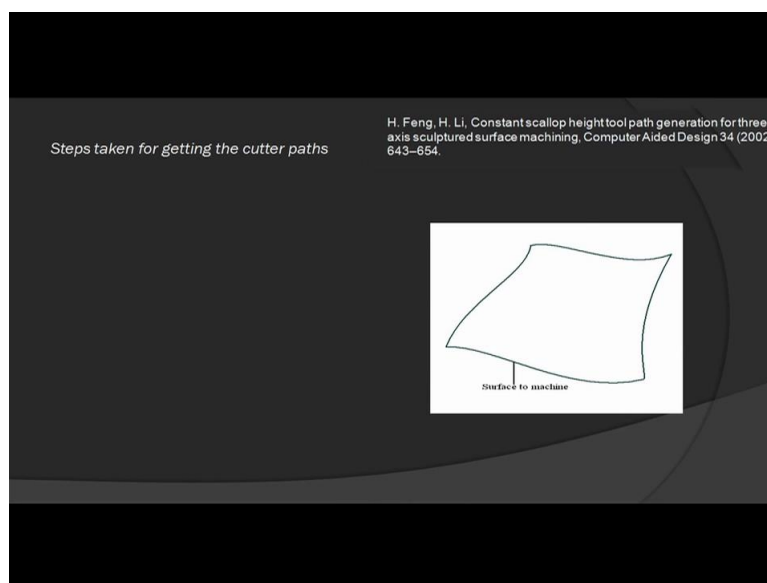
$$\Delta v^* = - \frac{\pm SS \left(E + F \frac{dw}{du} \right)}{\sqrt{EG - F^2} \sqrt{E + 2F \frac{dw}{du} + G \left(\frac{dw}{du} \right)^2}}$$

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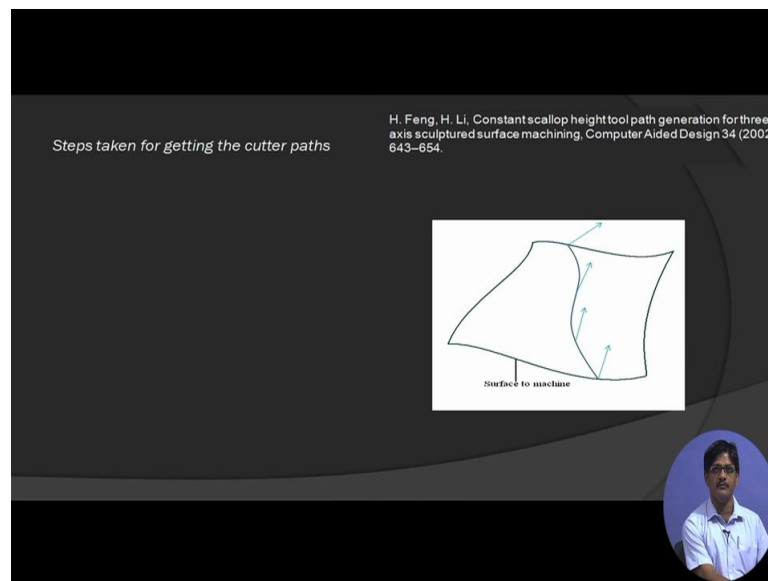
Here we put the magnitude of the sidestep because $P^* - P = \text{sidestep}$ and that is equated to Δu and Δv expression, so we get two equations and these are simple simultaneous equations and we can solve for Δu and Δv . Once we solve this, we get a magnitude we have also got the direction and we can pinpoint the sidestep positions. We pass the curve through this in the UW plane. And once we pass a curve through it, we truncated it or extend it in order to start and stop at the edges of the surface okay. You might wonder about the fact, why is this curve through all these cutter contact points done on the UW plane and not on the XYZ plane, please think about this.

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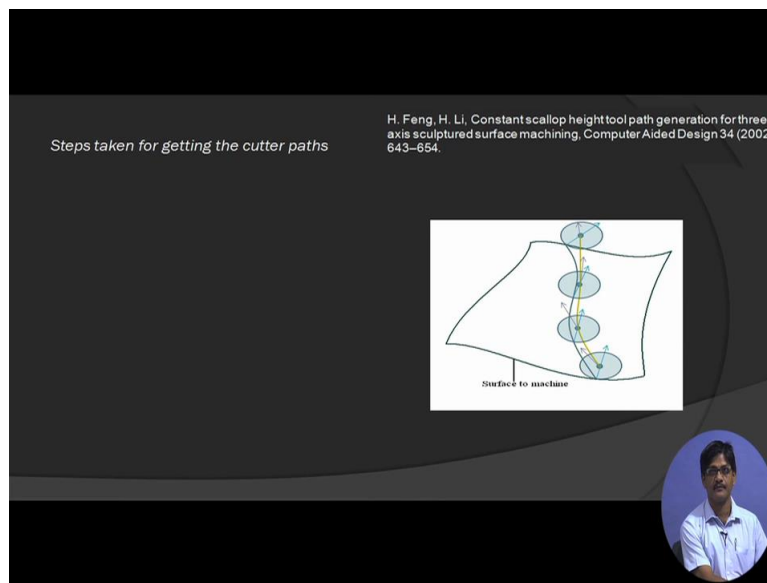
Now we will look at another method of cutter path generation for curved surfaces following the iso-scallop strategy in which it was observed that the cutter path generated by the method that we have discussed and some other methods, they have 2 dimensional considerations of geometries, which may bring in some errors. So we start this one, this discussion pictorially without resorting to those formulae and equations, this is the free form surface and say this is a cutter path, so this has been taken by a paper by Feng and Li came out in 2002.

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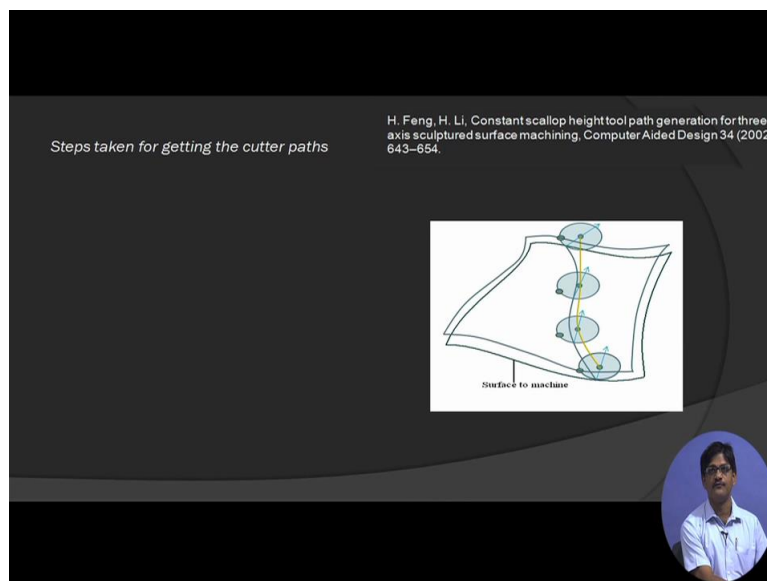
So this cutter path to start with, we mark at particular points which are forward steps adjusted, we determine the surface normals at these points. These points are the CL points, the centers of the nose portion of those ball end milling cutters. We join them by a smooth curve, we find out that tangents to this smooth curve hence we get the cutter directions at these respective locations.

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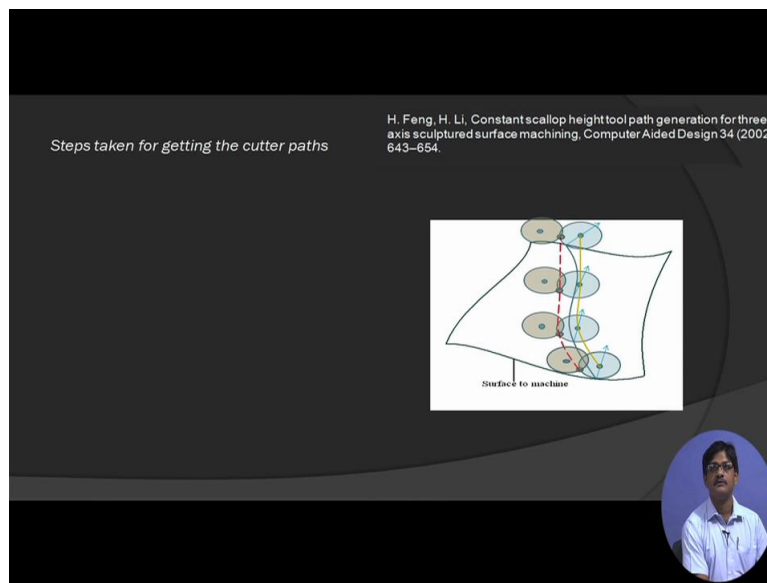
Next we draw the swept section of the cutter end cutter ball end at these respective locations and swept sections are the once which define the geometry of the groove being cut. We remove all the construction lines.

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Next we find out the intersections of the swept sections of the cutter with a surface offset from the design surface by the scallop height, so this surface is away from the design surface normal to it and all those respective positions by the scallop height and this way we are determining the tips of the scallop curve at the at these locations on the swept sections of the cutter, so these mark the tips of the scallop I mean the whole scallop geometry all through these particular position by the side of the 1st cutter path.

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Next we remove the scallop offset surface, we pass a smooth curve through these scallop tips points, we find out the adjacent position of the cutter in the adjacent cutter path from the consideration that it has to touch the surface and it also has to pass through the scallop points which are already determined and there is another condition that is the scallop tangent, scallop tangent means the those tangents, which are tangential to scallop, scallop tangent has to be perpendicular to the radius vectors of the swept sections of the second cutter path. So applying this condition we understand that the scallop tangent, it is not drawn in the figure but the scallop tangent has to be perpendicular to the radius vector of the swept section of the 1st as well as the 2nd path.

Does this make consideration that these two swept sections will be co-planar, no. While the radius vectors of those swept sections with the scallop tangent will be perpendicular, the swept sections themselves do not have to be perpendicular to this scallop tangent. We have shown one scallop tangent here and we are claiming that it is perpendicular to the swept tube, it is tangential to the swept tube and therefore it is perpendicular to the radius vector which is connecting this scallop point with the centre, it is perpendicular to this particular radial line but it is not perpendicular to these planes.

Had it been perpendicular to those planes, it would have again become the case of 2 swept sections being on the same plane and two-dimensional geometrical calculations would have been applicable, but it is not so this is the difference of this particular model with the previous one.

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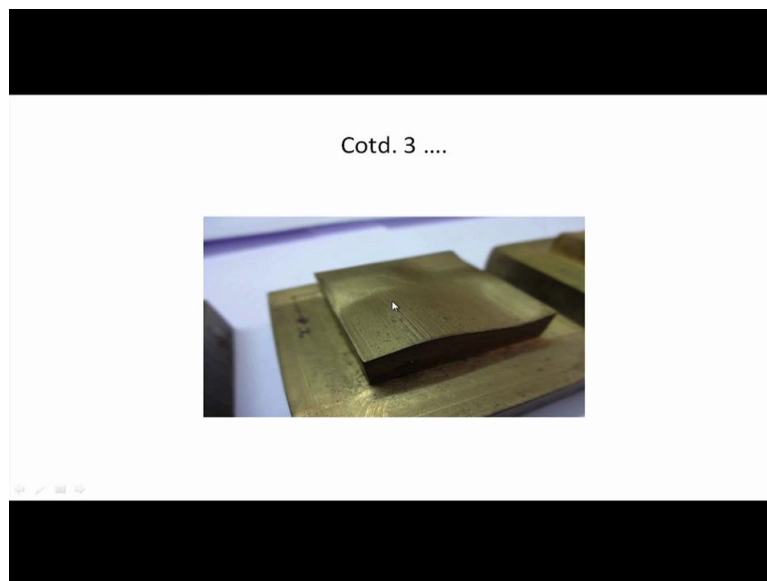
So at the end of this discussion, we come to 3 examples of jobs which have been cut with different forward steps and different side steps. As you can well identify, this one has large forward steps and also a very coarse side step pattern so that roughness will be visible to the naked eye while these are not so a closer look perhaps, yes.

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So you can see roughly those roughness marks appearing on the surface of this body.

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Yet closer look yes, those are the roughness marks which appear. What is the current status of CNC, is it restricted to all these machines machining and 3 axis or 5 axis maybe, etc, there are different fields or applications developing overtime for CNC. We have CNC being applied in case of say rapid prototyping, where the whole technology of rapid prototyping is dependent upon CNC. Unless the movements of the machine can be precisely controlled I mean the difference slides movements, different stages, we cannot have the implementation of different technology, as I discussed for example rapid prototyping and that way CNC dependent technology is developing very fast.

And ultimately it will move on to different other fields for example, in middle level production we are having flexible manufacturing systems, which is basically computer-controlled and only in mass production we have Special-purpose machines in in a there are other aspects of machining in which we have copy of different geometries still, there also we are gradually having the development of technology which can work without part specific tooling. Part specific tooling means basically tools which are dedicated to specific parts and that way we are very much depended upon special tooling physical devices, etc and CNC is targeted towards elimination of such technology, thank you.