Metal Cutting and Machine Tools
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Lecture-05
Geometry of Cutting Tools and Numerical Problems

Welcome viewers to the fifth lecture of metal cutting and machine tools. So, in this one we will be discussing about different cutting tools, taking up numerical problems and also one thing that I have left behind that is in geometry of cutting tools I will just touch the subject of master line system of tool angle reference master line. It is sometimes it is more often referred to as the maximum rake system.

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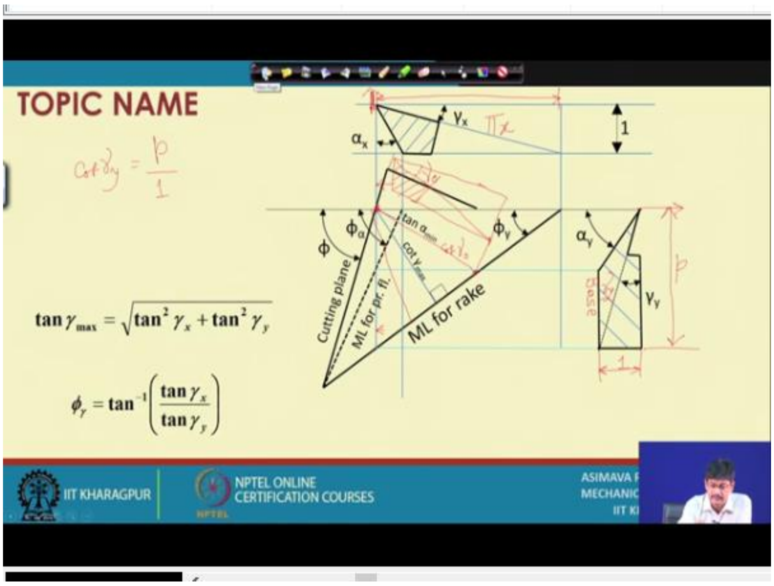

So, to start with; in this figure I have drawn the traces of if you look here the traces of the can you see this of the intersection of the rake surface with the base plane. Now, what do I mean by the base plane? By the base plane I mean that plane which is at the base of the tool and it is necessarily parallel to the reference plane. It might not always be parallel to the reference plane, but in this case we are assuming that. Which is the base plane here?

This is the in the sectional view of the tool in the machine longitudinal plane π_x. So, let me write down, so, this is π_x. So, in π_x what do we see, we see that γ_x α_x etcetera have appeared and this is the trace of the base plane. So, that means that I am assuming here that the velocity vector is exactly in this direction, it has not exactly fallen there.

I can draw it here; the velocity vector I am assuming to be like this. So, that this comes out to be the base plane or it is difficult to write this way. I have to have more practice base plane. So, if that happens in that case, this is a sort of representation of the reference plane and the rake surface as you can see here in this view, it is coming down, sloping down and intersecting the base plane at this point.

So, I take it to the plan view, this is the point representing the intersection of the rake surface and the base plane considered in the machine longitudinal plane. Similarly, in the machine transverse plane the rake surface is sloping down and it is reaching the base screen at this point and the corresponding point is here and this is the section that we have considered.

So, hence, what we have here is that, this line therefore will represent if we join these 2 points it will represent the intersection of the rake surface with the base plane. You might ask me, how did that this will be a straight line; it might well be a curved line. Here our argument is this that since we are having a plane intersecting another plane, the result should be a straight line.

So, this one we call master line for rake. What does this mean? This means that each and every point represents intersection of the rake surface with the reference plane if the reference plane in the basement they are considered to be the same here. So, that means that from here if we extend lines. These are going to represent this distance. For example, if I draw from here and I extend from here these distances are it is not coming very well in the upper view.

I can take this view; these are going to represent these distances on this particular figure, that is from here to this point is going to represent this one. If we take some other view, there also the same thing is going to happen. For example, if I take the orthogonal rake then orthogonal rake also will be represented that way where will the orthogonal rake view come it will be coming somewhere here that is if I take a section in this particular direction orthogonal and this will be optimal rake.

So, in that case what are we going to see this line is also going to slow down so, that it will be represented on this line somewhere here. That means, if you join this one this line is going to represent this very distance this distance. Now suppose I see that this height of this tool point where this point is being section. So, this point if I consider this height to be 1, in that case cotangent of this angle say γ_x.

Cot γ_x becomes equal to this distance divided by 1. So, let us have a calculation here done quickly. So, let me call this say P. And let me call this distance I set it to be equal to 1, that is good. And this I find is γ_y alternate angles. Therefore, I can say $\cot \gamma_y$ equal to what is this equal to this must be equal to this P divided by 1 that is it. P is equal to $\cot \gamma_y$.

So, I can say that this distance represents $\cot \gamma_y$, this distance represents $\cot \gamma_x$, this represents cot say, which one this one? Yeah, this one, this one represents $\cot \gamma_o$, so just draw a straight line in that particular section in which the rectangle is being considered. These lines will be representing the cotangents of those rake angles. That is good. That is interesting. So, that means all these lines emanating from here and coming up to this one, let me clean the view a bit.

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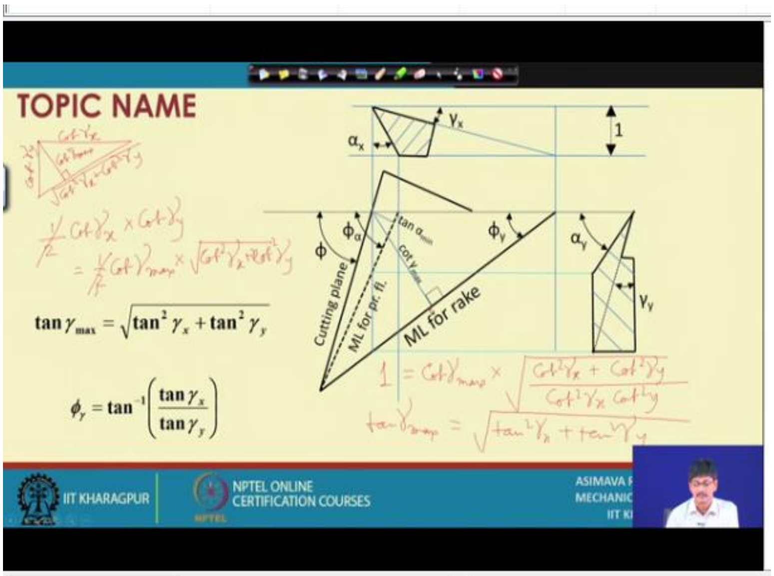

Yeah, all these lines are cotangents of the rake angle of that corresponding section. And immediately we come to the conclusion in that case, there must be a minimum distance here, which represents the maximum rake angle. This is it, this is the one and incidentally this particular angle, this maximum rake angle is represented by this relation.

How do we get this relation? We can have a quick look, if we take out this particular triangle. This is $\cot \gamma_x$ is it visible yes; this one is $\cot \gamma_y$. This one must be $\sqrt{cot^2\, \gamma_x + \, cot^2\, \gamma_y}$ and this must be equal to $cot\ \gamma_{max}$. Once we have this relation, we will be able to find out $tan\ \gamma_{max}$ to be equal to this. How do we do that? We equate any of the triangles.

I mean we equate the areas of the triangle considered from once from half the base into altitude taking this as the base for example, $cot\ \gamma_x * cot\ \gamma_y$ being equal to half $cot\ \gamma_{max}$ multiplied by $\sqrt{cot^2\gamma_x + cot^2\gamma_y}$. Once you have this if you divide the two sides by $cot\ \gamma_x$, $cot\ \gamma_y$ and cancel the half, let me cancel the half here itself.

So, if you divided by $cot\ \gamma_x$ on this side you will get 1, on the other side you will $\sqrt{cot^2\gamma_x + cot^2\gamma_y}$ divided by $cot^2\ \gamma_x * cot^2\ \gamma_y$, which will mean that you will get $tan^2\ \gamma_x + tan^2\ \gamma_y$. So, if you look at it, it will be ultimately from here you will directly get this relationship.

So, that ultimately we can say here we can consider this way, we can do it even in a quicker manner instead of cutting off $cot^2\ \gamma_x$, y we can send this to the other side, whichever way you do it, it will come out to be the same thing that is $tan\ \gamma_{max}$. I will do it here, here we have some space. So, we have $1 = cot\ \gamma_{max} * \sqrt{cot^2\gamma_x + cot^2\gamma_y}$ divided by $cot^2\ \gamma_x * cot^2\ \gamma_y$.

What do we have from here? We have one $cot^2\ \gamma_x$ cancelling out so, that this one can be transferred to the other side so, that we have $tan\ \gamma_{max}$ equal to root over of this one cancels out the other remains and we convert it to tan. So $tan^2\ \gamma_x + tan^2\ \gamma_y$ that is it. So, this way we get an expression of a maximum value of rake. Now, how is this useful to us?

This is useful to us in the sense that we can utilize this as information that if the rake surface orientation has to be expressed we are generally doing it as a pair of angles that there might be inclination angle an orthogonal rake, inclination angle and normal rake, back rake and side rake. Instead of doing that we can say that the orientation of the rake angle is maximum rake is equal to this much and the other side I mean in the orthogonal side then we are expressing the angles in mutually perpendicular directions, in the other direction the rake angle is 0.

So, we will say if you consider this particular orientation of the intersection of the base plane and the rake surface in this direction it is having no angle with the reference plane, but perpendicular to it is having γ_{max} as the inclination of the rake surface with the reference plane. So, if φ_γ and γ_{max} they are expressed, we get the orientation of the rake surface in terms of maximum rake and 0 as the 2 angles made by the rake surface with the reference plane in a mutually perpendicular direction.

So, that helps us during operations like grinding, when we take a tool grinding this will be made use of and we will really appreciate this particular application. And this φ_γ this particular angle which depicts the orientation of the intersection of the rake surface with the reference plane, this φ_γ can be found out very easily in this particular triangle that we are considering this one. It is simply tan inverse of $\tan\gamma_x / \tan\gamma_y$. I am not working it out, you can surely follow it.

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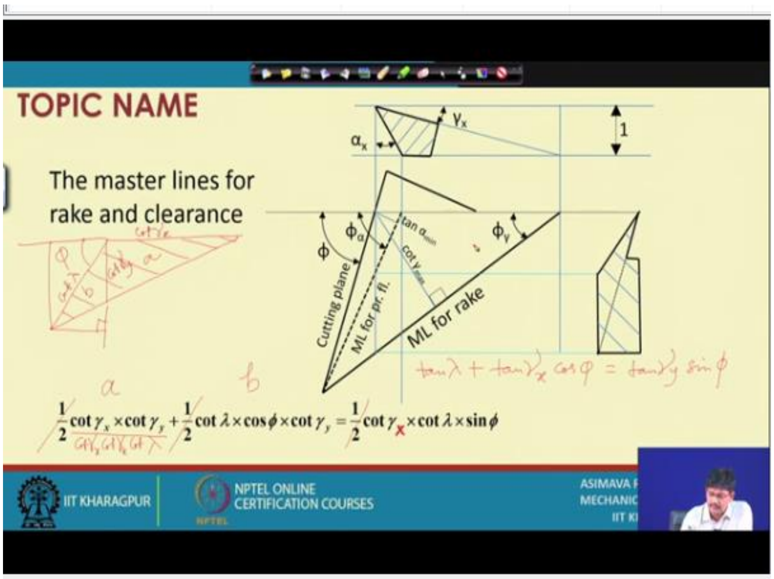

So, with this part you come to understand the usefulness of the master line method. And I will just give you one example through which you will be convinced that yes, it has some other uses also, all the tool angle calculations can be done very elegantly with the help of this master line principle. Let us have a quick look how this can be done?

For example, we can simply go back to the calculation of areas of triangles and make use of it, in order to find out the relations between different angles. We do not have to do with that altitude method that we have discussed previously. For example, suppose I can identify here some basic triangles, let me draw it, this is one triangle, this is another one, this is right. So, what do we have here?

We have here this to be φ, this to be $\cot\lambda$, this to be $\cot\gamma_x$, this to be γ_x, let me use the eraser that is it, $\cot\gamma_y$, in that case, suppose I want to establish the relation between γ_x, γ_y and λ. So, what do we do? We say that the area of this triangle say I name this a and I named this b, the area of this particular triangle is simply half I have already written here half $cot\ \gamma_x * cot\ \gamma_y$, that is it.

I think you have absolutely no problem in accepting this area a is equal to this plus suppose I add area b, how much is here b, area b is equal to this being the altitude, this altitude is nothing but $cot\ \lambda\ cos\ \varphi$. So, we use $cot\ \lambda\ cos\ \varphi$ into $cot\ \gamma_y$, $cot\ \gamma_y$ is the base, base into altitude to half. So, that way we get this expression b. Now, if I add a and b I must be getting the area of this triangle.

What is the area of this triangle? The base is $cot\ \gamma_x$. So, I write what have I written here, it should be please correct this $cot\ \gamma_x$. So, sorry but this is now correct. So, I have the base as $cot\ \gamma_x$ and the altitude as this one. So, this must be $cot\ \lambda\ sin\ \varphi$ and hence, from here if you multiply the equation I mean if you divide the expression by $cot\ \gamma_x\ cot\ \gamma_y\ cot\ \lambda$ let us see what we will get, let me utilize this space.

So, first of all half cancels out, if half cancels out and we divided by that thing we get from here $cot\ \lambda\ cot\ \gamma_x$. So, we are dividing this by $cot\ \gamma_x\ cot\ \gamma_y\ cot\ \lambda$ all of them. So, here only $cot\ \lambda$ will survive and I write $tan\ \lambda$ here because it is in the denominator. So, in the numerator I would write $tan\ \lambda$ plus here $cot\ \lambda$ will vanish, $cot\ \gamma_y$ will vanish and we will only have $tan\ \gamma_x$ surviving.

So, $tan\ \gamma_x\ cos\ \varphi$ equal to, so, here in the same way $cot\ \gamma_y$ will survive, so, we will have $tan\ \gamma_y * sin\ \varphi$. Therefore, we have established a relation between λ, γ_x and γ_y, using this method and totally circumventing the method that we had learned the other day and this seems to be more straightforward. Extremely simple.

So, with this knowledge, there will be some other exercises which I can offer you regarding master line method which I will be putting in the multiple choice questions or sharing with you through some uploaded multiple choice questions. Let us now move on to the next topic.

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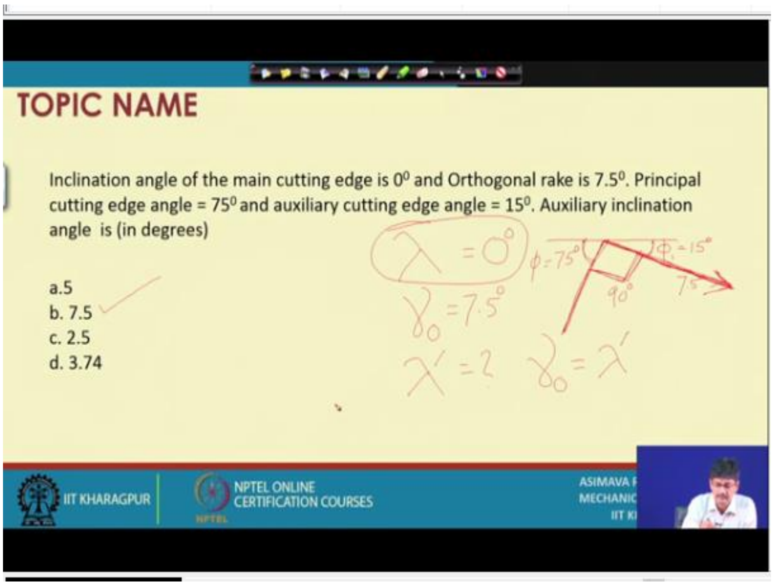

Right a multiple choice question what does it say? It says inclination angle of the main cutting edge is 0º. So, we write the information which is given is that right we were discussing this particular problem. So, here first of all what is given is λ equal to 0, orthogonal rake γ_o = 7.5º. Principle cutting edge angle is 75, this is the main cutting edge and actually the cutting edge is having an angle of 15º.

Once we have this auxiliary inclination angle that means, what is the value of λ'? This is the question. So, I know λ, I know γ_o, I know φ, φ_1, can I find out λ'. The answer to this question is very interesting; we need not go into any calculation whatsoever. We can argue that since φ is 75º, φ_1 is 15º. So, this must be a right angle.

If this is a right angle in that case, it is given that the tool is having 0º along this and the tool is having how much 7.5º along this direction. In that case, this must be how did I find out 7.5 because, since this is the cutting edge, if this is the cutting edge, I mean orthogonal plane must be at right angles to it. So, we find that the orthogonal plane and the auxiliary cutting plane they are becoming coincident.

So, if that be so orthogonal rake must be equal to λ' and hence auxiliary inclination angle or λ' is equal to 7.5. I just repeat λ is given to be 0º. So, along this direction it is 0º, γ_o is equal to 7.5º. So, γ_o is inclined this way 7.5 as 90º. But as φ is 7.5 and φ_1 is 15º, hence, this must be 90 and this must be the auxiliary cutting edge as well. So, this is the trace of the auxiliary cutting plane.

So, orthogonal plane and auxiliary cutting plane they are becoming coincident and therefore, this angle γ_o must be equal to λ'. These must be equal and therefore, λ' is 7.5. I have a question for you. If I had given λ to be nonzero, would the answer have been the same? That is this redundant please think and then come to a conclusion and then we will share our opinions in some later lecture. So, let us move on to the next one.

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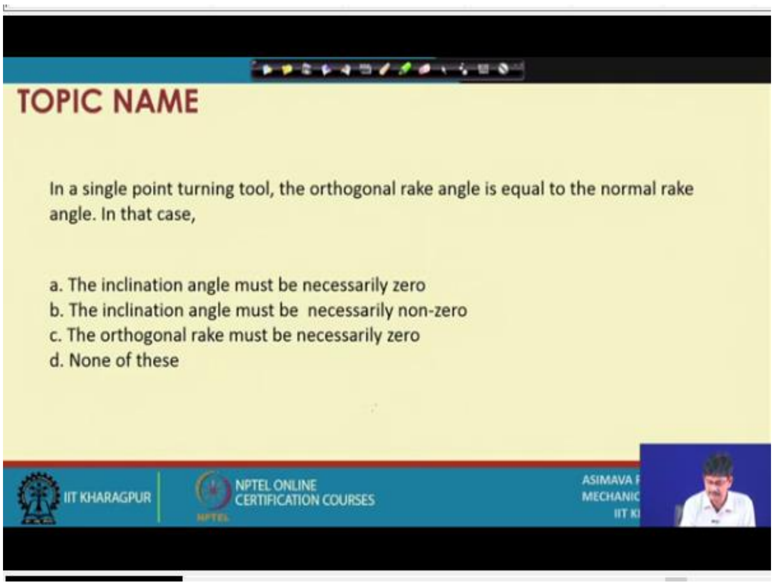

In a single point turning tool, the orthogonal rake angle is equal to the normal rake angle. In that case the inclination angle must be necessarily zero, the inclination angle must be necessarily nonzero, the orthogonal rake must be necessarily zero, none of these. So, first of all let us see what is given. We are given that the orthogonal rake.

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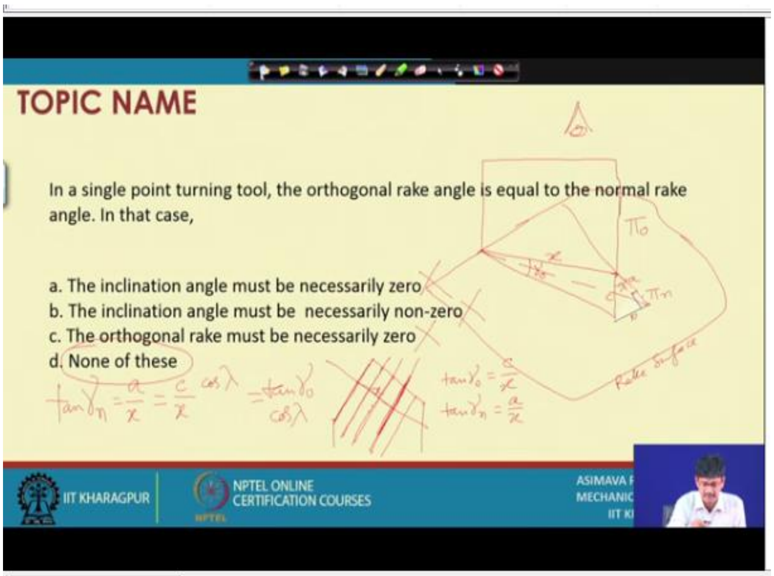

So, let us draw the relation between orthogonal rake and the normal rake if you have orthogonal rake coming out this way, this is the reference plane, this is the rake surface in the orthogonal plane, let me this is γ_o. In the same way the inclination angle is creating another intersection and not in inclination angle this is the incline in the normal plane. So, this is orthogonal plane.

This one is the normal plane. This one is the rake surface etcetera. On the rake surface orthogonal plane has come straight, vertical while the normal plane is likely inclined. Now, if you look at the plan view the rake surface comes has the cutting edge this way. So, that if I go on reproducing the cutting plane on the rake surface they will all come to be parallel and the cutting edge here makes right angles with the normal plane.

This is the reference plane, in this particular view this is the orthogonal plane, in this view the normal plane would not be visible. So, we are looking at like this. So, in that case we have a small triangle formed here, if we have a cut by the cutting plane a small triangle will be formed here and this is the intersection that we are talking about.

This line x is produced here. So, in this line what is this particular angle it must be 90^o because the cutting plane intersecting the rake surface will always produce intersections which are exact orientation of the cutting edge on the rake surface. These lines are all cutting edges. So, if this is the cutting edge, this must be 90^o because the cutting edge is always 90^o to the normal plane.

So, this is $90\,^o$. So, let us give it some names a, b, and c. In that case, we can quickly define $\tan \gamma_o$ equal to c divided by x and $\tan \gamma_n$ equal to what do we have here, a, a divided by x. Now, what about this angle? This must be λ because the inclination angle and the rake angle they are having this particular angle between them.

The normal plane, I am always using wrong terminology the normal plane and orthogonal plane are having angle λ in between them. Hence, this must be λ. So, we have a relation now between a and c I can write finally, let me write in bold here $\tan \gamma_n$ = a by x. Now, let us replace a, this a must be a component of c. $c \cos \lambda$ equal to $c \cos \lambda$ by x and c by x is nothing but $\tan \gamma_o$.

So, we have this as $\tan \gamma_o \cos \lambda$. Once this is proved we can tackle this question. Let us see the first one. The inclination angle must be necessarily 0. So, in a single point turning, the orthogonal rake angle is equal to the normal rake angle. The orthogonal rake angle where this one is equal to normal rake angle. So, $cos\ \lambda$ seems to be 1, if $cos\ \lambda$ is 1 then λ has to be a particular value. Now what how much is that? $cos\ \lambda$ is 1.

So, λmust be 0. But there is a big but here must be necessarily 0 not so. If the orthogonal rake and the normal rake they are both 0, that means $tan\ \gamma_o$ is 0, whatever be the value of λ, $tan\ \gamma_n$ will be 0. So, they will be equal. So, it is not correct. If rake angle is orthogonal rake angle is 0 normal rake angle will be 0, but inclination angle might not be 0, it can be any other angle.

So, therefore, the first one is wrong. The inclination angle must be necessarily non-zero. This is also not correct, because once again this might be having a value say 10^o. So, this will be having 10^o if this is zero. So, it cannot necessarily be non-zero. So, this can well be zero, this is also not correct. The orthogonal rake must be necessarily zero. This is also not correct.

Orthogonal might be 5^o, normal rake might be $5^o$and inclination angle might be 0. Therefore, this is also wrong. Hence, we come to the conclusion none of these. Actually it construes to this particular fact that if these angles are equal either the orthogonal rake is zero and the normal rake is zero and the inclination angle is just anything or the inclination angle is zero.

Orthogonal rake is nonzero normal rake is zero, nonzero. So, but they are equal or another case might be there orthogonal rake is zero, inclination angle is zero and normal rake is zero. So, the answer to this question is none of these. So, I did have some other questions also. But time is drawing here. Let me see, I think we can manage another one.

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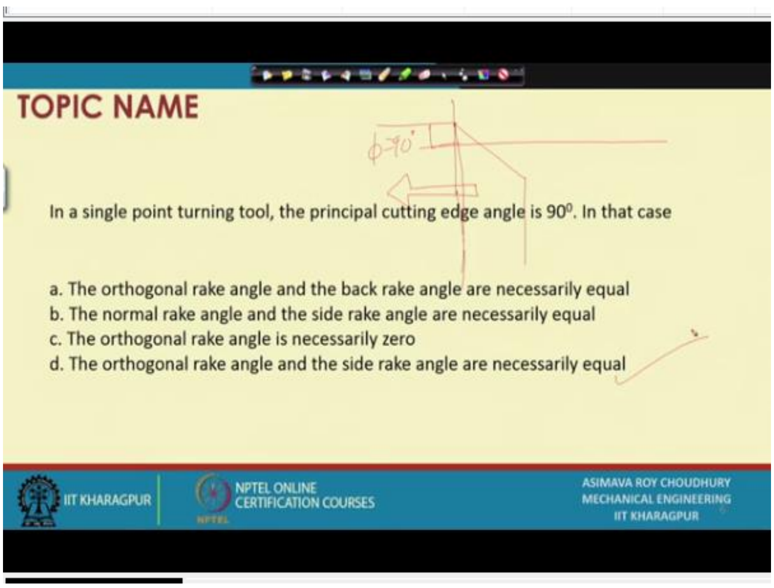

In a single point turning tool, the principal cutting edge is 90º. In that case, look. So let us draw a figure of a tool. This is it. A tool of this type, it is moving this way and this is 90º. That is it. So, in that case the orthogonal rake angle and the back rake angle are necessarily equal. Now what does this mean? Orthogonal rake is 90º to the cutting plane.

So, orthogonal rake must be in this section and the back rake angle is in this section. So, they are not going to be necessarily equal. They are completely different. The normal rake angle and the side rake angle are necessarily equal. Normal rake angle and the side rake angle, normal rake angle we do not have any clue, because inclination angle is not given. How much it is, we do not know.

So, this we cannot say they are necessarily equal, the orthogonal rake angle is necessarily zero. Now why should it be zero? We have not given any values. The orthogonal rake angle and the side rake angle are necessarily equal, this is correct. So, we put sign like this, this is correct. So, I think we have reached our limit. So, I will discuss some questions in the sixth lecture.

But unfortunately you would not be having access to that for answering the first assignment of the first week. So, in that case, I will upload them also. So, that if you want to have a quick look at them you can do that in the first week itself. Thank you very much.