Spur and Helical Gear Cutting
Prof. Asimava Roy Choudhury
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

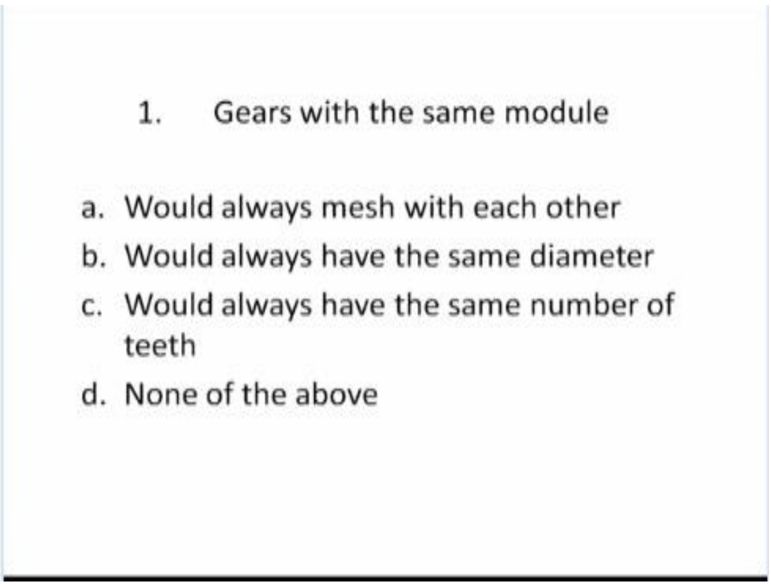
Lecture – 25
Numerical Problem MCQ

Welcome viewers to the fifth lecture of the open online course Spur and Helical Gear Cutting. So, we have finished 4 lectures in which we have discussed about gears in general their functions, what purpose they serve. And we have also discussed some of the calculational aspects of gears like what module.

And how to calculate gear ratios for obtaining a particular rotational speed from an initial rotating shaft by employing gears, what are the geometrical nomenclature of spur gears and helical gears, etc., and some special type of gearing like worm and worm gear, some mechanical machine element pairs which will be required for our subsequent lectures like screw and nut mechanism etc. **(01:34)**

All these things, we have discussed and today, we will take up some numerical problems, which will further help you to understand the way in which gears can be employed to suit our different requirements. So, let us start right away and look into some of the numerical problems, some multiple choice questions, both preliminary and difficult.

(Refer Slide Time: 02:13)



So, the first one, gears with the same module: Would always mesh with each other? Would always have the same diameter? Would always have the same number of teeth? None of the above. So, first of all, gears with the same module ‘m’ is the same for all these gears. And if you are dealing with a different system of units, for example, if you are expressing your diameter in inches, we would be saying gears with the same diametral pitch, which is basically Z by D, where D is the pitch diameter in inches and Z is the number of teeth.

So, what we are saying is that if such gears are taken which all of which have the same module, they would always mesh with each other. This is correct. So, in this multiple choice question, we have identified the first one is definitely correct. They would always mesh with each other. We are talking about basically spur gears at this moment, it can also be extended to helical gears, but let us restrict our discussion here.

So, we should, slightly modify the question this way is spur gears with the same module: would always mesh with each other? Would always have the same diameter? No, you are not supposed to have the same diameter because, in a family of gears with the same module, if you go on increasing the number of teeth, the diameter will increase. So, they are not necessarily going to have the same diameter. Would always have the same number of teeth?

No, not at all, because if you have all the gears of the same number of teeth, what purpose would they serve except for, transmitting power from one shaft to another. So, none of the above. So, a is correct; would always mesh with each other.

(Refer Slide Time: 04:21)

2. Gears having the same module

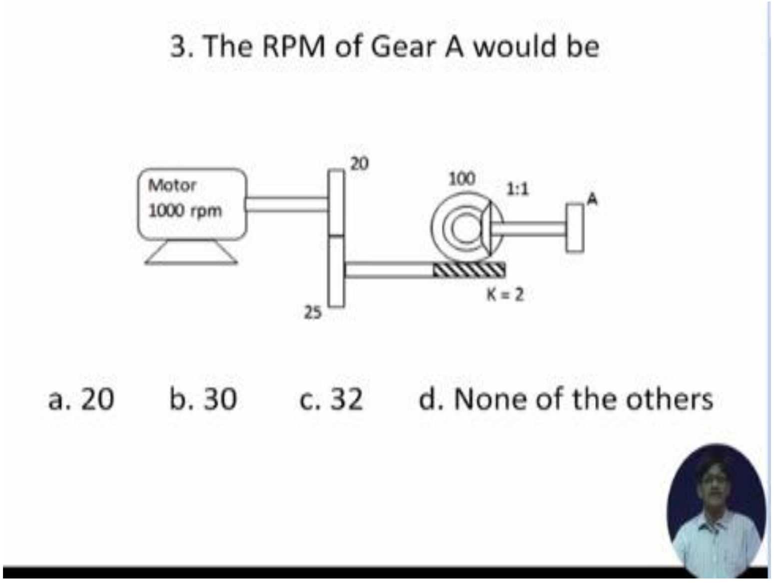
a. would have larger teeth for larger diameters
b. Would have smaller teeth for less number of teeth
c. Would have larger teeth for higher numbers of teeth
d. None of the above

Gears having the same module: would have larger teeth for larger diameters? So, all these gears have the same module, we have taken a family of gears. So, if we take larger diameters, we will find the teeth are being becoming larger and larger. Would have smaller teeth for less number of teeth? So, if you have less number of teeth, you will find that teeth are becoming smaller in size.

Would have larger teeth for higher number of teeth? Just the opposite. None of the above. So, let us see one by one. Would have larger teeth for larger diameters? So, first and foremost observation that we can make is that if you have the module to be the same, then the size of the tooth becomes defined. If module is the same, the size of the tooth is constant, be it addendum or dedendum or working depth or total depth or the chordal addendum or the chordal thickness, etc.

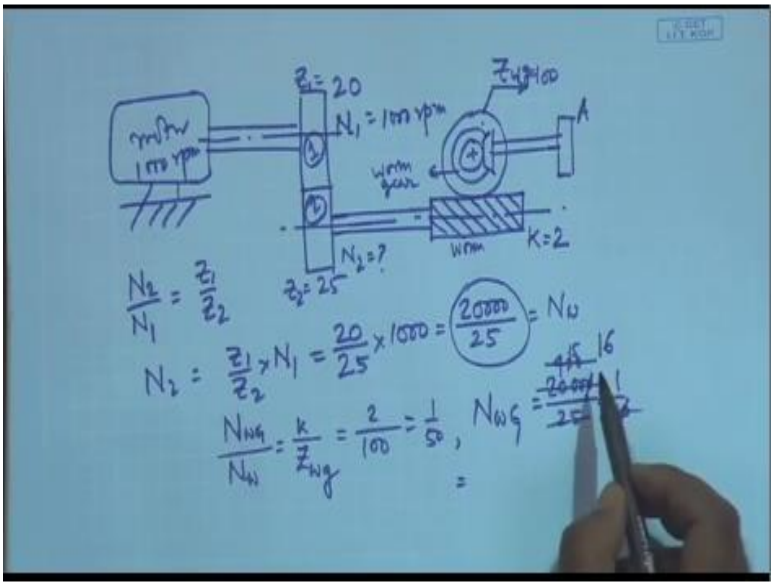
Everything is the same if the module is the same. So, none of the above? So, let us look at the question once again, there will not be larger teeth; they will not be smaller teeth and therefore, none of the above is correct.

(Refer Slide Time: 05:45)



Now, for a numerical problem. What is this numerical problems state? It states that the RPM of gear A would be 20? 30? 32? none of the others? So, let us start from the beginning. There is a motor with 1000 RPM and it is connected with 2 spur gears. Let us quickly present it on the sheet of paper. And let us see the calculations.

(Refer Slide Time: 06:21)



This happens to be the motor, let us draw quickly because it is already there in front of us. This is our motor which is rotating 1000 RPM. These are just hypothetical values just to frame a question. I am having 20 with that I am having 25. So, what do we do with this? We say, we already know:

$$\frac{N_2}{N_1} = \frac{Z_1}{Z_2}$$

Therefore,

$$N_2 = \frac{Z_1}{Z_2} \times N_1$$

N is the RPM. Z is the number of teeth.

So, if this be gear number 1 and if this be gear number 2, Z_2 = 25 teeth. And Z_1 = 20 teeth and N_1 = 1000 RPM because it is sharing its RPM with the motor and N_2 is not known. So, what is N_2 going to be? N_2 is going to be:

$$\frac{20}{25} \times 1000 = \frac{20000}{25}$$

Let us not cancel anything at this moment because there is something down the line after this.

This, we studied the last day that is how to read a drawing. What is this supposed to be? This is the symbol that we are using for our worm; there should be actually axis lines like this to show that they are having an axis of rotation and its axis symmetric. So, fine, this is a worm. How do we recognise a worm? k = 2 is a giveaway; number of starts of this worm is equal to 2.

And after that, let us identify the worm here. Yes, this is the worm gear. This is the worm and this is the worm gear. So, what is the worm gear going to do? How many teeth does it have? It has 100 teeth. So, we will have **(09:00)**:

$$\frac{N_{\text{worm gear(WG)}}}{N_{\text{worm(W)}}} = \frac{k}{Z_{\text{worm gear(WG)}}} = \frac{2}{100} = \frac{1}{50}$$

Do we know the worm rotation N_{worm}? Do we know? Yes, this is sharing its RPM with gear number 2. So, this must be equal to $N_{worm} = \frac{20000}{25}$, and $\frac{k}{Z_{WG}}$ is already found out and therefore, we have:

$$N_{WG} = \frac{20000}{25} \times \frac{1}{50}$$

And What is required? This one is again connected with a bevel gear pair. This is the symbol of the bevel gear. And it is taken out and given to gear A.

How many number of teeth does gear A have? It is not given. It does not matter. Whatever is the rotation of the worm gear, it is given to the bevel gear is given by 1:1 ratio to the other bevel gear and that is sharing its RPM with gear A. And that is it. If you can find out the worm gear rotation, you have found out the rotations per minute of A. So, this is the answer. Let us work it out:

$$N_{WG} = \frac{20000}{25} \times \frac{1}{50} = 16$$

Let us have a quick look at the calculations have we made any mistake.

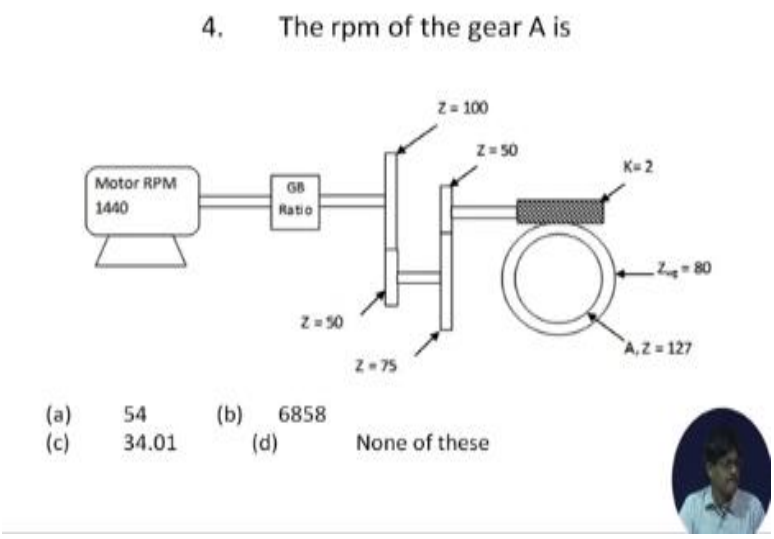
$$N_2 = \frac{Z_1}{Z_2} \times N_1 = \frac{20}{25} \times 1000$$

So, this is slightly less than 1000 and this is being given to the worm. So,

$$\frac{N_{WG}}{N_W} = \frac{2}{100} = \frac{1}{50} \Rightarrow N_{WG} = \frac{20000}{25} \times \frac{1}{50} = 16$$

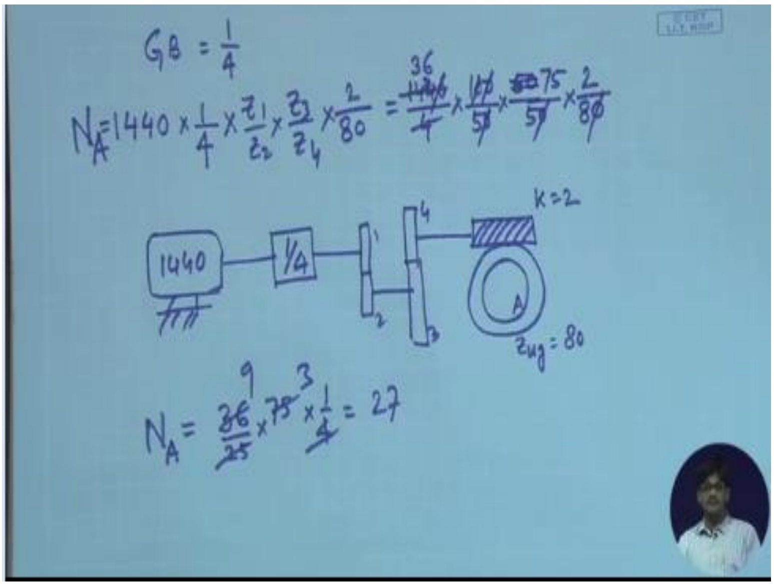
so, the answer is 16 RPM. That is good. So, none of the others is the answer.

(Refer Slide Time: 12:12)



Fourth problem, a similar problem, let us see. First of all, the RPM of the gear A is? The value for gearbox ratio is missing. Let us assume gearbox ratio (GB) = $\frac{1}{4}$.

(Refer Slide Time: 12:50)



In that case, let us find out what is the rotations per minute of gear A. So, we start with motor RPM =1440 RPM. When it passes through the gearbox, we simply multiply the ratio because this is equal to $^{\text{output}}/_{\text{input}}$. So,

$$\text{Output} = \text{Input} \times {}^{\text{output}}/_{\text{input}} = 1440 \times \frac{1}{4}$$

Output RPM of the gearbox must be equal to 1440 into one fourth.

This one multiplied by this gear ratio, which is coming after this, in these gear ratios that we have in the figure, the figure is somewhat like this, motor, gearbox and then gear 1, gear 2, gear

3 and gear 4, then comes worm and simply a gear mounted on the worm A and this is Z_{wg} = 80; k = 2. So, in this case, this we have defined as one fourth; this is 1440 and this is 1, 2, 3, 4.

Typically, this can show it by calculation as Z_1 by Z_2 into Z_3 by Z_4 multiplied by worm to worm gear. Once again this must be worm rotations per minute. So, k by Z multiplied by k by Z, k = 2 and this is 80. And this rotation is shared between gear A and the worm gear. So, this must be rotations per minute of A, which is N_A. So, how much is this? 1440 by 4 into, gear 1 has 100 by 50 into Z_3 = 75 by 50 into k is 2 and 80 is Z.

$$N_A = 1440 \times \frac{1}{4} \times \frac{Z_1}{Z_2} \times \frac{Z_3}{Z_4} \times \frac{2}{80} = \frac{1440}{4} \times \frac{100}{50} \times \frac{75}{50} \times \frac{2}{80}$$

This is the answer.

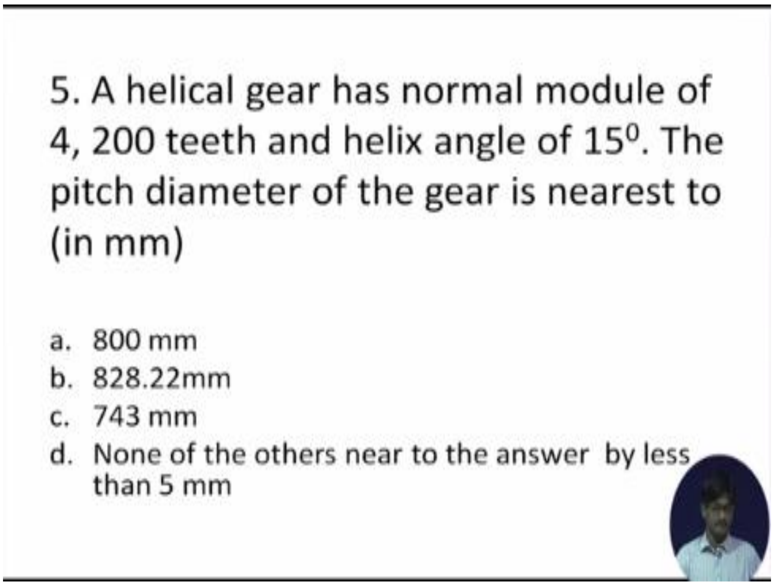
Naturally, it will not match with the answer because the gearbox ratio was not given, we have simply assumed some gearbox ratio and found it out. Anyway, let us do the preliminary calculations, after simplifying the above equation, we get:

$$N_A = \frac{36}{25} \times 75 \times \frac{1}{4} = 27$$

So, N_A is coming out to be 27. I suspect that the gearbox ratio in the actual problem which I had designed previously, it must have been half. In that case, it would have come out as 54. But anyway, whatever we have started with this thing stands by itself. The rotations per minute of gear A would be 27 if we take this gear ratio to be one fourth that is it.

So, even though, we are not able to do the problem, exactly as stated here, because gearbox ratio is not mentioned. I am sure you have understood this. Thank you. So, let us talk on to another problem.

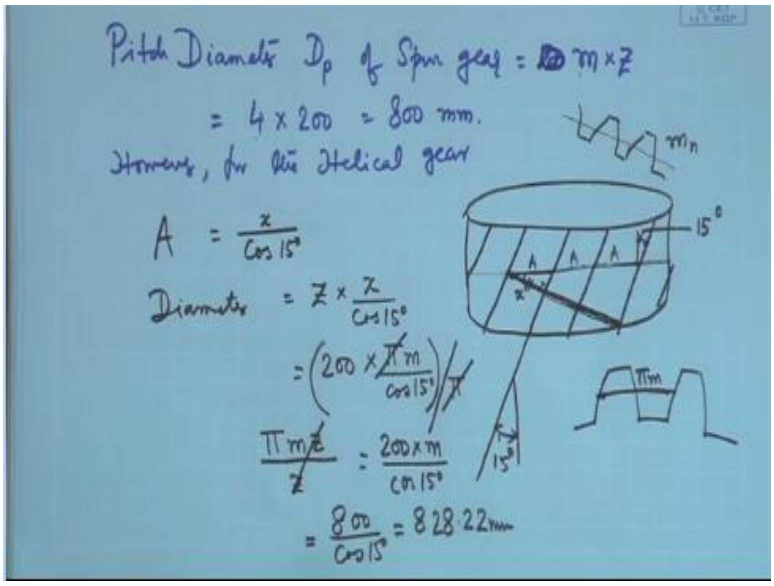
(Refer Slide Time: 17:20)



A helical gear has normal module of 4, 200 teeth and helix angle of 15 degrees. The pitch diameter of the gear is nearest to in millimetres? 800 millimetres, 828.22 millimetres, 743 millimetres and none of the others are near to the answer by less than 5 millimetres. That means none of them are correct basically. So, let us first see, upto till now, we have been discussing about spur gears.

Now, we are talking about helical gears. And in the last lecture, we had talked about a problem of this type. What was this problem? The problem was this that if you have a spur gear with module = 4 and teeth = 200, we can straightaway calculate the pitch diameter to be module × number of teeth = 800 in this case.

(Refer Slide Time: 18:25)



Let us write, pitch diameter D_p of spur gear = 4 × 200 = 800 millimetres. However, for the helical gear, this is the spur gear say, these other teeth are the helical gear. In a helical gear, what happens is that this distance taken normal to the tooth orientation, this one corresponds to these dimensions if you take a section here, it will be coming something like.

This section if you take it, if you cut it along this, you will see the teeth this way and this one corresponds to the measurements as per this module, this module that is why this is said to be corresponding to the normal module m_n. In this direction, however, it is slightly larger. Naturally, this distance is replaced by this distance if you take a section this way.

So, when we are mentioning this 4, we are talking about if you look at the problem once, we are talking about normal module. So, coming back to the hand drawn figure, we are talking about this distance, these distances they correspond to the normal module. Hence if that is so, the diameter which is existing here and which is the summation of A, A, A like that, it is slightly larger than the distances which are obtained by the calculation of the nominal module.

So, each of these sections, they are replaced by this value. And what is this angle? If you remember, this angle is equal to the helix angle. In this case, say this is 15 degrees; 15 degrees this angle. Let me draw it 15 degrees. So, if this is 15 degrees, this is a right angle and therefore, this angle will be 15 degrees. This angle will be 15 degrees and therefore, we find if this is equal to x.

This one, A is equal to; we can write:

$$A = \frac{x}{\cos 15^{\circ}}$$

And how many A is makeup the full diameter? If you go fully round as many number of teeth as many A values so, instead of having x values here, the diameter = Z × $\frac{\mathrm{x}}{\cos 15^{\circ}}$. Let us replace 200 multiplied by. Now, what is this x equal to? This x is equal to the distance covered by one tooth.

How much is that? That must be equal to π × m. So, (π × m)/ $\cos 15^{\circ}$. If you find this out, I think it will be coming out to be 828.22.

$$200 \times \frac{\pi \times \mathrm{m}}{\cos 15^{\circ}} = 828.22$$

So, please calculate this because, if you look at this figure, this distance which we are showing here, this distance is nothing but this total divided by number of teeth by $\pi \times m \times Z$ divided by Z, so, it is $\pi \times m$.

$$\frac{\pi \times m \times Z}{Z} = \pi \times m$$

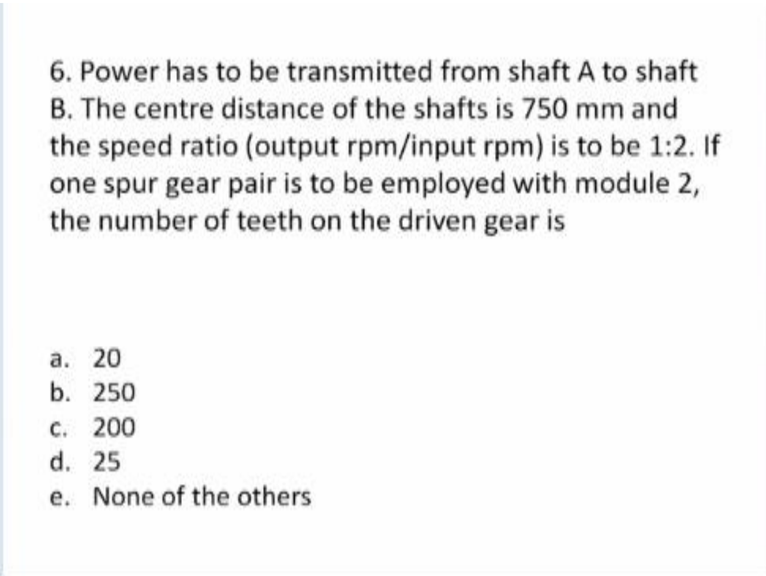
So, this thing is equal to the circumference. So, if you divide the circumference by π, this cancels out and this becomes:

$$\frac{200 \times m}{\cos 15^{\circ}} = \frac{800}{\cos 15^{\circ}} = 828.22 \text{ mm}$$

So, I hope this is alright. This is equal to $\pi \times m$. We go on adding $\frac{\pi \times m}{\cos 15^{\circ}}$, Z number of times, and we get this value. So, the answer is 828.22 millimetres.

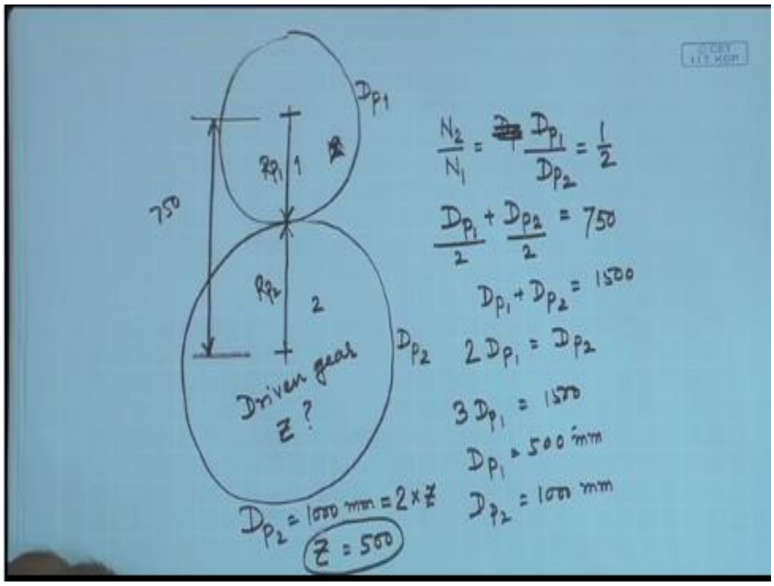
So, what we observe is that the pitch diameter of the helical gear is going to have higher diameter than the corresponding spur gear, if they have the same normal module.

(Refer Slide Time: 25:41)



Power has to be transmitted from shaft A to shaft B. The centre distance of the shafts is 750 millimetres and the speed ratio output by input ($^{\text{Output RPM}}/_{\text{Input RPM}}$ is to be 1:2 that means, the speed has to come down to half of its original value. If one spur gear pair is to be employed with module of 2, the number of teeth on the driven gear is? 22, 250, 200, 25, none of the others. So, let us try it out.

(Refer Slide Time: 26:17)



This is the centre distance. How much is this? This is equal to 750. I have to employ 2 gears say, hypothetically, let me draw them. This is one gear and this is another gear. I have to employ them. What are their diameters? I do not know. But, if these are representing the pitch diameters D_{p1} and D_{p2}, in that case, I can say that definitely the speed ratio:

$$\frac{N_2}{N_1} = \frac{D_{p1}}{D_{p2}}$$

that means, if N_2 is less, D_{p1} must be less.

So, how do we get this? From the basic definition of pitch diameters, we are obtaining this. What is $D_{p1} + D_{p2}$ equal to? Well, $(D_{p1}/2) + (D_{p2}/2)$ is simply equal to this distance plus this distance R_{p1} and R_{p2}. So, it must be 750. So, we write $(D_{p1}/2) + (D_{p2}/2)$ must be equal to 750 that is good.

$$\frac{D_{p1}}{2} + \frac{D_{p2}}{2} = R_{p1} + R_{p2} = 750$$

What is N_2 by N_1 supposed to be? (N_2/N_1) is supposed to be half.

So, let us send this to that side, $D_{p1} + D_{p2} = 1500$ and $2\,D_{p1} = D_{p2}$ that is good, let us replace D_{p2}. So, we get $3\,D_{p1} = 1500$. Therefore, $D_{p1} = 500$ and therefore, $D_{p2} = 1000$ so, the diameters have been found out.

$$D_{p1} + D_{p2} = 1500$$

$$2\,D_{p1} = D_{p2} \Rightarrow 3\,D_{p1} = 1500$$

$$\therefore D_{p1} = 500\text{ mm},\ D_{p2} = 1000\text{ mm}$$

So, let us see what is to be calculated. So, is this understood? That D_{p1} and D_{p2} can be found out from 2 relations that is the centre distance which is equal to the sum of the 2 radii that is equal to 750 and (D_{p1}/D_{p2}) is equal to the ratio of rotations per minute stated. By using those 2, we have solved D_{p1} and D_{p2}, but the question is different. Let us look at the question.

If one spur gear pair is to be employed with module 2, the number of teeth on the driven gear is? This is the driven gear. Is it solving the purpose? Yes, as it is connected with a smaller gear, if this has RPM X, this RPM will be less than X. So, because it is larger in size and in fact, we know their diameter values. Exactly, the diameter will be brought down by a factor of 2. So, everything is satisfied.

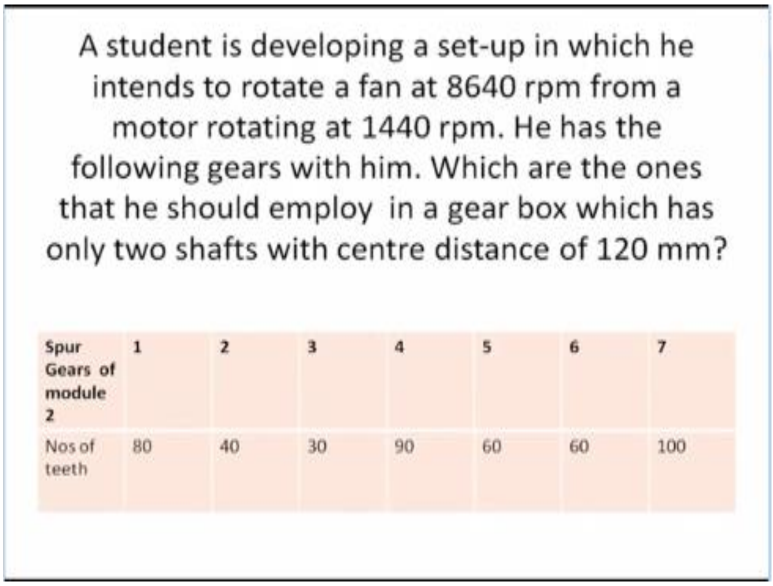
Now, we have to find out the number of teeth on the driven gear. So, Z has been asked for. So, what we can say is that since D_{p2} = 1000 millimetres and module = 2. So, we can write:

$$D_{p2} = 1000 \text{ mm} = 2 \times Z$$

Therefore, Z = 500. Answer is none of the above. Why have we given a question in which none of the above is the answer?

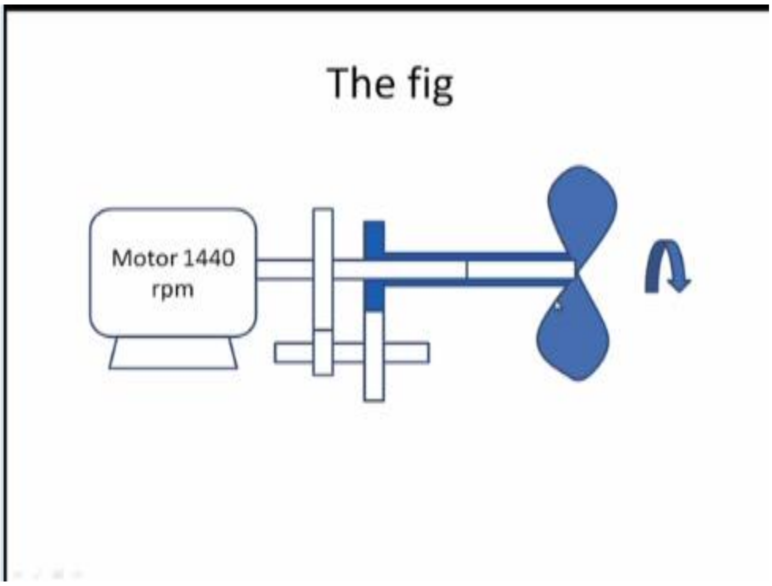
The reason is this very frequently students make a mistake. They put $D_{p1} + D_{p2}$ is equal to 750. And immediately, they will get an answer of 250. They will say 250 is there and they will take on 250. This is a tricky part of the question. You have to be extremely alert. So, answer is none of the others.

(Refer Slide Time: 31:44)



I think, we have just a 1 or 2 minutes, so that I will just introduce this problem and ask you to solve yourselves and I will provide the answer in one of the subsequent lectures and if time is there, I will discuss it as well. A student is developing a setup in which he intends to rotate a fan at 8640 RPM from a motor rotating at 1440 RPM. So, almost 6 times the RPM has to be increased, he has the following gears with him, which are the ones that he should employ in a gearbox which has only 2 shafts with centre distance of 240 millimetres.

(Refer Slide Time: 32:26)



So, let us have a look at the figure. This is the proposed figure motor 140 RPM gear with it; gear to, auxiliary shaft and this one sharing RPM, giving it to another gear, which is loosely fitted on the first gear, not having any rotational relation with the first gear, first shaft and this one is connected with the fan. So, with only 2 shafts, we are able to bring down the RPM once here and once there.

So, what should be the answer? You can think about this. We will discuss the answer in one of the subsequent lectures. Thank you very much.