Spur and Helical Gear Cutting
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Lecture – 21
Introduction

Good morning viewers. So, today, we are meeting for the first time for the open online course of 10 hours lectures on Spur and Helical Gear Cutting. So, basically in this course, we will be learning about a little bit of about gears and their uses to us in what we are making use of gears; what sort of gears that we are going to deal with and some basic calculations about gears and after that we will be directly going into the topic of gear cutting.

So, when we are talking about gear cutting, we are essentially referring to those methods in which extra material is removed from a blank and ultimately, finished gear is obtained. So, first of all, let us have a quick look what we are referring to as gears. So, first lecture we start today on spur and helical gear cutting.

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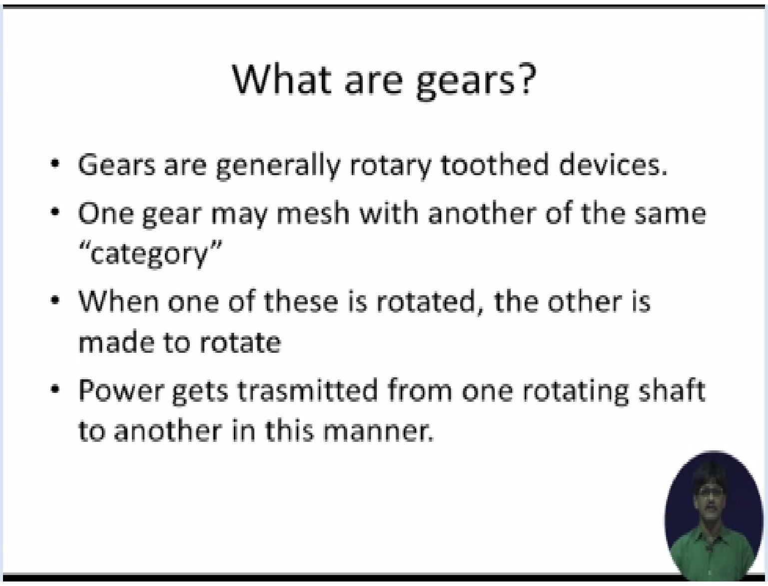

What are gears? Gears are generally rotary toothed devices. So, they are in most general cases, they are rotating. They have teeth on their periphery and 1 gear generally meshes with another. Meshes means gets connected intimately teeth to teeth with another of the same category. So, this word we are going to discuss later on in more detail. So, for the time being let us be satisfied that 1 gear of 1 gear may mesh with another one of the same category.

If they are not of the same category, they cannot mesh. And when one of these is rotated provided the other is free to rotate, it will be rotating about its axis of rotation and this way power can get transmitted from **(02:24)** one rotating shaft to another. So, let us quickly have a look at how we depict gears when we are dealing with calculations and representation of gears.

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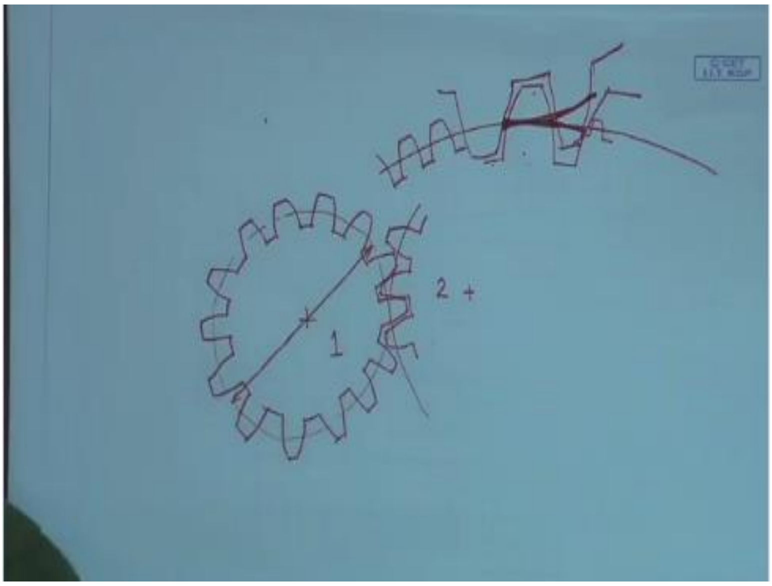

So, first of all, we might be having a circle. All around this circle in a gear, we have teeth of this type, all around. You will be seeing that I am drawing gear teeth, which are all of the same size, which means their height and their circumferential span, they are all the same for this gear. So, this way all around; we have teeth. So, if the gear teeth are of the same size, as we go on increasing the dimensions of a gear, that means, if I increase this particular diameter, this is referred to as the pitch diameter, if I increase this particular diameter, I will be getting proportionally higher number of teeth.

Now, you might say that, is that really necessary? That is if I have a large diameter, of course, I cannot draw it fully here. Does it mean that proportionately for if it has to mesh with this one, it has to have more number of teeth? Yes, essentially so. Then you might say that if it does not have to mesh with this one, does it have to have this sort of teeth? No, not necessarily.

It can have teeth of this type. It can have teeth of this type, etc. Different sizes of teeth, they are completely possible for whatever diameter you are choosing. There are of course, some limits on the least diameter etc. that you can add up for a particular category of gear teeth. But on the whole if you have chosen a diameter which allows for these teeth, all of them are possible.

And therefore, they will result in different numbers of teeth for a particular diameter. So in that case, is there any basic relationship that we can establish for gears with which mesh with each other? So, let us take a very simple example. Suppose there is another gear which is meshing with this one. And so, let us draw these teeth and immediately we get to understand that as the teeth of gear 1 have to be accommodated by the tooth spaces of gear 2.

Therefore, this particular circumferential distance something like this. The circumferential span of this tooth has to be accommodated by the tooth space of the corresponding gear with which it is meshing. This one has to accommodate this one. Likewise, this one will have to be accommodated by that one and immediately, we come to a particular condition that the circumferential span of a gear tooth and its corresponding tooth space. They have to be the same for meshing gears.

Basically, this plus this has to be equal to that plus that. Once we have established that, we come to a basic relationship between meshing gears. Let us have a quick look at that.

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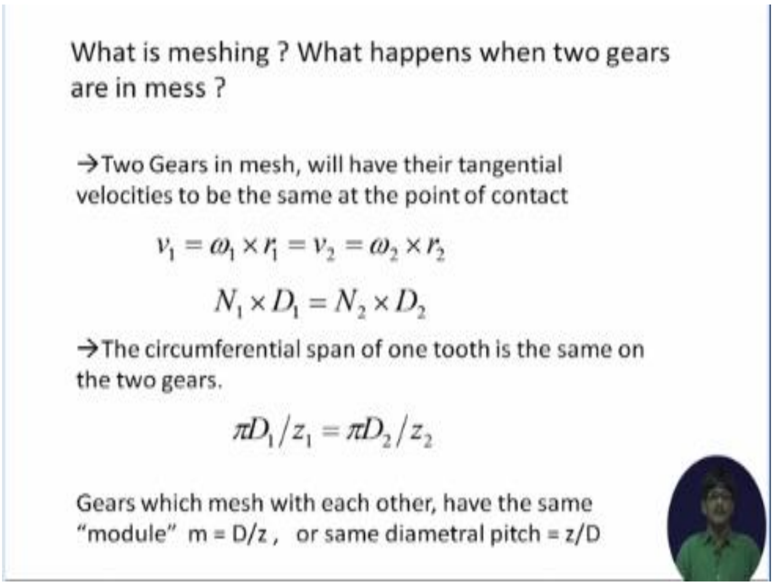

This one, I will come to these things later on, everything we will be covering, but first of all, we have πD_1 which basically means the circumference. What is D_1? D_1 is referred to as the pitch diameter of the gear. Pitch diameter is not the outer diameter, neither is it the root diameter, but it is something in between and it has a classical definition, which we will come across later on.

For the time being, just kindly accept that it is somewhere in between the outer and the root diameter of the gear teeth. So, pitch diameter divided by number of teeth. So, z_1 is defined as the number of teeth on gear number 1 must be equal to so, this is the span of the tooth, circumferential span of teeth on the gear 1. This is equal to circumferential span of teeth.

One tooth on gear number 2, so circumferential spans of one tooth on gear number 1 and number 2, they are the same. If they have to mesh and this particular equality is made use of to define a family of gears which have this value to be the same. And we can say that they can mesh with each other. And why keep the π which is appearing in this equation as it is on both sides.

So, let us cancel it out and we have:

$$\frac{D_1}{z_1} = \frac{D_2}{z_2}$$

which is equal to $\frac{D}{z}$, where D is the pitch diameter in millimetres and z is the number of teeth. And this one if you read the last line, it is mentioned that this one is called the module. So, we say that gears which have the same module, gears, you can say gear teeth, which have the same module or gears, which have the same module, they will mesh with each other and they form a family.

So, the category that we were talking about in the beginning, that actually boils down to this module, but mind you, since it is dependent upon the type of unit you are choosing, because D is the pitch diameter in millimetres, we also have similar parameters which are defined in other systems of units in a slightly different form like Diametral pitch which is equal to $\frac{z}{D}$.

It means the same thing. $\frac{z}{D}$ where z is the number of teeth and D is the pitch diameter in inches. So, first of all, we establish gears which mesh with each other have same module in the metric system and they have the same diametral pitch in the inch system and they would be able to mesh with each other. Once this is established, we can build on that further. So, let us look at the first equation that we have written down.

What is meshing? Now, we understand what is meshing. Yes, gears which are getting interconnected so that one tooth gets into the tooth space of the corresponding are the pairing gear. What happens when 2 gears are in mesh? Sorry, not in mesh. So, what happens when 2 gears are in mesh? This equality that the second equality can be said to have been satisfied.

So, when 2 gears are in mesh, we have stated in the first statement that the tangential velocities have to be the same at the point of contact. Let us have a quick look at that.

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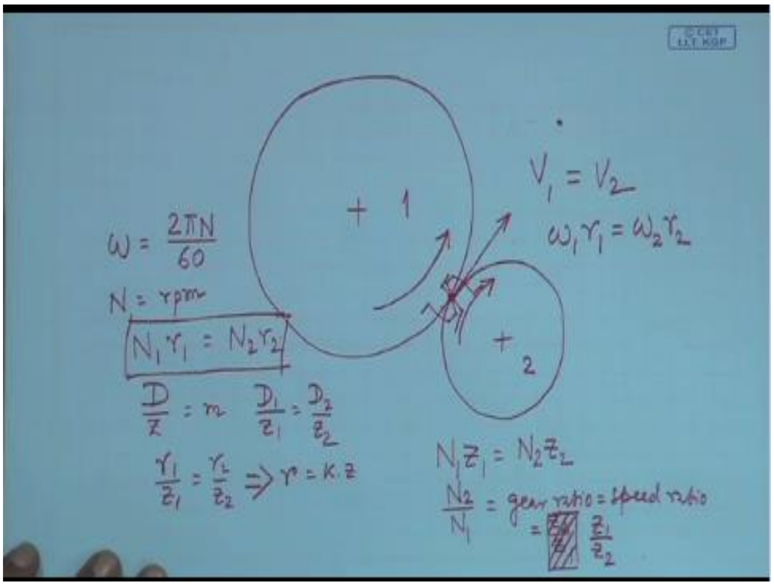

So, if you look at this particular figure, where we are basically representing the gears by circles. So, this is a sort of simplification of the figures. We are saying that if this gear is moving this way, it will make the other gear move this way if it is free to rotate about its rotational axis and therefore, we say that since at this point, gear number 1 is in contact with gear number 2, these 2 are the teeth which are in contact.

So, at this point this one and this one, they are moving together. This is not pushing into this, this is not pushing into this neither is this one losing contact with this one. So, as they are in contact and moving with each other, the tangential velocities which they have by virtue of their rotation respective rotations, that tangential velocity must be equal.

So, if this tangential velocity is equal, mind you here, I should point out one thing that this point of contact that we have drawn here in a simplified form to be on the pitch circle. It is not always so, but for all practical purposes for the gear cutting that we are going to deal with this sort of simplification is quite acceptable. If you have questions, we can definitely address them later on.

So, if they have a common tangential velocity, we can establish that if this is gear number 1, if this is gear number 2, definitely $V_1 = V_2$, these 2 tangential velocities. And what is V_1 after all? It is equal to angular velocity ω_1 into the radius r_1. So, this one must be equal to ω_2 into r_2 that is very good, but we already know that:

$$\omega = \frac{2\pi N}{60}$$

where N is the RPM, rotations per minute.

So, if that be so, we can simply replace ω_1, cancel out 2π and 60 and we can get:

$$N_1 r_1 = N_2 r_2$$

So, this means that N_1 in two pitch radius is equal to a sorry $N_1 \times$ pitch radius of gear 1 is equal to $N_2 \times$ pitch radius of gear 2. What does that ultimately provide us with? It gives us a relation between rotational speeds, rotational rates of the 2 gears with their respective radii.

But, in case of gears, we generally use terms like number of teeth, module, and terms like these generally not a pitch radius or something like that. So, can we relate the pitch radius that means, r_1 and r_2 with other terms that we are more interested to use. So, let us have a look at that. We have already established that $\frac{D}{z}$ is equal to module for gears which are meshing with each other. That means, we can write $\frac{D_1}{z_1} = \frac{D_2}{z_2}$.

Diameter of the gear 1 divided by number of teeth of gear 1 is equal to diameter of gear 2 divided by number of teeth of gear 2, which essentially means we can also simply write:

$$\frac{r_1}{z_1} = \frac{r_2}{z_2}$$

which means, r and z, they are proportional.

$$r = k \times z$$

Once we have established this, we notice that here $\frac{r_1}{r_2}$ can be replaced by $\frac{z_1}{z_2}$. Alright, $\frac{r_1}{r_2} = \frac{z_1}{z_2}$. So, $\frac{r_1}{r_2} = \frac{z_1}{z_2}$. So, we finally write this one:

$$N_1 z_1 = N_2 z_2$$

So, that if someone asks you, what is the ratio of the rotations of these 2 gears. You can simply say it is equal to $\frac{N_2}{N_1}$ output by input. Throughout these lectures, we will be referring to gear ratio, speed ratio, etc., terms like that as Output RPM / Input RPM. In many literatures, you will find that it is just the opposite. But to maintain one single definition of this particular speed ratio, we will simply take it as Output RPM / Input RPM.

So, $\frac{N_2}{N_1}$ is equal to gear ratio or speed ratio, equal to $\frac{Z_2}{Z_1}$. And hence we established the first law of the first law which will be helping us to calculate speed ratios.

$$\frac{N_2}{N_1} = \frac{z_1}{z_2}$$

Please correct this. So, once we have established that if we have a look at this, these we have now, we can confidently say yes.

We understand this; this comes from the constancy or equality of the tangential velocities at the point of contact and this is coming from equality of circumferential spans of the gear teeth on gear 1 and gear 2. So, from these we have $N_1 z_1 = N_2 z_2$.

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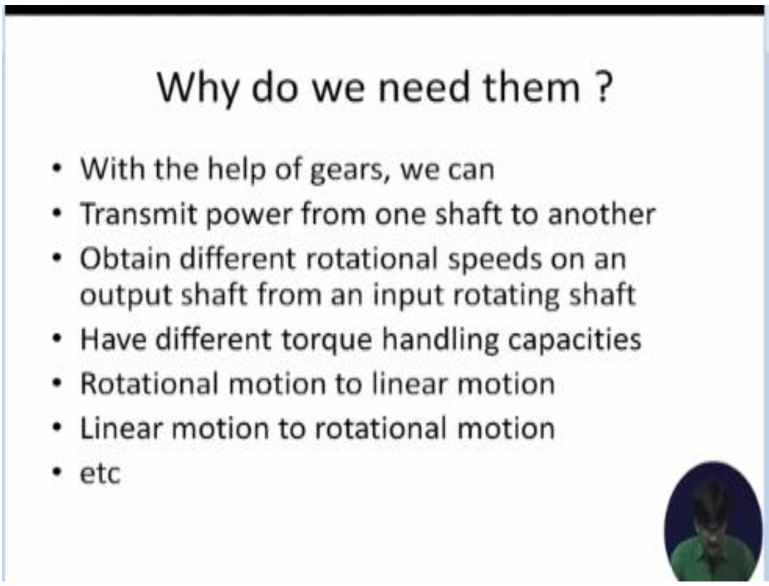

So, let us first of all so, before going into further calculations further calculations are required, let us finish of the introduction to gears that is okay. We understand. Now, we have a rough idea of what are gears and how they operate etc. But in the introduction, let us also establish why do we need them. Why do we need gears? So, for that we can identify with the help of gears, we can transmit power from one shaft to another.

So, that means if there is a question of some space existing between the source of power and ultimate point of application of power and if there is a gap, you can transmit the power with the help of gears from this shaft to that shaft and maybe there will be intermediate shafts also. Second, we can obtain different rotational speeds on an output shaft from an input rotating shaft.

So, if there are 2 shafts and suppose you have 1000 RPM on the input shaft and someone asks you, get the input power to drive the output shaft at 500 RPM. Mind from 1000 RPM in on the input shaft, you have to have 500 RPM on the output shaft. So, if you use gears, in that case, it is possible to drive the output shaft at this particular RPM.

So, in addition to transmission of power from one shaft to another, you can also dictate the rotational speed of the output shaft provided the input shaft is having a definite RPM. Have different torque handling capacities? So, on a particular shaft, you can have different torque handling capacities that means if the required output torque varies, you can make use of gears or gearboxes to handle that torque.

That means that if the torque becomes very high so, that you find initially that you cannot rotate that shaft, you can have an intermediate gearbox which will allow you to overcome that required output torque and make the shaft rotate. Next, rotational motion to linear motion. Let me give you an example by drawing a figure.

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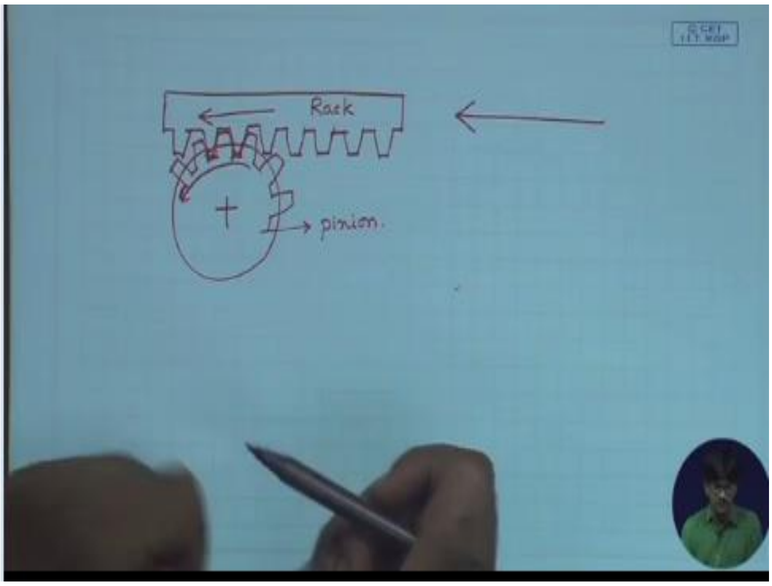

I have rotational motion here and I have put a gear here. This is rotating about this axis of rotation and I have teeth on this. So, I already put a gear on top of this rotating shaft and someone asks me to have linear motion like this. How can I do that? So, for that, one particular example is that I can have something called a rack, a straight sided gear. You have heard of the term rack and pinion, so, we are basically discussing that particular machine element pair.

The pinion is the circular gear. Heli pinion means a small gear in pair with a larger gear, so, it can also be rack and pinion. So, this one, we are calling the rack and this one, we are calling the pinion. So, if this rotates, this will move towards this side. So, with the help of this mechanism, which is basically a gear pair, we can have rotational motion getting converted to straight line motion.

So, naturally if you are able to convert rotational motion to straight line motion, you can also convert straight line motion to rotational motion. This is also possible. So, coming back to our discussion, we can easily identify these uses of gears and many more. We have not mentioned so many other functions of gears that we might be having, but these are quite simple examples very generic to make us realise that yes, gears are very much essential.

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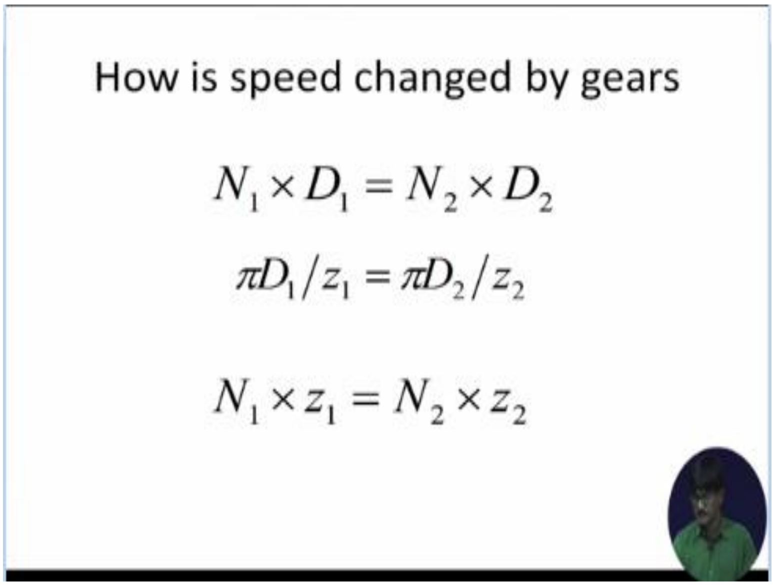

This one, we have already established. So, let us have a quick look at some numerical problems. It will be very interesting to discuss 1 or 2 numerical problems.

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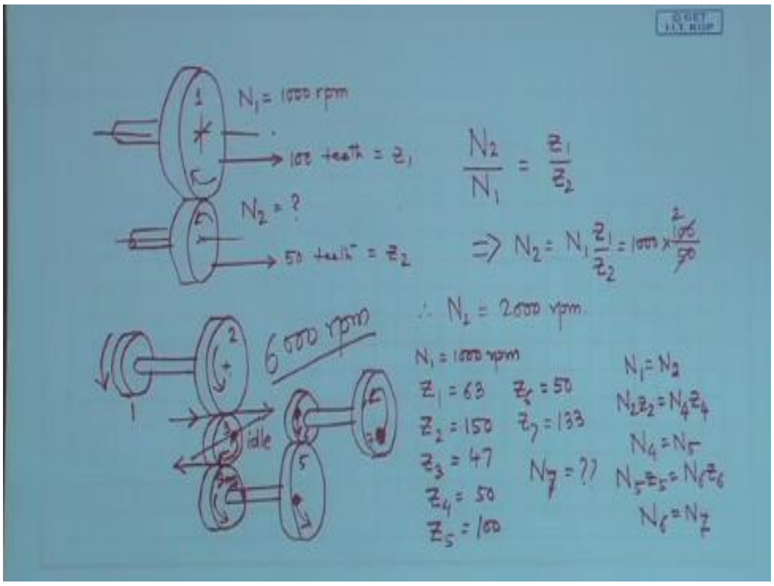

For example, say I have the problem that we were dealing with. This gear is rotating, this shaft is rotating at say let us call it one, $N_1 = 1000$ RPM and here, we have another gear which is rotating at, this is N_2, I do not know what is N_2. But I have been told that this has; just a moment, this has 100 teeth. This is 100 teeth and say this one has 50 teeth. So, what is given? N_1 is given; z_1 is given. So, this is z_1. N_2 is not given and z_2 is given.

So, find out N_2. So, we can say the answer is:

$$\frac{N_2}{N_1} = \frac{z_1}{z_2}$$

$$N_2 = N_1 \times \frac{z_1}{z_2} => 1000 \times \frac{100}{50}$$

$$\therefore\ N_2 = 2000 \text{ rpm}$$

Equal to, N_1 is 1000, z_1 is 100 and z_2 is 50. So, we have a 2 here, therefore, N_2 is equal to 2000 RPM. So, this is quite simple to understand. If I have 2 gears like this, if I am given 3 values, in that case, I can easily solve for the last value of the 4.

Let us have another example. Sorry. That is it. This looks quite impressive 1 2 3 4 5 6 7 and suppose, this is the data that I am providing you. N 1 is equal to once again 1000 RPM, z_1 = 63 that means the number of teeth on gear 1 is 63; z_2 = 150; z_3 = 47; z_4 = 50; z_5 = 100; z_6 = 50 and z_7 = 133. Find out N_7. Now, first of all, we should look at the problem this way that what are we supposed to find out? This rotational speed.

So, this rotational speed we are given N_1. So, this one rotates this, rotates that, rotates this, rotates this, etc., N_7 has to be found out. So, first of all, what we should establish is this:

$$N_1 = N_2$$

First, these 2 rotational speeds are the same and what about the number of teeth on one? It is absolutely not relevant to this problem. You can have any number of teeth. So, N_1 is equal to N_2, z_1 is not required.

Next, we understand that the rotational speed sorry the tangential velocity at the periphery at the pitch circle of 2, this velocity must be equal to, so, sorry, I think I made a mistake here. This rotates this way though this rotates this way, I am sorry. So, this velocity let us write properly, this velocity and this velocity, they are the same. This velocity and this velocity, they are the same and therefore, we can totally drop this one from calculations.

Let me draw a fine line to understand this is not required. It is called idle gear. What is its purpose? It does have a purpose. It does not affect the rotational speed ratio, but it will be affecting the direction of rotation. So, if it is rotating this way, this also will be rotating this way. Please understand that the rotational direction that I gave previously was wrong. So, this is this way, this is this way and this is therefore, this is rotating this way.

So, since, this velocity and this velocity are the same, we can establish the same relation between these 2. So, first of all, we can write that:

$$N_2 z_2 = N_4 z_4$$

Next, we again find out that this is since it is a rigid body with same rotational speed also. So, N_4 is equal to N_5. Next, if gear number 5 and gear number 6, they are connected like this, we again establish:

$$N_5 z_5 = N_6 z_6$$

and naturally N_6 is equal to N_7. That is it.

With this sort of relationship conditions, we can find out by solving these equations. Now, is it really painstakingly you have to go through this one? Once you are experienced with this and once you find out that the number of teeth that I have given. They are very simple values. You can quickly calculate as well. N_1 =1000 RPM; $N_{2=}$ 1000 RPM, what is the number of teeth ratio like in z_2 and z_4.

So, the number of teeth is becoming one third. So, the speed must be becoming triple. So N_1 =1000 RPM; $N_3 =$ 3000 RPM; $N_5 =$ 3000 RPM, z_5 is 100 and z_6 is 50. So, again, it becomes double $N_6 =$ 6000 RPM, so answer is $N_7 =$ 6000 RPM. So with this, we come to the end of our first lecture. Thank you very much.