Spur and Helical Gear Cutting
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Lecture – 24
Helical Gear Problems

Welcome viewers to the 4th lecture of the series Spur and Helical Gear Cutting. In the last lecture, we have discussed about some of the geometric aspects of spur gears and today, we will be continuing with that and also discussing something about helical gears.

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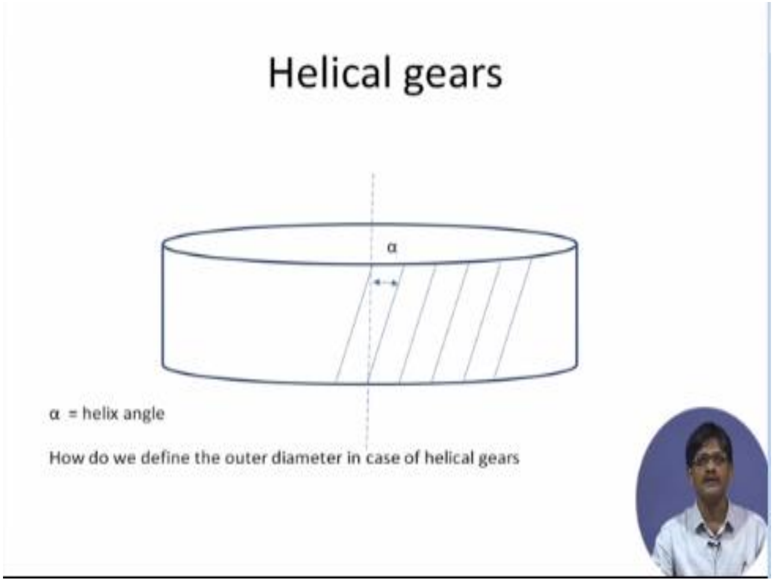

So, in case of helical gears, let us have a quick look at the depiction. The teeth are slightly inclined with respect to the central axis. So, we have drawn the central axis as a dot and dash line and the teeth are inclined at a particular angle α which we are calling as the helix angle. Now, you will notice that when we are depicting the helix angle of screw threads and the helix angle of helical gears, they are complementary.

So, let it please be noted that this is the helix angle of helical gears. Let me draw a figure and show you.

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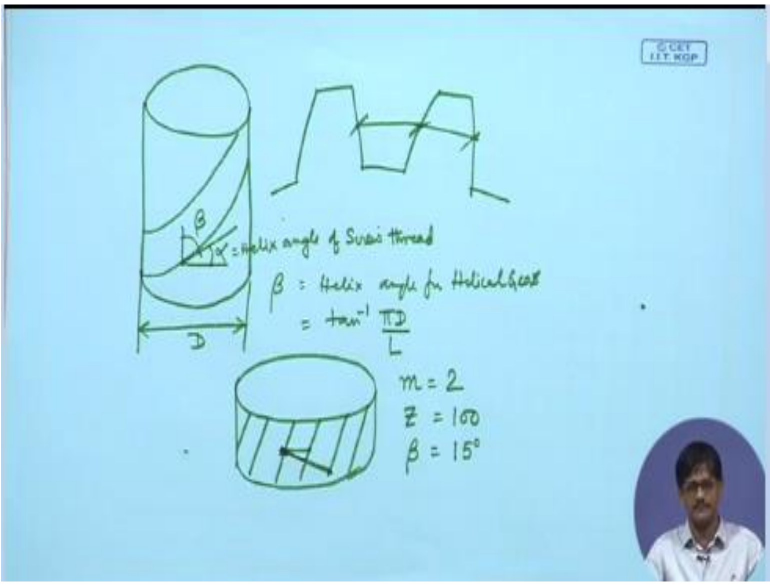

Suppose, these are helices on a screwed thread. In that case, we will express the helix angle as this one. And naturally if it is rising by p, in a single start thread, it will be:

$$\alpha = \tan^{-1}\frac{p}{\pi D}$$

and therefore, if we take the other angle for helical gears,

$$\alpha = \tan^{-1}\frac{\pi D}{p}$$

This diameter, we are taking as D. This one is p.

For multiple start threads, p will get replaced by lead (L) and therefore, β helix angle for helical gears is:

$$\beta = \tan^{-1}\frac{\pi D}{L}$$

So, coming back to our picture, how is the helical gear different from the spur gear of the same specifications? Let us take such a case, say, this is one helical gear. It is having teeth this way. First question is what is m?

Say, m = 2, and if you remember, m defines the distance between teeth as we have seen previously; this one distance is defined by m and this whole distance is nothing but $\pi \times m$. So, here also we will have the same idea. So, mind you, this distance we have to define this way perpendicular to these particular gear teeth orientation. This way, this distance is defined by m.

Therefore, this distance is larger than this particular distance which we have to note. Z = 100 and this particular helix angle that we are talking about, say, helix angle $\beta = 15^{\circ}$. So, in that case, our first question which comes is that: is the outer diameter different? Is the pitch diameter different? For that, I will leave the question to you with a hint. The hint is this. Suppose, I take a stack of books.

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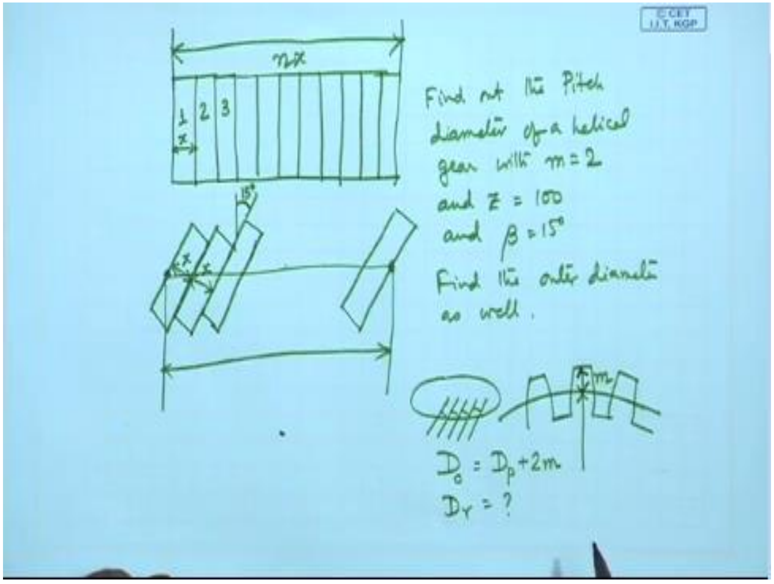

Suppose, I take a stack of books, some encyclopaedias and I keep it in a shelf, book 1, book 2, book 3 like that and say, I have filled up a particular stack this way. All these books are volumes of a particular encyclopaedia and they have the same width. Now, someone comes and disturbs this stack. So, that they get displaced and they are shifted. What is same between these books? This distance naturally remains the same.

Say, this is equal to x. This is also equal to x. What is the angle at which it has been disturbed? Say, this angle which is referred to in case of gears is 15°. So, the angle by which it has been inclined is 15° and x remains the same. Can you find out whether from this middle point to the middle point of the last book whether the shelf will be able to accommodate it or a larger shelf would be required middle point to middle point?

Once you find out the difference in this particular distance, if there are n number of books here the distance = nx. Will this be equal to nx once again? Obviously, not. This is x, so, it will be slightly more than x that is multiplied by n will be definitely higher. The moment you find out this increase, you have found out this difference in the diameters of helical gears and spur gears. Mind you, we are talking about pitch diameter up till now.

This is your assignment which we will be solving formally in later lectures. But you can have a quick look yourselves, find out the pitch diameter of a helical gear with m = 2 and z = 100 and $\beta = 15^\circ$. Now comes the interesting part. So, this is the problem that you are going to solve. And the idea that you are going to get it is embedded here. I am sure you can do it.

Next one is that find the outer diameter as well. Here, there is a particular catch. Is this one going to affect any increase in distance perpendicular to the plane of the paper? Mind you, all this is wrapped around a particular diameter. That means, I am accommodating a certain number of teeth on a circle. I am accommodating the same number of teeth, but inclined where the normal distance between these teeth, they are supposed to remain the same.

This is sometimes called as, referring to the system, same normal module. So, if this distance is supposed to remain the same, we will have to accommodate these in a larger diameter. The diameter of the helical gear with the same number of teeth, but with a helix angle has to be higher. But this, we are coming up to the pitch diameter.

Will the outer diameters be found out just by adding one module here and one module there? Remember, the addendum is equal to module. We have moved up to the pitch diameter by this calculation. What about this part? This part should be equal to module. Why should this part change? This is in a direction perpendicular to this particular dimension which has got affected. If this dimension is got affected, why should it affect distance perpendicular to it?

So, Outer diameter or Outside diameter (D_o) = still pitch diameter (D_p) + 2 module (2m). This is what we arrive at. So, please find out outer diameter this way, find out the pitch diameter, find out the root diameter also. And then we will compare our answers in the next lecture. So, with this one, let us take up a few problems, which will introduce you to use of gearboxes etc., when we are actually discussing cutting problems.

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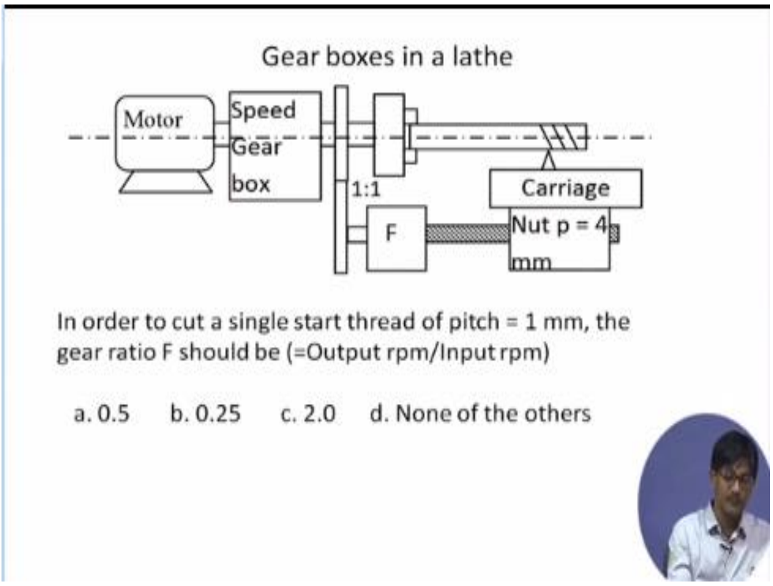

Have a look at this, the actual use of gearboxes in machine tools. First of all, why are we studying this? We are not supposed to become masters of gearbox calculations. No, but this would be required because we will be extensively using gearboxes and gearbox ratios, their calculations etc. in our, subsequent lectures. That is why 1 or 2 examples will really help.

What is this? This is the depiction of an ordinary centre lathe. There is a motor and from where we are leading on to speed gearbox, it changes the speed of the spindle. So, the spindle is rotating, you can change the rate of rotation of the spindle by changing the settings of the speed gearbox and after that a tapping has been taken that means power has been taken from the main spindle by a 1:1 speed ratio to box called F. This is the feed gearbox.

So, we become conversant with the idea that there are 2 gearboxes on the lathe and one is called a speed gearbox, which changes the speed of the spindle and one is called a feed gearbox, which changes the speed of the feed rod or the lead screw, whichever might be the case. So, in this case, we are showing the lead screw and it is connected to the half nut of pitch = 4 millimetres and that is connected to the carriage and it is cutting a thread.

The question is, in order to cut a single start thread of pitch one millimetre, the gear ratio F is equal to ${}^{\text{output RPM}}/_{\text{input RPM}}$ which should be equal to one of the following: 0.5, 0.25, 0.2 and none of the others. First of all, we have taken up this question from 2 aspects. One is to understand the positioning of gearboxes and to understand the way its gear ratio is to be calculated. Please look at this drawing, which I am going to do on the piece of paper and try to answer these questions.

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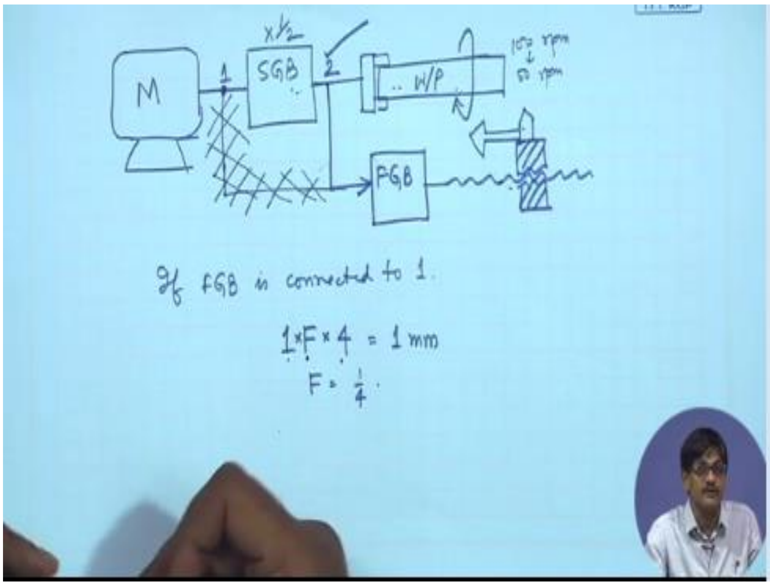

If I have a motor and then I am claiming that I have a speed gearbox here, from which I am rotating the workpiece. Now, you will find in the figure, we have taken a tapping from here and then I am putting another gear box called feed gear box. And then I am rotating something. On that, I have put a nut and moving it. Now, first of all, I could have connected it up here.

Let us call this point 1. And let us call this point 2. Do you think this would have been better? So, let us have a look how is location of a gearbox decided on a machine tool. If I had connected it here in that case so, suppose I do not connect it. So, if FGB which means feed gearbox is connected to 1, then what would have happened? Actually, it is connected at 2. But, if it is connected 1, then?

In that case, whatever changes you make in the speed gearbox would not be registered by the feed gearbox. So, suppose you make the speed of this one half; half of what was previously existing. So, this slows down say from 100 RPM, it comes down to 50 RPM. What happens to this one? This rotates undeterred. It is not affected by any changes, because it has taken the tapping of power upstream of speed gearbox.

So, even the speed gearboxes, slowed down by half of its previous value, this one is undeterred and rotates at whatever speed it was rotating previously and this moves at the same speed and hence, the relationship of movement gets disturbed and you will be getting a different thread.

Because thread is defined by movement of this one per rotation of the workpiece. Workpiece is moving slower now.

And you will cut smaller pitch thread, half the pitch will be cut now. So, it is a problem, if you connect it here and if you are doing something to the speed gearbox, be very alert because your thread which you are cutting, it will be changed. You will say I am simply changing the speed because I want to cut faster. No sir, if you make any changes here and you are having the tapping of the feed gearbox here, you are in trouble.

So, by logic, it has to be here because whatever changes you are making here should be shared by this line also, otherwise, their relationship will be disturbed. Wherever you are doing something where relational motion is important, you really cannot do this. So, that is why you have to be very alert whenever you are doing something.

What are you doing here? You are cutting a thread. How is thread pitch defined? Thread pitch is defined by the linear motion of this one per rotation of this, here is that relation that we are talking about. So, first thing, we notice about location of gearboxes is this. Whenever you are relating some motions, always have the bifurcation of power after the gearbox, which is defining the motion of the first one.

So, that is why we have taken the power tapping here. Change the speed now, this will become faster or slower, this will also become correspondingly faster or slower, proportionally faster or slower. So, there will be no change in the relational movement which was taking place which defines pitch. So, having understood the reason for putting a gearbox at a particular location, we now go for the calculations.

Coming back to this figure. Let us have a look at it. In order to cut a single start thread of pitch = 1 millimetre, so, by the time the spindle rotates once, the carriage should be moving by one millimetre. The gear ratio F should be ${}^{\text{output RPM}}/_{\text{input RPM}}$ equal to one of the given options. So, let us have a look. The nut is having 4 millimetres pitch. Here, there is 1:1 ratio.

So, by the time the spindle rotates once, this particular shaft will also rotate once. So, once it passes through F, if this ratio of ${}^{\text{output RPM}}/_{\text{input RPM}}$ be say F only so, this will be $1 \times F$

as the output. So, $1 \times F$ is the rotational rate of this lead screw and therefore, if pitch = 4 millimetres, the movement of the carriage will be $1 \times F \times 4$. Because one rotation gives 4 millimetres; F rotations give $1 \times F$ rotations $1 \times F \times 4$ millimetres of movement.

So, we have:

$$1 \times F \times 4 = 1 \text{ millimetre}$$

because we have started with a consideration that I am rotating, that is why I have written this one every time, I am rotating the spindle by one rotation 1:1 ratio makes the shaft just before the feed gearbox rotate by one rotation multiplied by F which is the factor $\text{output RPM}/\text{input RPM}$ of the feed gearbox and multiplied by 4 because 4 is the pitch of the nut thread this must be equal to one millimetre that is the requirement.

So, F is nothing but $\frac{1}{4}$. Let us look at the problem now. Yes, b is the correct answer. So, we will have lots of applications of this particular thing in our subsequent lectures that we actually discuss machining of gears. This is just for practice.

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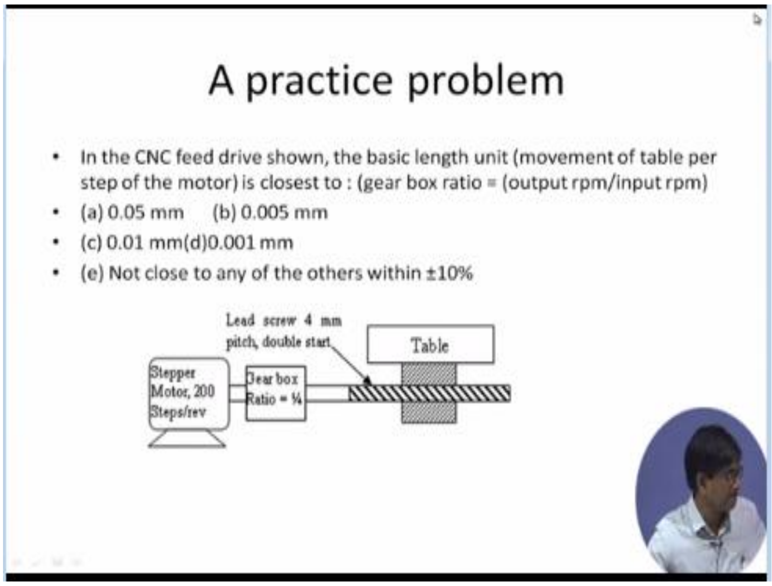

Another one slightly different configuration, but this will also help you to understand the application of gearboxes later on. In the CNC feed drive shown, the basic length unit that is the movement of the table per step of the motor. So, what do we have? We have a CNC system in which there is a stepper motor which rotates in steps to cover 1 rotation in 200 steps. So, 1 rotation is 360° and 200 steps are making up that one rotation.

Therefore, per step, it is moving by 1.8°. So, for that 1.8° movement, we are asking what the table movement is. So, what is given? If you notice for stepper motor 200 steps/revolution is given. Basic length unit is defined and given here, movement of table per step of the motor; 200 steps making up 360° of movement and gearbox ratio which is always $\text{output RPM}/_{\text{input RPM}}$ that is also given $\frac{1}{4}$.

Lead screw pitch is given 4 millimetres; double start mind you, we have been discussing about double start, single start, multiple start. So, here is an actual application. So, let us calculate this and find it out.

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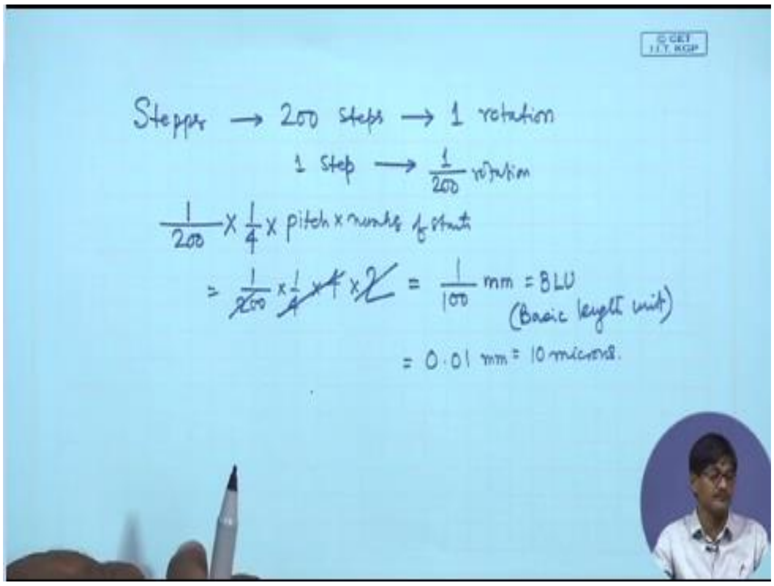

So, we have a stepper with 200 steps which gives 1 rotation. So, for 1 step we get $\frac{1}{200}$ of a rotation. Therefore, I start with $\frac{1}{200}$ of a rotation multiply it by the gearbox ratio one fourth multiply it with the lead screw lead; lead screw lead is Pitch into number of starts equal to $\frac{1}{200}$ into $\frac{1}{4}$ into pitches 4 millimetres and it is double start so, multiplied by 2 equal to $\frac{1}{100}$ millimetres.

$$\frac{1}{200} \times \frac{1}{4} \times \text{pitch} \times \text{no. of starts} = \frac{1}{200} \times \frac{1}{4} \times 4 \times 2 = \frac{1}{100}\,\text{mm}$$

What is $\frac{1}{100}$ millimetres? The basic length unit that means this smallest distance or rather the distance moved per step of the stepper motor. How much is this? Therefore, this is equal to 0.01 millimetres = 10 microns. So, this way, we understand how the location of the gearbox can be decided and how calculations can be done in order to find out the output which particular gearbox is providing.

If you have a look at the typical problems that we are going to discuss. I will discuss a very short problem which you can solve yourselves.

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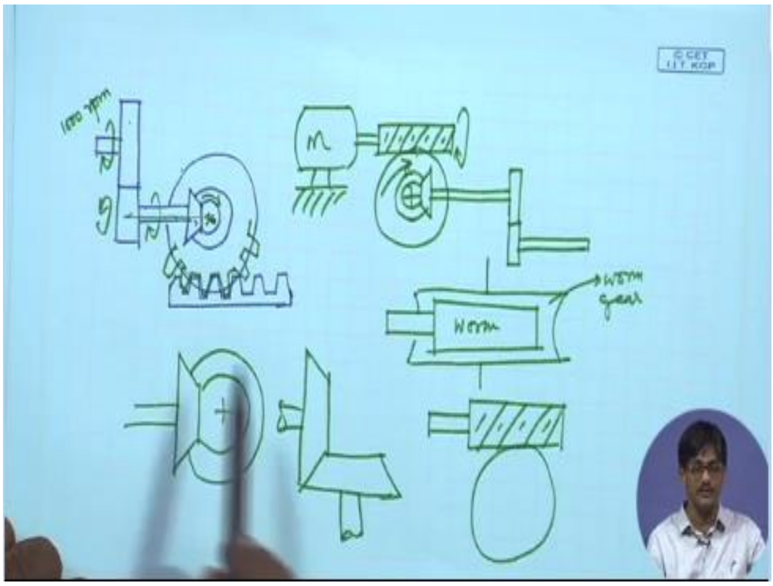

So, say here, we are having a gear connected with another one. So, let us see what sort of configuration that we are talking about. I might say that here, I am providing you 1000 RPM. These are spur gears. So, they are rotating say this way, this one is rotating that way. Now, this one, I wanted to introduce some typical figures, these are 2 bevel gears which are connected with each other.

Bevel gears will transfer power from one axis to another axis, which are generally intersecting and while transmitting power from one axis to another, these might be intersecting at 90°. So, that when this is rotating this way, this is rotating say this way etc. And this one is having on the same shaft another gear which is in connection with a rack.

So, I wanted to introduce these figures so that you are converse with this sort of configurations. There might be yet another one where I can have, a motor, shaft is coming out. I put a worm gear here. With a worm gear, I get a configuration like this. I can have a bevel gear here, a bevel gear pair. This one is taken on this side. So, this motor is rotating this worm gear.

So, with this worm, this worm is rotating this worm gear; this worm gear is rotating on its own shaft, a bevel gear. This bevel gear is rotating this bevel gear. This one is rotating this shaft this gear and together with that, that gear. So, as you can see, bevel gears can be drawn this way; one bevel here, another bevel here. They can be drawn this way, one bevel and another one.

Worm in worm gears can be drawn this way. This is the worm. This is the worm gear. They can also be drawn this way. This is the worm and this is the worm gear. Many times, what happens is: we include these drawings in a figure and ask a question. And the student in the very beginning is at a loss to understand what has been depicted. So, in this case, now after seeing these, you can easily a recognise.

Yes, this is a worm and this is a worm gear, this is a bevel and its pair. This is a rack and it is connecting pinion. That pinion is connected to the bevel gear on the same shaft. That bevel is connected to this bevel. This bevel is connected to this spur gear. This spur gear pair and then it is ultimately getting some input of 1000 RPM. So, when we discuss further problems, we will have applications of these figures very frequently. So, with this, we come to the end of the 4th lecture. Thank you very much.