Elements of Metal Cutting, Machine Tools, Gear Cutting and CNC Machining
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Lecture-30
Numerical Problems on Gear Milling

Welcome viewers to the 10th lecture of the course spur and helical gear cutting. And in this 10th lecture we will be solving some numerical problems on the subjects that we have covered up till now.

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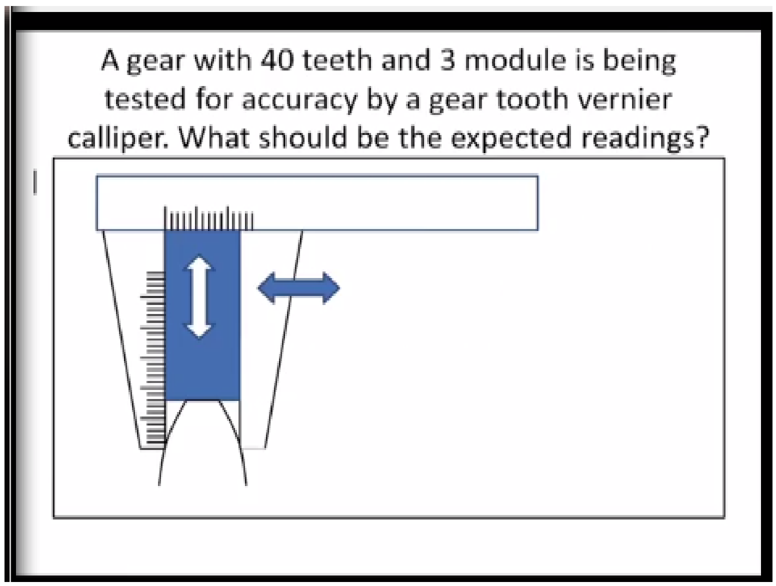

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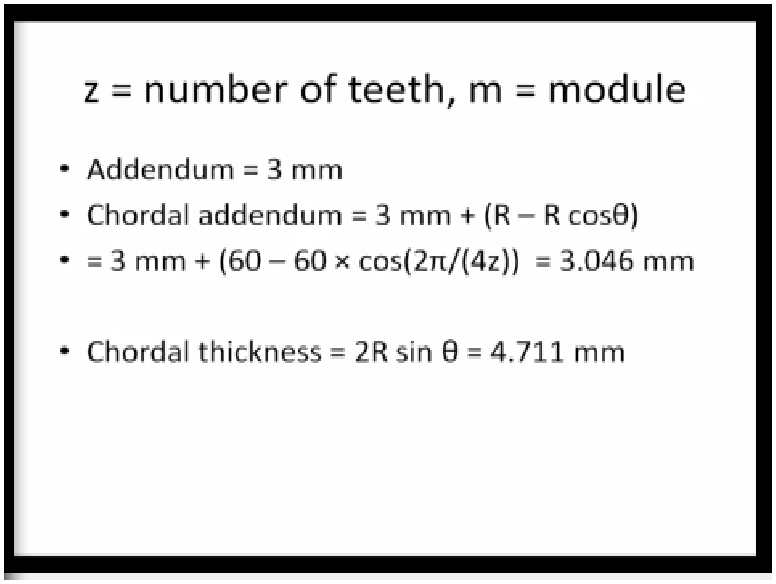

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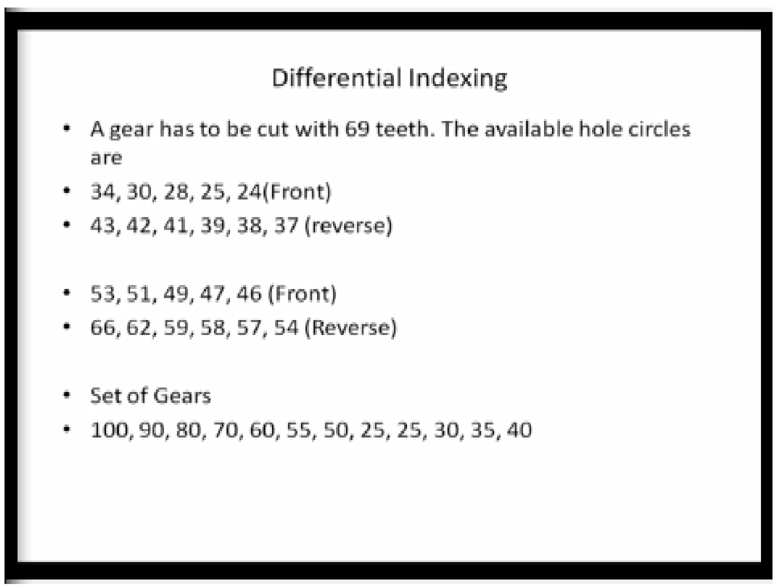

So, let us move on directly to those subjects. First comes the question of differential indexing. A gear has to be cut with 69 teeth. The available hole circles, that means on the index plate are having some holes present. And the hole circles which are available they are put here. So, the thing to be noticed is that the highest value is 66 and the set of gears which are available they are 100, 90, 80 etc.

Now what are we supposed to do with these gears? If a change gear ratio is to be developed by setting up a gearbox between the worm gear and the index plate as is generally the case in differential indexing then we can make use of these gears. So, this is the statement of the problem, I have to cut a gear with 69 teeth, available hole circles are provided and the set of gears i.e change gears are also provided. Now let us see how we can solve this problem.

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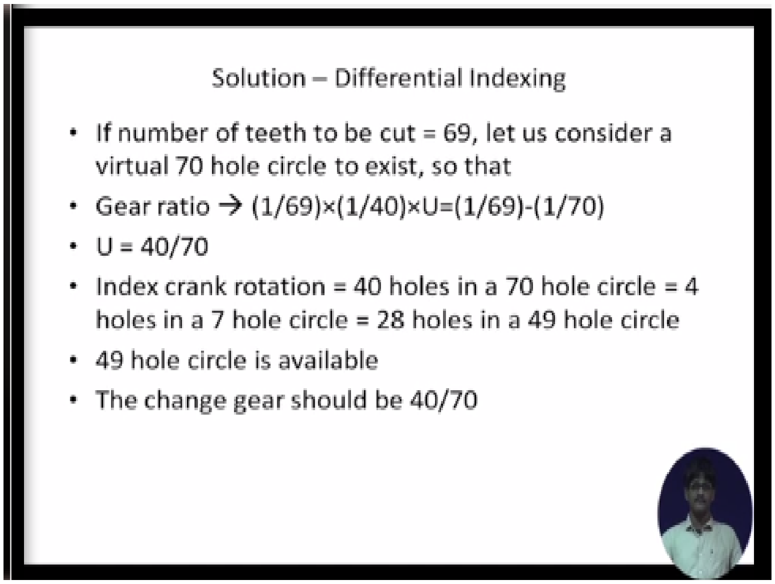

First of all, if 69 teeth are to be cut, we can move by the method of virtual hole circle. Now what is that? We say that 69 teeth are to be cut and we do not have some hole circle which is very close to it, say 70 hole circle, so we assume that 70 hole circle is present. Now why are we doing that? Because in that case the gear ratio becomes very simple but if you do not have something how can you proceed with the calculations?

Because many times it is seen that the fractions which will be appearing in the calculations they can be realized by other gear boxes or other hole circles as well. So, let us proceed, frame the method and see whether it is a giving acceptable answers. So, the gear ratio, how do we calculate the gear ratio?

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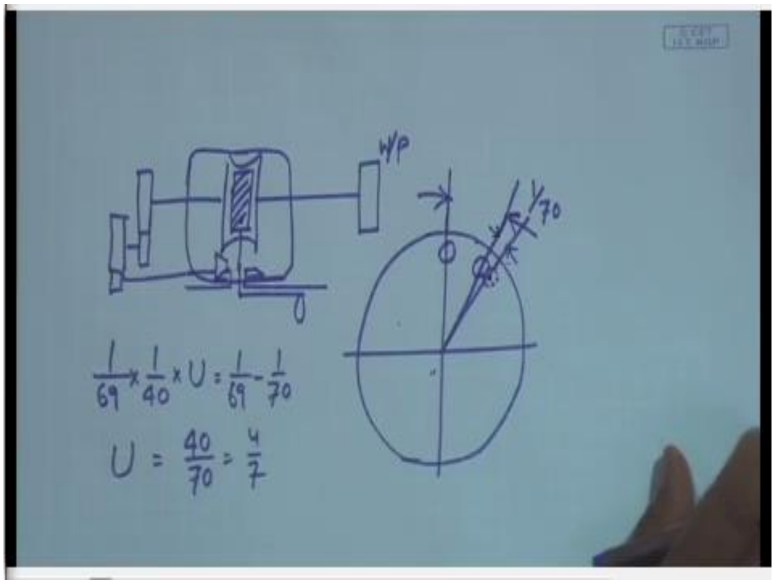

For that please have a look at the paper, what we generally do? Just to remind you is this, that we have this dividing head. On the dividing head we have this particular index crank sticking out, this is the index plate. And this gets connected to the worm; this one to the worm gear, worm gear feeds the workpiece, that is your blank. On this side we have those gears etc. This one connects up through a bevel gear to the index plate.

The drawing hopefully is clear I can show you another drawing but you will find in the previous lectures, we have already discussed it. So, index crank rotating moves right through, rotates the worm, rotates the worm gear and that rotates the job and the worm gear feeds back its rotation to this change gears and they connect up to the index plate and index plate rotates.

Now this index plate when it is rotating it creates a differential motion between the rotating crank and the rotating index plate, this is what we are interested in. And how does it occur here? This is the figure, in the figure this is the existing first hole, existing second hole and therefore we can say that this distance must be $\frac{1}{70}$, this is what is physically provided to you, first hole, second hole, third hole, fourth hole like that.

Now if 69 hole circle is not there, had it been there from the first hole the second hole would have been slightly away here. And therefore, as per method of differential indexing while I moving the crank from here towards this side. This hole, that means this hole which is on the plate, the plate should travel, so that this hole comes here. And therefore, the movement on the plate required is this one, how much is this?

This must be $\frac{1}{69} - \frac{1}{70}$, so now we move from the index crank this way. That the movement that I am ultimately providing therefore by index crank must be $\frac{1}{69}$, I am travelling right up at this point, plate is traveling from here to here. So, $\frac{1}{69}$ rotation has been given, it goes here and suffers a reduction of $\frac{1}{40}$ gets multiplied by U, rotates the index plate, index plate is rotating by this much amount, $\frac{1}{69} - \frac{1}{70}$.

$$\frac{1}{69} \times \frac{1}{40} \times \text{U} = \frac{1}{69} - \frac{1}{70}$$

So, that basically we have derived the equation once again, 69 will cancel out and we will have $\frac{40}{70}$, this will yield a numerator of 1, denominator 69 will cancel out and 70 will remain. So, the change gear has to be $\frac{40}{70}$, having a virtual hole circle of 70 holes, it is not there. This is the gear ratio connected up with this particular hole circle. Now this means that the gear ratio is basically $\frac{4}{7}$, so if you can provide a gear ratio of $\frac{4}{7}$ at this point.

The index plate will have the required motion for indexing for a 70 hole circle, this is understood. For a 70 hole circles it will have exactly the motion required if you put here $\frac{4}{7}$, so how much is the movement of the crank which is required? As we know the magic number is 40, let me choose a different piece of paper.

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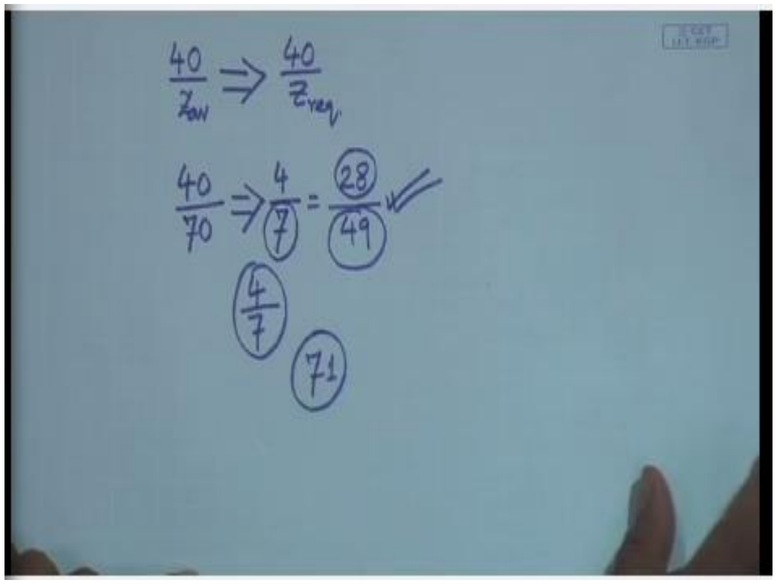

As we know the magic number is $\frac{40}{Z}$ and in differential indexing what happens? The $Z_{required}$ is not there, so move by $\frac{40}{Z_{available}}$ and the machine will adjust through the mechanisms that change gear and all those things. This will be converted to $\frac{40}{Z_{available}}$, gets converted to $\frac{40}{Z_{required}}$, that is a function of that gearbox and all those things. So, in this case how much do we have to rotate the crank?

We have to rotate the crank by $\frac{40}{70}$, that means had you not had a 70 hole circle, but a 7 hole circle you would have had to rotate the crank by 4 parts out of 7 parts, out of 1 rotation $\frac{4}{7}$ of rotation would have had to be done. So, in that case we understand that all that virtual 70 hole circle etc., that is not important but what is important is this ratio. This rotational amount has to be provided, that is all, you might not have a 70 hole circle.

It does not really matter. Rotational amount is this $\frac{4}{7}$ of a rotation has to be provided by whatever means. So, that means that if I have a 7 hole circle or any multiple of it then that will do. And we look up that particular table let us have a quick look at this table. We have multiples of 7. 49 is there, 42 is there, ok all sorts of multiples of 7 they are present.

So, coming back what we do is we multiply by 7 the numerator and denominator we come up with $\frac{28}{49}$. On the 49 hole circle if you move by 28 holes that will suffice, you need not have 70 hole circle.

But mathematically we see the only restriction is $\frac{4}{7}$ of a rotation as we provided and differential mechanism will take care of everything else.

So, this is the answer 28 holes on the 49 hole circle have to be moved through and you will be cutting 69 teeth. The change gear ratio has to be $\frac{4}{7}$, that is it. If you do these 2 things your problem is solved. Next, let us take a corollary, if you have this problem on the same machine how would you be cutting 71 hole circle? The 71 hole circle can be cut with this same setup only an extra pinion would be required to change the direction of rotation.

Instead of the rotations taking place in the same direction, crank rotation and plate rotation taking place in the same direction, crank rotation and plate rotation will be opposing each other if you cut 71 holes up, other things will be remaining the same. So, this is one problem in differential indexing, let us move to yet another problem.

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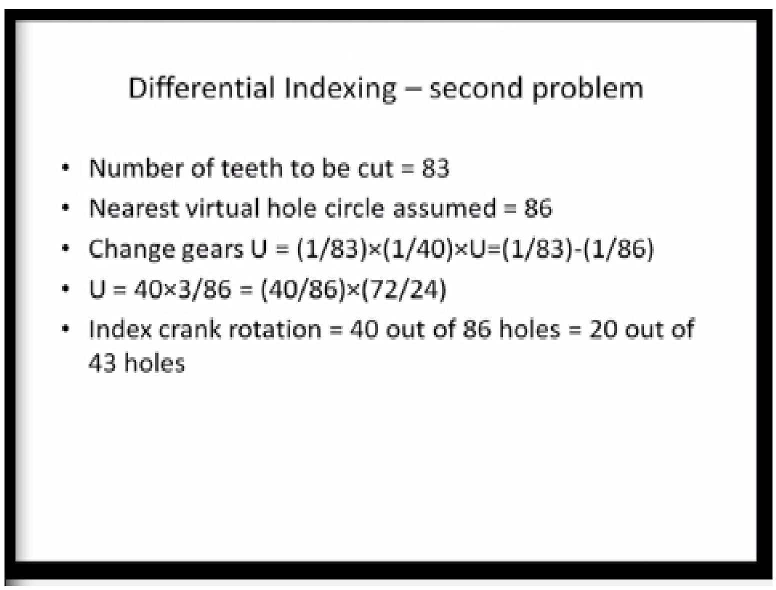

Say number of teeth to be cut is 83 and nearest virtual hole circle which is provided, I am saying, use 86 hole circle. So, this time I am not leaving it to you where you assume 70 hole circle but this time we are providing assume a virtual hole circle of 86 and solve the problem. So this time the change gears we can directly apply the formula instead of deriving it.

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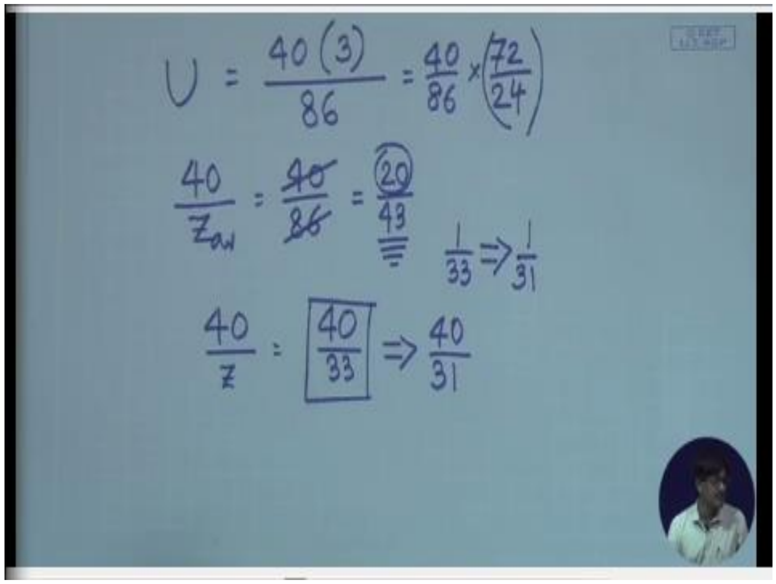

$$U = \frac{40 \times (N_{required} - N_{available})}{N_{available}} = \frac{40 \times 3}{86}$$

From the change gears available you will find that you can use $\frac{72}{24}$.

$$\frac{40 \times 3}{86} = \frac{40}{86} \times \frac{72}{24}$$

So, these change gears in the gearbox they will provide you with the required particular gear ratio and what about the actual rotation of the crank?

Once again $\frac{40}{Z}$ is the magic number.

$$\frac{40}{Z_{\text{available}}} = \frac{40}{86}$$

That means we can cut it at least once and we can get 20 by 43, that means if you have a 43 hole circle. In that case if you move by 20 holes, that will also suffice, so coming back to here if you use a 43 hole circle by 20 holes on that, that will suffice. But why not use 86? Because it is mentioned already 86 does not exist.

So, how do I know 43 exists? 43 has to exist otherwise this problem can only be solved. So, in the statement of the problem we should have said that 43 hole circle is existing and solve the problem by assuming 86 is the nearest virtual hole available. But that would have made it a fully foolproof problem.

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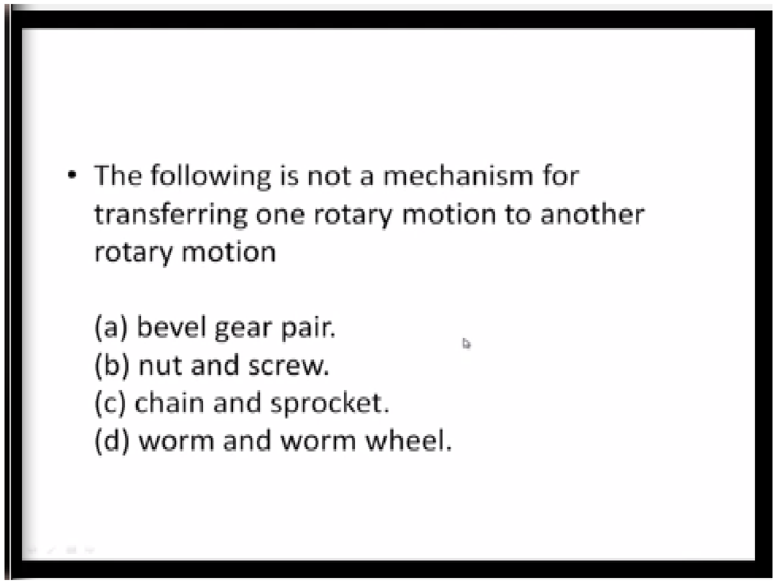

So, let us try out some problems of this type which will be helping you to apply logic to find out the answers to several multiple choice questions. The following is not a mechanism for transferring one rotary motion to another rotary motion. Bevel gear pair, nut and screw, chain and sprocket, worm and worm wheel, so does the bevel gear pair transfer one rotary motion to another? Yes, it does, so this is not the answer.

Nut and screw, yes, provided the screw is restricted from translation and nut is restricted from rotation, transfer from rotary motion to linear motion takes place. So, this mechanism is not for transferring rotary motion to rotary motion. You might say that if I lock the nut with this screw, that means then screw is rotating on top of that the nut is mounted therefore nut is also rotating. So, we have to add another statement to the question that nut is restricted from rotation with the screw but that will be a giveaway, you are already telling the answer.

Anyway, let us go through the other options chain and sprocket. Chain and sprocket is basically used for rotary motion to rotary motion because the power ultimately gets transferred from sprocket to sprocket. But suppose I argue this way that at least from the chain to the sprocket, since it is mentioned as the pair, sprocket to chain rotation to translation is taking place.

So, that way this will also qualify, strictly speaking in the chain sprocket mechanism when the sprocket is rotating the chain is undergoing linear motion, so option 'c' also qualifies. Worm and worm wheel, if the worm rotates the worm wheel rotates, so this is for rotary to rotary. So, here if we look at the answers strictly from a logical point of view nut and screw qualifies, chain and sprocket qualifies.

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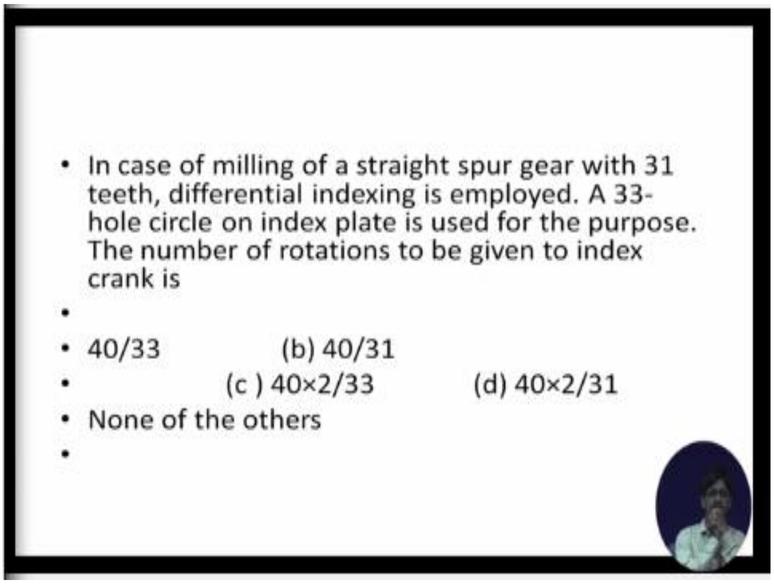

In case of milling of a straight spur gear with 31, teeth differential indexing is employed. A 33 hole circle on index plate is used for the purpose. The number of rotations to be given to index crank is? So, this is now very simple for you people because you have already done 2 problems. So, what is asked for?

The number of rotations to be given to index crank, what is the number of rotations? Mind you, there will be one question in which you will be asking for the gear ratio, another question where you will be asked for the number of rotations of the crank. So, in this case let us find out what is the number of rotations of the index crack? If the rotations of the index crank, they are taking place as let us write it down $\frac{40}{Z_{available}}$. What is the $Z_{available}$?

The $Z_{available}$ is 33 and we are cutting 31 teeth. So, if we provide $\frac{40}{33}$ amount of motion ultimately it will be converted to $\frac{40}{31}$. How is this made possible because ultimately if we are providing $\frac{1}{33}$ of a rotation it is getting converted to $\frac{1}{31}$ of a rotation. So, $\frac{40}{33}$ will be getting converted to $\frac{40}{31}$?

Here I should mention that if gearbox had been asked for. It would have had this particular answer:

Gear box = $\frac{40}{\mathrm{N_{available}}} \times \mathrm{N_1} - \mathrm{N_2} = \frac{40}{33} \times 2$

In that case option ‘c’ we would have been correct but gearbox has not been asked for and mind you there is a sign which is appearing here which can change indicating direction of rotation. We will come to that in some problem which is given later on. So, the answer to this is $\frac{40}{33}$, that is option ‘a.’

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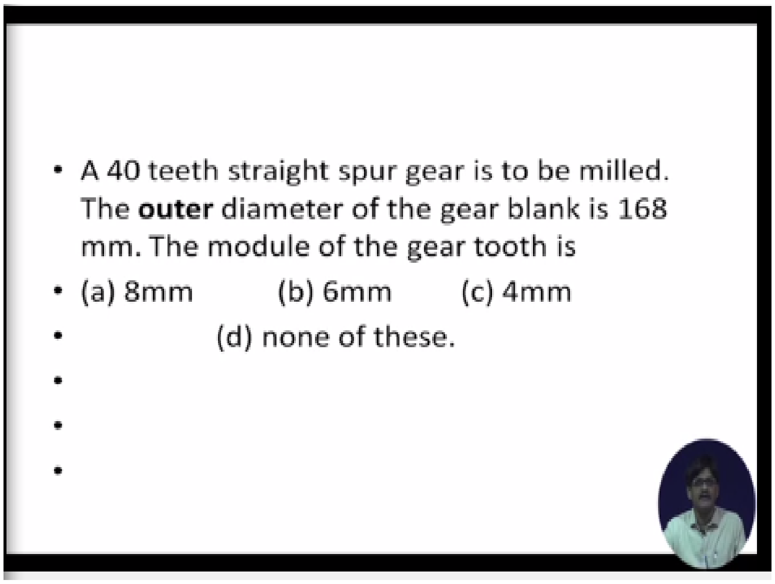

A 40 teeth straight spur gear is to be milled. The outer diameter is 168 millimeters; the module of the gear tooth is, so how do we solve this question? We say that the outer diameter of the gear blank in case of straight spur gear is equal to we will write it down here.

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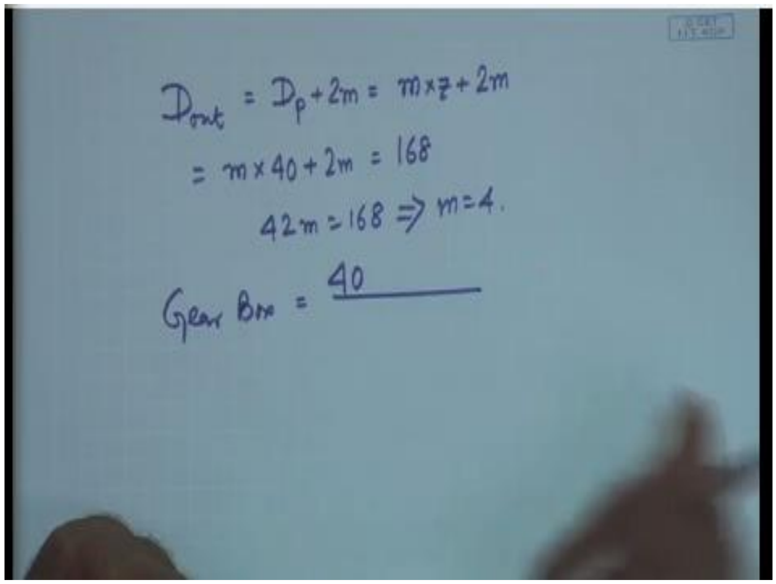

$$D_{out} = D_p + 2m = m \times Z + 2m$$

$$= m \times 40 + 2m = 168$$

$$42m = 168 => m = 4$$

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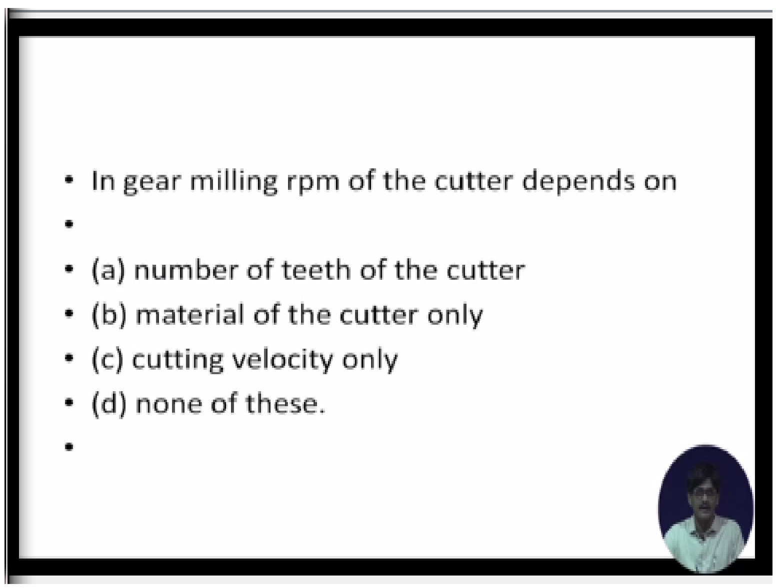

In gear milling RPM of the cutter depends on? Number of teeth of the cutter, material of the cutter only, cutting velocity only and none of these. So, let us look at them one by one. In gear milling RPM of the cutter depends on the number of teeth of the cutter, why should it? There is no reason because of which it should depend. RPM which it is developing, it should be related to the cutting speed not the number of teeth.

So, it depends upon the material of the cutter only, it depends on the material of the cutter but it does not depend upon it only there are other factors also on which it will depend. Cutting velocity only, now if the cutting velocity is mentioned for the cutter does that define the RPM? No, cutting speed is defined as $\frac{\pi DN}{60}$, where D is the diameter of the cutter and N is the RPM.

So, if velocity is equal to diameter × RPM, so RPM depends both upon the cutting velocity as well as upon the diameter. So, RPM depends both on cutting velocity as well as on it is diameter, so it does not depend upon cutting velocity only. So, here we have the answer to be none of these. It does not depend upon number of teeth, it does not depend upon material of the cutter only, it does not depend upon the cutting velocity only, therefore the answer is none of these.

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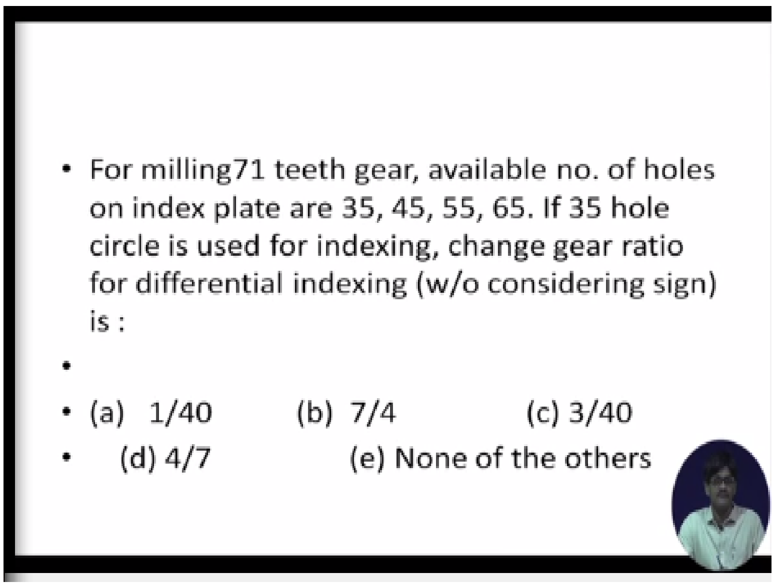

For milling 71 teeth gear, available number of holes on the index plate are 35, 45, 55, 65. If 35 hole circle is used for indexing, change gear ratio, now we are asking for change gear ratio so we have to be very alert, change gear ratio for differential indexing without considering sign. For the time being we are not bothered about the sign is, so some options are given. So, let us quickly apply the equation for gearbox. So, gearbox = 40 by available number of holes, let me write down the problem on the fresh piece of paper.

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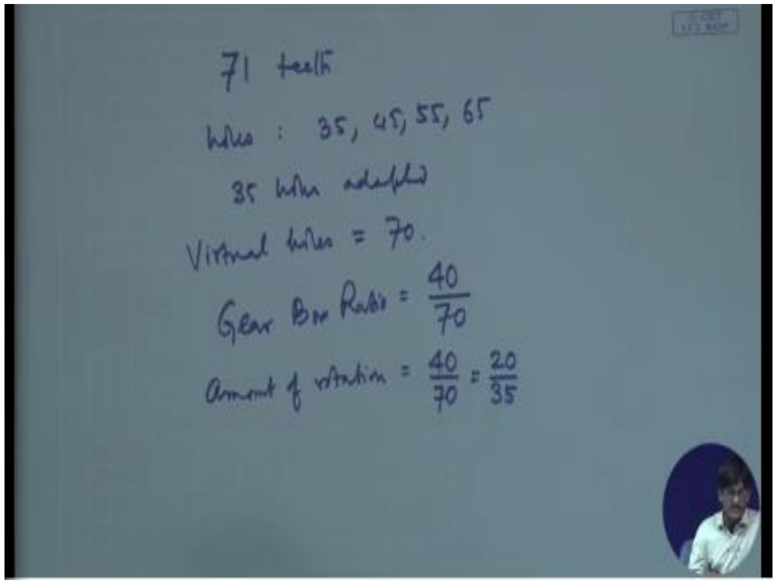

71 teeth, holes available 35, 45, 55, 65. 35 holes adapted and with this how much should be the change gear ratio? So, let us take a virtual number of holes = 70. Immediately we can understand that is 70 holes are present, the gearbox ratio becomes $\frac{40}{70}$, that is it, $\frac{4}{7}$. So, let us see whether that answer is there. You might say that what about that virtual thing that has been mentioned here?

It means that if I can apply this particular gear ratio and work with the 70 hole circle, we have nothing to worry about. But then you will be saying that 70 hole circle is not there. But if with the hole circle our main headache is whether I can execute the required amount of rotation on that hole circle. So, how much is the amount of rotation? The amount of rotation is, if we look at the piece of paper.

Amount of rotation must be equal to $\frac{40}{70}$, that is converted to $\frac{40}{71}$. So, if it is $\frac{40}{70}$, it is as good as 20 holes on the 35 hole circle, so we have nothing to worry about. We will be having this gear ratio, it will be taking care of the conversion and the amount of rotation which is $\frac{40}{70}$, it can as well we obtained by $\frac{20}{35}$. It is going to give rise to the same fractional movement.

$$\text{Amount of rotation} = \frac{40}{70} = \frac{20}{35}$$

So, both these things are satisfied and therefore answer should be $\frac{4}{7}$. So, let us go back to the problem and have a look at the options. The change gear ratio for differential indexing is $\frac{1}{40}$, $\frac{7}{4}$, $\frac{3}{40}$, $\frac{4}{7}$, so d is the correct answer the gear ratio $\frac{4}{7}$.

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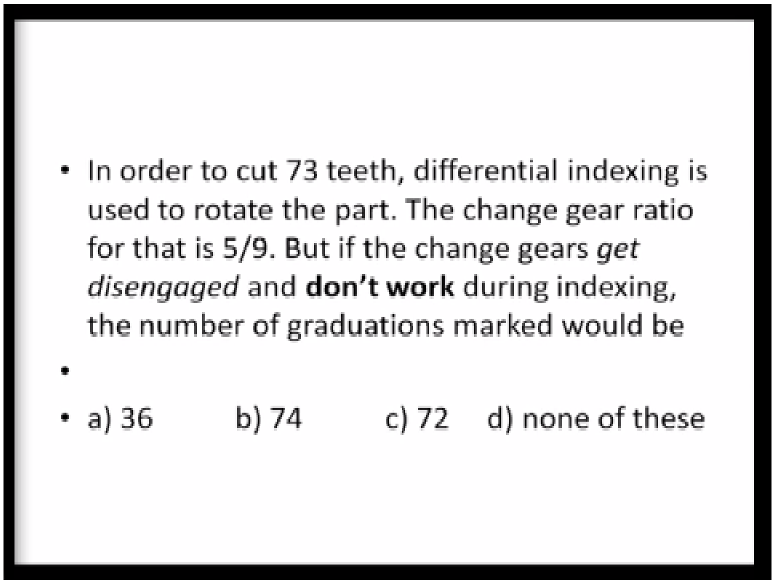

Taking the next problem let us see what it says. In order to cut 73 teeth, differential indexing is used to rotate the part. The change gear ratio for this is $\frac{5}{9}$, so you have put a particular gear ratio from this we can back calculate what is the number of holes that you are employing, but if the change gears get disengaged and do not work during indexing, the number of teeth cut would be cut is. So let us solve this problem if you look at this piece of paper.

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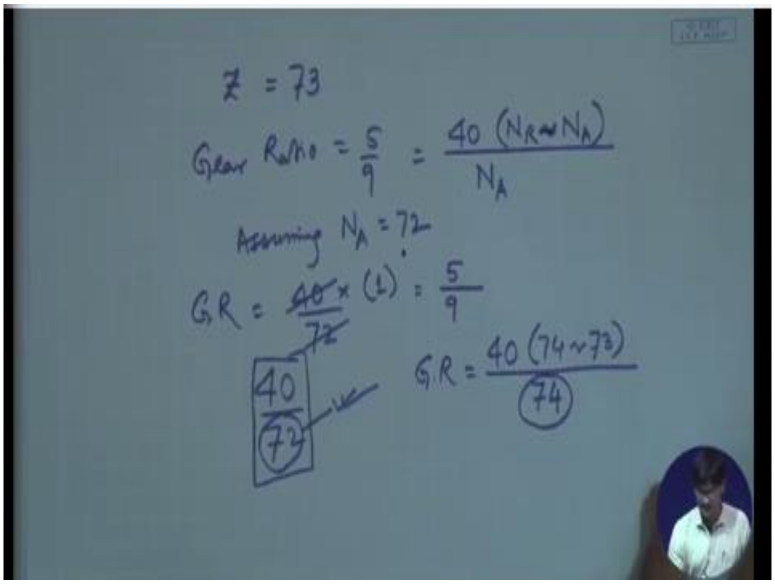

So, we write here:

$$Z=73$$

$$\text{Gear Ratio} = \frac{5}{9} = \frac{40 \times (N_R \sim N_A)}{N_A}$$

$$\text{Assuming } N_A = 72$$

$$\text{Gear Ratio} = \frac{40}{72} \times 1 = \frac{5}{9}$$

So, it must be working with of a hole circle or a virtual hole circle of 72. But what is the problem? Problem is the gears suddenly get disengaged and when the operator goes on using the setup, he is basically not doing differential indexing but he is doing simple indexing with 72 hole circle. And therefore, the number of teeth which are being cut they are defined by $\frac{40}{72}$.

Number of teeth which are being cut is 72 if it gets disengaged from differential indexing. So, 72 teeth will be cut in that case. What about 74, can 74 be an answer? Let us see, in that case:

$$Gear\ Ratio = \frac{40 \times (74 \sim 73)}{74}$$

74 is going to drive the solution somewhere else, $\frac{40}{74}$ is not equal to $\frac{5}{9}$. So, we have this to be the answer is 72 is the correct answer.

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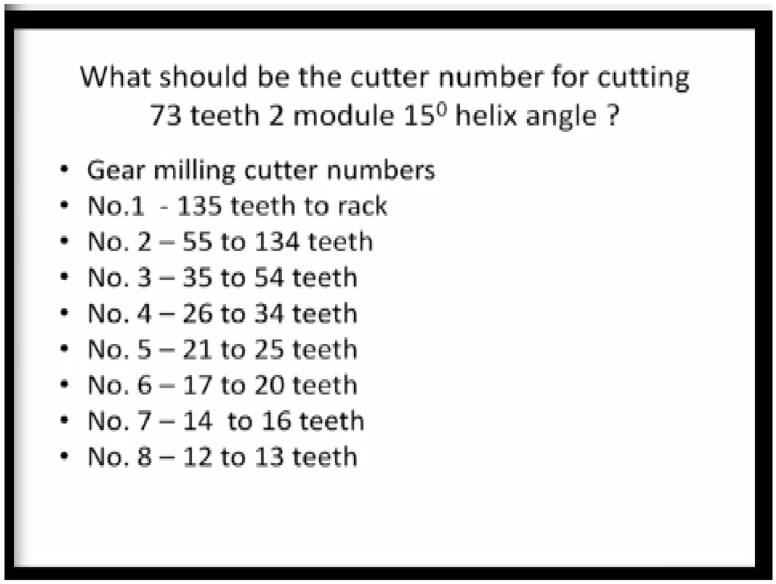

Coming to the next problem, what should be the cutter number for cutting 73 teeth 2 module 15° helix angle gear? So, what is the information that is provided? First of all once again the statement of the problem is what should be the cutter number for cutting 73 teeth 2 module 15° helix angle gear teeth by milling? Obviously because some milling cutter numbers are given, what is that information?

Cutter number 1 cuts from 135 teeth to the rack, cutter number 2 from 55 to 134 teeth, cutter number 3 from 35 to 54 teeth, cutter number 4 from 26 to 34 teeth, cutter number 5 from 21 to 25 teeth, cutter number 6 from 17 to 20 teeth etc. Now what is basically the thing that we notice? We find that for small number of teeth the cutter numbers are rapidly changing. As the numbers of teeth become larger the tooth profile becomes flatter. And ultimately, we find that the cutter numbers can handle huge numbers of teeth single handedly. Cutter number 2 for example can handle 55 to 134 teeth. So, with this idea in mind let us proceed.

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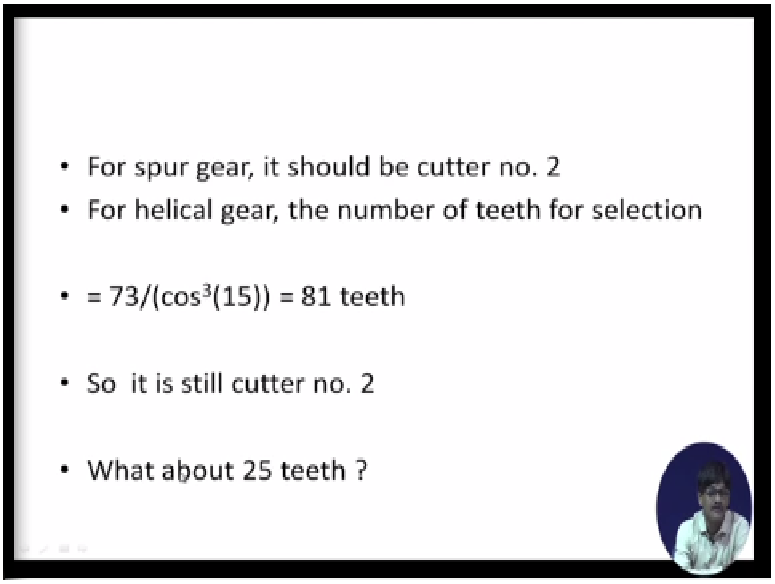

First of all, our first thing that we notice in the calculations is that, for spur gears what should be the cutter number? The cutter number should be 2, why? Because if we go back, if we are cutting 73 teeth it falls in this particular category, 55 to 134 teeth and cutter number 2 should be used. So, for helical gear the number of teeth for selection should be given by:

$$\frac{N}{\cos^3 \propto} \text{ or } \frac{Z}{\cos^3 \propto}$$

So, $\propto$ is given to be 15 degrees and we do this calculation:

$$\frac{73}{\cos^3 15} = 81$$

So, 81 means that it is still cutting number 2, why? Because the cutter number 2 can handle from 55 to 134 teeth, so if you looking for any change in cutter number most of the cases where you have a small cutter number, you will notice that in that case there is frequent chance of a cutter number getting changed.

So, if you have 25 teeth with helix angle 15 degrees, you are surely going to have a change. I have not included this calculation but you can do it very simply, that is the effective number of teeth for selection of the cutter will be $\frac{25}{\cos^3 15}$. That will give you the number of teeth which will be used for selection of the cutter, thank you.