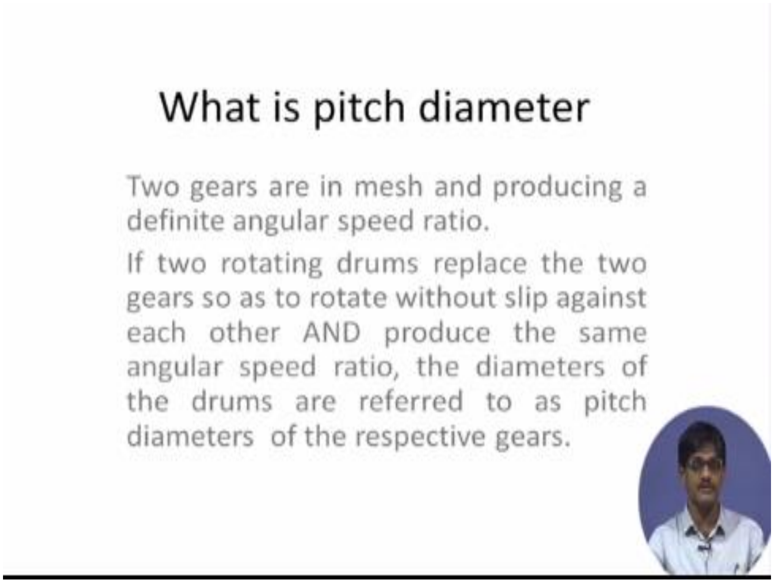
Spur and Helical Gear Cutting
Prof. Asimava Roy Choudhury
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture – 23
Gear Geometry

Welcome viewers to the third lecture of the series Spur and Helical Gear Cutting. So, last time in the first 2 lectures, we have discussed at length some of the calculations of gears, which involve module, number of teeth, RPM, etc. and we have had introduction to gear trains, worm and worm gears, then rack and pinion and after that a bit of discussion on spur and helical gear geometry which we will be continuing today.

And also, we have done some very primary calculations in order to find out RPM or gear ratios, etc. So, today we will take up the loose ends that we left off in the last lecture and continue with our discussions on geometry of gears. So, that it will be easy for us to take up calculations for actual gear cutting practice on machine tools.

(Refer Slide Time: 01:42)

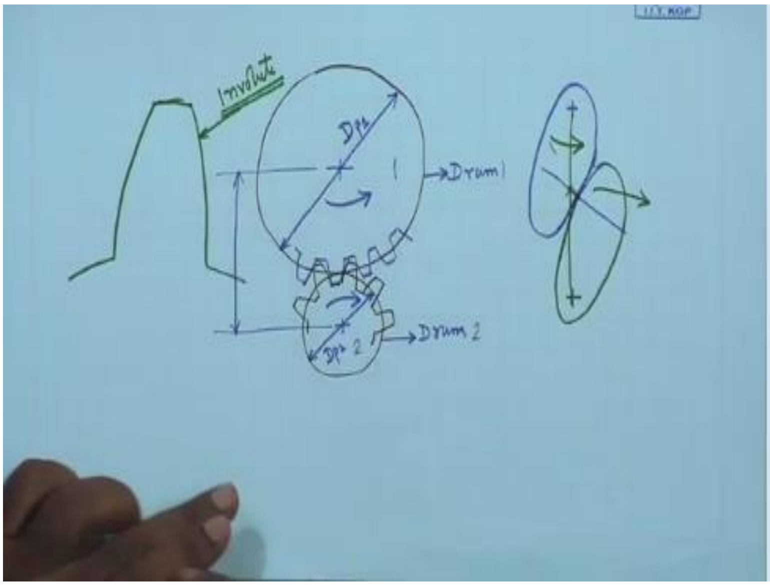


So, to start with, we had discussed about the pitch diameter. What is after all the pitch diameter? Pitch diameter is a virtual diameter like physically it cannot be tangibly identified on the gear. That means a gear, if we say that, what is the outside diameter of a gear you can actually pinpoint it, this is the outside diameter which is existing, but pitch diameter that way does not exist.

So, suppose 2 gears are in mesh and they are producing a definite angular speed ratio that means, there is a particular RPM ratio, rotations per minute ratio existing between 2 gears as they are rotating. So, this ratio is not affected by the speed values that these gears take up, that is, if I speed up one gear, the other gear will also speed up. If I make one gear slow, the other gear will also become slow.

The thing that will remain constant is the speed ratio. In that case, if we replace the 2 gears by 2 virtual rotating drums, rotating against each other without slip and replacing the 2 gears and producing the same angular speed ratio that means RPM ratio or speed ratio in that case, the diameters of those drums would be referred to as the pitch diameters of the respective gears.

(Refer Slide Time: 03:24)



Let us have a quick look at a depiction of this diagrammatic depiction of this particular case. Say, this is one particular gear and this happens to be another one and the teeth which are existing on these gears, say, this one is this way, etc., and that one is that way. Let me take another colour and this gear is having its teeth this way. So, first of all physically these 2 gears are existing as that toothed profile. This one is physically existing.

This one is also physically existing. Now, we are making them rotate together. Say, if this is the driver, this is the driven, they are rotating. We now replace them with these 2 rotating drums or discs. So, the restriction is that this will have to be in contact with that one that means the centre distance which the 2 gears were maintaining before; the centre distance will have to be the sum of the 2 radii of the 2 drums.

So, that they will always remain in contact and one would be driving. Say, this one is rotating; this will be driving this one by friction without slip. In that case, if the rotating discs produce the same RPM ratio as that obtained by the rotating gears. In that case, the diameters of these 2 drums would be referred to as the pitch diameters of the 2 gears. So, this is what is our definition of pitch diameter.

Why do we bring in the idea of the pitch diameter, because it makes our calculation is very simple, we do not have to think of instantaneous points of contact, which might vary with time and the instantaneous contacting radii might be different at different times. We do not have to bother about that. Now, you might raise a very pertinent question that if the point of contact is shifting and it moves either this way or that way, the instantaneous contacting radii will be different, than how come they are going to produce the same RPM ratio.

Now, in this case, there is a law of gearing which says that if any 2 smooth profiles contact each other. So, that the common normal, this is one profile rotating about this point, I have drawn an irregular profile to emphasise the point that it need not be very nicely shaped surface, but this profile is important. And if there is another profile, this one is driving and this one is getting driven and they are undergoing contact here, this point of contact is varying.

But if always the common normal to the point of contact always cuts the line of centres at a definite point. In that case, we will find that the speed ratio will remain constant. So, we choose gear profiles in this manner that means gear tooth profile. When we are talking about a gear tooth, you might have seen that it looks somewhat like this and this profile is very important.

Now, when we are not knowing much about gears, we might be thinking that maybe; they are parts of circles or they are aesthetically made or they have come down to us through evolution of cogwheels to give us a definite shape, not really so. They are coming from this case, those profiles which satisfy this condition can be used as the profile of the gear teeth here.

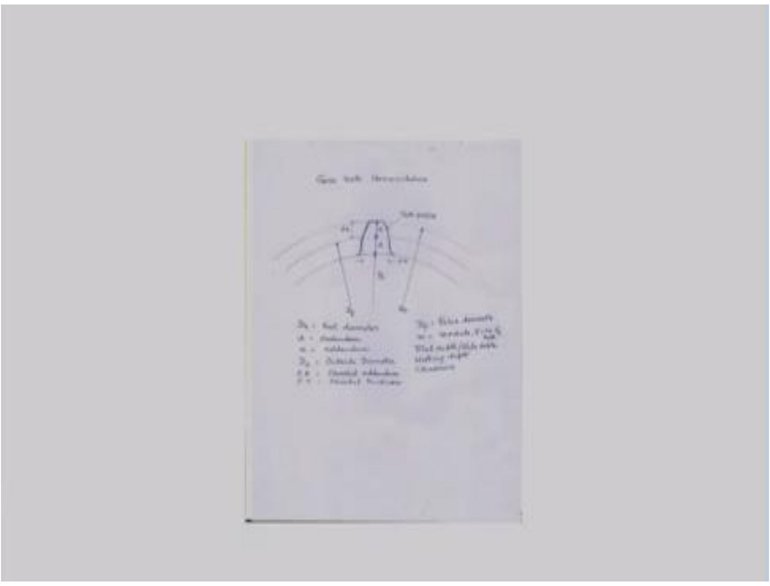
So, naturally, what are these particular profiles? One example is the involute. If we get time, we will discuss more about the involute. But, are the other profiles also? Yes, cycloid or cycloidal gears are there. But you will find. In most cases, we are using the involute. Why? Because the involute has some advantages, which are not available with other methods. Other methods have their respective advantages, but involute advantages proved to be the winning factor for it.

For example, in case of the involute, if the centre distance slightly changes, the speed ratio does not change. Now, what is that? It means that say, you are having an automobile gearbox. In a car, you have a gearbox with the help of which you can change the rotational rate of the wheels. Even though you are not doing anything with the engine. There is a gearbox in which you can have different output rotations per minute, which is ultimately given to wheel.

Now, in that case, it is quite obvious that the car when moving over road might be facing lots of jerks and impacts and there is a possibility that the gearbox will be experiencing lots of jerks due to which the centre distance of some of the gears might be changing. The involute is not affected by this centre distance change. So, we understand that this particular profile can be different types and involute is one of the most popular ones.

So, let us now look at a little more detailed dimensions and nomenclatures used in connection with spur gears.

(Refer Slide Time: 11:01)

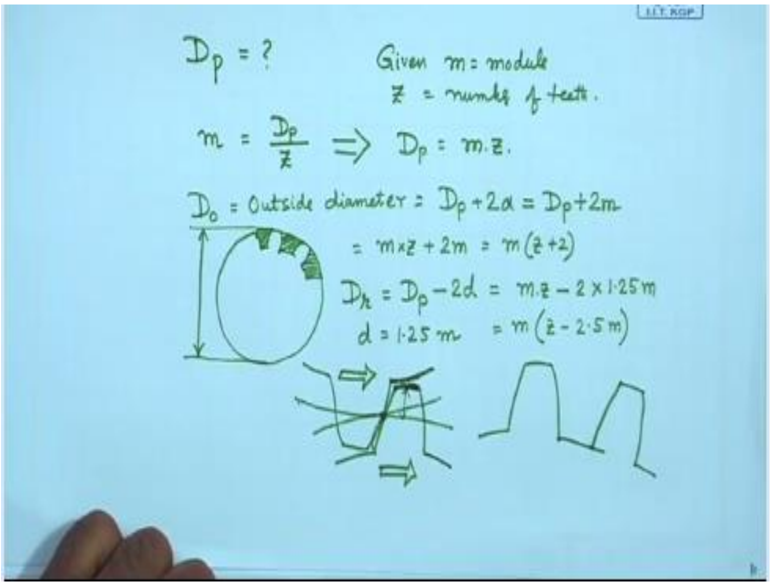


This figure is not so large, but I hope you can follow it. For example, let us look at first of all, what is drawn on the piece of paper. We have a gear tooth shown here. The gear tooth is having an outside diameter ‘O’. It is having a pitch diameter given by D_p. D_p is a pitch diameter. It is having a root diameter given by D_r. It is having other things like chordal thickness ‘C.T’, it is having something called ‘E’ addendum and it is having something called ‘D’ dedendum and it is having chordal addendum. Chordal addendum and addendum seem to be very close to each other.

We will see what is the difference. And what do we know about this gear? In this example, we know the module and we know the number of teeth. Can we find out all of these values from here? And of interest would be our total depth or whole depth, which means addendum + dedendum, the working depth that means the maximum depth up to which it can go in a mating gear and the clearance that would exist between the 2 teeth in that case, so, let us take them up one by one.

So, starting with let us now look at the piece of paper; starting with pitch diameter .

(Refer Slide Time: 13:01)



Pitch diameter has to be found out and what is given m = module; z = number of teeth. This is straightforward. We have the definition of module as:

$$\mathrm{m} = \frac{\text{Pitch diameter } (\mathrm{D_p})}{\text{number of teeth } (\mathrm{Z})}$$

Therefore, pitch diameter = m × z. I can find out the pitch diameter if given the number of teeth and the module. Can I find out the outside diameter? Now, let us go to the figure once again.

The outside diameter is shifted from the pitch diameter by addendum on this side and there will be another addendum on the other side of the gear. So, we write that outside diameter, we come back into the paper now, Outside diameter (D_o) = pitch diameter (D_p) + twice addendum (2a).

Now, what is the value of addendum? I do not know that. Well, those people who have been specifying gears have made a very simple type of relationship. That is addendum is equal to module.

Therefore, it might be slightly different in special cases, but in the most general case, pitch diameter (D_p) + twice module (2m). We get outside diameter. How is the outside diameter necessary? What is the importance of the outside diameter? Well, when you are cutting a gear. You start from what is called a blank. If you are subtracting material and attaining a particular shape, you will be removing this material; these amounts of material would be removed.

And you will be ultimately getting the shape which results from it. Hence, the blank diameter, the finished blank diameter which you ultimately start cutting of the tooth spaces; these are called tooth spaces; these will be cut off. So, the blank diameter is required so that the machine operator can check and ultimately start on that directly. So, blank diameter that is very relevant. Pitch diameter is formed by m × z.

So, for Outside diameter, we can put in m × z, which gives:

$$D_o = m \times Z + 2m = m(Z + 2)$$

Now, comes the question of the root diameter. What is the root diameter like? Let us come to the figure once again. The figure you will find, the root diameter is related to the pitch diameter by subtracting dedendum on both sides; dedendum on one side and dedendum on the other side of the gear. So, let us write down:

root diameter (D_r) = Pitch diameter (D_p) - twice dedendum (2d).

Now, what is dedendum? Maybe it is equal to Module? No. Dedendum is equal to 1.25 × module (m).

$$d = 1.25 \times m$$

You might ask why is this so. Because or nearly it should have been dedendum equal to the module following the definition of addendum. It is not so because we want some clearance to exist between gear teeth, when they are in mesh. What do we mean by this?

But before that, let us write it down the root diameter:

$$D_r = m \times Z - 2 \times 1.25m = m(Z - 2.5m)$$

So, module can be taken common. Now, why is this so? So, for that, let us see a figure. This is one gear and this is another gear tooth coming in contact with it and maybe it is getting driven. So, this is rotating this way; this is a rotating this way. So, these 2 gears have a contact here and say somewhere this is the pitch diameter of one gear. This is the pitch diameter of the other gear, etc.

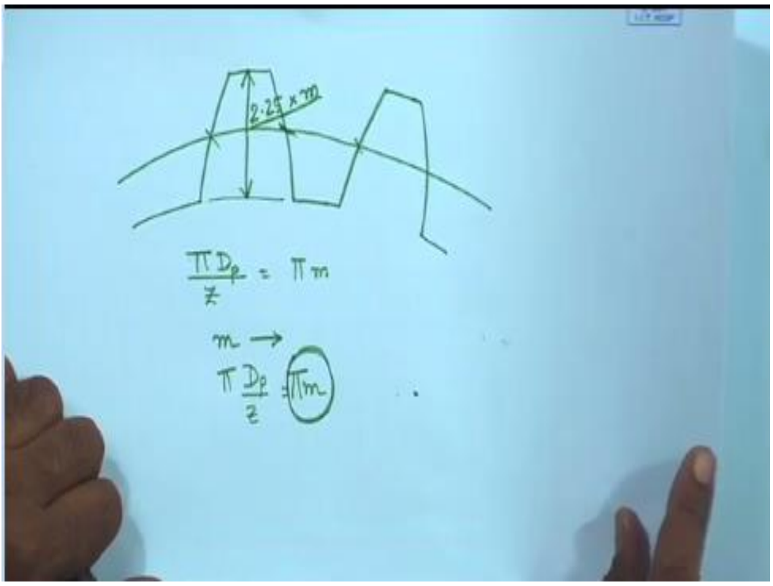
Problem is from the pitch diameter this one gear has depth equal to dedendum and this other gear from the pitch diameter, it has a depth of addendum. If addendum and dedendum had been the same, in that case, there would have been contact in some cases between the outside diameter cylindrical curved surface of one gear and the root diameter cylindrical curved surface of the other gear, these 2 would have got contacted.

This would not have given the speed ratio that is existing between these 2 gears. The speed ratio which is existing between these 2 gears is defined by the contact between these 2 involutes. These are not involutes. Their speed ratio is simply defined by their diameter ratio and these 2 diameters are definitely not going to get the same ratio as the pitch diameter ratios.

So, the ratio of speed defined by the 2 pitch diameters is not going to be realised by contact between these 2 and these 2 should never get in touch with each other. So, provide a clearance in between which is 0.25 module that is why dedendum is larger than the addendum. The tip of these gear teeth can never contact the roots of the other gear. So, we now understand why dedendum is more than addendum and what is the expression of root diameter; what is the expression of the outside diameter etc.

Now, let us look at some other and what is addendum and what is dedendum. You might say why is addendum proportional to the module. Let us have a look at the gears. Now, if this be a gear, how is module affecting its shape or size.

(Refer Slide Time: 21:41)



Let us take a fresh piece of paper if this be, how is module affecting its shape or size. We find that if we draw the pitch circumference from here to here, this distance is constant for all gears having same module. So, module defines this distance on all gears having the same module. So, this distance is equal to $\frac{\pi \times D_p}{Z}$. So, module is relevant to the gear teeth by this dimension.

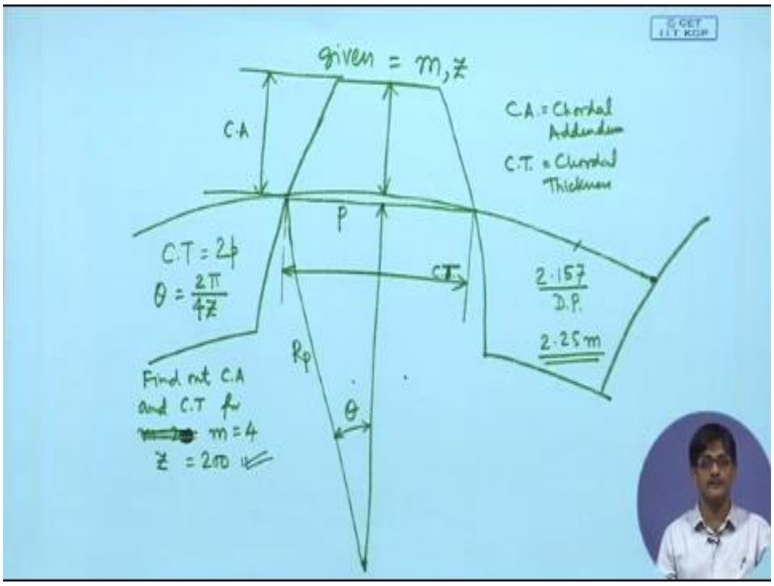
And if we assume that these 2 dimensions, I mean, this is equal to this, then this one is also perfectly defined by module. Once you give me the module and the number of teeth, I can perfectly define this. I can exactly define how much this should be. If that be so, that means, the width of the gears, they are defined by the module and the number of teeth. So, if we should say that whatever be the; this is equal to $\pi \times$ m.

And therefore, we can say, I should correct this particular statement. If module is given, we can say that this distance will get defined. If model not the number of teeth, the number of teeth will simply go on adding these distances to the circumference. So, if module is given, this thing gets defined. So, if the width is defined by the module, this should be proportionate to it and from the idea of dedendum means slightly more than addendum means this 1.25 and that is why this thing becomes 2.25 × module, proportionate to the module.

And why is this distance only dependent upon the module because, since, $\frac{D_p}{Z} = m$, this distance is nothing but multiplied by Pi so, it is fully defined by the module. This distance is fully defined by the module I made a mistake, it is not dependent. We did not say it is dependent upon the number of teeth. No, it is fully defined by the module and we simply make now the width is defined by module.

We make the height also proportionate to it, this one has to be slightly more and therefore, from that it comes to be 2.25 module. You might say that why is it exactly equal to module; why not 0.75 module or 1.2 module? Those gears are also available like you can have this to be 0.8 module which are called stump teeth etc., but the most widely used one is the simplest one equal to addendum equal to module. So, once we have understood this let us move on to the other definitions which are present here.

(Refer Slide Time: 25:50)



For example, you will find that in this figure if we make a larger depiction of this one, if this be one gear tooth, you will find that on the pitch diameter, we have depicted 2 dimensions. The bottom one being called as Chordal Thickness (CT). And this one being, which is joined by straight lines, this is equal to Chordal Addendum (CA).

So, these are very much relevant in different applications like measurement of the accuracy of gears, if you geometrically want to test whether the gear that you have cut is correct or not in the use of gear Vernier tooth calliper, this is very much relevant. Can these be measured? Suppose, I know the number of teeth and the module. By the way, I think I should mention here that apart from module, you also have D .

And in case of D_p, the dimensions are slightly different in that case the total depth = $2.157/D_p$ just because D_p is the reciprocal of module, of course, the units have to be kept in mind. So, it comes at the denominator just like so, we have 2.25 times module as the total depth in case of module gears. In case of D_p, we have $2.157/D_p$ and the working depth is equal to 2 times module and here the working depth will be equal to $2/D_p$.

Let us come back to our discussion of chordal thickness. Chordal thickness can be found out this way if we can draw a right angled triangle right up to the centre. This is the centre which you cannot see, but, if I see that I know this angle θ, I know this pitch radius R p. Can I find out this 'p'?. Chordal Thickness = 2 × p.

Let us quickly have a look whether we can find out p. Can I find out θ? Yes, θ is equal to 2 π that means 360 degrees divided first of all by z number of teeth. This will give you 4 times theta, right up to the second time that this is rising, this full angle is $\theta + \theta$ plus middle point θ plus θ this way. So, if we join this to the centre, this will be 4 θ. So,

$$\theta = \frac{2\pi}{4Z}$$

So, I am going to supply you the number of teeth, Z is known to you. So, that way θ can be calculated. Can R_p be calculated? Yes, basically in all problems, we will be providing you given m, z. From this R_p can be found out, because m × z = D_p, half of that is R_p therefore p can be found out this way.

Why? Because once θ, R_p are known, you can find out $R_p \times \cos\theta = p$. And 2p = chordal thickness (CT). Can you find out chordal addendum? Yes, this distance is equal to addendum and this small distance is equal to $R_p - R_p\cos\theta$. So, the next day when we are taking up the subsequent lectures, we will solve a problem as mentioned below.

Find out chordal addendum and chordal thickness for m = 4 and Z = 200; find out chordal addendum and chordal thickness for the same.

Why are we doing this? Because as I said, this will be useful in the measurement of accuracy of gear geometry after a gear has been manufactured. So, this, we will be solving in our assignments what we call it, solution, maybe in the fifth lecture, so, you can have some practice, get it done yourself and then we will compare notes. Last of all, let me just add, you might think of some problems like if I give you 2 gears. One is having module 3 and another is having module 10, does it mean larger module will have larger teeth or smaller module will have larger teeth. So, with this, we come to the end of the third lecture. Thank you very much.