Spur and Helical Gear Cutting

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Lecture – 22
Simple Gear Calculations

Welcome viewers to the second lecture of our open online lecture series on Spur and Helical Gear Cutting. So, last time we were discussing the speed ratio of whole gear train which means that if we have number of gears forming our train from the input side to the output side how their numbers of teeth etc. and their configuration that means the way in which they are connected, how it will be affecting the speed ratio.

And we solve a small numerical problem and obtained an answer. Now, when we are dealing with gear cutting through say milling gear, hobbing gear, shaping some methods like these, we will invariably come across some calculations which will be involving either gear trains like these or there will be some mechanisms like screw nut mechanisms, worm and worm gear mechanisms, then bevel gears etc., which I think I should introduce at this point.

So that in the introduction itself will be quite confident that yes, I understand about the basic mechanisms which are used in case of gear cutting and therefore, I can follow when it is being discussed in a more advanced form. So, let us have so, in the second lecture, let us first start with some of these mechanisms.

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We also have, apart from gear frames, extended gear trains, screw nut mechanisms, worm and worm mechanisms, worm and worm gear mechanisms and rack and pinion. So, first of all, let us have a quick look at what is this screw nut mechanism? Let us see.

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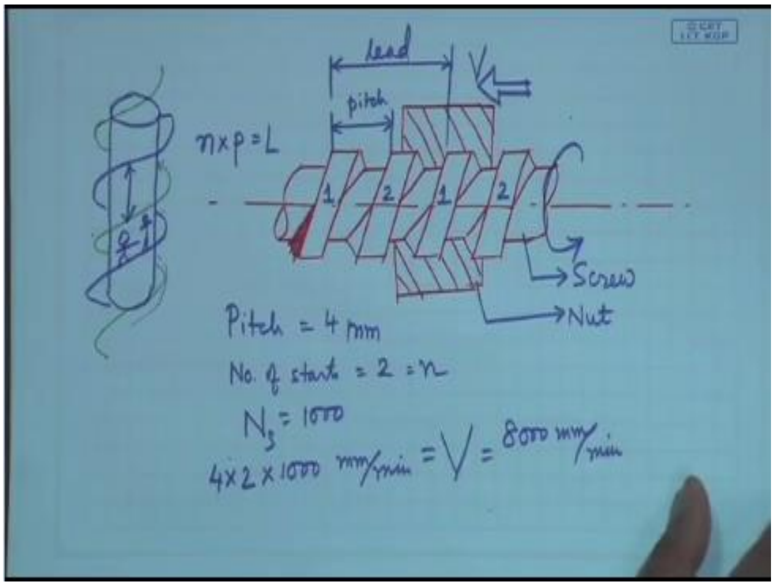

First of all, if you kindly have a look here, I draw an axis line to mean that whatever I am drawing it has its axis symmetric. And this is basically a screw, that is it. So, we have a screw to start with. So, it is basically a helix wrapped around a cylinder going this way. So, when you have such a member, it has a corresponding body called nut which can be fit on top of this.

So, if I fit it here now, we can depict it this way in an engineering drawing. So, this particular member, we named as the screw and this member, we call the nut. And what is the specification that we give here? Pitch equal to say, let us put a numerical value so that we can correspondingly solve a problem as well say 4 millimetres. Number of starts equal to 2. What is number of starts?

Number of starts is, you know, this looks like a helix. And what is the helix? To give a very, very simple everyday example, you might have seen against the round object, a spiral stairway. What is the idea of the spiral stairway? You can move up the spiral stairway, up or down. So, this is what is this spiral all around this particular cylinder. Now, the question is if someone said, just like you have 2 way highways, that is, there is a divider.

On one way, you go say forward and on one way you go, not backward, but you go the other way. So, in the same way, in order to have better traffic control on this spiral stairway, build another one, build another one parallelly. So, think of the advantage you will have on the spiral phase. Spiral phase are very much constructed in space, constrained in space, so that if you have someone moving down, you have a problem, you have to shift and give him or her some space to move.

And already the space is constrained. So, it will be a problem. So, someone says, why do not we have a parallel spiral here? Why do not we have it? We generally do not have it at all. We could have had it. Is there going to be any clash in space? No, they are all moving in the same direction, then can you really come down? Yes, why not? You can do, go down and along this, will you have a clash? No. Never.

So, why do not we have it? Because of possibly this head space. If you put another spiral stairway, you would have had to move with bent down. So, you do not have it here. But on the screws side, there is no constraint of accommodating someone here in between. No. So, we many times have 2 or 3 or even more such spirals moving parallel. So, it may be spiral 1, moving all around and coming up spiral 1 and this is a completely different one, spiral 2, moving around like this and coming back again in spiral 2.

In this case, if these helices are belonging to separate threads, these threads are belonging to separate helices and there are 2 such helices, we will say a number of starts is 2. Then how does the definition of pitch still remain relevant? It remains relevant as this distance is the pitch and the nut now, if there is number of starts is higher than one and nut movement will be correspondingly higher.

So, the nut will be moving actually twice of pitch and it is called the lead and we have the equation:

$$\text{Number of starts (n)} \times \text{pitch (p)} = \text{lead (L)}.$$

So, if $N_s = 1000$ and the nut is made to move; the screw is rotating and not allowed to translate, no and then nut is allowed to translate but not allowed to rotate. In that case, what will be the movement of the nut? Say, what will be the velocity of nut in millimetres per minute?

So, we can say per rotation of the screw. The movement of the nut will be one lead. So, this will be 4×2 per rotation and if there are 1000 rotations per minute, per minute it will be $4 \times 2 \times 1000\ \text{mm/min} = \text{V}$, So, that comes out to be 8000 mm/min of the nut. So, this way we understand the screw works, we have to keep a lookout for the pitch.

We have to keep a lookout for the number of stars and sometimes it is referred to as **k** and we have to keep remain alert for the number of rotations suffered by the screw and that combined will give us ultimately the movement of the nut. So, this is about screw and nut mechanisms. So, we have learned now about gears, we have learned about gear trains, we have learned about screw and nut mechanisms. And now, we are going to have a combination of the 2.

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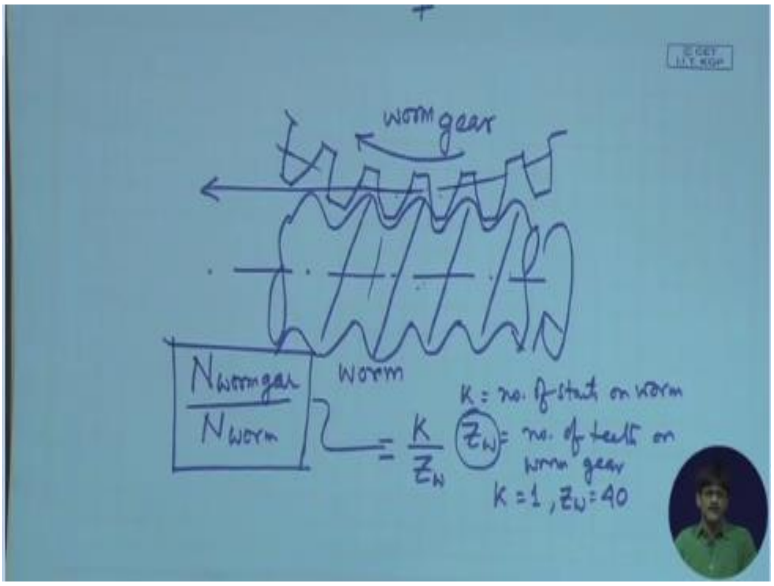

Let us have a quick look at what we mean by that. Let us represent the screw once again. Let this be our screw and some innovative person say things that; let me combine the gear and the screw and he puts it here; he or she puts it here. But you will say what is going to give you. The screw which is rotating here that is simply going to carry out rotation. The screw is rotating. And have there been a nut? It would have been pushed to one side.

So, it will do that only. All these teeth which have got engaged in these thread roots, they will get pushed to one side. But how are you constraining this particular gear? So, we call this the gear and this screw is given a specific name, the worm. Why? Because worms, those earthworms, they are annelids. So, this is roughly looking like annelid that means rings, one after the other as if there are rings, really not so but anyway. So, this called worm.

Now, in this gear, what is the way in which we are constraining this gear, we are simply having a pivot and the centre is the pivot point. That means it is able to rotate. So, if you give it a push here, this way, it is going to suffer rotation due to that. So, the gear suffers rotation. So, if the worm rotates, this is now getting a name of itself worm gear, the worm gear will rotate to one side.

And therefore, we have a mechanism in which you can transfer motion from rotating screw thread to here. This is a big achievement. Previously, we were transferring motion from one gear to another gear, we were transmitting motion from screw threads to nut, etc. But we were not able to in any way move from a screw thread to a gear directly. This is being done here.

And there is a very important function which is being served by this worm and worm gear mechanism. Incidentally, the speed ratio that means $N_{\text{worm gear}}/N_{\text{worm}}$. This is the speed ratio. That means the worm gear is being rotated and worm is the driving member. This one can be shown to be equal to:

$$\frac{N_{\text{worm gear}}}{N_{\text{worm}}} = \frac{k}{Z_w}$$

Where, k = number of starts on worm, and

Z_w= number of teeth on worm gear

So, from this, we understand that very useful purpose will be served by this mechanism. What is that? Jelly the number of starts is equal to 1, 2, 3 like that. And the number of teeth on the worm gear can be typically say 80, 40 like that. So, let us take an example in which k = 1 and $Z_w = 40$. What is the speed ratio we get? The speed ratio will be a thumping $\frac{1}{40}$. So, just imagine the rotation on a shaft can be reduced to 40 times its original value.

Why is this so important? And why is this so useful? Let us have a look here. If I am having say an input shaft with rotational speed (N) = 400.

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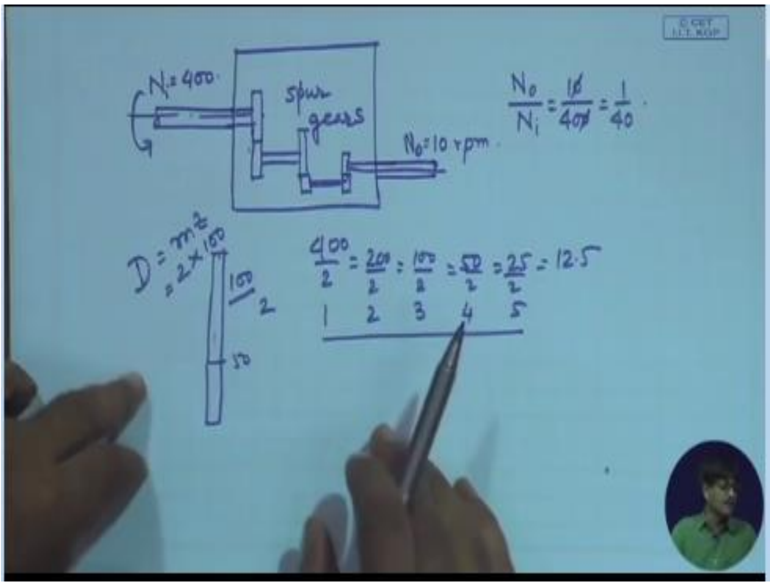

And if I tell you that make a gearbox into which it is moving in, put whatever ordinary gears that we have discussed up till now spur gears, I will come to this term spur gears, put whatever gears that you can put in and get me a speed of 10 RPM. So, you immediately find out what is the speed ratio required.

$$\frac{N_{output}}{N_{input}} = \frac{10}{400} = \frac{1}{40}$$

So, you decide fine, I can put gears of this type maybe and maybe reach $\frac{1}{40}$. Let us see how difficult it is. Whenever we were talking of gears, which are having say, 100 teeth, 50 teeth what sort of speed ratio is possible here, 2. So, if you are going on putting gears of this type as an example.

So, 40 divided by 2, in one stage, you can get 200 RPM, second stage, you can get 100 RPM, another stage 50 RPM, another stage 25 RPM, another stage, how much 12.5 RPM, this is more like the 10 RPM that we were requiring. So, you would have had to use stage 1, stage 2, stage 3, stage 4, stage 5, just imagine 5 steps in here.

Here, I have only drawn 3 steps. And that too when you are accommodating gears of 100 numbers of teeth and it is the module = 2 therefore, the diameter which you would have had to accommodate would have been:

$$D = m \times Z$$

m = 2 and number of teeth = 100. So, it would have been 200 millimetres just imagine, this way, you would have a cover 200 millimetres, this would have given rise to 100 millimetres, 300 millimetres in every stage, 5 steps such stages.

So, in the most general case, it would have a gear box requiring a huge space. Now, who would be interested in that sort of gearbox? If just to get a reduction of 400 to 10, you would be requiring say, 2 metres × 2 metres of space, no one will interest. Here, in comes the worm and worm gear, application of a worm and worm gear, you can notice here by the worm and worm gear that we had just discussed just now.

In one step itself, you can reduce the speed from 400 to 10 by employing k = 1 and Z = 40 teeth as simple as that. This is the advantage of a worm and worm gear. Drastic speed reduction in a single stage, drastic speed reduction, there are problems associated with worm gears and sometimes those problems are made use of, for example, for specific helix angles and coefficients of friction, this is self locking that means, by rotating the worm, you can rotate the worm gear, but by rotating the worm, you cannot rotate the worm.

Some particular properties like that they are associated with worm and worm gears, but this is the big thing. You can get drastic speed reduction in one stage.

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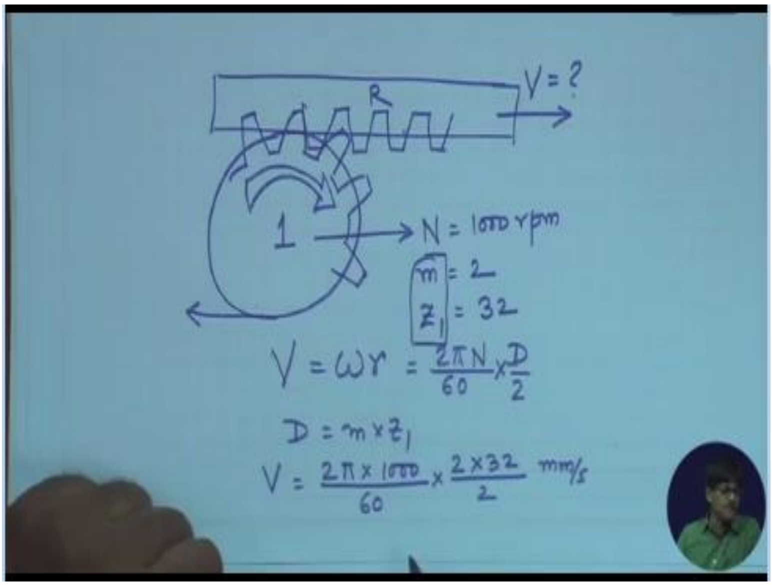

Now, let us take the case of rack and pinion. Let us take a simple problem also. Say, I have this as a depiction of a gear and this as the rack. Suppose, I give this information. This one is rotating and this is called 1 and this is called R. For R, I am interested to find out the velocity it will develop and here, I am having N = 1000 RPM, module = 2, number of teeth = 32.

What will be the speed developed by the rack? So, in this case, what do we do? In this case, we can say that I can easily find out the peripheral speed of this gear at the pitch circumference. Once I find out the peripheral speed of the gear, I can equate it to the rack speed and that will be my answer. How do I find out the peripheral speed? If you move from first principles, you can say velocity has to be found out which can be equated to $\omega \times r$.

$$V = \omega \times r$$

This can be equated to $\frac{2\pi N}{60} \times \frac{D}{2}$. Now, this N is in rotations per minute, is it given? Yes, it is given. Is the diameter provided? No, I do not know the diameter. So, how do we proceed? Here, you will find once again if N and Z they are provided, you can use:

$$D = m \times Z_1$$

and you can put it here.

Therefore,

$$V = \frac{2\pi \times 1000}{60} \times \frac{2 \times 32}{2}\ \mathrm{mm}/_{\mathrm{s}},$$

and this is rotations per second and this is in millimetres, we have millimetres per second. This is the answer. Some calculations are required. I am sure you can do this and solve the problem. So, now we have covered gear trains, screw nut mechanisms, worm and worm gear mechanisms, rack and pinion and therefore, we can carry out all the calculations which are required to understand how machining or cutting of gears takes place.

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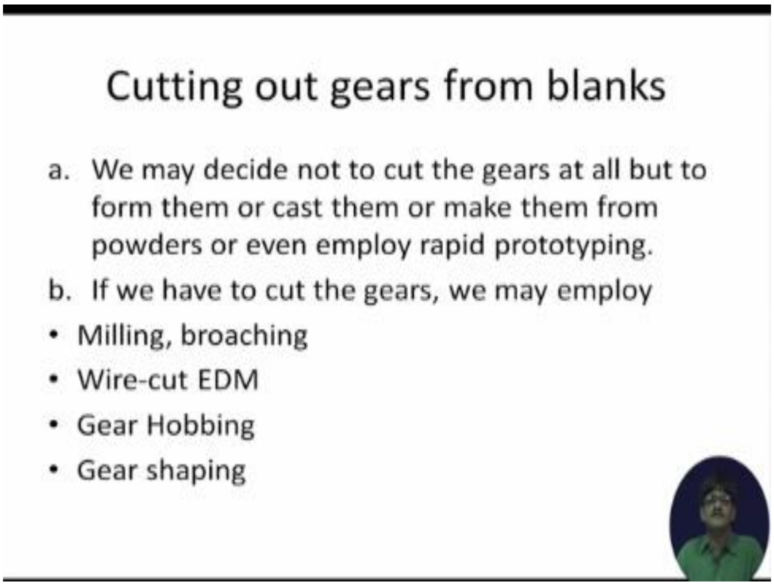

Now, coming back to our original discussion. Let us now take up the methods by which gears are cut out. But before that, we should notice that gears are not produced only by cutting or machining, machining basically means cutting with the removal of unwanted material in the form of chips from a larger sized blank, we might not always cut gears out of blanks. We might get gears made by completely different means.

We can go for forming them like die casting, stamping, etc., or even blanking. So, we might also cast them or we might be getting them by means of powder metallurgy or even employ rapid prototyping. All these methods are involving either chipless manufacturing or they are primary methods of manufacturing or powder metallurgy practice or rapid prototyping, etc.

So, they are completely different from cutting of gears, but a whole family of methods of cutting out of gears have evolved over time and they have their own areas of specialisation. Like, there will be some cases in which they are very much essential to be used. And we will be studying about those things. So, what are the typical methods of cutting of gear? We might have milling, broaching, we can also have wire cut electrical discharge machining. We can have gear hobbing. We can have gear shaping and we can have yet different other methods, but these are roughly the most generalised processes and they have their respective areas of special applications. For example milling, what do we exactly mean by milling? I can cut out a gear by say for example a rotary two third cutting, what we call it rotary cutting tool.

So, what do we exactly mean by this rotary cutter? This is basically a form cutter and let us have a look at that what is its area of application.

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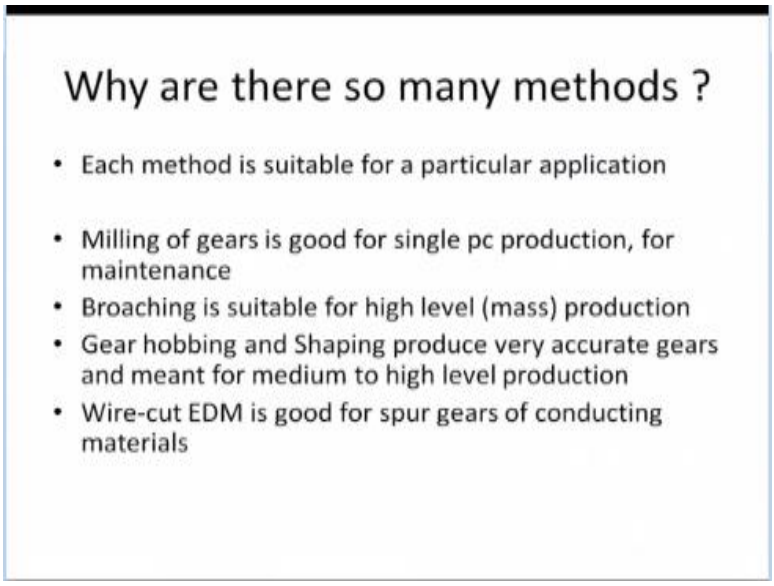

So, milling of gears is good for single piece production for maintenance, etc. Suppose, you are working in a factory in a night shift and as one gear on a machine which fails gets damaged, so that it has to be replaced immediately. So, what do you do? Do we go to a shop? No, because in the night-time, most of the shops are closed. We go for online ordering it. So, that you could be mailed to you and you will be receiving it through courier. No, because that takes certain amount of time.

You cannot manage to get it immediately. But you can have your own machine shop and you can have a milling machine which can be set up very fast and rotary form disc type cutters which I have referred to or end-mill cutters which once again they are form cutters, they can be employed with a method of indexing, so that you can easily get such a gear

Or there is another example. Suppose, you are having a job shop in which you are depending upon orders which are coming from time to time and somebody places an order for 2 such gears, 5 such gears, etc. So, for that you cannot have a specialised machine, you have to have a machine which can be quickly set up for that kind of production and maybe it will even be manually operated.

And though it seems to be not very significant in this sort of production might, it have a considerable volume. That means there might be a huge number of customers who would be approaching you with orders of say 2 to 3, 5 to 7 pieces of such gears for which you cannot spend money to have a specialised machine dedicated for that purpose.

So, instead you have generally a manually operated machine in which setup can be done very fast for doing that job and then you can change the setup for some other job that you might be having in line. So, milling is good for that. So, milling is used in such cases. Broaching is suitable for high level production and it is a single pass method, so, that it is very much desirable that way.

We also have gear hobbing and gear shaping, which can result in quite accurate gears because if they do not have the errors and inaccuracies, resulting inaccuracies of milling, etc. And therefore, they can be used for medium to high level production. Wire cut EDM: if you have conductive materials and if you are having spur gears to be made, then wire cut EDM is quite good.

But if you have helical gears, it cannot be handled by wire cut. At this juncture, I think I should introduce the difference between spur gears and helical gears.

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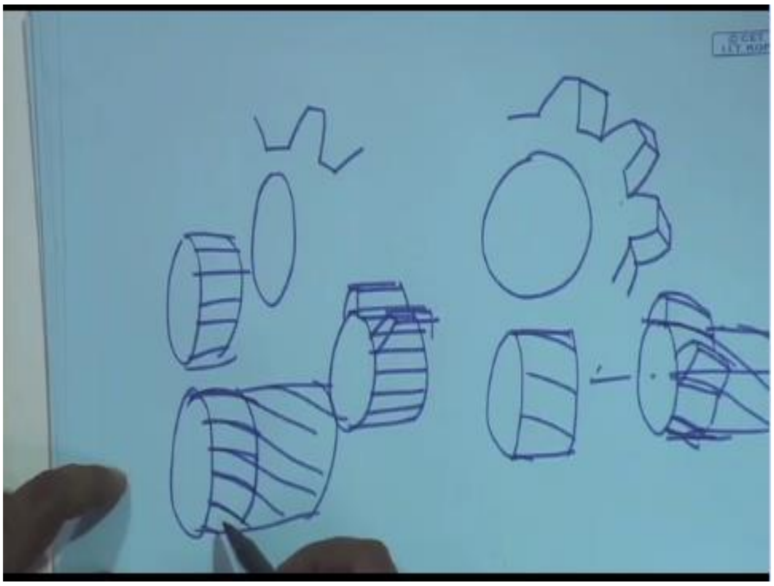

If you have a quick look here, I can have a gear of this type where the teeth are of this type as we have been drawing. That means if you look along the axis of the gear, the teeth are straight. So, that when you draw them, you can draw them this way, but you can also have gears in which the teeth instead of being straight. Here, we have drawn gears which are having straight teeth that means these teeth are this way.

Instead of that, they might have this way. That means these teeth will be looking this way which actually means that these are basically parts of helices about this particular central axis. So, it will be ultimately going this way. So, if I am talking about a helical gear, it will look like this. All these helices are there and I cut it from here, I get my particular helical gear.

So, while spur gear, the teeth will be like this parallel to the axis. The helical gears will have the teeth in the form of small portions of helices about that particular cylindrical portion. So, this is the difference between spur and helical gear. Tomorrow, I have a plan to show you some actual helical and spur gears by bringing them here and putting them on display. So, that you will be convinced they are definitely different.

So, with this one, we come to an end to the second lecture. We will meet again for the third lecture. Thank you very much.