Elements of Metal Cutting, Machine Tools, Gear Cutting and CNC Machining
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Lecture-29
Helical Gear Cutting on Milling Machine

Welcome viewers to the 9th lecture of the course spur and helical gear cutting. So, today we will continue our discussion on differential indexing for gear cutting and continue with helical gear cutting.

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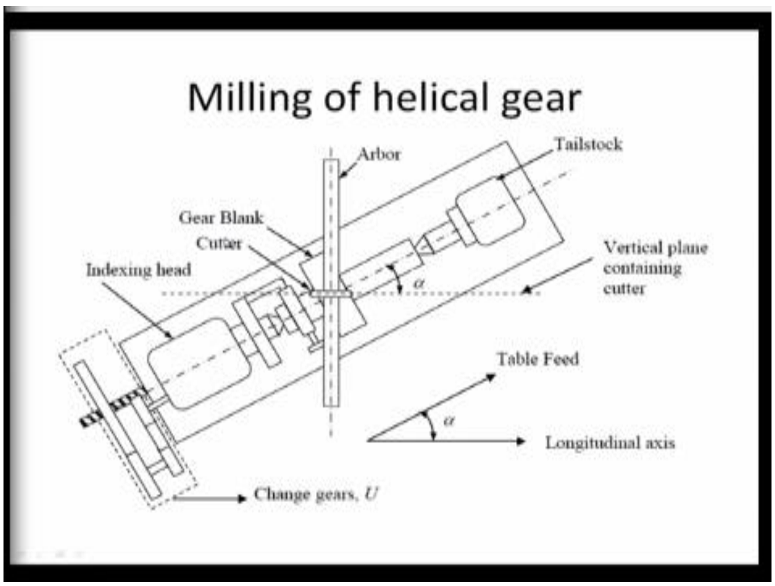

So, we have finished most of the discussion on differential indexing, so, let us start with helical gear cutting on the milling machine. What do we have here? We are seeing the milling machine from the top, the milling machine table, you have already seen this one in the previous lecture and let me quickly identify the various parts which are present here. For example, this is the gear blank, it is basically a disk that means cylinder shaped job with a short height.

And it is rotational axis is this one; this one is the rotational axis of this gear blank. Where is the cutter? The cutter happens to be this one and the rotational axis of the cutter is here. So, the rotational axis of the cutter and the rotational axis of the workpiece are inclined to each other at a particular angle, there is an angle existing between the 2. This angle is $\propto$ the helix angle, this is the indexing head you are already conversant with the working of the indexing head.

And we are utilizing this indexing head for our purpose of helical gear cutting, in what way? First of all, it is providing the support on one side of the gear blank, the other side is supported by a tailstock, the workpiece may be held on a mandrel and this is the dong which drives the mandrill. And therefore, if the indexing head suffers rotation the workpiece is going to rotate, how is the indexing head set up?

You can see at the bottom, there is this lead screw of the machine table sticking out; it can be seen from the top. Now this dotted line is showing a gearbox, it is called change gears U, so it is taking its input from the lead screw. For the time being we will look at it this way, it is taking its input from the lead screw and through a set of change gears the power is ultimately coming to the index plate.

The index plate has not been drawn here but the index plate is here, index plate is getting rotated. So, if we lock the index crank with the index plate the index crank is also going to rotate and this rotation will ultimately be transferred to the worm and then to the worm gear and then on to the workpiece. So, if the lead screw rotates these change gears will pass on this rotation, changed rotation of course, to the index plate then on to the index crank which is locked with the index plate and then to the worm, to the worm gear and ultimately to the workpiece.

So, now we have connection between the rotation of the workpiece on one side and the longitudinal motion of the workpiece, this is the longitudinal motion and the rotational motion they are connected up. Why are we connecting it up? Because last day as we discussed whenever if you have a look at this piece of paper.

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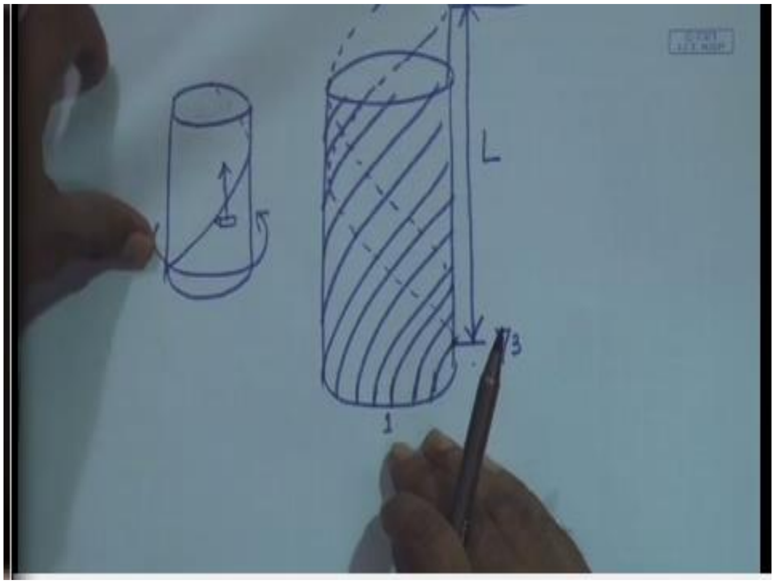

If we have a particular cylinder and on that by some means if we are tracing a line. This line will be executing a helix, I mean the tracer will be executing a helical line if you have circular motion of this disk combined with straight line motion of this pencil. If they are proportionally moving, that means if this is moving a certain number of units, this moves by a proportional number of units and this proportionality is maintained.

Even though this might be slowing down, I mean the rotation or so transferring might be slowing down if they are proportional to each other this will be describing a helix. This is the principle we are using for cutting out a helix in case of helical gear cutting. So, if we come back to this figure, we have rotational motion of the blank connected with translational motion of the blank and we know that in one rotation a helix climbs up by the lead for multiple start threads.

Let us come back to this one and let us have a quick look here. So, let us draw a larger figure, so in a gear you have multiple starts because you have so many teeth. So, these are the helices and they are belonging to different helices, they are multiple starts. How many starts does this threaded element have? If you are cutting 73 teeth, it is having 73 starts, so each of these are individual helices and therefore say if you name this one as number 1, this climbs up this way.

And just imagine what distance it is going to climb up? It is come back to the same position and this is equal to the lead. The climb up in one rotation, so if this be the climb up, we have to ensure that the tool I mean the cutter moves this much longitudinally while the workpiece suffers one rotation, so that is it, that is how we are going to move.

So, first, the thing that we must do is calculate the lead. How much is the lead for our particular job, particular gear to be cut. So, let us have a look at that, so this is understood, job connected through change gears to the lead screw, so that longitudinal motion and rotational motion of the job they are connected together. Last day which is also discussed one point, why have they inclined the job at a particular angle in about a vertical axis? This is because once we have understood that the cut is taking place along an inclined line.

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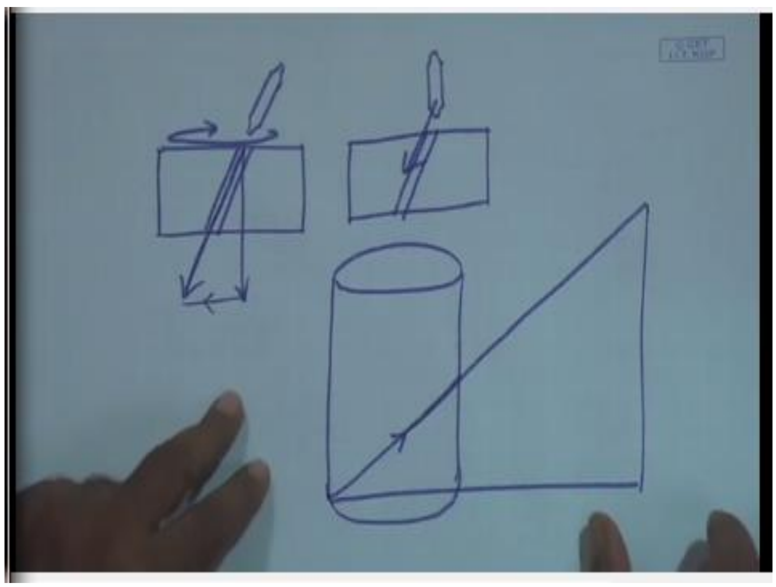

If you have a look once again, once we have understood that even though this is the job the cut is going to take place this way. So, the cutter which is once again that same cutter that we have used for spur gear milling, that same cutter has to pass this way now. We have ensured the motion to be in this direction by the combination of rotational motion and straight-line motion.

We have already ensured that the motion is going to be this way, so you have to orient the cutter physically in this direction. Otherwise, you will not have a cutting action but you will have a slapping action, what sort of? In that case had you not oriented it, this would have been the case. The cutter would have hit the job on it is side, so it is a sort of slapping action, it would not have cut at all.

It would have cut and brushed and the whole thing would have been spoiled, this trying to move this way, just imagine. So, orientation of the cutter, so that it follows through the cut, it is very important and that is what we are doing by rotating the table about a vertical axis.

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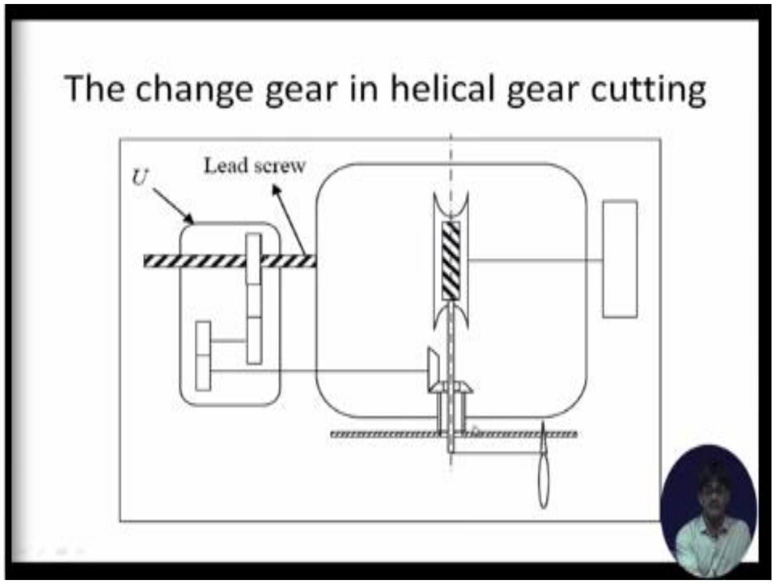

So, now for the calculations, now those who had problems in understanding what we mean by locking of the index plate with the index crank, here is a quick view at that. This is our job, this is our worm and worm gear and this is our index plate and this is our index crank. Right from the lead screw if you follow up, lead screw, gearbox connected with the index plate through these 2 bevel gears and you climb up to the index plate.

The index plate is rotating due to the rotation of the lead screw now. Now you have locked the index crank with the index plate, so that if the index plate rotates, the index crank will rotate and the worm will rotate and the worm gear will rotate and job will rotate. So, this is the connection seen in two dimensions, I hope it will be now easy for you to follow.

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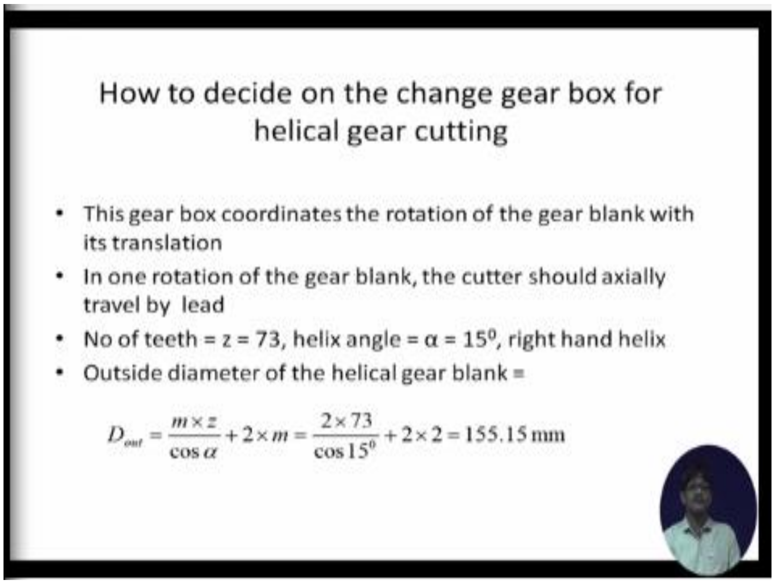

Now for the calculations, what are the calculations? How to decide the change gear box for the helical gear cutting? First of all, this gearbox coordinates a rotation of the gear blank with it is translation, this we have already discussed it at length. In one rotation of the gear blank the cutter should axially travel by lead, this is also accepted; we have already gone through this number of teeth being = 73, helix angle being equal to $\propto$ = 15 degrees.

Right hand helix, we will come back to right hand helix for this calculation it is not that important. Outside diameter of the gear blank is being found out, now why suddenly we are finding out the outside diameter? This is because the operator has to choose the correct gear blank size. So, that the gear is correctly made, if you make a mistake in the outside diameter, it is with respect to the outside diameter that you are going to apply depth of cut.

So, if the outside diameter is not correct, everything will be affected, so outside diameter has to be found out very carefully. And as we have calculated previously it is equal to $\frac{m \times Z}{\cos\propto}$. You come up to the pitch diameter + 2 $\times$ m you go up to the outside diameter, so it is coming out to be 155.15 millimeters.

$$D_{out} = \frac{m \times Z}{\cos \propto} + 2 \times m = 155.15 \text{ mm}$$

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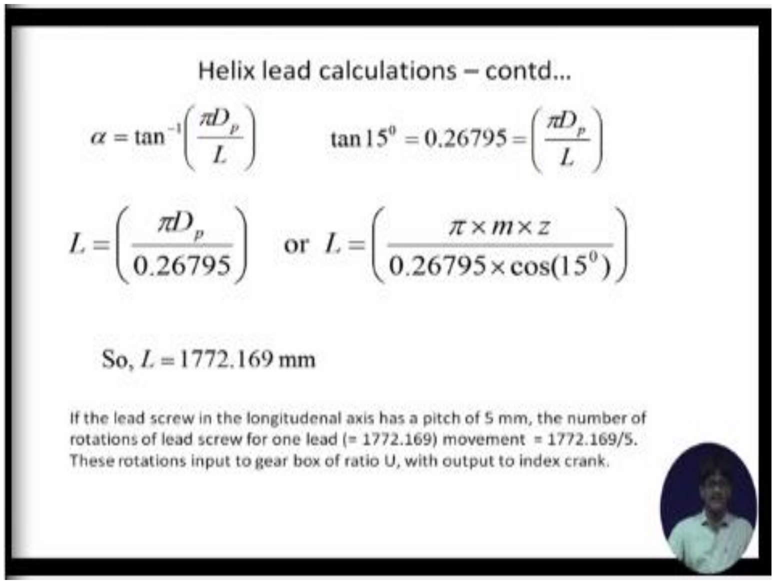

Next, we understand that ∝ the helix angle is equal:

$$\propto = \tan^{-1}\left(\frac{\pi D_p}{L}\right)$$

Let us have a quick look at this. If we come back to our page, this is it, this is the direction of teeth. So, if you unfold this helix and get a triangle, you are unfolding it, sorry I should have drawn it from here. Let me choose a fresh piece of paper, so that there is no misunderstanding.

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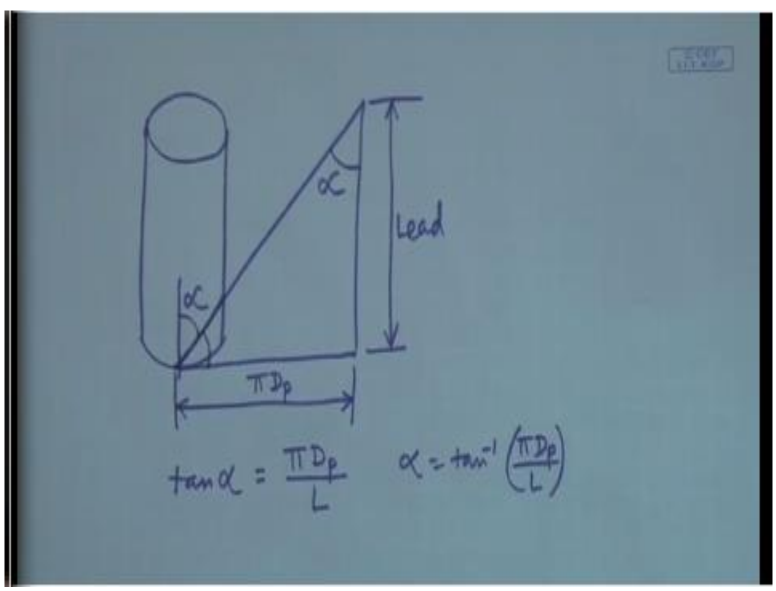

This is your cylinder, this is your helix and you unfold it, so that you get a triangle, what is this triangle having? It is having this side equal to πD_p that means the circumference of the pitch circle. It rides up by the lead in one rotation, so this must be equal to lead. And what is the helix angle?

As we discussed before, this angle is the helix angle for screw threads, this is the helix angle $\propto$, sometimes in some literature is called β, this is the helix angle for gear teeth.

If that be so, we can write:

$$\tan \propto = (\frac{\pi D_p}{L})$$

$$\propto = \tan^{-1}(\frac{\pi D_p}{L})$$

that is it. So, once we have established that let us come back to this, so that is what we have written out here, this is the relation which defines $\propto$. Now do we know this side and that side? Yes, $\tan 15^{\circ}$ is known to us, so I have simply calculated and written it down here.

Now do I know D_p? Yes, I know D_p,

$$D_p = \frac{m \times Z}{\cos \propto}$$

So, I can write that down also, so that I can calculate L. So, that way L becomes defined, so I have taken L to the numerator. So,

$$L = \frac{\pi D_p}{0.26795} = \frac{\pi \times m \times Z}{0.26795 \times \cos 15^{\circ}}$$

And therefore, I have found L = 1772.169 millimeters, just imagine a small gear of 155 mm outside diameter it is having a lead more than 1 meter almost 2 meters, 1.772 meters.

So, lead has been found out, once we find out the lead let us see what is written in the small print. If the lead screw in the longitudinal axis has a pitch of 5 millimeters, this is important, this says that, we plan to get a longitudinal motion of 1772.169 millimeters in the same time that we rotate the job once. So, how can I achieve? How many rotations of the lead screw would be required to get 1772.169 millimeters.

Naturally, divide this distance by the pitch of the lead screw and you will get the number of rotations that is what I which has been said here. See, if you read this one the number of rotations of lead screw for one lead movement, lead being 1772.169, is equal to $\frac{1772.169}{\text{pitch of the lead screw}}$. Now what is the pitch of the lead screw equal to?

That we are providing you, if the lead screw in a longitudinal axis has a pitch of 5 millimeters. So, we get this as the number of rotations of the lead screw. These rotations are input to gearbox ratio U with output to index crank. If you follow that the lead screw was giving up it is rotation as input to the gearbox. So, this rotation is input to the gearbox, so let us see what happens.

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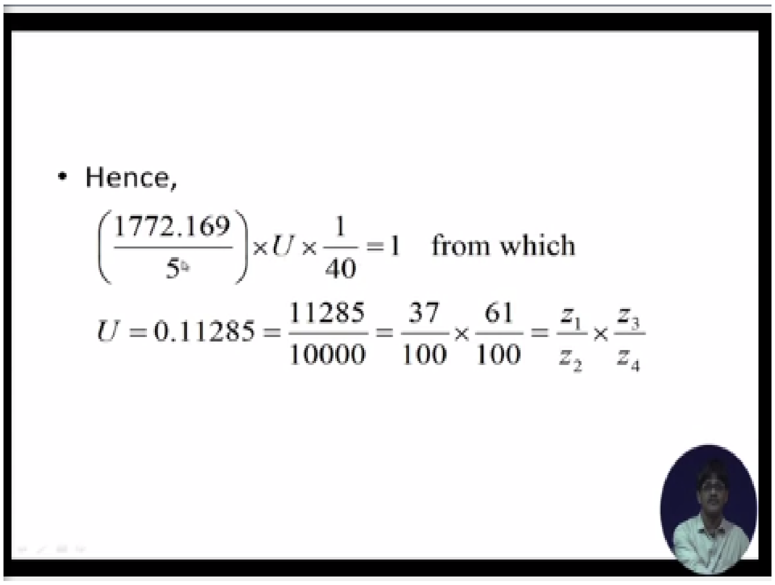

So, this rotation is input to the gearbox, so multiplied by U gives you the output from the gearbox that enters the index plate, index crank and goes to the worm, worm gear where it suffers a reduction due to worm and worm gear rotation of $\frac{1}{40}$. And this is given to the workpiece, so if this must be equal to 1.

$$\frac{1772.169}{5} \times \mathrm{U} \times \frac{1}{40} = 1$$

So, I hope this is you fully agree with this and therefore from here we can get U = 0.11285 and I have divided it into 2 fractions, where I am getting the numbers of teeth of those gears which can form this particular gear ratio.

So, leading from the lead screw to the index plate I can have $\frac{Z_1}{Z_2} \times \frac{Z_3}{Z_4}$ to be equal to this one. So, this way I can calculate the gear ratio for helical milling and set up the machine, so that a helix is cut.

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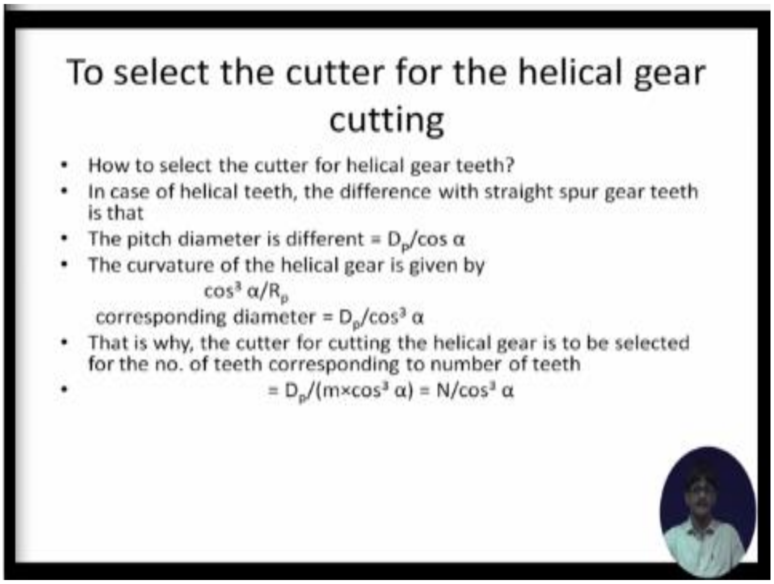

However, this is not all for helical gear cutting. In helical gear cutting, selecting the cutter is a headache, why? Because the same cutter as used in case of spur gears will not do. What do we exactly mean by that? We mean that suppose you are using the same cutter as used in spur gear cutting, if you are using the same cutter then something must be the same for the 2 gears. Yes, the value $\pi \times m$ must be the same for the 2 gears, however for the helical gears it does not occur in the same plane as that of the spur gears, let us see what we mean by that.

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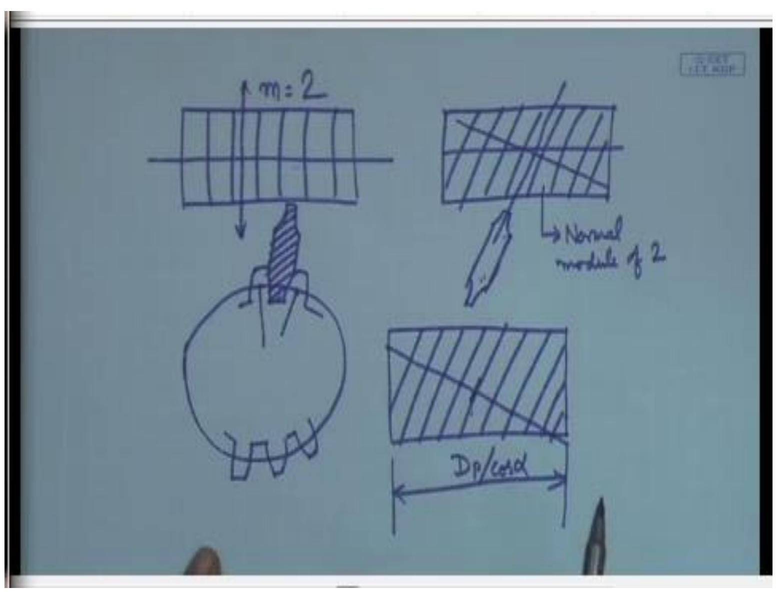

If you have a look here, in case of spur gears this is the direction in which their teeth align and on the other view if you look at this, the teeth are this way. So, this is basically the cutter, the cutter is physically defined piece. And therefore, it gets defined by this particular gap and as we have studied previously this gap is equal to this particular distance not the angle**.** This particular distance is going to π × m. So, the cutter is having inside it, this particular information.

And this information is connected with the direction perpendicular to the axis, this is the axis, it is imprinted perpendicular to the axis. However, if you are using the same cutter, in that case if these be the teeth direction the same cutter is used this way. And that information of the cutter is imprinted normal to the teeth that means normal to the teeth you can say that the distance will be π × m from this point to this point.

So, that is why we say that this helical gear has a normal module of 2, so here the module is 2, for the spur gear module is 2, this is the cutter with module = 2. The same cutter used here defines the normal module of the helical gear. So, we say that the normal module of the helical gear is 2, that is fine. So, in this direction the distances between the teeth will be different if you cut it along this direction.

Having understood this thing, now comes the question how do we choose the number of the cutter? For this if you look at the helical gear these are the teeth, already we know that this diameter is equal to $\frac{D_p}{\cos\propto}$, where D p is the pitch diameter of the corresponding spur gear. That means same module as the normal module of the helical gear and same number of teeth.

So, that is divided by cos ∝ and that way we get the pitch diameter of the helical gear. Now if you cut it this way, since we are doing all the cutting perpendicular to this direction of the teeth if you are doing the cutting this way. Essentially you are working on if you take a section of a cylinder at an inclined plane; you are essentially developing an ellipse. In this ellipse at this point, there is a question of major axis and minor axis, the minor axis is definitely $\frac{D_p}{\cos\propto}$. So, if we draw now another figure.

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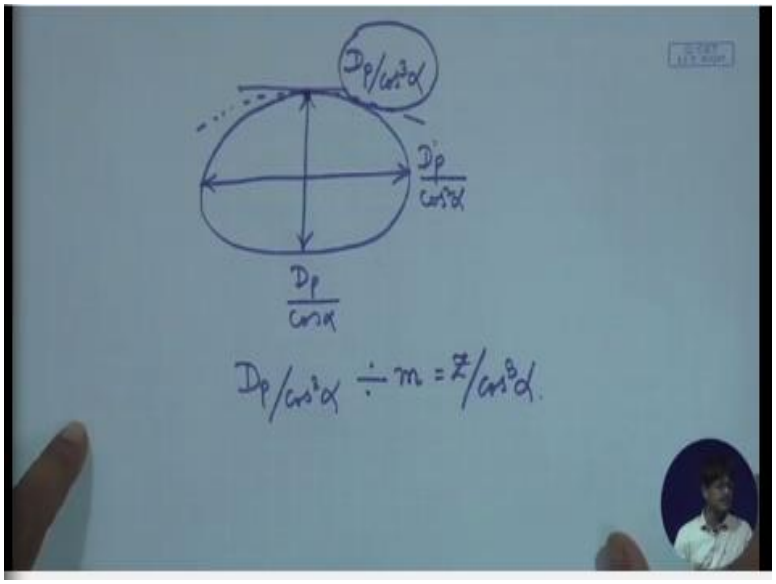

This is what it looks like now, this one is twice be = $\frac{D_p}{\cos\propto}$, this one if you consult the figure, you will find it will be equal to $\frac{D_p}{\cos^2\propto}$. And we have to find out at this point what is the curvature, what is the radius of curvature of the gear? That radius of curvature will have a corresponding diameter. Radius of curvature double of that diameter of curvature, that diameter will be able to accommodate a certain number of teeth.

That number of teeth will define the particular cutter number that we have to choose. Now what do we mean by this? As we discussed before gear milling cutters, they have a particular number associated with them corresponding to the number of teeth that they can handle, this thing we have discussed in detail previously. So, let us see at this point what happens is the diameter of curvature is $\frac{D_p}{\cos^3\propto}$, now how do I get that?

I can easily differentiate the equation of the ellipse here and find out the double derivative and from the double derivative I can find out the curvature. Naturally here the first derivative will be 0 because it is reaching maxima point. But the second derivative will not be 0, the second derivative can be found out and it will lead to this particular diameter, I am not working this out. I will try to provide you with supplements which will be available as attachments with this particular study and you can open it up and study it yourself.

In many cases as much as possible I will try to provide you with figures, videos etc., which will make this thing much clearer. So, $\frac{D_p}{\cos^3\propto}$, this is now understood to be the effective diameter of the gear at this particular point because it defines the curvature here.

So, if $\frac{D_p}{\cos^3\propto}$ is the effective diameter, the number of effective teeth will be diameter divided by the module. Because m × Z = diameter, so this thing divided by the module will give us the effective diameter. So, D_p/m is nothing but equal to number of teeth, so

$$\frac{D_p}{\cos^3\propto} \div m = \frac{D_p}{m\times\cos^3\propto} = \frac{Z}{\cos^3\propto}$$

So, if you have a certain number of teeth divided by the cube of the cos of the helix angle and you will get the number of teeth which you have to refer to for the selection of the cutter.

Because the cut is taking place at an inclined angle and therefore the profile of the teeth is slightly changing. Because they are getting defined at a different curved surface, different curvature. This curvature defines the diameter; this defines a particular number of teeth. So, with this idea in mind we will solve some numerical problems which will make this thing clear. So, this is what I have written down here all the things that we have discussed.

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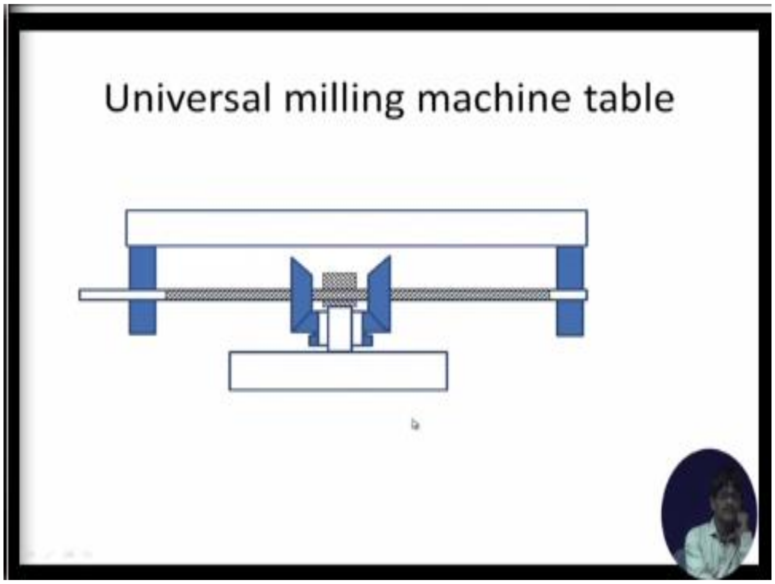

This we will come back if we need to. And therefore, we are coming to the end but we will have 5 minutes of discussion which I want to utilize for discussing certain aspects of helical milling. For example, let us take a particular case.

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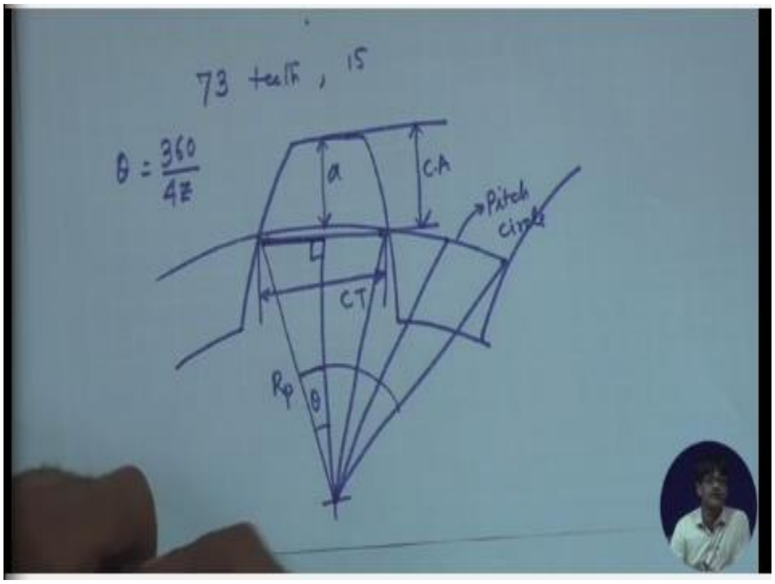

Suppose let us take a particular numerical example. Suppose you are having 73 teeth, you are having 15 degrees; I think we have solved this problem.

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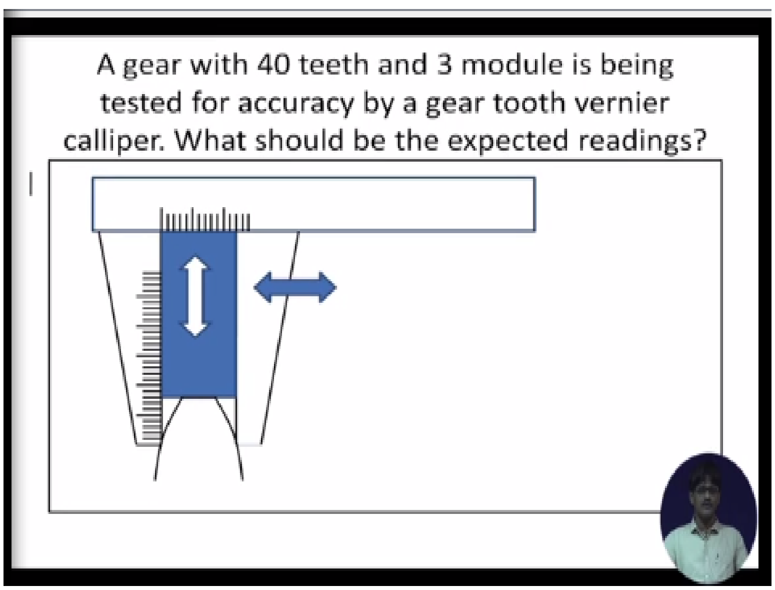

So, I will take this opportunity to introduce one gear measuring method I mean testing of gear teeth whether it is made well or not by discussing about the gear tooth vernier caliper. We had slightly referred to this in our previous lectures but not solved any particular numerical problem, so we can do it now itself. What is the gear to tooth vernier caliper? The gear tooth vernier caliper as shown has 2 calipers connected up into one.

That means this particular caliper jog and move and accommodate different dimensions between these 2 jaws. At this moment it is measuring this particular distance, this is a gear tooth. Corresponding to these 2 jaws there is another moving slide moving in between which can be moved out or moved in to measure precisely this particular distance. And therefore, if we know a particular point on the gear where to these 2 distances can be calculated, we can open up these jaws to the lateral dimension.

And expect the other dimension to be recorded in the vertical scale. So, we will quickly do the calculations and touch the gear tooth in whatever place it accommodates that particular distance. And read of the vertical scale reading and compare it with the calculated value to see how accurately this has been made? So, let us have a quick look at the calculations.

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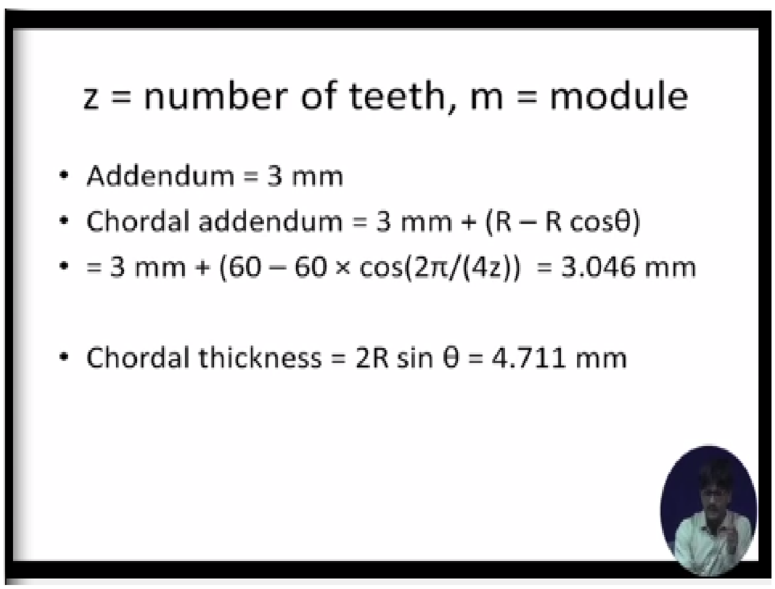

For example, if you look at the figure, suppose I am having a gear tooth, this is one gear tooth and suppose I say that if this is the pitch diameter, I say this is pitch circle. On the pitch circle if I take this distance, we can call it chordal thickness. And this distance we can call it chordal addendum. Now we can make exact calculations for these 2 values, in what way chordal addendum is very simple, it is simply equal to the addendum, so we call it ‘a’ plus this small distance.

What is this small distance? If we have this one as the pitch radius, yeah if you have this much as the pitch radius and if you join this distance this being a perpendicular. We can say that this angle θ has a definite value, in what way? We have worked this one previously,

$$\theta = \frac{360}{4Z}$$

360 is the whole if you divided by Z, you get up till this point you divide it into 4 parts 1, 2, 3 and 4 and therefore this is $\frac{360}{4Z}$. Once Z, you can find out θ, so θ is known R_p can be found out from module and number of teeth.

And therefore, this triangle is completely solvable, so that we can find out this distance, how much will this be? This is equal to $R_p \sin\theta$, so this one is $2 \times R_p \sin\theta$ and this one is $m + R - R\cos\theta$. Once we are equipped with these 2 values, we can employ the method of gear tooth vernier caliper; let us have a quick look at the calculations now. The calculations say that Z = number of teeth and m = module, so let module = 3 millimeters and number of teeth = 20.

So, if number of teeth = 20 and module is 3 millimeters, addendum = module, so addendum = 3. So, chordal addendum must be decided upon the number of teeth by back calculation. Have I mentioned the dimensions in the previous case? Yes, a gear with 40 teeth and 3 modules.

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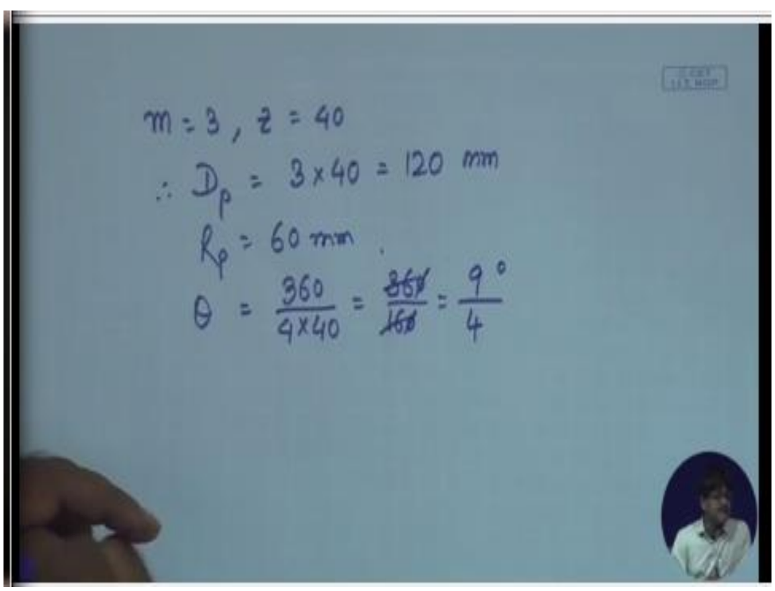

So, if there are 40 teeth, we can write that

$$m = 3, z = 40,$$

$$\therefore D_p = 3 \times 40 = 120 \text{ mm}$$

$$\text{So, } R_p = 60 \text{ mm}$$

Once we know R_p = 60 mm and we also know:

$$\theta = \frac{360}{4 \times 40} = \frac{9}{4}^{\circ}$$

So, once the angle is known we can easily find out and what we have done is we have found out addendum this way.

Chordal addendum = addendum + R - R cos θ =3mm + (60 - 60 × $\cos(\frac{2\pi}{4Z})$) = 3.046mm.

And what is the chordal thickness?

Chordal thickness =2 R sin θ = 2 × 60 × $\sin(\frac{9}{4})$ = 4.711 mm

So, if you open this one to 4.711 and put it on the gear tooth and make this slide move away by just the amount which is given by the pushing of the tooth tip outwards.

And it will register a particular distance along this scale, this should be very close if not exactly 3.046. If it is away from 3.046 all these terms which are there one of them must be at fault. So, we can go directly to the manufacturing either if θ is not correct, there was some problem with the indexing. If R is not correct there is some problem with the adoption of either the depth of cut or the outside diameter.

So, as it is made up of R and cos θ at least one of them must have been at fault. So, with this we come to the end of the 9th lecture, in the 10th lecture we will have discussion about a few numerical problems on all the subjects that we have covered up till now, thank you very much.