Metal Cutting and Machine Tools
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Lecture-14
Calculations on Mechanisms in Machine Tools

Welcome viewers to the 14th lecture of the online course metal cutting and machine tools. So, today after finishing the previous lecture on lathe I could feel the requirement for some discussion on the typical mechanisms that we would be coming across on the lathe, on the milling machine etcetera. So, I have slightly changed the course of the lectures and included a small discussion on different types of mechanisms.

So, some simple calculations are included, let us move a right away into the calculations. So, 14th lecture calculations pertaining to machine tools.

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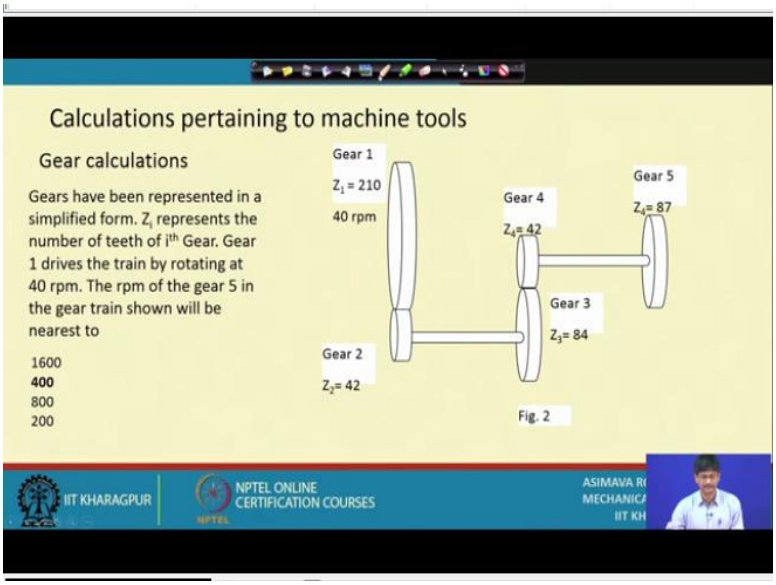

First of all calculations pertaining to gears, we have a gear train and what are we given? Gears have been represented in a simplified form, that is understood, they are represented as circles seen you know from an angle. So, in this particular view we are seeing gear numbers mentioned to you know identify them and numbers of teeth have been mentioned as Z_i.

So, Z_1 represents the number of teeth of gear 1, Z_2 represents the number of teeth of gear 2 and their rotations per minute of the first gear has been mentioned. So, this is the input, so we understand that this is the input rotation 40 rpm. So, if 40 rpm is the input rotation the

question is what is the rotations per minute of the 5th gear? The last one in the train so gear 1 is rotating gear 2, gear 2 is on the same shaft with gear 3, gear 3 is rotating gear 4 and gear 4 is on the same shaft as gear 5.

If 2 gears are on the same shaft and they are integral in that case, they will share the same rotations per minute their rpm will be the same or their angular velocities will be the same. What about 2 gears which are connected like this in mesh? They will have the same tangential velocity. With respect to this tangential velocity let us finish a quick calculation.

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First of all, this is one gear and say this is another one, these are their pitch circumferences. So, they are in contact with each other here and I draw the teeth of one gear this way and I draw the teeth of the other gear here like this, though the drawing is not very perfect, I think we can continue our discussion with these geometry elements in view.

These are the teeth of the gears, if this gear is rotating the other gear is made to rotate and we understand that in order to have this meshing or this interconnection between 2 gears on the pitch circumference this distance from here to here, this one distance has to accommodate or rather this one gear tooth has to be accommodated by this particular corresponding distance on the other gear. And this distance has to accommodate the gear on the other side.

So, we can say these 2 distances have to be the same on these 2 gears and what is this basically? It is pitch circumference divided by the number of teeth. So, we can write pitch circumference equal to πD_p where D_p is the pitch diameter, what is the pitch diameter? Pitch

diameter of a gear is defined as that diameter of 2 gears in mesh. So, let me draw this. If 2 gears are replaced by 2 circles rotating against each other without slip to produce the same rpm ratio as that of the 2 gears.

In that case those 2 diameters are referred to as the pitch diameters of the 2 gears. So, I have 2 gears, I replace them by drums, rotating against each other without slip and producing the same rpm ratio. Then those particular diameters are called the pitch diameters of the 2 gears. So, I am saying on the pitch circumference this distance is nothing but $\pi D_{p1}/Z_1$.

Z_1 is the number of teeth on the first gear 1 and 2. This must be equal to from whatever we have discussed here this must be equal to $\pi D_{p2}/Z_2$, so that π, π cancels and I have D_p/Z is a constant. $D_{p1}/Z_1 = D_{p2}/Z_2 = D_{p3}/Z_3 = D_p/Z$ a constant for meshing gears and this one we give a name.

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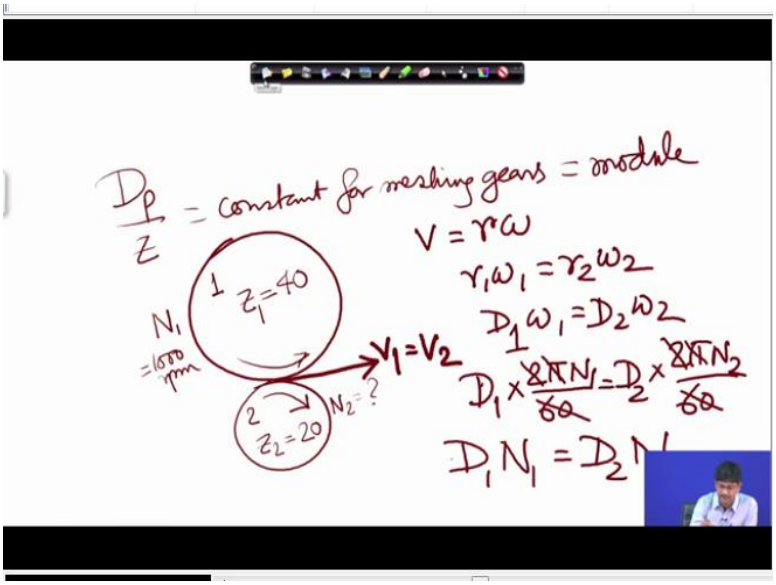

So, let us select a new page D_p/Z is constant for meshing gears and it is given a name called module. So, if you go to the market and say that I want a gear of 32 teeth the question that the shopkeeper will ask you is that what is your module? Module in system is slightly defined slightly differently, but teeth you know the discussions clear and easy to understand we will stick to basically module.

There we have the existence of diameter pitch which is equal to Z/D_p where D_p is in inches, but we will do restrict our discussion to module. So, this point is understood gears which mesh with each other have a constant module or equal module and module differentiates

families of gears which mesh with each other. So, now that we have understood let us go back to our discussion of gears.

So, we express 2 gears this way and then we say that suppose this 1 gear and this is gear number 2, there in mesh with each other and this has say 40 teeth, Z_1 equal to 40 and Z_2 = 20 and in that case if N_1 is given to be 1000 rpm what is N_2 equal to? What is the value of N_2? So, in that case how do we solve it? We first of all say that as we observe last time this particular velocity is shared $V_1 = V_2$.

If $V_1 = V_2$ we can say V is nothing but equal to rω. So, naturally we have $r_1\omega_1 = r_2\omega_2$ and you know for our convenience let us replace r by D. $D_1\omega_1 = D_2\omega_2$ which means you know D_1 into and we replace ω by $2\pi N/60 = D_2 * 2\pi N_2/60$. All these things will cancel out and we have D_1 is equal to and we have $D_1N_1 = D_2N_2$.

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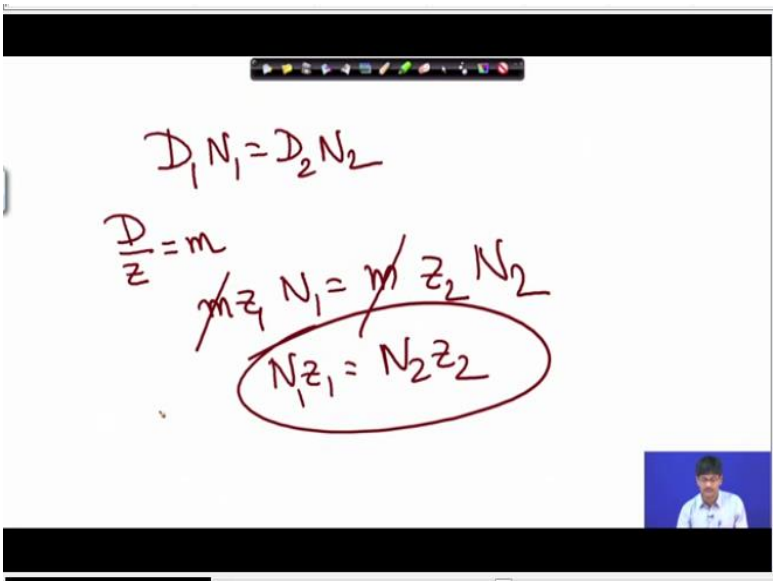

So, let us select a fresh page and write here $D_1N_1 = D_2N_2$ and therefore since we have found previously D/Z you know D_p and D here they are the same things, D/Z is equal to module, I can replace here D with $m*Z_1$ So, $m*Z_1*N_1 = m*Z_2*N_2$ module is the same for machine gears. Hence I cancel out module and I have $N_1Z_1 = N_2 Z_2$. And therefore if we go back to the problem so this is what we are going to use now.

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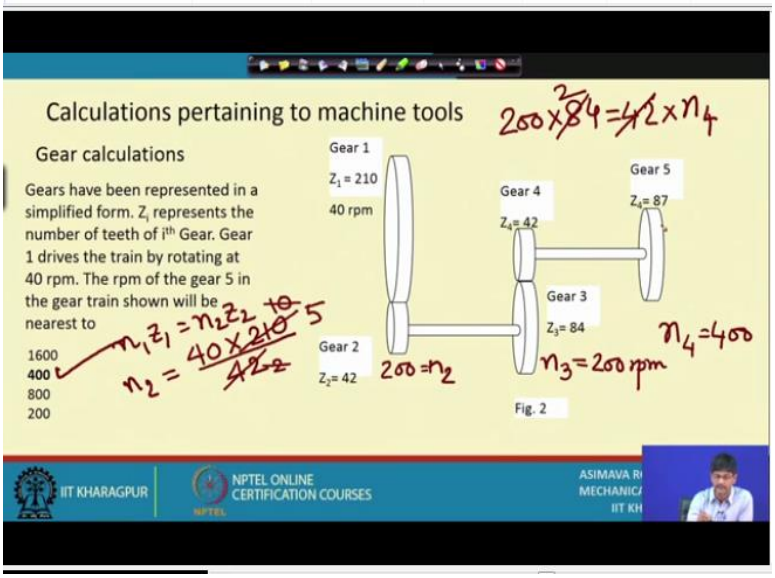

And if we go back to the original discussion we find Z_1 is given, N_1 is given. So, let us solve it right away, we have $N_1Z_1 = N_2 Z_2$. For these 2 machine gears; therefore, I can say that N_2 can be found out as $N_1 Z_1/Z_2$, 40*210 / Z_2, how much is Z_2? Z_2 is 42, sure 42 goes into this 5 times I hope 42, 21 goes into this twice and on this 10 times and you can cancel it further 5 yes, 5 into 40.

So, this is going to rotate at 200 rpm, N_2 is 200 rpm. So, this is also going to rotate N_3 is going to be the same as per our previous discussion 200 rpm. Let us apply the same thing here. N_3Z_3 that means 200*Z_3, 84 must be equal to Z_4, 42*N_4 that is good. So, we have this one cancelling out twice and therefore we have N_4 = 400 and you know they are sharing the rpm.

So, this will also have rpm equal to 400, N_5 is 400, the rpm of the gear 5 is 400. This one is correct you might say what about Z_4? Did not you have some role in the deciding of the rpm, no it is rotating in air not in connection with anyone? So, it has absolutely no role here. So, just one thing I would like to mention here.

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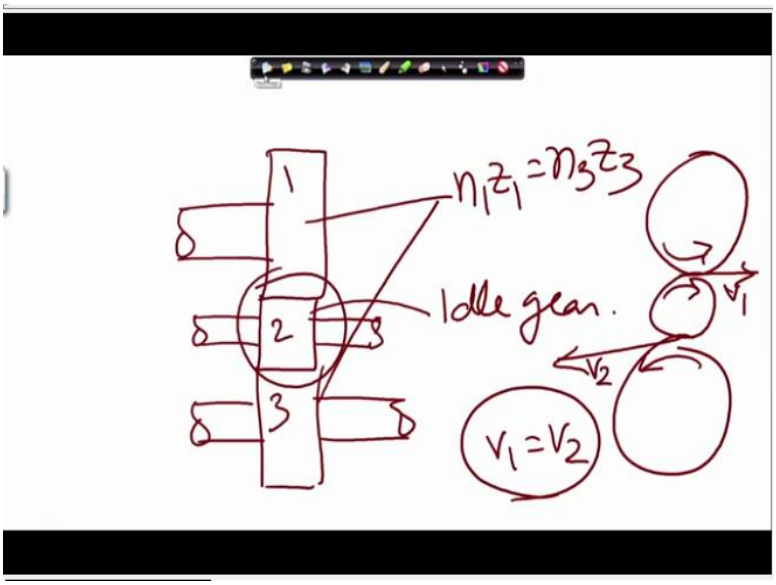

Before we move on to the next subject if you have a shaft in which you have a gear and you have another gear, another shaft and you have yet another gear another shaft. In that case what happens is if this is number 1, number 2, number 3, you can connect up $N_1Z_1 = N_3Z_3$, directly instead of considering about 2, because 2 in this case is called an idle gear because in the other view it will look like this gear number 1, gear number 2, gear number 3 and they are rotating this way.

And therefore they have the same tangential velocity. This tangential velocity, let us call it V_1 and V_2 they are the same. V_1 and V_2 they are the same. What does that mean? That means that I can directly establish the equality of velocities between these 2 elements. That is it. So, having established this one we will be utilizing all these results extensively in our discussions. So, this answer we have already established is 400, let us move on to the next subject.

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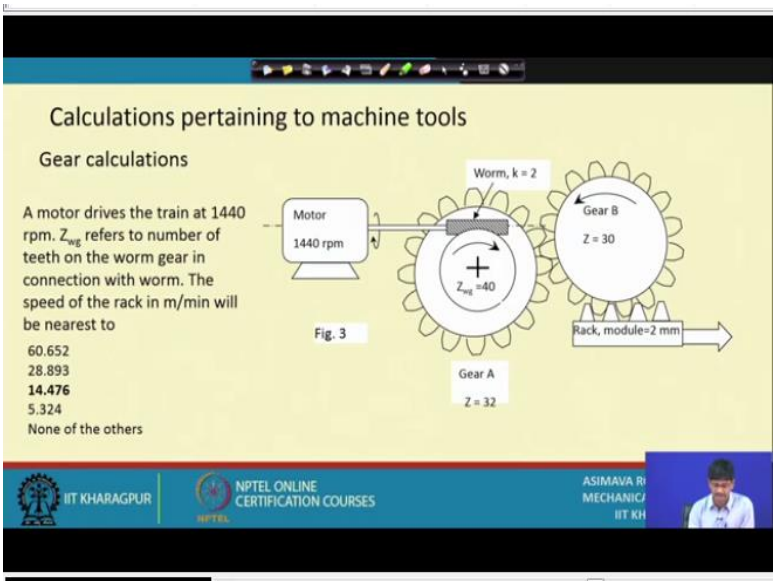

What an elaborate example? So, I will come back to this one. I will come back to this because at this moment we are not having any knowledge of what should be the speed ratio between worm and worm gear. Let us discuss all those aspects.

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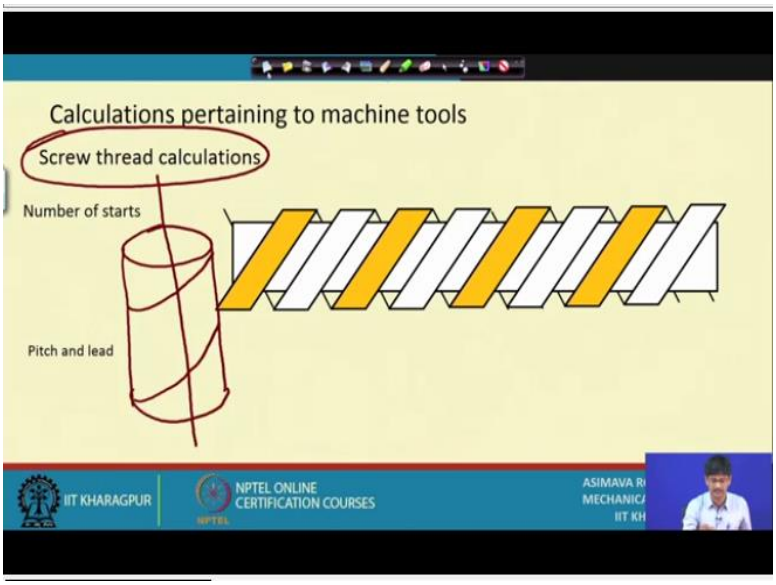

This is a threaded element. What is the meaning of a threaded element? It means that on the circumference of a cylinder outer wall of a cylinder suppose I am climbing up, you all seen spiral stairway, maybe all of you have visited places like the Qutub Minar, I do not know during your time whether you are allowed to move up the Qutub Minar.

But in our time we were allowed up to a certain I think the first storey and it was a helical climb. There was a helix, that means a helix goes all around the cylinder and it rises while its rotating all around, it rises while it rotates, that is a helix. So, whenever we are having such

helices in machine tool elements I mean machine elements they serve some purpose. They can be used as clamping devices, they can be used as devices for deriving motion etcetera and therefore we come across today's discussion about screw threads. Let us see how they help us.

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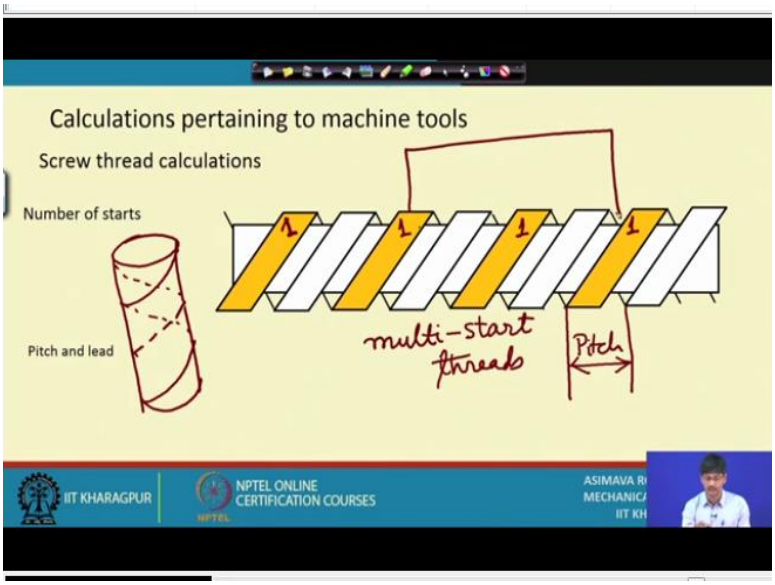

First of all, what is this screw thread doing, what information does it give us? For example, in a screw thread you will find that there are some typical definitions attached to it, what sort of definitions are these. For example, distance between similar successive points on a thread. So, first of all this is a helix, similar successive points on a thread it is called pitch. This one is called pitch. So, what is so great about that?

You will find surprisingly that there is not a single helix in this one but there are 2 parallel helices. This is helix 1, they given a different color, helix 2, helix 3, helix 4 like that, what does it mean? It means that this is helix I should not have written helix 1, helix 2, this will unnecessarily confuse you. This is helix 1 only continued. What does it mean? It means that this helix is absolutely independent from that one.

They are climbing parallely, but how does it show up on the cylinder? If this is the cylinder it shows up this way, this is one helix climbing up etcetera helix 1. So, it moves all around this way and there is yet another helix, so let me give it a different color, it is the same color. So, I can change at least the font, there is yet another helix moving this way.

So, parallel helices just like your spiral stairways 2 parallel spiral stairways. What purpose does it serve? In this case it is always referred to you know if you have 3 helices or 4 helices or 5 helices it is called multi start threads, what purpose do they serve? In that case if you have a nut connected that**.**

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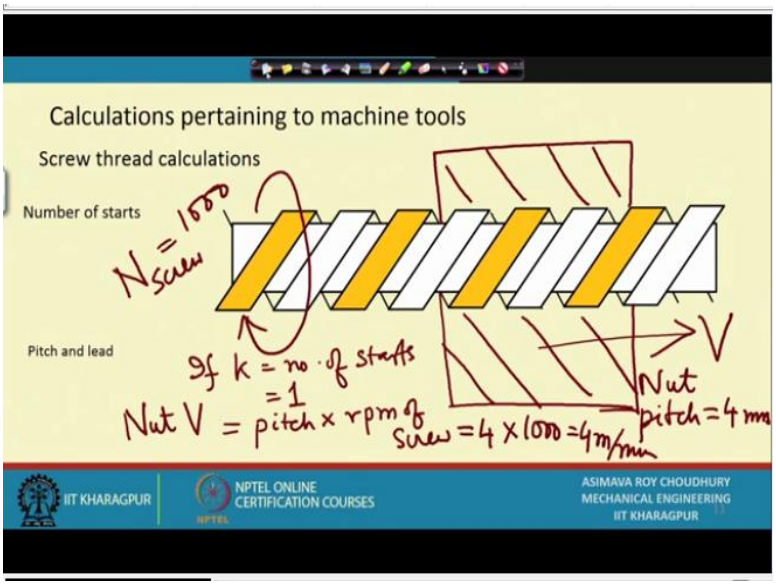

Nut is a device which has fitting grooves on it into which these threaded elements can be fit in. So, this is the nut, if you rotate the screw and if you hold the nut steady so that it cannot rotate, it will execute translational motion. Now if for single start threads if let me write if k equal to number of such helices, separate helices it is called number of starts equal to 1.

In that case the movement of this nut past this screw I mean if it is not allowed to rotate with the screw and if the screw is not allowed to translate in that case the movement is pitch into rpm. So, we can write nut velocity, velocity of the nut will be equal to pitch multiplied by the rpm. Let us have an example. Suppose the rotation of the screw, rpm of screw is equal to 1000 rpm. So, in that case if pitch be equal to nut pitch and screw pitch has to be the same otherwise they would not match.

Nut pitch is say 4 millimetres, in that case this will be equal to 4 * 1000 = 4 m/min just imagine, 4 m/min will be the movement of the screw, if number of starts equal to 1.

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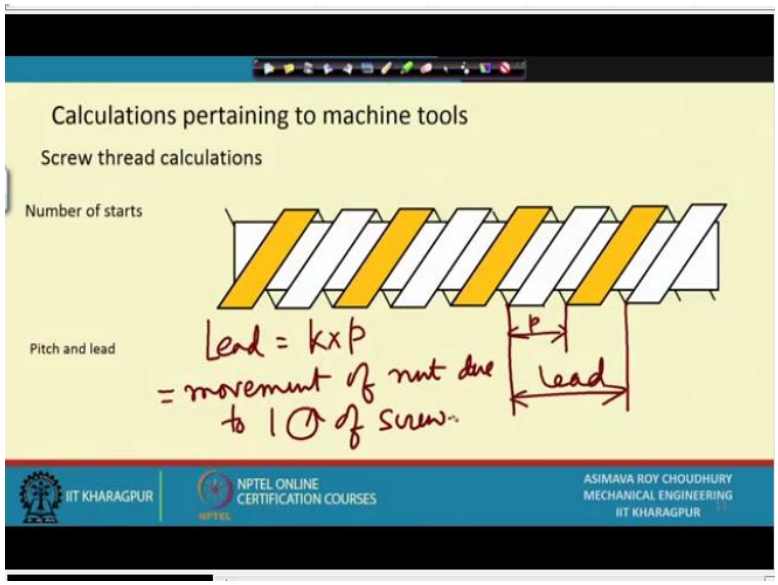

However, if there are 2 starts as shown in this figure there is another term defined which is called lead while this was your pitch. So, we have lead equal to k into pitch, k is the number of starts and equal to movement of nut due to 1 rotation of screw. So, ultimately it boils down to the fact that if the nut and screw they are of multi starts say they are of double starts.

In that case if you rotate the screw once the nut I mean does not move by the pitch but it moves by the lead where lead is equal to number of starts into the pitch having established that we understand that this is the way in which the nut and screw work.

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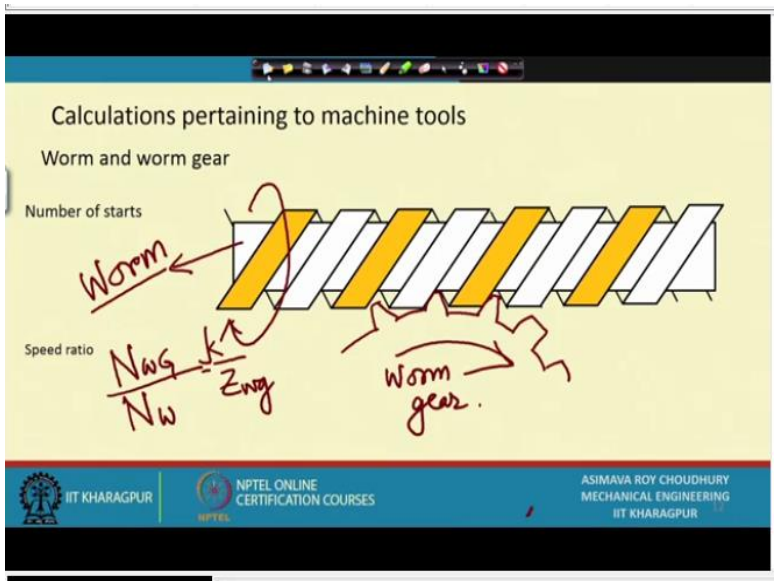

Now let us move on to the last machine element pair that we have to discuss the worm and worm gear, what is the worm and worm gear? Now at this moment let us identify the worm by simply referring to it as a threaded element. So, this is our worm. So, why are we suddenly

bringing in this term because when it is coupled by another machine element it forms the very popular device for speed reduction called a worm and worm gear?

So, what is the worm gear in connection with the worm, you know the moment people started thinking about different machine events at one point in time they connected up 2 different machine elements. The screw and the gear. So, that they made this sort of a connection, like that which means that if the screw is rotating, the worm gear will have to rotate with it in which way that is tooth by tooth this particular helix as it is slightly you know inclined.

These points will be coming sequentially and they will be pushing the wheel toothed wheel to one side and this is called the worm gear. So, what is so great about it? The thing which is great about this machine element pair is that it can bring in huge reduction in speed only in one stage. Generally, if you are trying to reduce the speed maybe with the help of sliding clusters you can reduce the speed by say a factor of 1, a factor of 2, maximum maybe 3.

But not more than that at one stage, but at one stage itself the worm and worm gear can reduce the speed by a factor of 4, just imagine factor of 40, what is the speed ratio? So, the speed ratio of the worm and worm gear is equal to N_{wg}, why is it called the worm because it looks like an earthworm that is why. N_{wg} divided by the rpm of the worm is equal to k/ Z_{wg}.

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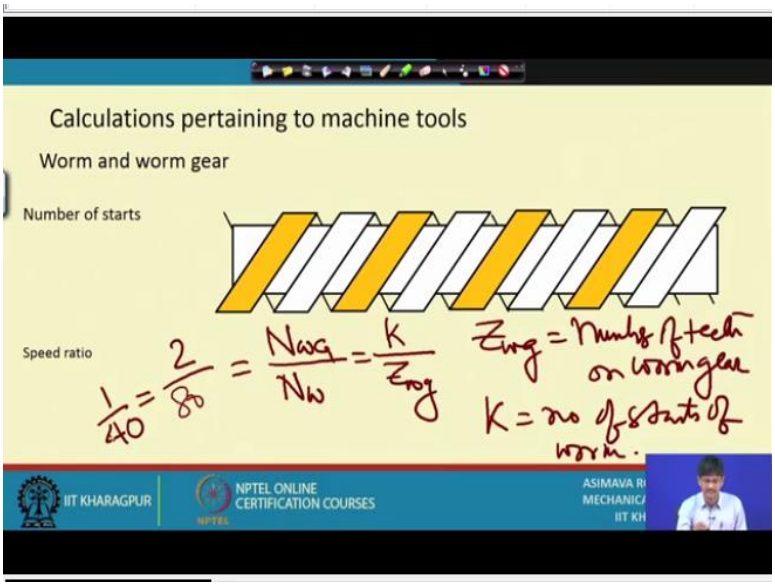

Let me draw it a bit more clearly N_{wg} / N_w = k/ Z_{wg}, where Z_{wg} is equal to number of teeth on worm gear, what is the worm gear? The one which is connected to the screw and k is already

defined as number of starts of worm. So, having understood this we see that the ratio of these 2 speeds can really be very, very small in what way suppose k is equal to 2.

And suppose Z_{wg} is equal to 80. So, this will be equal to 2/80 = 1/40. So, this is the capability of this device and we will have extensive use of it in our mechanisms. So, with this one let us quickly go to the previous slide in which we had some discussion on this.

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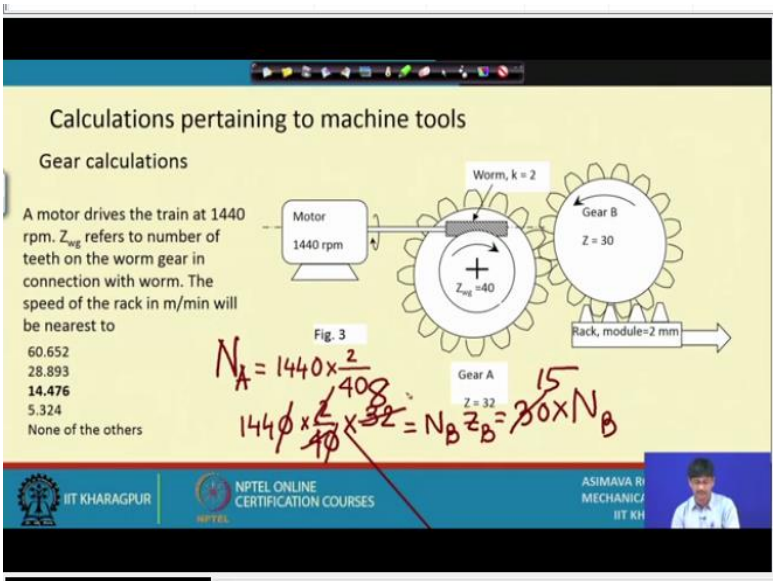

This is the one let us quickly solve this and end our class today. The motor rotates at 1440 rpm; Z_{wg} refers to the number of teeth on the worm gear in connection with the worm that is understood. The speed of the rack in meters per minute will be nearest to. So, what do we have here? The motor connected to the worm, number of starts is equal to 2, this is the worm gear, its teeth have not been shown.

Number of teeth on the worm gear is 40 and on the same shaft we have another gear where the number of teeth is equal to 32 that is in connection with a gear B whose number of teeth is equal to 30, that is in connection with a straight sided gear called a rack with module 2. We have to find out the speed of the rack in meters per minute. So, let us move right away, 1440 rpm is the rpm of the motor we start from the motor.

Next we come to the worm gear. Worm gear converts this rpm I mean the worm and worm gear pair changes this rpm as per its speed ratio. So, speed ratio is already known k/z which means this is nothing but 1440 multiplied by 2 divided by 40. So, this part so far so good. So,

this is now can be removable, we have moved up to the worm gear shaft. That shaft is containing this gear.

So, it must be having the same rpm, so this must be equal to the rpm of gear A. So, this is also equal to N_A that is good. Therefore, we move 1440 into 2 by 40 multiplied by N_A Z_A will be equal to N_BZ_B equal to N_AZ_A 32 must be equal to N_BZ_B = 30 multiplied by N_B. Now how much is that? Let us have a quick cut 0, 0 cancels, this 1 cancels with this, 4 8 are 32, 16, this 2 let it go with this one 15. So, we have how much 8 here, 1440, 144 * 8 / 15.

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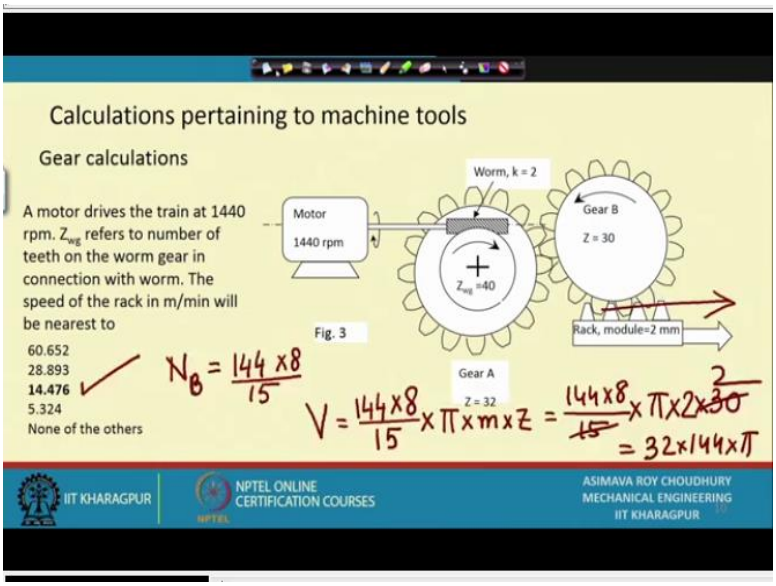

N_B = 144 * 8 / 15, what do we have here? After this the rack is being moved due to its connection with Z_B. What is the speed of the rack? Now they must be sharing the same velocity here. What is this velocity? It must be number of rotations multiplied by the circumference. In one rotation it moves by the circumference. So, in n number of rotations it must be moving by n into the circumference.

So, n is this, so we have the velocity to be velocity = 144 * 8 / 15 into as we discussed into the circumference, circumference is π * D. What is D? D is equal to module * Z. So, module is 2, m * Z = 144 * 8 / 15 * π * 2, module is 2 * 30 that is it. This should be the answer 2, 4. So, 4 8s are 32, once again 32 * 144 * π. So, this has been named as the correct answer in meters per minute.

All these things were in millimetres. So, we will have divided by 10^3, I leave it to you to check whether this one is correct, let me write it down 32 * 144 π.

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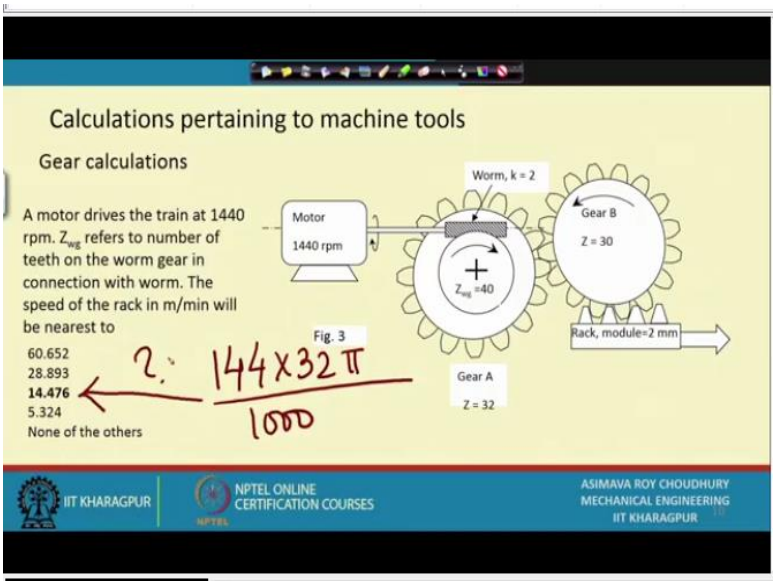

Please check yourselves 144 * 32 π / 1000 is this equal to this one, you have to check. So, with this we come to the end of today's discussion, thank you very much.