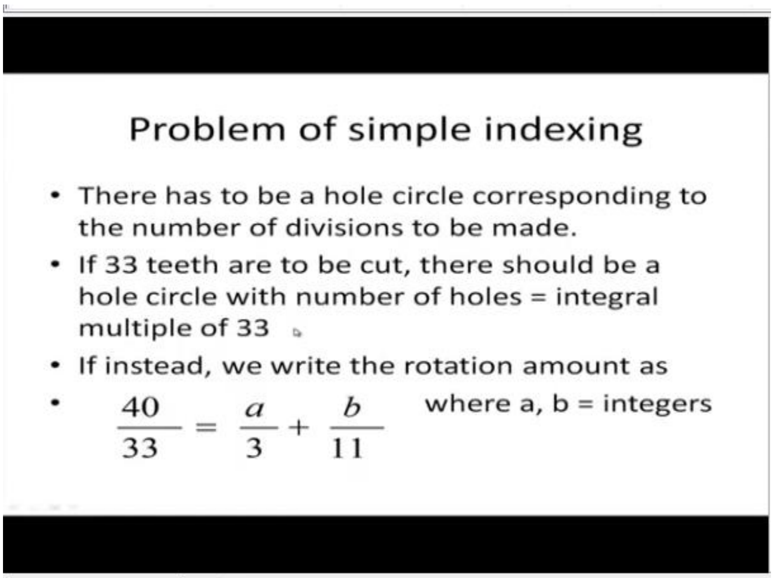
Elements of Metal Cutting, Machine Tools, Gear Cutting and CNC Machining
Prof. Asimava Roy Choudhury
Department of Mechanical Engineering
Indian Institute of Technology-Kharagpur

Lecture-28
Differential Indexing

Welcome viewers to the 8th lecture of spur and helical gear cutting. So, up till now we have discussed simple indexing and identified some problems of simple indexing like there might be some numbers of teeth for which simple indexing would not really work or you would require an assortment of index plates in order to achieve simple indexing in such cases. So, there can be other solutions to these problems and let us quickly go through such examples.

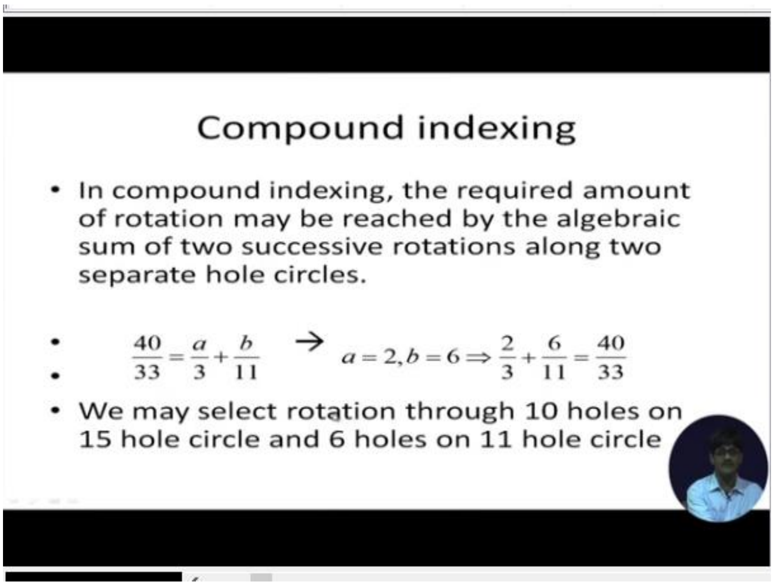
(Refer Slide Time: 00:56)



We had been discussing about $\frac{40}{33}$ being unachievable that means suppose I do not have a 33 hole circle, so you might say that is it really possible that you do not have 33 hole circle? This is just by way of an example, there might be other numbers of teeth, suppose we have 87 or 265 or something like that which is not there. So, we are taking a problem where this particular hole circle is not there.

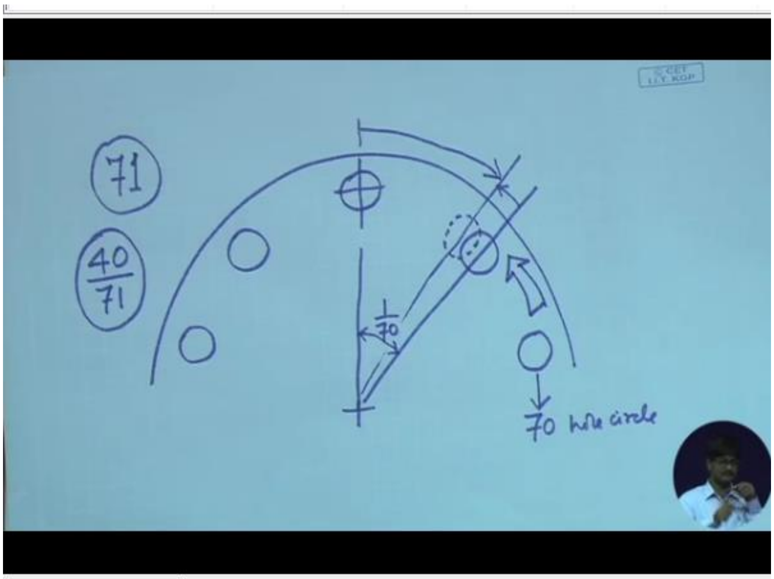
And we say that if that denominator can be broken down into its respective factors then we might be able to employ the method of partial fractions and achieve something called compound indexing. That means, this rotation which is required which is $1\frac{7}{33}$ of a rotation that will be equal to the algebraic sum, they might be having different signs also, algebraic sum of these 2. What is the restriction of a and b? First of all a and b are whole numbers and they have to satisfy this particular equation. So, in that case they are integers they have to satisfy this.

(Refer Slide Time: 02:26)



So, what we can say is that, if this be so we can easily let us look at the piece of paper.

(Refer Slide Time: 02:39)



We are having:

$$\frac{40}{33} = \frac{a}{3} + \frac{b}{11} = \frac{11a + 3b}{33}$$

There are elaborate ways of finding out the possible solutions of a and b and finding out the number of whole circles which will be employed in these cases, but at this moment we are not going for such an elaborate discussion, compound indexing is not that popular because it essentially combines 2 different successive rotations.

We would always be interested to work with a single movement; less number of movements to achieve the required rotation would result in less amount of errors. So, this means that

$$(11 \times a) + (3 \times b) = 40$$

and by the method of trial and error you can easily hit upon some solutions. This is basically the equation of a straight line. a and b have to be integers and so let me suggest suppose a = 2,

$$(11 \times a) + (3 \times b) = 40 \rightarrow (11 \times 2) + (3 \times 6) = 22 + 18 = 40$$

So this means that if a = 2, $\frac{2}{3}$ rotation plus $\frac{6}{11}$ of a rotation, these 2 separate rotations will give rise to 40 by 33 of a rotation.

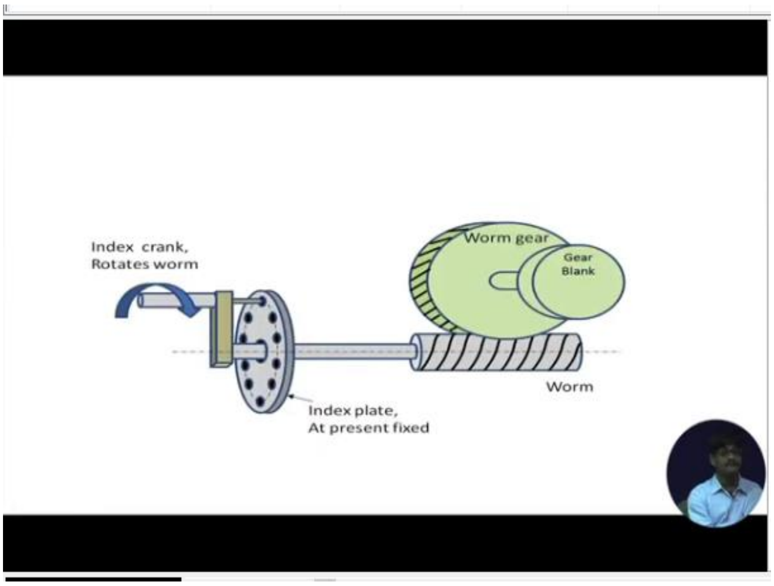
$$\frac{2}{3} + \frac{6}{11} = \frac{40}{33}$$

What does this mean? This means that suppose I employ any particular value say multiple of 3 how much say 30? So, 20 hole on a 30 hole circle plus 6 holes on an 11 hole circle, this will be the same as 40 by 33 of a rotation.

$$\frac{20}{30} + \frac{6}{11} = \frac{40}{33}$$

So, that means first of all I rotate. Let us go back to the previous just a moment.

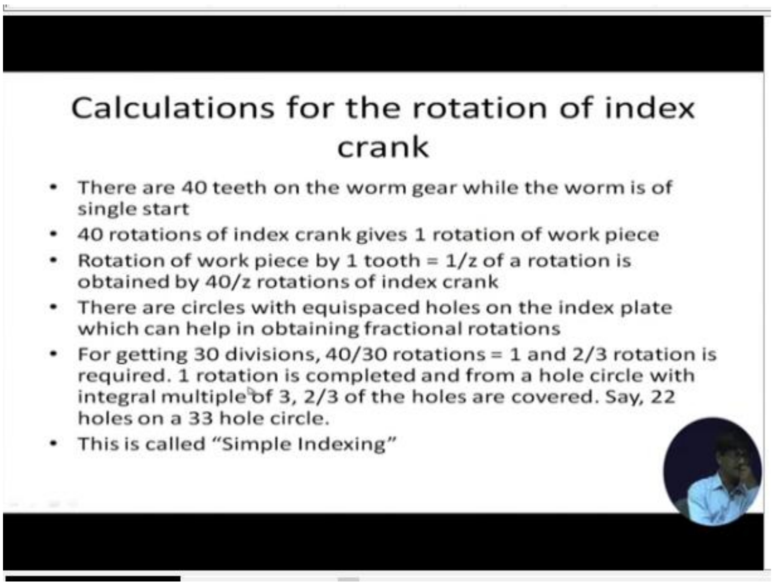
(Refer Slide Time: 05:42)



First I take out this pin from the index plate and rotate this by 20 holes on a 30 hole circle and put the pin down that means I am sealing this particular initial rotation 20 holes out of a 30 hole circle; put the pin down and after that if I still rotate the index plate is going to rotate with us, why are we doing this because, now I am going to keep track of the rotation by the help of some other fixed device.

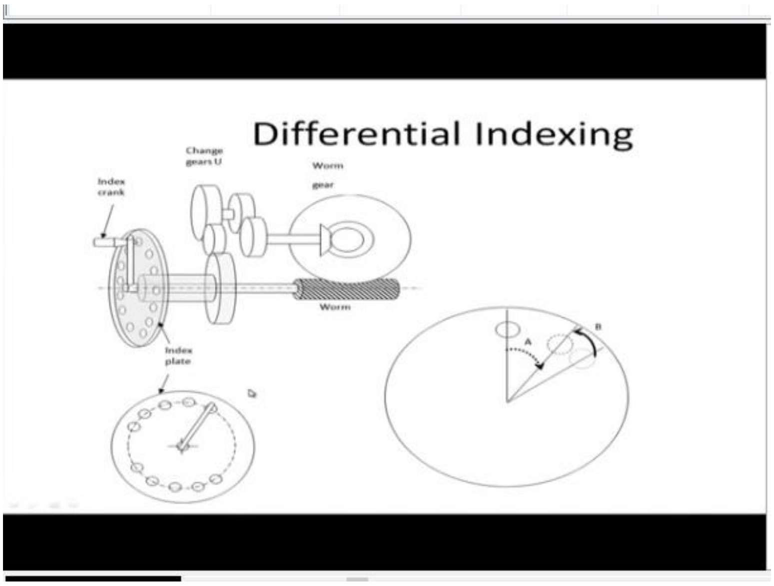
Suppose I put a pointer here with respect to this pointer now this particular index plate is going to rotate with respect to this say fixed pointer the index crank is now going to rotate. So, first move by 20 holes on a 30 hole circle, now with respect to the pointer with the pin inside the index plate, in this way it also rotates after this and with respect to this particular pointer make sure that you move by 6 holes on an 11 whole circle. So, both these whole circles have to be present on the same side of the index plate. After this is done the net rotation will be 40 out of 33. So, this is called compound indexing.

(Refer Slide Time: 07:07)



Now there is yet another method in which we might be having the use of feedback.

(Refer Slide Time: 07:18)



So, indexing with feedback is called differential indexing. And let us have a quick look at it. What does this figure show us? This figure shows us once again, so let us go through things that we already know this is our index crank, this is the worm axis, this is our worm gear, this is a pair of bevel gears which turn the rotation by 90 degrees. And we find that there are some gears now existing which rotate the index plate. In simple indexing we had not rotated the index plate.

Index plate was remaining stationary, with respect to the index plate we were counting off our rotations of that particular index crank. Now what we do is we have differential motion of the device. Differential motion means that the ultimate motion is the difference between 2 motions. One motion is the direct motion of the index crank and other motion is the fed back motion of the index plate.

Due to the difference of these 2 motions we will be achieving our required amount of motion. So, let us see that in this particular view. This is the front view of the index plate, this is a starting hole, from here this is the next hole which is actually existing let me draw a figure. So, this would be easier to follow. If you look at this index plate, on the index plate this is say a starting hole and this is the second hole, this is the third one etc.

Now you might say which hole circle are you talking about, so let us define the problem. Previously if you remember we were talking about cutting 70 teeth. Now the problem is slightly different and more difficult. 71 teeth have to be cut, have there been 70 teeth people would have used simple indexing. There would not have been any problem with that. Now the problem is 71 teeth have to be cut, what is the problem? The problem is 71 teeth hole circle is not there.

So, we know the magic number $\frac{40}{71}$ of a rotation has to be achieved on the crank handle in order to achieve $\frac{1}{71}$ of a rotation of the gear. $\frac{40}{71}$ of a rotation has to be achieved on the index crank. If you do not have 71 whole circle this is near next to impossible. So, what do we do? We find out some hole circle which is quite near to71.

So, for that suppose this is the 70 hole circle not the 71, 71 hole circle is not there. And suppose we define this one as our starting point, say this is the center, 70 hole circle first circle 70 hole circle second one. So, if I now ask you what is this amount of rotation you will say why this is $\frac{1}{70}$ of a rotation. $\frac{1}{70}$ of one full rotation, yes I accept it.

Now had there been 71 holes, then the second hole if this is the first hole of that virtual 71 hole circle had it been present. The second hole would have been somewhere very close and ahead of the second hole, so this I am drawing as the virtual 2nd whole which would have existed, had I had the 71 whole circle, first hole, second hole of the 71 whole circles it is not there.

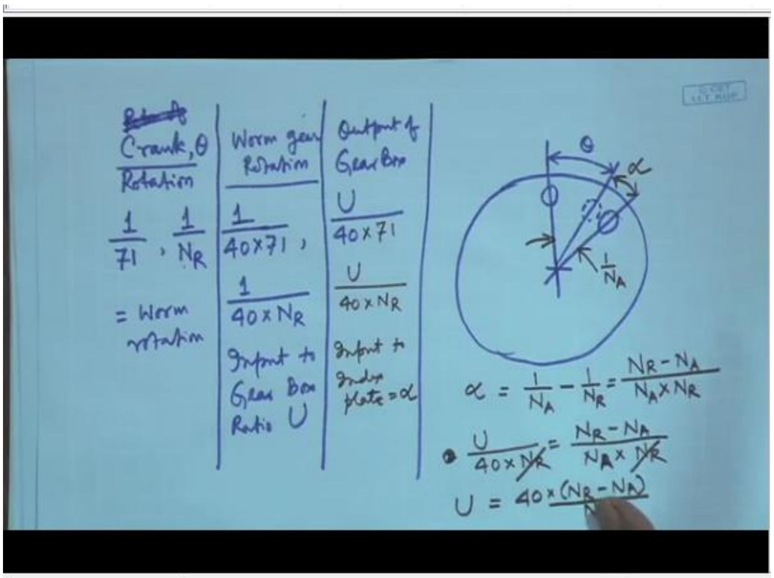
So, my plan is, in order to get it here it is virtual, things which are virtual they are not tangible you can cannot touch them, you dream of things and then they disappear something like that, but I want it, you say I want it here what do you, you bring this one here? While you have started your movement from this point as you just reach this point with your crank handle this much amount of rotation.

In that time you import it to this point. So, you will find a hole and it will get inside it, this is differential indexing the whole index plate will be rotating backwards while you have put up your crank and your moving like this, you are moving like this in that time this whole plate will be moving this way. So, that this one comes to this point that is it, that is what we are going to achieve.

This is the basic ideas of differential indexing, make the plate move so that you achieve what you want you get a hole in the location that you want. Coming to the figure now, let us see how this is physically implemented, physically we see that there is an addition of an gearbox. Now all those discussions that we had about gears and gearboxes etc. will now become quite relevant.

What we have done is since I want to rotate this index plate I am tapping motion from the worm gear, putting in a gear box, some ratio I do not know that, have you find out what should be its ratio and I am simply rotating the index plate backwards or forwards whichever be it is. How much is the amount of rotation required from the index plate? This much, that is what we have calculated. How much is the motion of the index crank during that time? This one and totally they make up $\frac{1}{70}$, in the general case $\frac{1}{n}$ available. So, let us do the calculation.

(Refer Slide Time: 15:06)



We do the calculation this way. The crank how much does it rotate? First hole, second hole, virtual hole, this is the crank rotation θ, how much is it? This must be equal to 1 by the number of teeth we cut, this is the virtual hole position and this must be equal to $\frac{1}{71}$ in our case.

And in the generalized way we write $\frac{1}{N_{required\ (R)}}$ both are correct this is specific case, this is the general case, next from this one where are we going if you look at the figure this is going to directly this amount of rotation is just going to the worm. So, we need not worry about the worm rotation, but we can worry about the worm gear rotation, worm gear rotation must be related to this.

This is basically equal to worm rotation. So, worm gear rotation must be equal to a fraction of this $\frac{1}{40}$, so we have $\frac{1}{40\times71}$ and the generalized case is $\frac{1}{40\times N_R}$. From the worm gear we are moving on to the change gears. So this must be input to the change gears, input to gearbox and gearbox is having ratio unknown (ratio U).

And therefore the output of gearbox must be equal to, just multiply U with that because we know that this must be equal to output by input. So, this one into input must be equal to the output, so we write $\frac{U}{40\times71}$, general case $\frac{U}{40\times N_R}$. And this is input to index plate in a 1:1 ratio, so see this same value is input to the index plate and what is the index plate rotation, this amount.

So, this must be equal to this, say let us call this ∝, equal to ∝. So we make this particular equation now, this thing is equal to ∝. And what is ∝? ∝ must be the difference between $\frac{1}{N_{available\ (A)}}$ and $\frac{1}{N_{required\ (R)}}$, so let us put that here.

$$\propto = \frac{1}{N_A} - \frac{1}{N_R} = \frac{N_R - N_A}{N_A \times N_R} = \frac{U}{40 \times N_R}$$

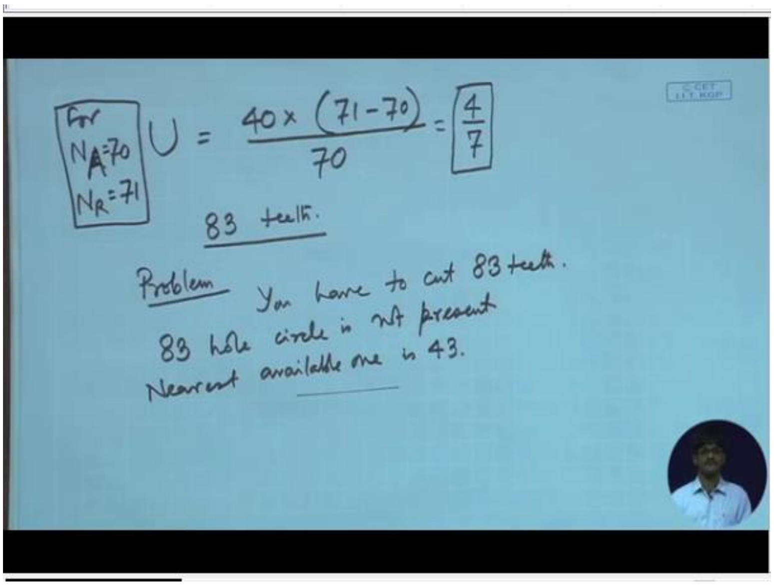
And therefore, from this you get the relation:

$$U = \frac{40 \times (N_R - N_A)}{N_A}$$

So value of U has been exactly calculated and therefore we can state, if we come back to the figure once again, if I set the change gear ratio this being the input side and this being the output side.

If I set the change gear ratio to a value $\frac{40\times(N_R-N_A)}{N_A}$, in that case I will exactly achieve a resultant rotation of the index plate. So, that I will be achieving exactly $\frac{1}{N_R}$ rotations on the gear blank. This is the most general statement that we can make. That is this change gear ratio if it is set to $\frac{40\times(N_R-N_A)}{N_A}$, in that case it will result in rotation of the index plates so that a rotation of $\frac{1}{N_A}$ on the index crank will result in $\frac{1}{N_R}$ rotation of the gear blank and that is what exactly we want.

(Refer Slide Time: 22:24)



Now for the specific calculation let us apply this therefore for $N_A = 70$, $N_R = 71$,

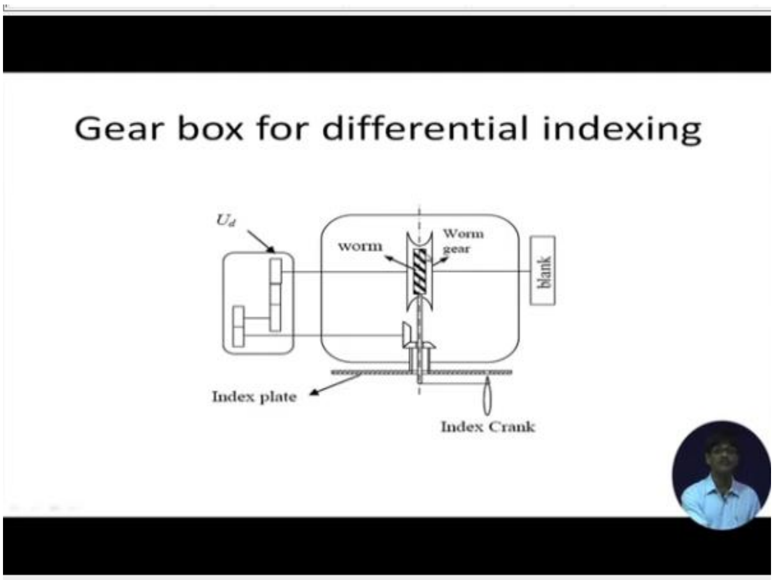
$$U = \frac{40 \times (71 - 70)}{70} = \frac{4}{7}$$

If you can put such gears with the ratio $\frac{4}{7}$ in between the worm gear shaft and the index plate shaft, you will be achieving differential indexing in this case. So, we will be definitely doing some problems during the problem session and let me define them so that you can think out the idea and we can compare our results during our questionnaire session.

For example, you have to cut 83 teeth, 83 hole circle is not present and the nearest available one is 43, why 43 I could have given it 80. Here the idea is if 43 is present you can do the calculations with 86 in mind and after that you will find that 2 will cancel out so that ultimately you can work with 43.

So, this is also another thing that we will be learning exactly the nearest whole circle might not actually be present but with its virtual presence you can do the calculations and after that you might find that a fraction of it, if it is present that we do the trick. So, this one please work this out we will carry out the calculations during our questionnaire session. So, after this so I will be giving you more problems of this type when we even discuss further.

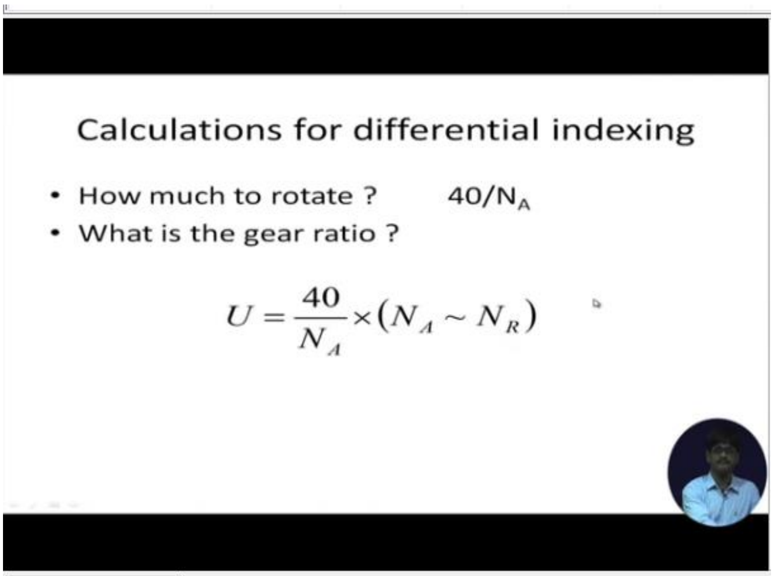
(Refer Slide Time: 25:57)



Now for a quick look at what we have discussed so that our doubts if any will be clear. To those people who have had troubles in following the 3-dimensional picture I have tried to give the explanation of the thing flat two-dimensional drawing, let us have a quick look at it. This is our index crank; when you are rotating the index crank this is the shaft of the index crank which goes right through to the worm.

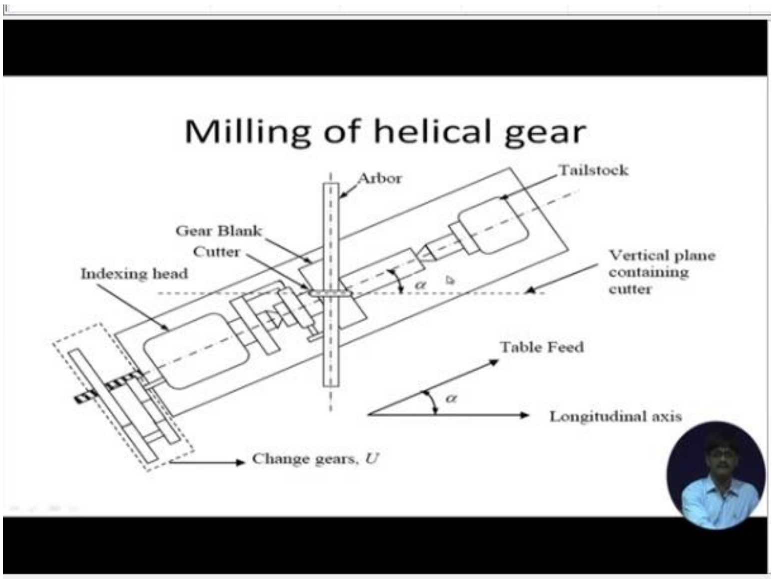
And the worm gear is shown here like this and it has its shaft on one side it is carrying the blank, on the other side it is scanning the change gearbox and the change gearbox output is given to the index plate through a pair of bevel gears and the index plate rotates due to that. So, this is a typical figure which has the details of a gearbox for differential indexing.

(Refer Slide Time: 27:06)



This is the calculation that we have worked out, so I am not doing it once again.

(Refer Slide Time: 27:13)



There might be one difference in sign, which I have written $N_A \sim N_R$ to generalize the case. Now question is the operator who is carrying out the differential indexing he might say that so far so good you have put a particular gear ratio and we are expecting that cutting will be done quite properly with that, but how much do I rotate the index crank?

If 71 teeth are to be cut and 70 hole circle is available, how much is the rotation that I give? The rotation will be $\frac{40}{Z}$ that means $\frac{40}{N_A}$, if 70 hole circle is available you will be moving by 40 holes and automatically the machine due to the presence of the gearbox will be converting it into $\frac{1}{N_R}$ on the workpiece. So, $\frac{40}{N_A}$ will give rise to $\frac{1}{N_R}$. So, on a 70 hole circle move by 40 holes and the rest will be taken care of all those gear boxes and other things that we have put in between that is it.

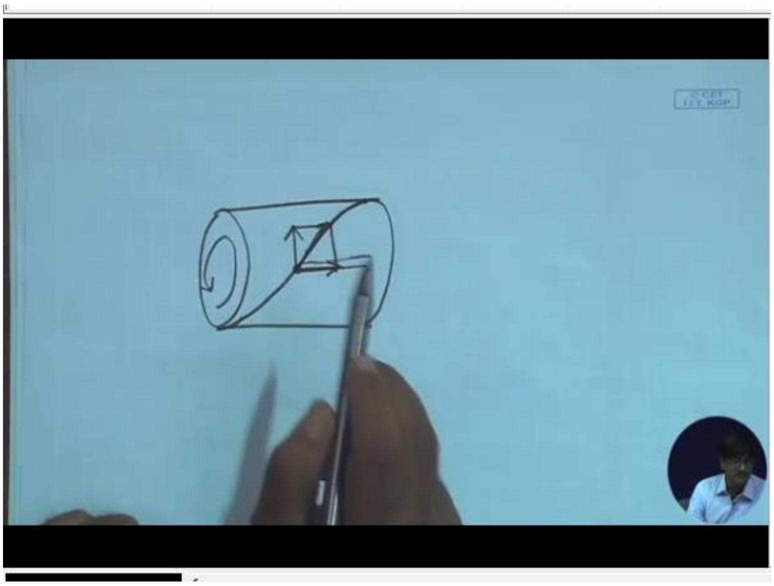
So, the rotation is 40 holes out of 70 holes level. Now I will just introduce the idea of milling of helical gears, we have studied about differential indexing instead of going for solution of numerical problems I will introduce the idea of helical gear and later on in our 9th and 10th lecture we will solve numerical problems. Let us have a quick look what we are trying to convey through this particular figure.

This is the table of a universal milling machine seen from the top; this is a horizontal column and knee type universal milling machine. Why horizontal? Because this is the axis of the cutter, axis of the arbor which is holding the cutter. Arbor is the shaft which is holding the cutter, this is the cutter. Now how does it correspond to the cutter that we have seen in our drawings, this is that rotary disc type cutter the disc is being seen from the top, not the circular part, but the width of the cutter is seen in this direction.

In the other view if you look along the arbor you will see a circular part with the teeth on its periphery. So, how does the cutter move? It rotates about the arbor to develop cutting speed . And what are the other types of movement that it has? Relative to the table the table is going to have movement this way. The table has movement this way, what other movements are being provided here?

The cutter is being rotated. Previously we were having the cutter here and the table was straight and the table was simply moving to and fro so that the cutter cut a groove right across on the circular or curvilinear surface of the workpiece the cylindrical surface of the workpiece. At this moment the vertical plane containing the cutter is at an angle defined by the helix angle is at an angle with the workpiece. Why is this so? Because we have to cut a groove which is a helical groove on the surface, so for that let us have a quick look at this figure.

(Refer Slide Time: 31:39)



The basic idea is if you have to say put a pencil mark on a cylinder in the form of a helix; the helix is moving this way which means that at a particular moment it can be defined by a definite velocity this way and a definite velocity this way, so that the resultant is always tangential to the helix. How is this possible? You can put a pencil here, suppose I stick a pencil here give a rotation to this particular cylinder and move the pencil actually at the same time, I am holding the pencil, I am moving the pencil this way.

And at the same time I am rotating this particular body you will have the tracing of a helical line on this one on this particular cylinder, same thing for the cutting action during helical milling of gear teeth. We have rotation of the workpiece, giving the rotation for the helical motion and straight-line motion of the workpiece. These 2 combine to provide relative helical motion of the cutter with respect to the workpiece.

But then, why are we rotating the whole table about a vertical axis? Why is this particular thing inclined? We could have done could well have done the whole thing with the cutter in this state with the table in this direction. That we will discuss the next day, please have a thought about it. So, this way we come to the end of the 8th lecture. In the ninth lecture we will discussing the rest part of helical milling and all the other things that are left behind for milling of spiral helical gears, thank you very much.