Spur and Helical Gear Cutting
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Lecture - 39
Gear Hobbing - V

Welcome viewers to the 19th lecture of the course Spur and Helical Gear Cutting. So, last day when we were discussing, we were talking about the hand of the helix and relation between the circular pitch of gears to be cut and the pitch of the hob etc. So, let us start right away with those discussions.

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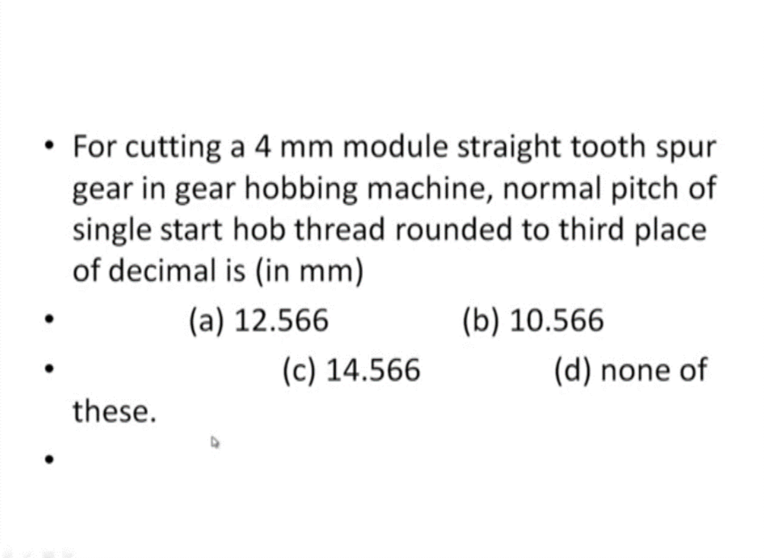

We were discussing last time this one. For cutting a 4 millimeter module straight tooth spur gear in gear hobbing machine, normal pitch, I made a small correction here, it was written previously pitch. So, I have made a change. Normal pitch of the single start hob thread rounded to third place of decimal is in millimetres. There are 4 options given. What do we exactly mean? If you have a look at the sheet of paper?

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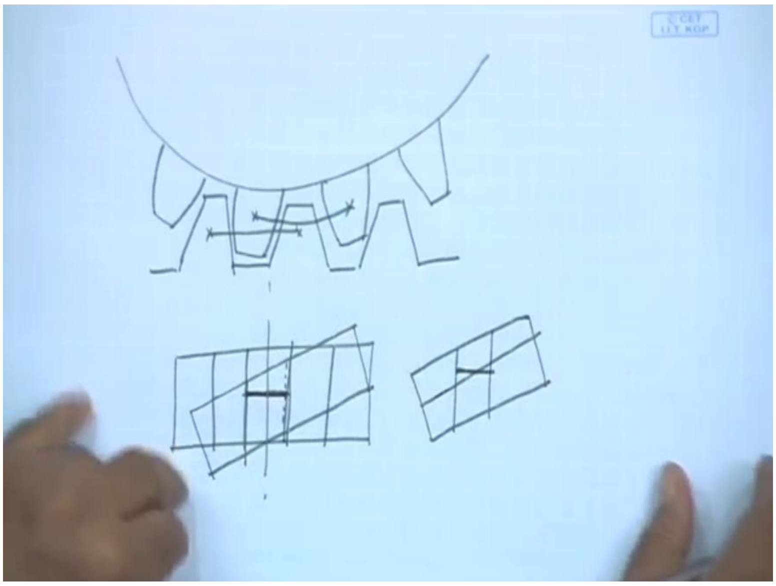

We have this sort of a relation existing; say this is the basic gear. And with this one we are drawing, these being the gear teeth, we have the hob which is actually a worm with cutting edges. This sort of a relation is existing, which means the circular pitch is equal to the linear pitch, but if you remember, in the other view, the worm gear being like this is being a spur gear, the teeth are of this type and the hob has to be at an angle.

So that on that side the thread matches in direction in alignment with the teeth of the gear. So, basically this distance that we have shown as circular pitch, circumferentially this distance has to be equal to the distance between the threads and that means the normal distance between the threads. So, if we look at the hob alone, it is this distance not the actual distance.

So, we are not talking about actual pitch, we are talking about the normal pitch. Normal pitch has to be equal to the circular pitch of the gear. This is the equality which has to be maintained so that they will be in mesh. This particular equality does not necessarily have to be satisfied in all cases, but it is the general case. So, we have the circular pitch equal to $\pi \times$ module, how do we get that?

Let us take a fresh sheet of paper.

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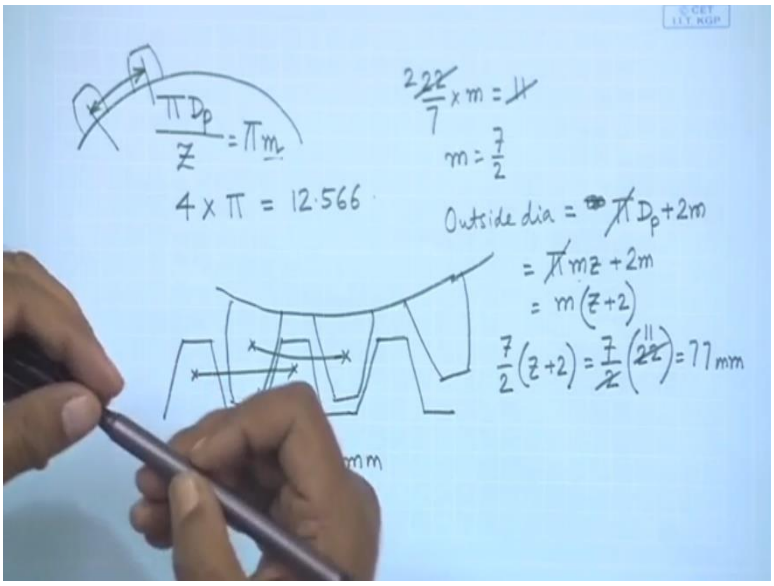

So, circumference being equal to:

$$\frac{\pi D_p}{Z} = \pi m$$

So, that means this being a spur gear, this is equal to simply the module and this is multiplied by π. This is the distance that we are talking about these other 2 teeth. This has to match with the normal pitch of the hob.

And therefore, we have. What is the module? Module is 4. So,

$$4 \times \pi = 12.566$$

and some trailing numbers. So, we find if we come back to the question now. The options are 12.566 millimetres, 10.566, 14.566, none of these. Therefore, a 12.566 is the correct answer. Normal pitch of a single start hob thread. Once we have understood this one, let us move on to other questions. Linear pitch of a rack is 11 millimetres.

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- Linear pitch of a rack is 11 mm. It is in mesh with a pinion having 20 teeth. Outer diameters of the pinion is
- (a) 67 mm (b) 77 mm (c) 87 mm (d) 70 mm

So, let us draw a rack. It is in mesh with a pinion having 20 teeth. Outer diameter of the pinion is? So, this is the pinion etc. So, we need to find out what is the outer diameter of the pinion? Now, as they are meshing therefore, linear pitch of the rack must be equal to the circular pitch of the gear. Circular pitch once again is $\pi \times$ module.

$$\pi \times \mathrm{m} = 11\mathrm{mm}$$

$$\frac{22}{7} \times \mathrm{m} = 11$$

$$\mathrm{m} = \frac{7}{2}$$

$$\text{Outside diameter} = \mathrm{D_p} + 2\mathrm{m}$$

$$= \mathrm{mZ} + 2\mathrm{m}$$

$$= \mathrm{m}(\mathrm{Z} + 2)$$

$$\frac{7}{2}(\mathrm{Z} + 2) = \frac{7}{2}(22) = 77\ \mathrm{mm}$$

So, coming back here, let us see, b is the correct answer, 77 millimetres. So, once we arrive at that let us move on to the second question.

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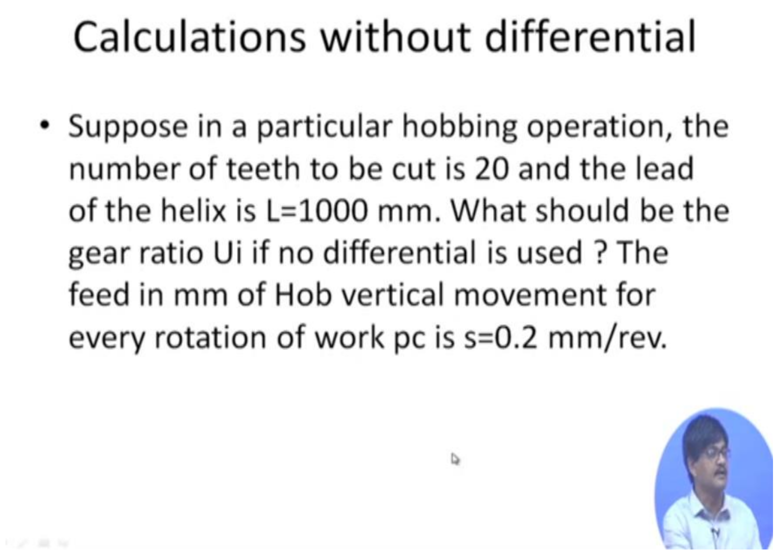

I mean third one, calculations without differential. Last day we had identified a problem that is the differential really necessary and we had a theoretical discussion at the end of which we had reached the conclusion that if we have a machine without differential there will be gear ratios which will be very difficult to implement. So now, we will follow this up with a numerical problem that is why is the differential becoming absolutely essential for the gear hobbing machine.

Suppose in a particular hobbing operation the number of teeth to be cut is 20 and the lead of the helix is 1000 millimetres. What should be the gear ratio U_i if no differential is used? The feed in millimetres of hop vertical movement for every rotation of the work piece is s = 0.2 mm/rev of the work piece. So, what do we have? We have a helical gear being cut its number of teeth is 20.

The lead of the helix is 1000. And U_i has to be found out, no differential is used and the feed and millimetres of hop vertical movement is 0.2 millimetres per revolution.

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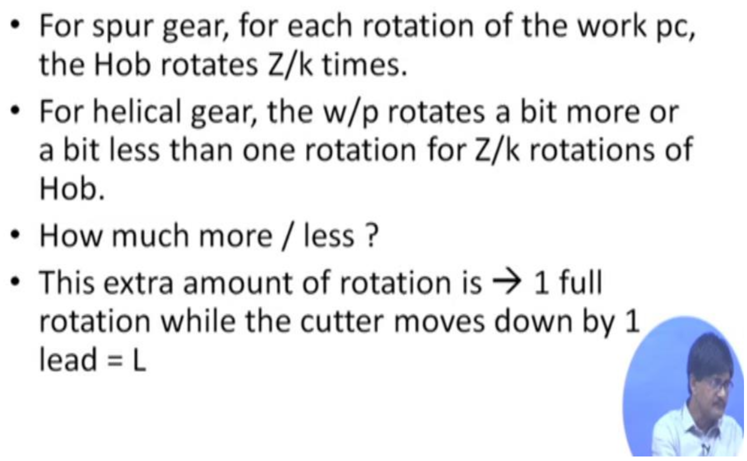

So, what we say is that if we are cutting a spur gear of 20 teeth, then for every rotation of the work piece, the hob rotates $\frac{Z}{K}$ times.

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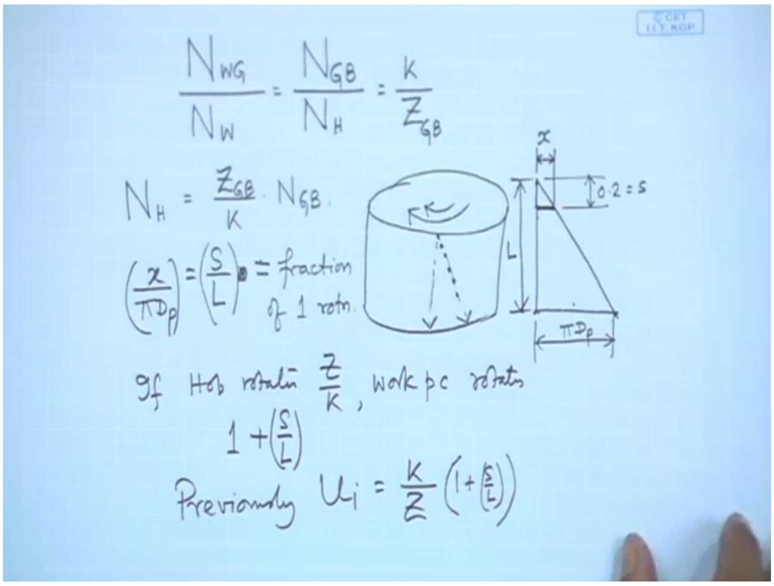

Now, you might ask me. What is that? How do we get that? So, if we come to the piece of paper, we will have:

$$\frac{N_{WG}}{N_W} = \frac{N_{GB}}{N_H} = \frac{K}{Z_{GB}}$$

This is accepted. $\frac{K}{Z}$. Now, if we are cutting a spur gear of Z number of teeth with K number of starts of the hob. In that case, I can say that therefore:

$$N_H = \frac{Z_{GB}}{K} N_{GB}$$

So, if this is 1 for every rotation of the gear blank. N_H will then definitely be equal to $\frac{Z}{K} \times 1$. So, if I have to have 1 rotation of the gear blank, I have to rotate N_H that is the hob rotation has to be equal to $\frac{Z}{K}$ .This is understood for spur gears. We have done it so many times. But now, the situation is changing in this manner that in order to cut a helical gear, we are deciding that every rotation has to have an additional rotation of the blank.

So, that this cutting will not proceed this way, but the cutting will be proceeding say this way. And in that case, we will be putting some extra rotation of the blank and that will lead the cutting action, the downward motion, and it will be proceeding next point of cut will be here like that. What is this extra rotation? In the theoretical discussion that we had we came to the conclusion that this extra rotation is related to the downward motion in this manner. This being equal to πD_p, this being equal to lead (L).

If you move down by S that means a 0.2 millimetres per rotation, 0.2 = S. Then this much is the amount of rotation that is required to be added. So, with the downward movement taking place this way, the extra rotation is this much. I am moving down this you maintain this extra rotation. So that at the end of the day I moved down by L and the extra rotation is 1 rotation. So, mind you this is extra rotation.

This is already rotating to maintain N = $\frac{Z}{K}$ and work piece rotation is equal to 1 that is already maintained. The extra rotation takes place this way or rotation that is taking place, I am moving down this much, you add this much. So, how much is this? So, from similar triangles we can easily find out this x amount of rotation must be equal to $\frac{S}{L}$.

$$\frac{x}{\pi D_p} = \frac{S}{L} = \text{fraction of 1 rotation}$$

This is a fraction see this is one full rotation a part of it. So, the fractional amount of rotation is nothing but $\frac{S}{L}$. So, if this be so, therefore, now, if we can make a statement, if hob rotates $\frac{Z}{K}$, work piece rotates $1 + \frac{S}{L}$ that is it. This is the extra amount of rotation in expressed in fractions of rotation, one full rotation, fraction of it. Hence, the gear ratio previously $U_i = \frac{K}{Z}$.

U_i was equal to, if you started from hob and you went through all the gearings and say all the other gears and belts and bevel gears etc., they were 1:1. You would have had this to be $\frac{K}{Z}$, $\frac{\text{Output}}{\text{Input}}$. Now, there is an extra rotation added because the output is not 1 but 1 + $\frac{S}{L}$. U_i is equal to output rotation which means gear blank rotation, by hob rotation and gear blank and tradition is not 1 anymore, it is 1 + $\frac{S}{L}$.

$$\text{Previously } U_i = \frac{K}{Z}(1 + \left(\frac{S}{L}\right))$$

Previously we were putting 1. So, in that case, let us take a practical example, this particular numerical problem.

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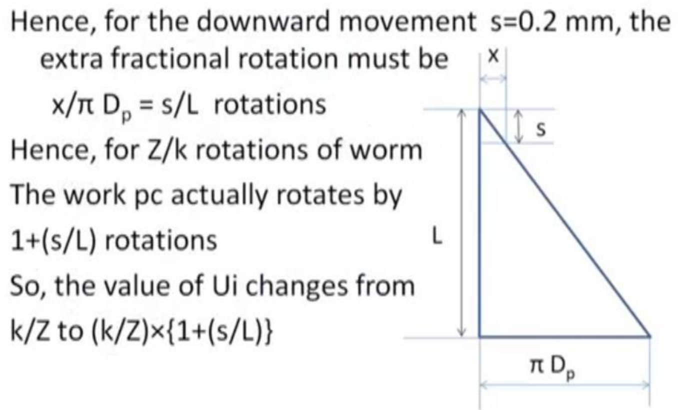

All these things we have already discussed.

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- Ui = (1/20)×{1+(0.2/1000)} = {1000.2/20000}
- = 10002/200000=10001/100000
- This ratio is quite difficult to achieve through ordinary gear box and use of differential becomes necessary

$U_i = \frac{1}{20}$. That means, $\frac{K}{Z}$, K single start worm and number of teeth to be cut is 20. This is 20, $\left(1+\frac{S}{L}\right)$, so 1000 comes in the denominator. So, that we have this one to be equal to 1000.2 in the numerator and 1000 at the denominator, this 1000 gets multiplied by 20.

$$U_i = (\frac{1}{20}) \times (1 + \frac{0.2}{1000}) = \frac{1000.2}{20000}$$

I cut it. I remove the decimal so it becomes:

$$U_i = \left(\frac{1}{20}\right) \times \left(1 + \frac{0.2}{1000}\right) = \frac{1000.2}{20000} = \frac{10002}{200000}$$

which is equal to 10,001. Wait a minute, have I made a mistake? Yeah, there is a small mistake here. Please look at this.

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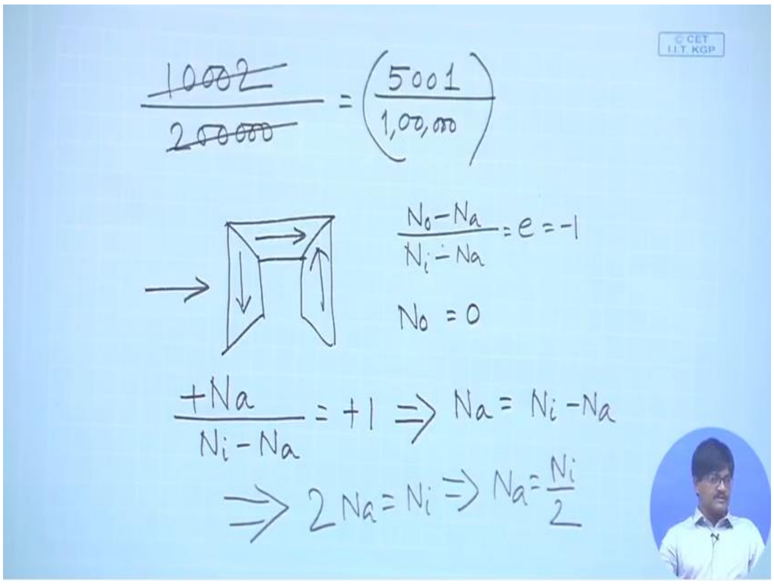

$$\frac{10002}{200000} = \frac{5001}{100000}$$

This is quite a difficult fraction to attain. How do we attain this fraction by gear ratio? First of all, we will try to divide it into products, if they allow and after a lot of permutation, combination, the gear ratio it will be difficult to obtain.

Instead of that if you employ the differential system that means there will be lead change gears and the differential. The problem of say achieving a particular number of teeth and problem of achieving a particular helix angle they become totally different. They do not remain an integrated problem but they become totally different from each other. They become mutually exclusive.

So, this is the reason to avoid such difficult gear ratios. We go for finally the differential instead of non-differential system.

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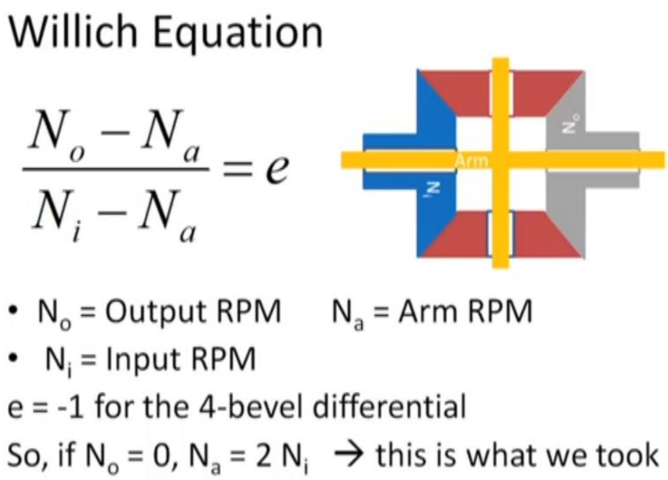

Next, the Willich Equation. The 4-bevel differential. What did we do when we came across this particular mechanism? We said that we were be divided it up into 2 different cases where we said that if this be, this is written slightly upside down, N_i input, I am giving the input here.

I am giving the second say input through the arm rotation and I am getting an output here. I can also give this one as a second input; this one is the first input and get the arm rotation as an output. This is what we had done in the actual numerical problem on gear hobbing that we had discussed. Now what are we trying to do? We are saying that at that time we had roughly estimated the rotation of the arm by thumb rule method.

We can use the Willich equation in order to actually calculate what will be the arm rotation if one of them is held steady. Suppose this N_i is equal to 0 and N_o is equal to some particular value. The arm rotation in our numerical problem was the output. So, the Willich equation says that the output rotation with respect to the arm rotation divided by the input rotation with respect to the arm rotation will bear a constant ratio which is obtained by keeping the arms steady and rotating the system.

So, if I rotate N_i looking from this particular direction, suppose it is rotating clockwise and this is rotating counter clockwise. Let us draw a figure and show it. Suppose this is gear one, this is gear 2 and this is gear 3. So, if it is rotating say this way, if it is rotating this way, this will rotate this way. So, that if I look from this direction, I will find that these 2 are having different sense of rotation.

So, coming back to this problem, e is defined to be if the arm is kept steady, they will simply have the same rate of rotation, but the sense will be opposite. So, that e = -1 for this particular 4-bevel differential, where the bevel gears are all of the same number of teeth. So, if e = -1 and if we keep 1 of the bevel gears to be fixed, so, I put $N_o = 0$ for example.

So, I am giving an input N_i, I am keeping N_o to be 0. And I am trying to find out the arm rotation and you will find it will be $\frac{-N_a}{N_i - N_a} = -1$. Therefore, if you solve it, it will become let us solve it quickly.

$$\frac{N_o - N_a}{N_i - N_a} = e = -1$$

$$N_o = 0$$

$$\frac{-N_a}{N_i - N_a} = -1 => N_a = N_i - N_a$$

$$2N_a = N_i => N_a = \frac{N_i}{2}$$

That is what we had taken up at that time and by applying Willich equation we can actually calculate it out to be this much. So, this is the bevel differential and this is what is used there and when you have 2 different rotations given. Suppose N_o is not 0. In that case what will be the arm rotation as a result?

It will be simply equal to $\frac{N_i+N_o}{2}$. So, it will be basically as we had stressed at that time an algebraic addition of the inputs divided by a particular factor as we have understood here the factor is 2. That means $N_a = \frac{N_i}{2}$. I have written it wrongly here please correct in the calculations that we have done it has been done correctly. When I distribute this PPT in PDF form I will make this correction.

So, we understand that arm will be always equal to half of this input and in the same manner arm will be equal to half of this input. If this is put on and this is put off and both of them when they are acting it will be the algebraic addition of the arms of these 2 input RPM's. So, as we had claimed this does a sort of addition, this bevel differential. We will take a short break from here to have look at how this actually operates.

(Video Starts: 24:48) So, let us see. This is the arm and that is one of the bevel gears at the top. This is put off and the bottom one is rotating. Once again let us see, it is rotating. See the arm, this is rotating and the bottom one is not rotating with it but it is having a separate rpm. Unfortunately, I could only get hold of this particular differential where the bevel teeth they are not the same.

That means the bevel gear diameters are not the same but anyway something is better than nothing. So, this is what we had taken consider in our setup. **(Video Ends: 25:34)**. Coming back, let us start again. So, this one is now understood and accepted.

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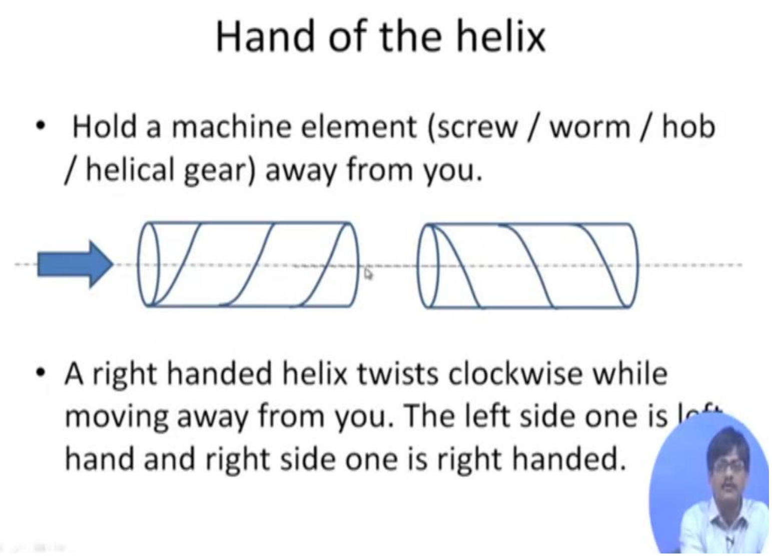

Now comes, we had a discussion on the hand of the helix. How do we decide upon the hand of a helical member? The basic procedure is, hold the machine element, it might be a screw, a worm, a hob, a helical gear away from you that means this particular arrow is designating that you are here and you are holding this away from you. It is going away from you.

If it is a right handed helix, it twists clockwise while moving away from you. So, this is moving away from you but in which direction is it twisting? Obviously, it is twisting counter clockwise. So, this cannot be a right handed helix. What about this one? This one is rotating this way, clockwise while moving away from me. So, this must be right handed. So, the left side one is left-handed and the right side one is the right handed and they cannot be mutually changeable.

If you put it the other way around, it does not become right handed. No. So, this one is right and this one is left hand helix. Let us take a problem.

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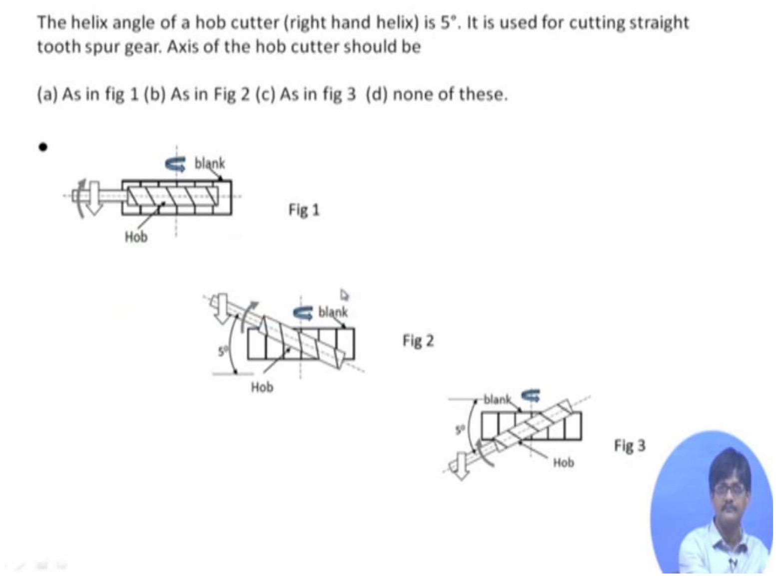

I hope you can read it. The helix angle of a hop cutter a right handed helix is 5 degrees. It is used for cutting straight tooth spur gear. Axis of the hob cutter should be? As in figure 1, as in figure 2 and as in figure 3. Which one is correct or none of these? So, let us look at these. This one the axis of the hob is crossed with the axis of the blank that means at 90 degrees. But this will not do because the thread of the hob has to align itself in the direction of the teeth being cut.

Here, they will be at an angle. Here, the thread appears to be aligned with the teeth of the gear but the problem is, these are the threads that you can see so they are away from the gear. The other side of the thread which is facing the gear teeth, that is not aligned with the gear teeth. Why? Because it is moving this way, it is crossed, it is not vertical. So, this is also not correct.

This one is correct, because here this is the part of the thread which is not visible to us, but it is definitely in line with the teeth, where it is meeting the teeth in contact with the teeth. And hence, the answer is as in figure 3.

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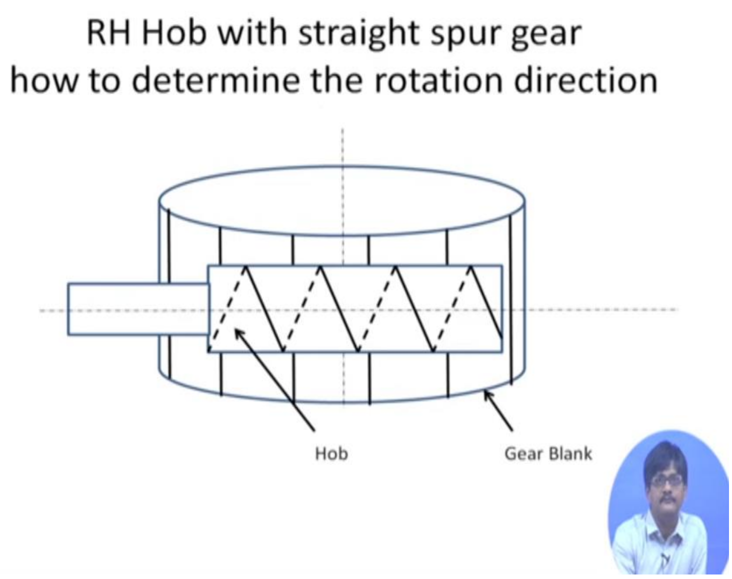

Now coming to this. This problem also we had identified. How do we decide, whether a particular hob is correctly aligned with the work piece or gear blank? For that let us see we are taking the case of a spur gear with straight teeth and this is a right handed helix. So, the dotted one is the one which is touching, on the side of the gear teeth in contact with the gear teeth.

The basic rule is that these dotted threads, traces of the thread, which is in contact with the teeth after proper positioning or orienting of the hob. These dotted lines should align themselves with the potential teeth. The teeth which are going to be cut the thread has to get aligned, the dotted thread it has to be aligned with those teeth.

So, here obviously the dotted thread should be vertical. Now, let us try this out. Is this direction correct? No, not at all. It is becoming more and more horizontal, the dotted threads. So, this is not allowed.

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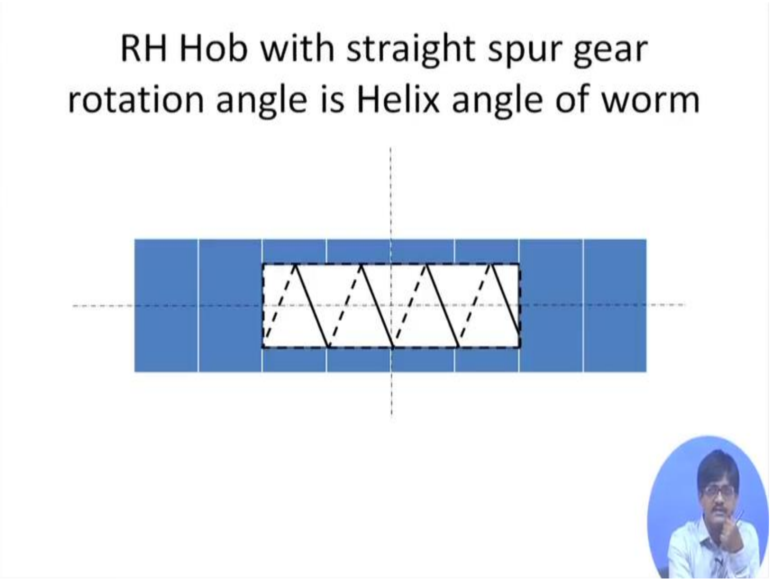

Second case, let us try it out. The blue one is the gear to be cut. And these white vertical lines, they are the traces of the teeth. Now, let us give it a rotation and see what happens. Yes, this seems to be it. Therefore, the dotted lines are matching with the direction of the teeth being cut. And therefore, we say that for cutting a spur gear with right hand hob, the rotation angle is simply the helix angle of the worm.

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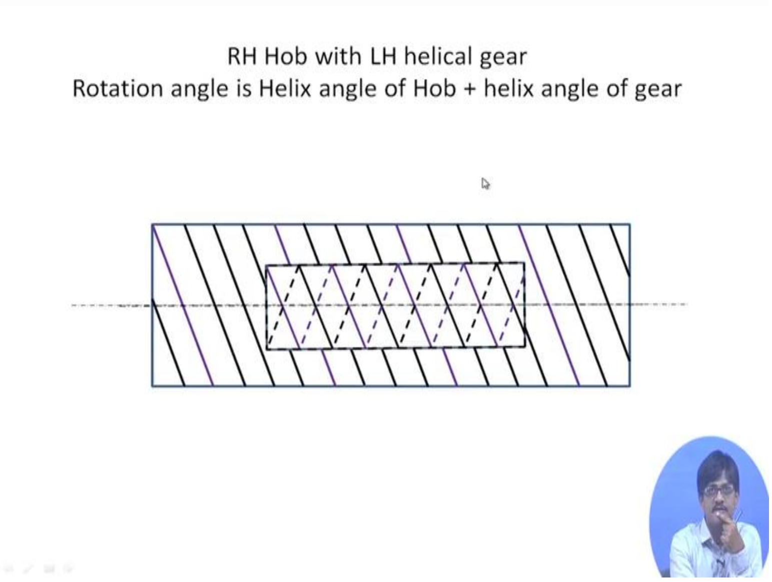

Second case, there can be so many combinations. I am having a right handed hob. And I want to cut a left handed helical gear. So, this is the axis of the hob, hob is rotating about this axis. And what are these threads, they seem to be different? Yes. To add to the variety, I have incorporated a hob with 2 starts. Can you see there are 2 starts? It hardly makes a difference.

You can definitely use a 2 starts hob, there is nothing wrong with that. And in fact, you will be more confident after going through that sort of a problem. So, is this right handed hob? Yes, it is moving away from you as it rotates clockwise. The dotted part is on the other side of the hob. What about the gear? If you are here, as it moves away from here, it rotates counter clockwise.

So, this must be a left handed helical gear and the axis of rotation which has not been drawn, it is in this direction. So, how do I rotate it? The golden rule once again is that get those dotted lines aligned with the direction of the teeth. So, this is the correct rotation. So, you have to give large amount of rotation here which happens to be equal to as written above helix angle of the hob + helix angle of the gear.

Please remember helix angle of gears, they are expressed with respect to the slant of the teeth with the axis of the gear. So, this is for right handed hob with left handed helical gear.

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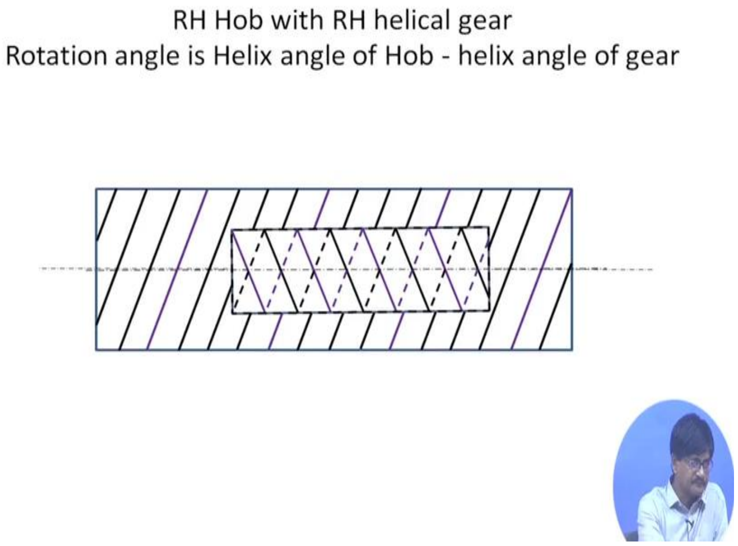

Another combination right handed hob with right handed helical gear. Here, the teeth appear to be almost aligned. And if they helix angles are equal, they will be perfectly aligned. But that might not be the case, the helix angle of the hob might be 2 to 3 degrees or something like that while the helix angle of the gear might well be different.

So, the angular rotation ultimately is the difference in these helix angles. See the slight amount of rotation which is required to make these teeth exactly get aligned with the gear teeth that is it. So, it is a difference, which has to be the amount of rotation.

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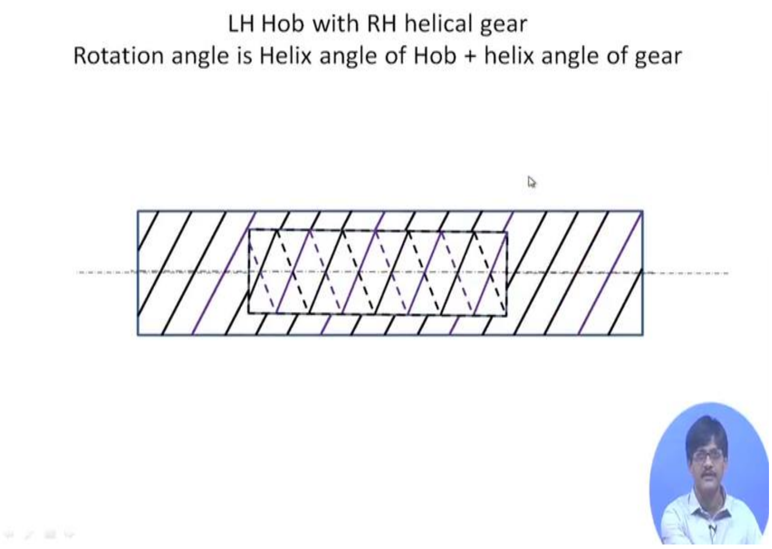

Taking another case. What is that? Left handed hob. How do I know it is a left handed hob? While it is rotating and moving away from you, it is rotating essentially in a clockwise direction that is good. And what about this particular thread? That is right hand thread, because while moving away from you, it has to have rotation in a clockwise manner when you are looking at it from this side that is good.

So, how much is the rotation? Let us try this out. So, once again, quite a bit of rotation is required and this comes out to be the helix angle of the hob + helix angle of the gear.

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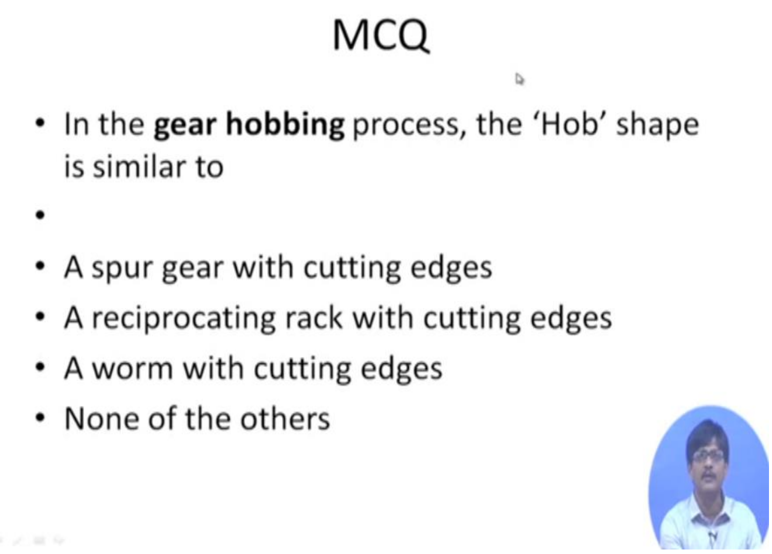

So, let us end up with a couple of multiple choice questions. In the gear hobbing process, the hob shape is similar to a spur gear with cutting edges. No, a reciprocating rack with cutting edges not exactly, a worm with cutting edges. Yes and of course, none of the others, in that case, is not applicable. So, in this case, the correct answer is a worm with cutting edges.

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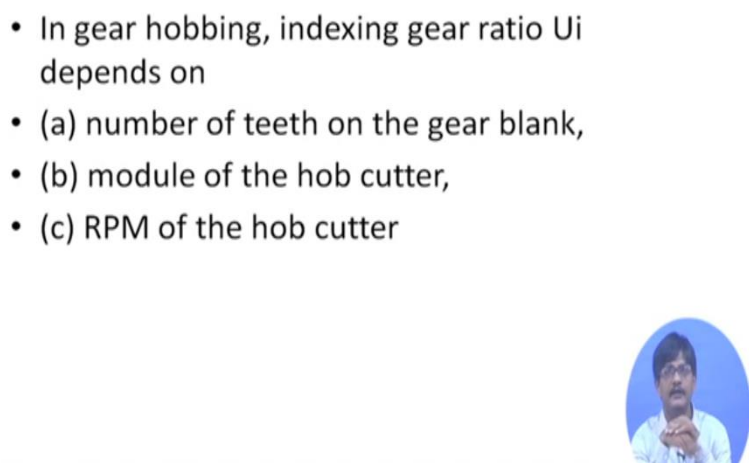

In the gear hobbing indexing gear ratio U_i, depends on? Number of teeth on the gear blank, module of the hob cutter and rpm of the hob cutter. Let us see. What is it dependent upon? Let us write down here.

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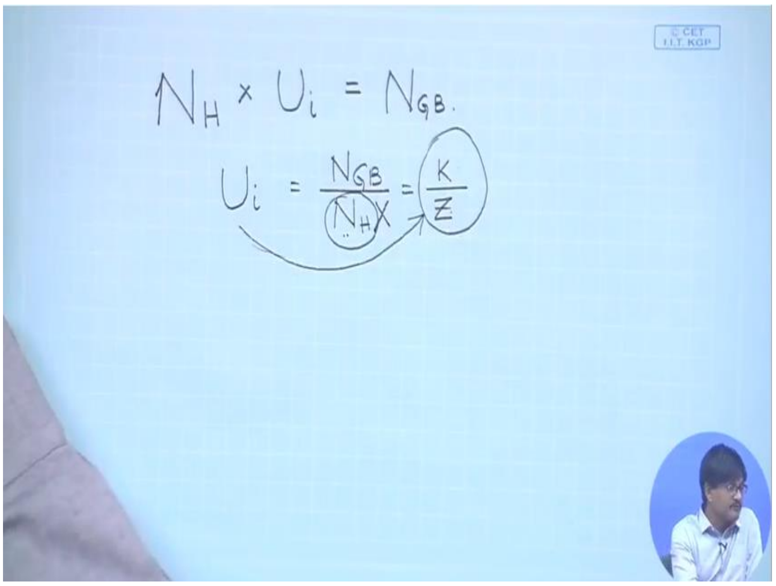

We know that starting from the hob rotation, suppose how much, rotation of hob (N_H) × U_i must be equal to, we are considering that there are no other machine elements in between no gears, no belts, no bevel, no worm, worm gear etc.

$$N_H \times U_i = N_{GB}$$

$$U_i = \frac{N_{GB}}{N_H} = \frac{K}{Z}$$

So now, we understand that U_i is equal to this one. It is a constant. $\frac{K}{Z}$. So now, if I say that it depends upon the number of rotations of the hob, I can definitely say, No, it does not. You change the rotations of the hob; N_{GB} will also change so that this will be untouched if you double N_H. Do you mean to say that $\frac{K}{Z}$ will change? No. So, it does not depend upon the number of rotations of the hob.

Does it depend upon Z? Yes, if you are cutting a different number of teeth, you will find that U_i will change. So, let us come back to the problem. Does it depend upon the number of teeth on the gear blank? Yes, module of the hob cutter. Well $\frac{K}{Z}$ did not contain module of hob cutter, rpm of the hob cutter just now we showed that rpm of the hob cutter is not going to affected. It is only equal to $\frac{K}{Z}$.

Therefore, the correct answer is number of teeth on the gear blank. So, with this, we come to the end of the 19th lecture. We have one more lecture left and we will pick up our loose ends in that one. Thank you very much.