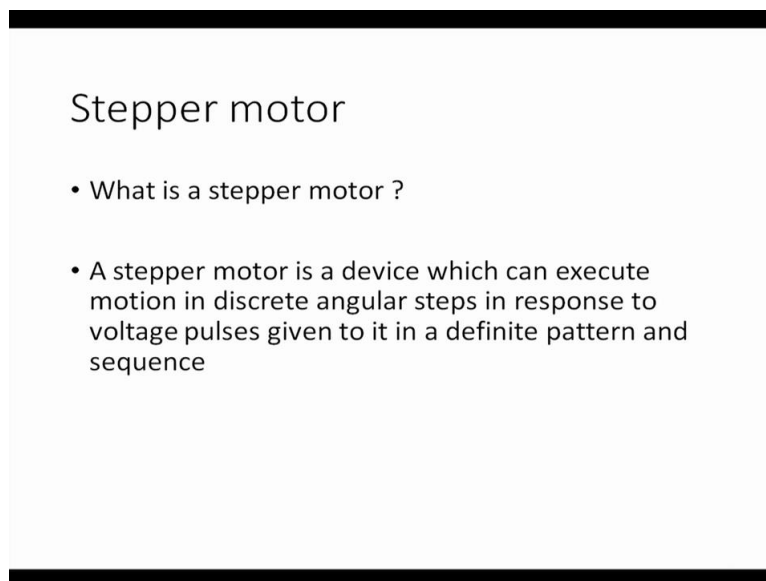


Computer Numerical Control of Machine Tools and Processes
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Lecture 06
Stepper Motor, Permanent DC Motor

Welcome to the 6th lecture of the online course Computer numerical control of machine tools and processes. Today we are going to discuss something about the prime movers stepper motor and permanent magnet DC motors. So to start with just have a look, what do we exactly mean by stepper motor in this case?

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Stepper motor


- What is a stepper motor ?
- A stepper motor is a device which can execute motion in discrete angular steps in response to voltage pulses given to it in a definite pattern and sequence

In this case, the stepper motor is a device which can execute motion in discrete angular steps in response to voltage pulses given to it in a definite pattern and sequence. So voltage pulses that mean voltage which is remaining high for a particular time and then going low this way, we are having voltage pulses and we will be giving them in a definite pattern that means several cables will be there, they will be carrying voltage pulses and in different combinations with respect to time. So if such a voltage pattern is given in a definite sequence the stepper motor will be able to take discrete angular steps that mean its output shaft will be rotating in definite angular steps.

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Stepper motors contd...

- In commercially available stepper motors, one rotation of the motor shaft is generally carried out in 200 steps
- Hence, 1 step is equal to 1.8 degrees



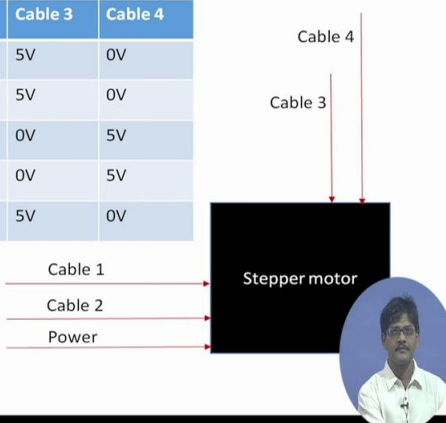
So we generally have stepper motor available which can carry out 1 complete rotation in 200 equal angular steps therefore, one step is = $360 \text{ degrees divided by } 200 = 1.8 \text{ degree}$, this is generally the standard angular step available with commercially available stepper motor. Other options are definitely there, but we will be generally dealing with this particular value.

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Input given to a stepper motor

Voltage pulses along cables in this pattern and sequence

	Cable 1	Cable 2	Cable 3	Cable 4
Signal	5V	0V	5V	0V
↓	0V	5V	5V	0V
	0V	5V	0V	5V
	5V	0V	0V	5V
	5V	0V	5V	0V



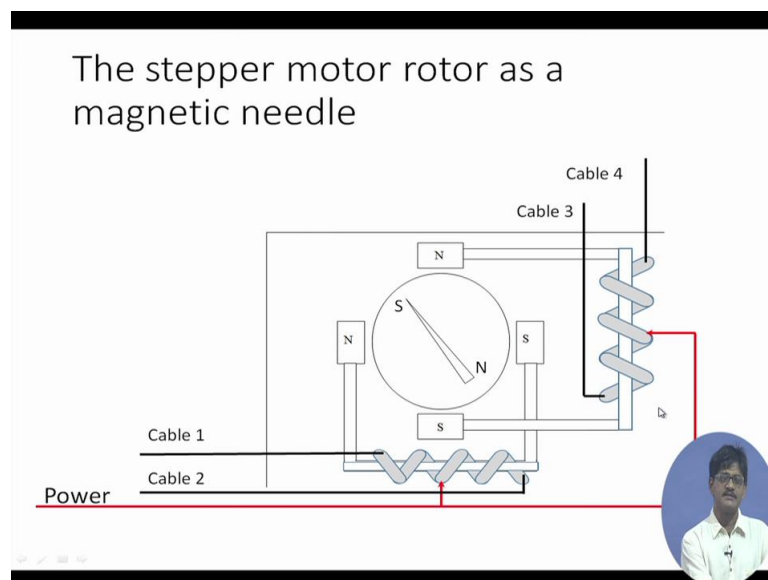
So the input given to a separate motor as we can see in the figure, there are 4 input cables and one power cable, what are these 4 input cables? We can see that these cables might be containing 5 volts signal or 0 volts. That means, first the pattern of voltage values given to the cables they are; cable 1 containing 5 volts, cable 2 containing 0 volts, cable 3 containing 5 volts and cable 4 containing 0 volts. This remains for a certain amount of time after which

they are changed to the second pattern that means cable 1 goes to 0 volts, cable 2 goes to 5 volts, et cetera et cetera.

We will notice that in this particular pattern one thing which is always remaining constant is that cable 2 is always containing the complementary function of cable 1. If cable 1 is 5 volts, cable 2 is 0 volts, so they are always having either 5 volts value or 0 volts, so they are binary signals and more over cable 2 is always the complement of cable 1. In the same manner, cable 4 is always the complement of cable 3, so we could have had only two cables carrying 2 basic signals and then we could have taken their complements and got the 4 signals shown here.

So this is the pattern, if we read it in say binary variables of 1 and 0 and designate 5 volts as 1 and 0 volts as 0, we would be reading this as 1010, 0110, 0101, 1001 and then again the 1st signal will be repeated and this will move on repeating, this cycle of 4 signal patterns will be going getting repeated over and over again. And what would be the response of the stepper motor? The stepper motors simply would be taking one step after another corresponding to every change of this particular pattern. So what is there inside this black box? Let us have a quick look.

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This is a simplified representation of what is going inside the stepper motor. So what do we have here? The rotor of the stepper motor is represented by this magnetic needle, south and north poles are shown here on the figure. The cables 1, 2, 3 and 4 which we had shown which we had seen in the previous slide, they are shown here once again and they are seem to be

connected to 2 coils, cable 3 and cable 4, they are connected to the ends of a coil wound around an electromagnet that means this is the horse shoe, complete horseshoe magnet which is shown here and there is yet another magnet shown on this side.

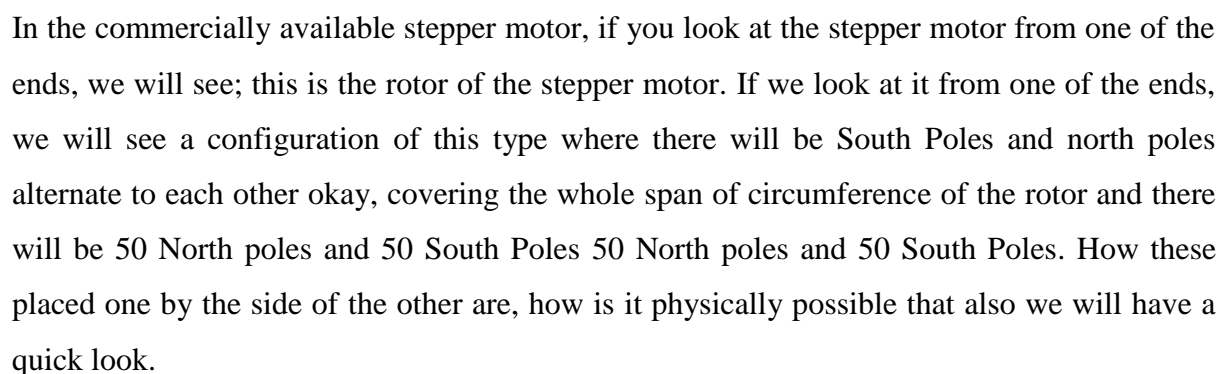
So basically there are 2 horseshoe electromagnets making up the stator poles. What cable 1, cable 2, et cetera, what are they doing? They are energizing the ends of this particular coil and the power connection, power means the one which is actually containing 5 volts and can deliver current that is connected in the middle. So what does it all amounts to? It amounts to this that if cable 1 is given 5 volts, there will be no flow of current in this arm of the coil. But if cable 2 at the same time is 0 volts, this arm of the coil will be energized and therefore it will create a definite kind of polarity in this particular horseshoe magnet.

So if cable 1 now changes the voltage state and becomes 0 volts, cable 2 will become 5 volts because they are complementary and therefore, at that time where in this arm there will be no passage of current while in this arm the current will flow and its direction will be opposite to the one direction which it had in the 1st case, so the polarity will switch and we will have south pole at that time here and north pole at that time there. So we understand that this way the polarity of these stator poles can be controlled simply by switching the voltage states of cable 1, cable 2, cable 3 and cable 4. How does the rotor which is a permanent magnet represented by this needle, how does it respond to this sort of voltage change I mean polarity change?

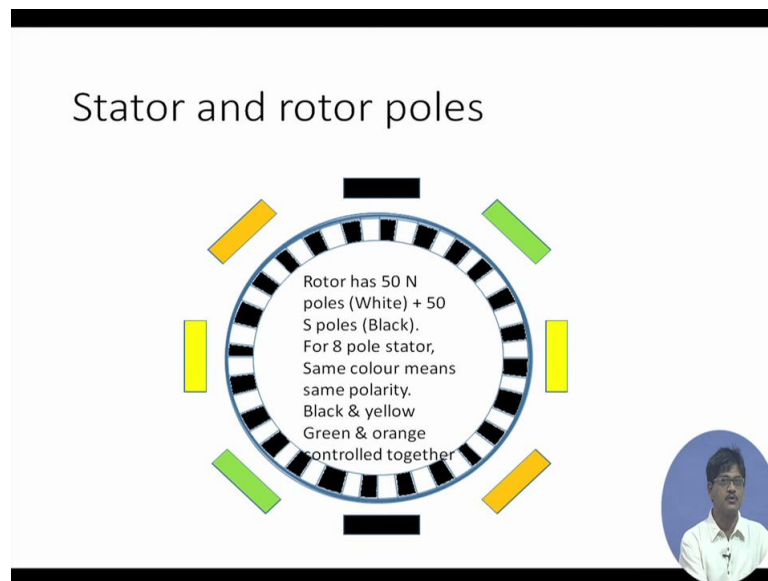
In the case shown here, when this one is the North Pole and that one in the North Pole, they will equally attract the South Pole which will be taking up a position exactly 45 degrees at with the vertical and horizontal and be electromagnetically locked in space, there is the most stable state of the needle in this configuration. After a switch of polarity due to the 2nd signal in which cable 1 and cable 2 have switched voltage levels, this will become a South Pole and this will become a north pole so that the same same condition of 2 north poles being side-by-side will occur here and the South Pole will have the most stable state in this position, okay.

South Pole here and North Pole here, that means there will be a switch of the needle to this position, which means that there will be a 90 degree step executed by the magnetic needle or the rotor. So this way if we constantly have a change in the signal which was shown there 4 signal patterns, the needle will go on taking 90 degree step and cover one rotation in 4 steps, but where does it lead us to? It leads us to the case of a simplified design in which that the signals are working for execution of rotation of the rotor, but with with large steps of 90

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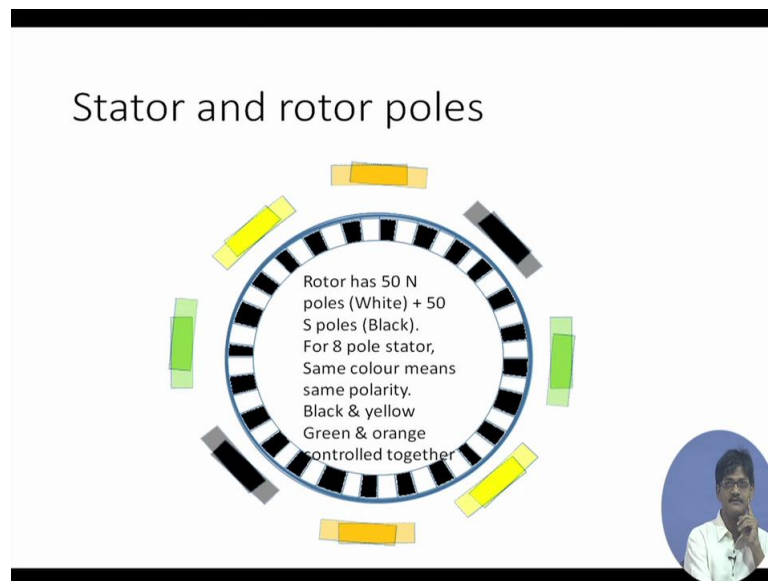
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So this shows that rotor has 50 north poles and the north poles are designated the white colour and south poles are designated black colour, I have not exactly drawn 50 north poles and 50 South poles in the drawing, but for the time being this explains how they are positioned. And on the stator we are having 8 poles, these poles are just like the previous case of horseshoe magnet, these poles are electromagnetically controlled by the 4 cable signals, in what way? The upper black and the lower black signals, they are always going to have same polarity that is why I have colored them with the same colour upper black and lower black.

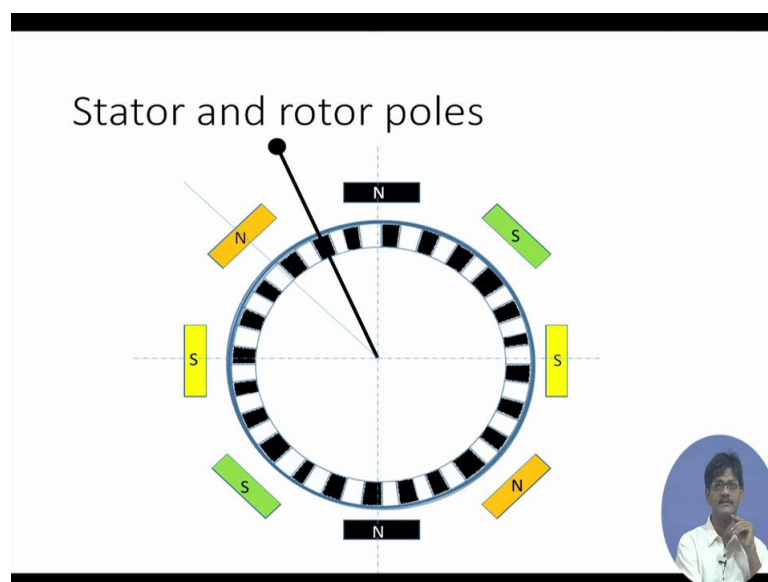
The side, that means the peripheral yellow colored boxes I mean the poles, they are always going to have the same polarity and further and they are going to have opposite polarity of the black ones, so black and yellow they are complementary in the same way, green and orange they are also going to be complimentary that means just the opposite of the other, so in this case we are going to have ultimately on the stator at any point in time if the cables are given the signals that we have discussed, there will be 4 north poles and 4 south poles so that we can well have say this one to be north, this one to be north in that case this would be South and that would be South and these 4 are controlled together.

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This say is north, in that case this would be north, that would be south and this also would be south and these 4 are controlled together. So the stator poles are controlled by those signals and they constantly shift in polarity so they would be looking some somewhat like this okay if they are changing very fast, they will seem to be like this. And the rotor will be following and having a rotation of this type. How this rotation is made possible because they seems to be more complex in comparison to the previous case.

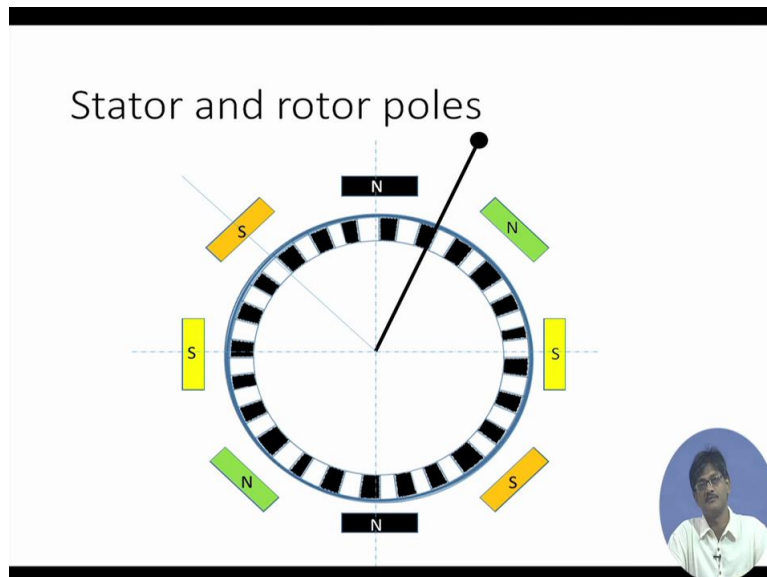
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What exactly happens this, at a particular configuration I mean particular moment in time, suppose this configuration is prevailing and we have 2 north poles at the top so that so this orange north pole is at 45 degree with the vertical, therefore at 22 and half we find the effect

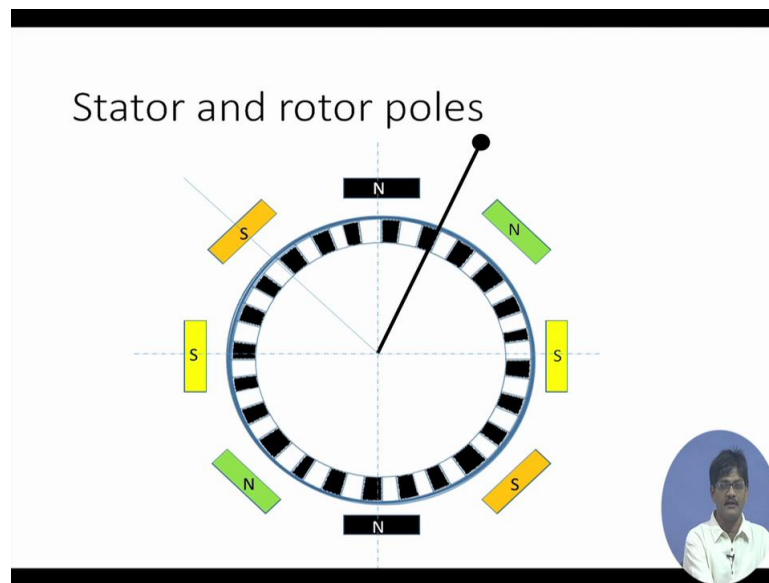
of the 2 north poles creating a stable position for the south pole on the rotor that means the rotor is equally attracted by these 2 north poles and it aligns itself to the field direction so that the black south pole shown on the rotor is exactly aligned meridionally with the black line shown, so this is a rotor is now electromagnetically locked in space.

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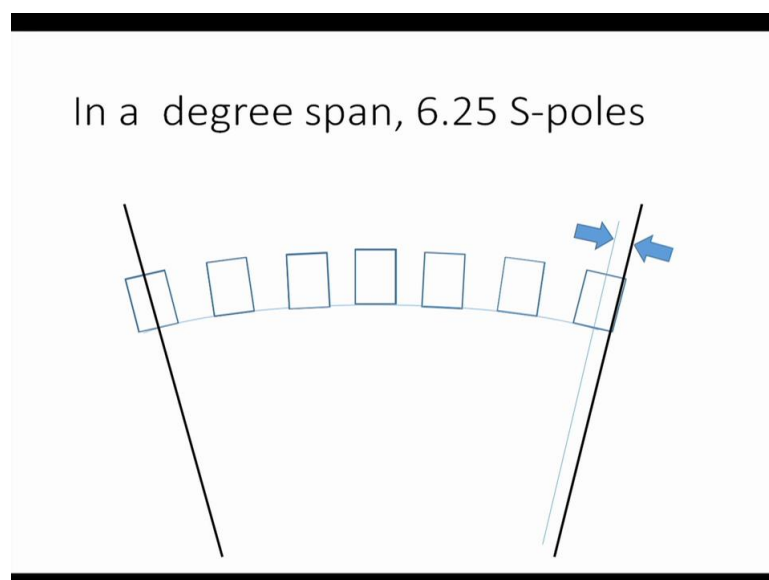
After this, suppose we have a change of polarity so that this thing occurs that means the north poles or the orange north poles have now changed to green north poles and the and the green north poles have shifted, the orange ones have become south poles. Naturally, the most stable direction for the south pole has shifted between the 2 north poles and we see that the nearest south pole on the rotor is not exactly aligned in this direction, there is a definite angular phase difference between the present location of the south pole in the vicinity of this particular black line and the orientation of the black line at 22 and half degrees with the vertical.

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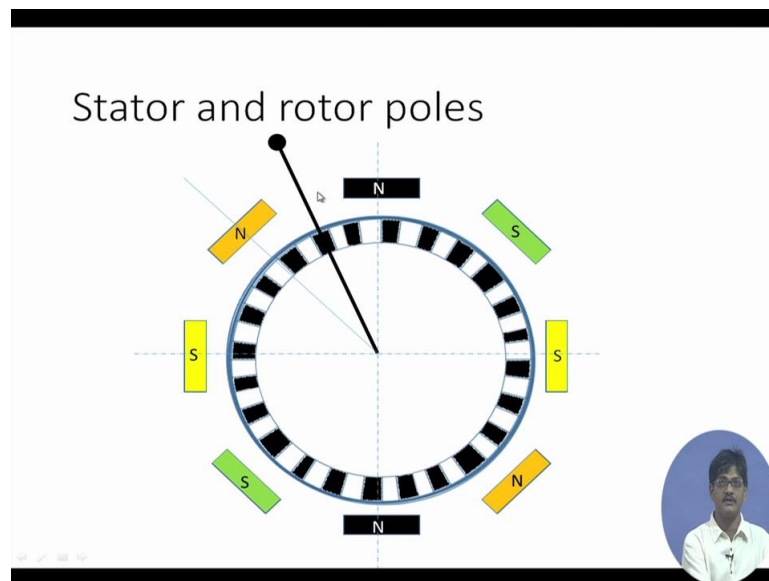
So what will happen is a nearest south pole to this black line will shift a bit so that it is again meridionally placed that means centrally located with respect to this line, a slight shift let us watch that is it that is the slight shift, let us go through a rewind. This is the position where the black South Pole is not exactly centrally located with this black line and therefore it shifts this much that is the shift and that is the step of this particular rotor. So let us have a quick look, let us have a calculation, how would be the calculation?

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In a sorry this particular, in a degree span it means in a 45 degree span if you notice the meridional line was shifting, the meridional line had a shift of 45 degrees just a minute.

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This particular line okay, the resultant field direction had a shift of 45 degrees, 22 and half on this side, 22 and half on that side so it encompasses 45 degrees in between these positions. So, in this 45 degrees span how many such south poles are residing, so 50 south poles are covering 360 degrees therefore, 6.25 South poles will be covering 45 degrees by unitary method. And therefore we can say that 0.25 that means one fourth pole of a pole will be shifting in order to centrally locate it with respect to the resultant field direction in the vicinity of those 2 north poles.

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Handwritten calculation on a blue background:

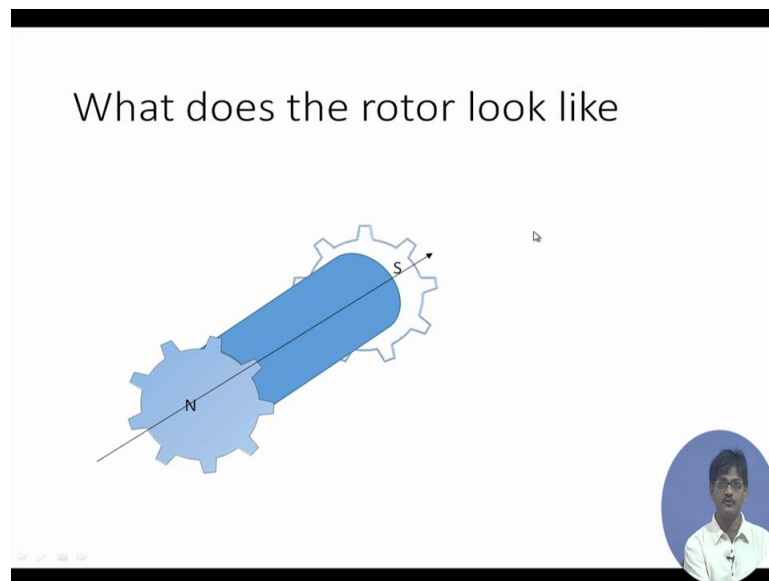
$$\begin{aligned} 360^\circ &= 50 \text{ poles.} \\ 1 \text{ pole} &= \frac{360}{50} \\ \frac{1}{4} &= \frac{360}{50 \times 4} = \frac{360}{200} \\ &= 1.8^\circ \end{aligned}$$

A small inset shows a person speaking.

Having understood this, how much is one fourth of pole? Well if 50 poles are making up 360 degrees, a single pole is 360 let us have a quick look on the calculation, so 360 degrees which

are making up 50 poles therefore, one pole will be $(360 / 50) \times 4$ equal to $360 / 200$ and there you have 1.8 degrees. So we have essentially established the fact that if I have rotation in this sort of a stepper motor, we are essentially going to have steps of 1.8 degree. Coming back to our discussion, but how do we attain that physical configuration of poles being alternate in position to each other in the circumferential direction?

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For that, if we have a look at the rotor of a stepper motor, it is basically a permanent magnet with a with 2 heads on 2 sides looking just like 2 gears with 50 teeth on each and they are having half a tooth stager okay. There is one angular stager of half a tooth so that in the line of sight if you look at the rotor from this side, you will find 1 North Pole to be residing exactly in the middle of 2 south poles on the other side of this particular rotor and the whole thing is a permanent magnet. So that this side as a whole is north, this side as a whole is south.


And therefore if you look at it from this side, you will see basically a sort of gear with north poles regularly placed or spaced and with south poles alternate to these north poles all round the circumference, this is how the shape which was previously shown is attained.

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Stepper motors – contd...

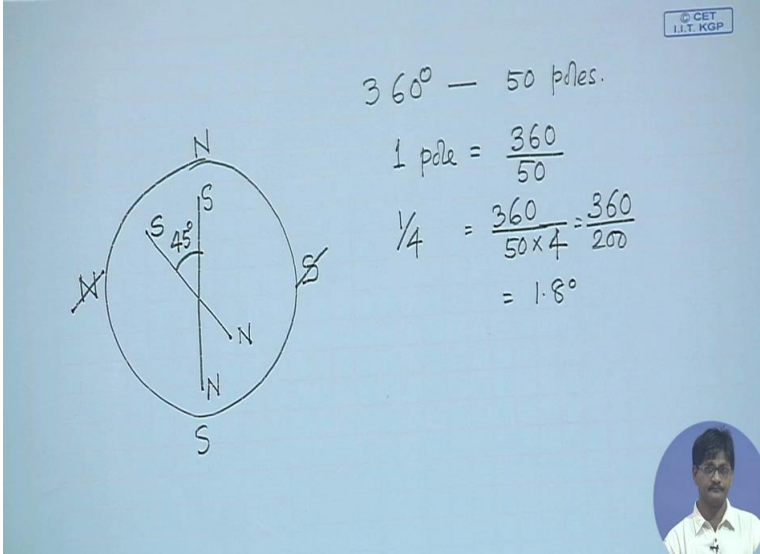
- How can we make a stepper motor rotate by 0.9 degrees ?
- By sequentially putting one signal pair off

	Cable 1	Cable 2	Cable 3	Cable 4
	1	0	1	0
	0	0	1	0
	0	1	1	0
	0	1	0	0
	0	1	0	1
	0	0	0	1
	1	0	0	1
	1	0	0	0




The stepper motor can also be rotated by 0.9 degrees. How do we do that, we can do it by sequentially or alternately switching off one pair of signals. As we have discussed cable 1 and cable 2, they are complement to each other so they can well be represented by 1 signal. So, after we have the 1st signal 1010, we can have the switching off of one pair of signals 0010. After that, 0110 and then again this pair is now switched off, so this way we can switch of alternately one pair of signals and achieve this way 8 signals, so that these 8 signals will be producing half a step.

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360° - 50 poles.

$$1 \text{ pole} = \frac{360}{50}$$
$$\frac{1}{4} = \frac{360}{50 \times 4} = \frac{360}{200} = 1.8^\circ$$


Let us have a quick look at simple model of 4 steps. This is my magnetic needle, this is south pole, this is north pole and this one the stator is going to have say at one point north and

north, south and south therefore, this is the starting position and after this I want to switch off one of the pairs of these poles, so out goes this north, out goes this south and therefore, this is simply going to take up the vertical position because the south is attracted directly to the only existing north pole. And therefore, instead of 90 degree movement which was being achieved by this simplified model, I am going to have a 45 degrees movement that means I am effectively dividing the steps into 2 and effectively achieving half a step.

But the problem is that the power to the motor is being half, and therefore the torque that it can handle becomes less, this is going out of the picture at that moment and therefore the torque that the motor can handle becomes much less, almost half of the previous case. But we can achieve a smaller basic length unit in this manner. Now for some discussion on permanent magnet DC motors.

For example, when a DC motor is rotating, we can handle some torque at output shaft and achieve a particular angular speed of the motor shaft and that angular speed is the parameter of interest to us, how much speed we are attaining. The problem with CNC machines is that whenever we are giving a command to the CNC machine that move along X axis by say 20 centimeters, the motor might be starting from stalled condition that means from completely static condition, it is not rotating at all but suddenly it has to start moving and say in the program we have we have given a command that move the table at say 50 millimeters per minute.

So in that case if you are moving at 50 millimeters per minute then it is expected that the motor should be speeding up and attaining this particular speed right from $t = 0$ from the 1st movement itself. Now that is very difficult because the motor has some inertia, it cannot be attaining 50 millimeters per minute from time $t = 0$, it will be requiring to accelerate and attain this particular moment. But if x motor and y motor they are working together, and if their response time are not the same that means the time required to achieve a particular fraction of the recommended speed or the final speed are different.

So if those respond times are not the same, we are going to have some deviation from the programmed path in the beginning and that means we will have some profile error. If we get profile error, the problem is that it is going to lead to some inaccuracy. Since we are not using some physical devices and depending upon ability of the of the motors or the prime movers to make the table move or make the cutter move along the along a particular predetermined path, if there is error or deviation due to unequal time response of the 2 axis, we are going to

have trouble. So let us have a quick look how this time response can be controlled and how it can be reduced because lower time constant will restrict the error to a very small portion of the profile.

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Basic definitions for DC motors

$\tau = \text{mechanical time constant} = \frac{RJ}{k_1^2}$ $T = \text{torque at the output shaft}$ $I = \text{armature current}$ $R = \text{armature resistance}$ $V = \text{Input voltage}$ $E_b = \text{Back emf}$	$k_1 = \text{motor constant}$ $\omega = \text{angular speed of the motor shaft}$ $\dot{\omega} = \text{angular acceleration of the motor shaft}$ $J = \text{polar moment of inertia}$
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These are the definitions like this T is torque, I is the armature current, R is the armature resistance, V is the input voltage to the two motor and the motor develops back EMF due to electromagnetic induction, etc, all these things we will be able to understand while we are going through the analysis.

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PMDC – Response to step voltage


$V - E_b = IR \quad \dots\dots\dots(1)$
 $E_b = k_1 \omega \quad \dots\dots\dots(2)$

Now, output power delivered at the motor shaft is input power less joules heating

$$T\omega = VI - I^2R = I(V - IR) = IE_b = Ik_1\omega$$

Which means $\rightarrow T = k_1 I \quad \dots\dots\dots(3)$

Last of all, $T = J \dot{\omega} \quad \dots\dots\dots(4)$



First of all, whenever a motor is rotating the voltage which is applied to the ends of the terminal or the armature that should obey Ohms law and therefore V should be $= IR$, but the problem is as the armature is basically a coil rotating in a magnetic field, it develops back EMF whose direction is always such to oppose the very cause to which it is due that means the voltage, so it develops a direction opposite to that of a voltage so this back EMF tends to reduce the effect of voltage on current so that it is just its direction is opposite and so the resultant voltage is $V - \text{back EMF}$ and this is current and this is armature resistance.

And the back EMF by Lenz's law will be directly proportional to the rate of change of flux that means the rate of change of the number of lines of magnetic force cut by the coil and that will be proportional to the speed at which the coil is rotating inside the magnetic field, which means ω angular speed. So back EMF is proportional to angular speed therefore, we can replace that and we can write $V - k_1\omega = IR$

Further the output power just like we have power is $= \text{force into velocity}$, in angular form we can write it as torque into angular velocity that must be $=$ the total power which I am investing in the system. So it can be deduced as follows

$$V - E_b = IR$$

$$\text{or, } V - IR = E_b$$

$$E_b = k_1\omega$$

$$T\omega = VI - I^2R = I(V - IR) = IE_b = Ik_1\omega$$

$$\text{Or, } T = Ik_1$$


So, we get T is equal to the torque is proportional to the current.

It is quite obvious, if you if you hold if you take a small motor and hold the shaft with force and put it on, you will find that current will rise and ultimately might burn off the armature. So we find the torque is proportional to the current and last of all we know torque is $= J$, polar moment of inertia into angular acceleration okay, Newton's 2nd law.

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PMDC motor – contd.

- Hence, by substitution, we get
- $T = k_1 \left(\frac{V - E_b}{R} \right) = k_1 \left(\frac{V - k_1 \omega}{R} \right) = J \dot{\omega}$
- $\frac{RJ}{k_1^2} \dot{\omega} = \frac{V}{k_1} - \omega$
- Re-arranging
- $\frac{RJ}{k_1^2} \dot{\omega} + \omega - \frac{V}{k_1} = 0$


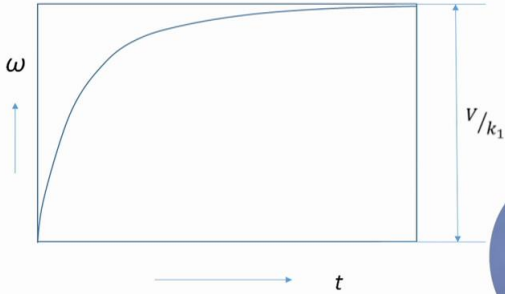


So after we substitute okay so we are simply substituting the value of $I = \frac{V - E_b}{R}$ and here from E_b we are getting $k_1 \omega$ and then we are utilizing the last equation that we discussed $= J \dot{\omega}$, so that finally after rearranging we get a differential equation of which describes the dependence of angular speed on the initial condition that means the initial voltage okay and armature constant, etc.

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Build-up of angular speed with time for step voltage

- If this is integrated with the initial condition at $t = 0$, $\omega = 0$ and $V = V$, we get

$$\omega = \frac{V}{k_1} \left\{ 1 - e^{-(t/\tau)} \right\}$$


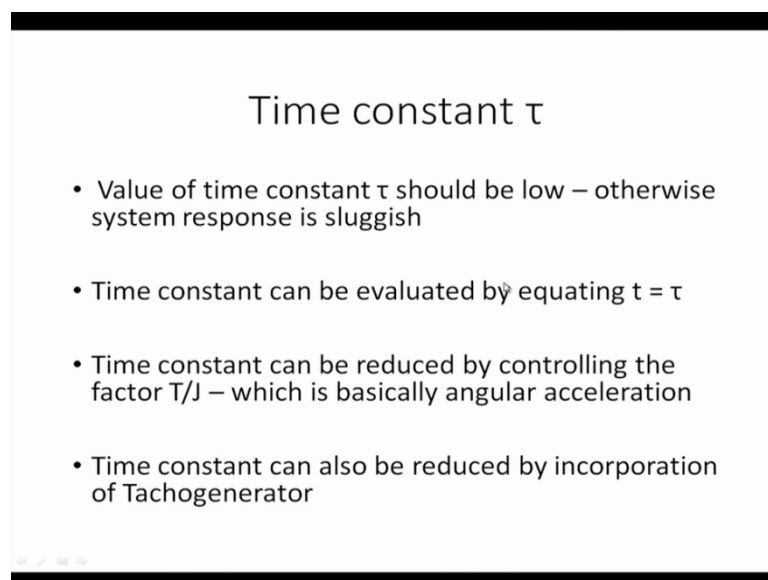
Rearranging the equation can be deduced as follows

$$\frac{RJ}{k_1^2} \dot{\omega} + \omega - \frac{V}{k_1} = 0$$

If this is integrated with initial condition that $t = 0$, the motor is in the stalled condition that means $\omega = 0$ and suddenly voltage is a step voltage applied at the input is V , we will find that the behavior of the motor is described by this particular relation. ω ultimate value is V/k_1 and with time it has an exponential growth okay and it is governed by this particular equation.

In this equation, this τ which is appearing here which is RJ/k_1^2 square, it is called the time constant and this time constant represents the time response of the system to an input. If this is low, we have a very fast response and if this is high, we have a sluggish response and we always want the response to be as fast as possible warning the case that it does not overshoot that is it should not overshoot, but it should be as fast as possible, so that is what we have just not discussed.

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Time constant τ

- Value of time constant τ should be low – otherwise system response is sluggish
- Time constant can be evaluated by equating $t = \tau$
- Time constant can be reduced by controlling the factor T/J – which is basically angular acceleration
- Time constant can also be reduced by incorporation of Tachogenerator

And time constant can be evaluated by putting $t = \tau$ which will give us e^{-1} and time constant can be reduced by 2 methods basically, one is by controlling the factor torque by polar moment of inertia or it can also be controlled or reduced by the incorporation of a tachogenerator in feedback, we will discuss this in the coming lecture, thank you very much.