

Metal Cutting and Machine Tools
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Lecture-12
Wear and Life of Cutting Tools - II

Welcome viewers to the 12th lecture of the course metal cutting and machine tools. And today we are going to take up the last part of the discussion on wear and life of cutting tools. On the previous day we have understood how a wear and tear can reduce the working life of a cutting tool and ultimately make it render it to be unusable. What exactly happens is wear takes place mainly on the principle flank and on the rake surface.

On the rake surface it manifests as a crater and on the flank surface it removes part of top of the flank, so that a land forms and when this the average wear extent of this land reaches around 0.3 millimeters we name that tool, I mean we state that that tool has become unusable due to wear and tear. So, it gives rise to an expression of tool life and we have already studied about Taylor's tool life equation not the extended one, the simple one.

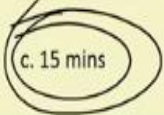
Taylor's tool life equation we have already studied in which a relation is established between the cutting speed in meters per minute and the tool life in minutes. So, that if a tool is used at different velocities, we can establish a relation between all of them. So, let us go on to the solution of a few problems, so that we understand how the operation, how problems can be solved in this particular direction?

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WEAR AND LIFE OF CUTTING TOOLS - 2

In a machine shop, single point turning tools are used at 75m/min to yield a tool life of 30 min. However, war breaks out and the turning tools have to be used at 100 m/min to yield a tool life of 20 mins. Last of all, the machine shop passes onto the hand of the enemy army, who start running the tools at speed of 125 m/min. They will get a tool life nearest to

a. 1 min b. 10 mins⁺ c. 15 mins d. None of these



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Let us see in a machine shop, single point turning tools are used at 75 m/min to yield a tool life of 30 minutes. So, you are using it at 75 m/min and the tool life is 30 minutes. However, war breaks out and the turning tools have to be used at 100 m/min to yield a tool life of 20 minutes. Last of all the machine shop passes on to the hands of the army, who start running the tools at a speed of 125 m/min.

They will get a tool life near as to some options are given. So, what exactly is the case? First data we get a set of values I mean a pair of values of velocity and tool life 75 m/min and 30 minutes. So, war accelerates a number of activities you need more spares, maybe for different war related equipment and for that you have to get them fast, so you do not really care about economy and meaningful tool lives.

And therefore may be due to some government applied command sort of they have to be operated 100 m/min. So, obviously the tool life will be less and we will be operating with the Taylor's tool life equation. So, let us just write it down, so that we are remind that is alright. $V \cdot T_n = \text{constant}$ and V is in m/min and T is in minutes. Now, so war breaks out, so you have to make get things done fast and you increase it to 100 m/min, so another set of values they are given.

Last of all the machine shop passes on to the hands of enemy army, this happens very frequently. In the second world war what happened was that the Germans took over some countries very fast, almost overnight and I remember having read somewhere that in Czechoslovakia the machine tools were of high repute. And the Germans started using them, so what did the Czechoslovakians do?

They put sand into the cutting fluid at all the machines were rendered useless. So, this thing will happen very frequently enemy army will use them not even bothering about how much expenses are incurred but they just have to get some of their parts made and the earliest. So, another value is given 125 m/min, velocity but the time is not given, what is the tool life? And the tool life has to be decided. So, the tool life options are 1 minute, 10 minutes, 15 minutes and none of these, so let us see how we can proceed?

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WEAR AND LIFE OF CUTTING TOOLS - 2

Handwritten notes on the slide:

- $V_1 = 75 \text{ m/min}$
- $T_1 = 30 \text{ min}$
- $V_2 = 100 \text{ m/min}$
- $T_2 = 20 \text{ min}$
- $\frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^n$
- $\ln\left(\frac{V_1}{V_2}\right) = n \ln\left(\frac{T_2}{T_1}\right)$
- $V_1 \times T_1^n = V_2 \times T_2^n$
- $75 \times 30^{0.709} = C = 836.25$
- $V_3 \times T_3^{0.709} = 125 \times T_3^{0.709} = 836.25$

Logos at the bottom: IIT KHARAGPUR, NPTEL ONLINE CERTIFICATION COURSES, ASHMAV MITAL IIT

So, first of all what we have written here is $V_1 \times T_1^n = V_2 \times T_2^n$, let us see what are the values that we know. Do we know V_1 ? Yes, $V_1 = 75 \text{ m/min}$. Do we know T_1 ? Yes, we know T_1 , $T_1 = 30$ minutes. Do we know V_2 ? Yes, $V_2 = 100 \text{ m/min}$, so this also we know. T_2 do we know? Yes, T_2 also we know, what is to be formed out? N we do not know?

So, $T_2 = 20$ minutes, so in that case what we can do is that we can write $V_1/V_2 = (T_2/T_1)^n$. And after that we can find out $\ln(V_1/V_2) = \ln(T_2/T_1)$, does it seem to be alright? Yes, so V_1/V_2 what is V_1 ? V_1 is 75, V_2 is 100, so I think better let me just clear it.

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WEAR AND LIFE OF CUTTING TOOLS - 2

$$V_1 \times T_1^n = V_2 \times T_2^n$$

$$75 \times 30^{0.709} = C = 836.25$$

$$V_3 \times T_3^{0.709} = 125 \times T_3^{0.709} = 836.25$$

Handwritten notes on the slide:

$$\frac{100}{75} = \left(\frac{30}{20}\right)^n = \left(\frac{3}{2}\right)^n$$

$$\frac{4}{3} = \left(\frac{3}{2}\right)^n$$

$$\ln\left(\frac{4}{3}\right) = n \ln\left(\frac{3}{2}\right)$$

$$n = 0.709$$

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Let us put in the values actually, $75 \times 30^n = 100 \times 20^n$, which means we can write $100/75 = (30/20)^n$ which means $(3/2)^n$, and this is $4/3$. So, $4/3 = (3/2)^n$ and after that we can take $\ln$, $\ln 1.333 = n \times \ln 1.5$, we can find out these values very easily and n can get solved. So, I have found n to be 0.709, so please check, n comes out to be 0.709, does this solve our problem? No, it does not solve our problem, because if we try to find out T_3 .

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WEAR AND LIFE OF CUTTING TOOLS - 2

$$V_1 \times T_1^n = V_2 \times T_2^n$$

$$75 \times 30^{0.709} = C = 836.25$$

$$V_3 \times T_3^{0.709} = 125 \times T_3^{0.709} = 836.25$$

$$T_3^{0.709} = \frac{836.25}{125}$$

Handwritten notes on the slide:

$$\frac{100}{75} = \left(\frac{30}{20}\right)^n = \left(\frac{3}{2}\right)^n$$

$$\frac{4}{3} = \left(\frac{3}{2}\right)^n$$

$$\ln\left(\frac{4}{3}\right) = n \ln\left(\frac{3}{2}\right)$$

$$n = 0.709$$

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Suppose we put $V_3 \times T_3^n = C$, I do not know C till now, but I do know n, I do not know T_3 , I know this, I do not know this, I do not know this, I know this, so it still cannot solve my problem. So, what we need to do is, we need to find out C and that is what we have done here. In one of the known cases say the first case we know all the values, and therefore C can be found out, 75 velocity into T^n , n is known now is equal to C.

And therefore C comes out to be 836.25, I just at this moment I am not writing the units, I think for this problem you can still this will be clear to you. So, $C = 836.25$, so that now if I apply it in case of the third equation that means V_3 and T_3 we have 125 m/min multiplied by $T_3^{0.709} = 836.25$, which means I will have $T_3^{0.709} = 836.25/125$. And therefore once again by taking log I can solve it.

That means it will be $0.709 \log T_3 = \log$ of this value and therefore I can send 0.709 to this side and then take anti log and solve for T_3 , let me see what value I had obtained.

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WEAR AND LIFE OF CUTTING TOOLS - 2

Find the value of $n = 0.709$
 Find the value of $C = 836.25$
 Find the value of $T_3 = 14.595$ mins

$$V_1 \times T_1^n = V_2 \times T_2^n$$

$$75 \times 30^{0.709} = C = 836.25$$

$$V_3 \times T_3^{0.709} = 125 \times T_3^{0.709} = 836.25$$

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I obtained a value of 14.595; I obtained this particular value of 14.595. So, if we look at the solutions 15 minutes, this is then the correct answer. 14.595 that is as good as since they will get a tool life nearest to, this will be taken as the correct answer. Now let us pass on to the next problem.

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WEAR AND LIFE OF CUTTING TOOLS - 2

Find the value of $n = 0.709$
 Find the value of $C = 836.25$
 Find the value of $T_3 = 14.595$ mins

$$V_1 \times T_1^n = V_2 \times T_2^n$$

$$75 \times 30^{0.709} = C = 836.25$$

$$V_3 \times T_3^{0.709} = 125 \times T_3^{0.709} = 836.25$$

a. 1 min b. 10 mins c. 15 mins d. None of these

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WEAR AND LIFE OF CUTTING TOOLS - 2

The following data is received for a new carbide tool machining low carbon steel. Determine the n and C of the Taylor's tool life equation given as below

$$VT^n = C$$

$V(\text{m/min})$	40	50	60	70	80
$T(\text{min})$	40	32	26	20	17

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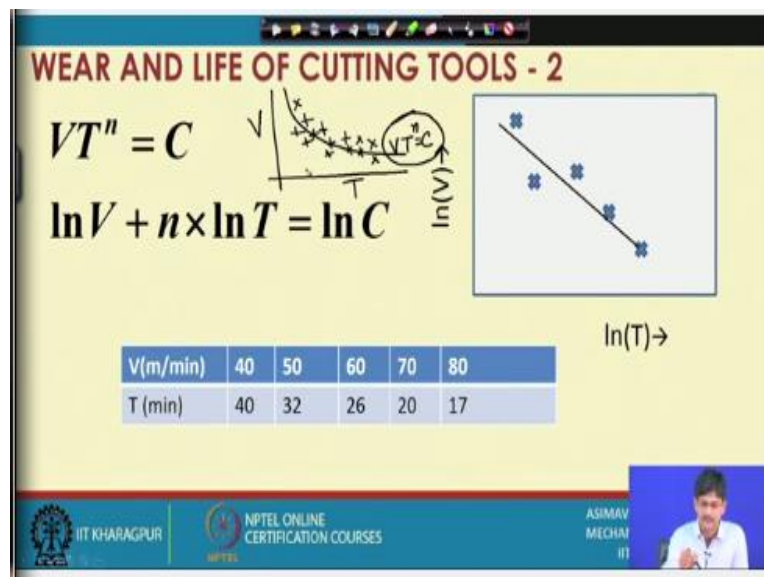
So, the following data is received for a new carbide tool machining low carbon steel. Generally, carbide tools are not used for steels, because diffusion wear can take place and it can destroy the tool within a few minutes of it is run, first run. So, generally we take precautions like we can put a coating, say titanium nitride coating is put there, so that it deters any such diffusion wear.

So, in this case we assume that some problem if it is occurring like that, it is not existing here. So, we are now supposed to determine n and C of the Taylor's tool life equation given as below. What is the data given get to us? That is it has been run at several cutting velocities and we have

the corresponding tool lives in minutes, so what is the problem? We will say that yes just put these values in and find out n and C, there is a problem, what is that?

So, let us quickly have a look at the data 40 m/min gives a tool life of 40 minutes, 50 m/min give a tool life of 32 minutes, 60 m/min give a tool life of 26 minutes. So, as the velocity is increasing the tool life is decreasing, that is understood. Now let us see what is the way in which n and C can be determined?

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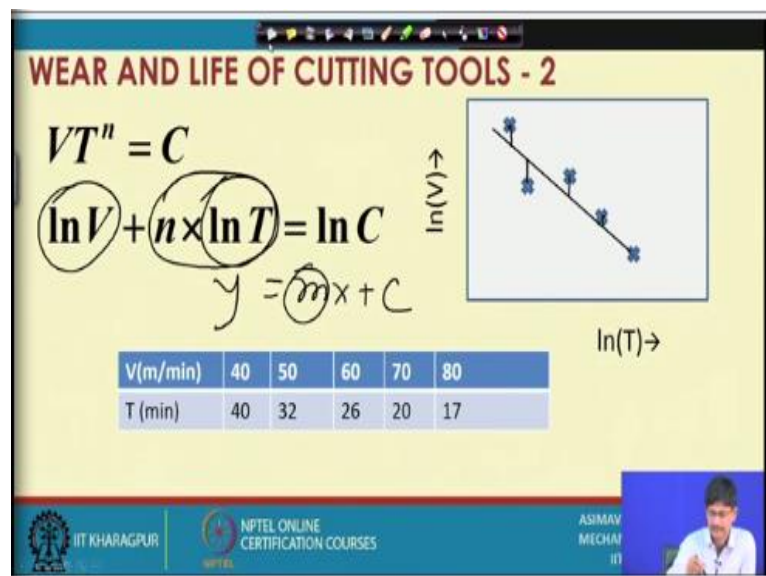
n and C you know if we plot the values of n and C in an ordinary paper, ok let us say this is our graph paper. And on this side I have tool life, on this side I have velocity, I am supposed to have a curve which obeys $V * T^n = \text{constant}$. But you know this is a relation between velocity and tool life in a sort of generalized manner, a single point might not be lying on it, a point might be here, you will say why?

It is supposed to obey this particular rule but this rule is a sort of fitted behaviour to general data. The points might be clustered around it, and then the general behaviour has been predicted by fitting a curve, this is fitted curve. And all sorts of data points experimentally obtained, it will be clustered around it. Because of which this particular behavioural pattern has been identified and mathematically expressed.

So, if we have this data, it is not always true that all the points will be very comfortably lying on a curve which will be obeying this one. No, rather it will be like this, so in that case how will you find out n and C ? In that case first of all what we can do is once we notice this particular form of the equation we can write it out as taking log on both sides, I have $\log ab = \log a + \log b$.

So, $\log V T^n = \ln V$ that means natural $\log V + n \log T = \log C$, I have taken natural log. So, I might be using log directly, please understand that I am always referring to natural logarithm. If this happens we quickly notice that this represents nothing but a straight line, in what way?

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We might say $y - mx = C$, which means it corresponds to $y = mx + C$, this mx term happens to be on this side, but we write it here it will gain one minus sign but that is mx , I mean that is the general form of the straight line. What is y ? We will say y is nothing but the natural log of velocity. What is x ? x is nothing but the natural log of tool life in minutes. What is n ? n must be equal to m .

If it is going to that side there must be a negative sign, let it be, no problem. So, we have here if $n \log T$, if natural log of time is this axis if natural log of velocity is this axis. And if the points obtained experimentally fall like this, we can draw a line which is sometimes called the best fit. Now what does the best fit mean? It means that whatever errors we are incurring from the actual

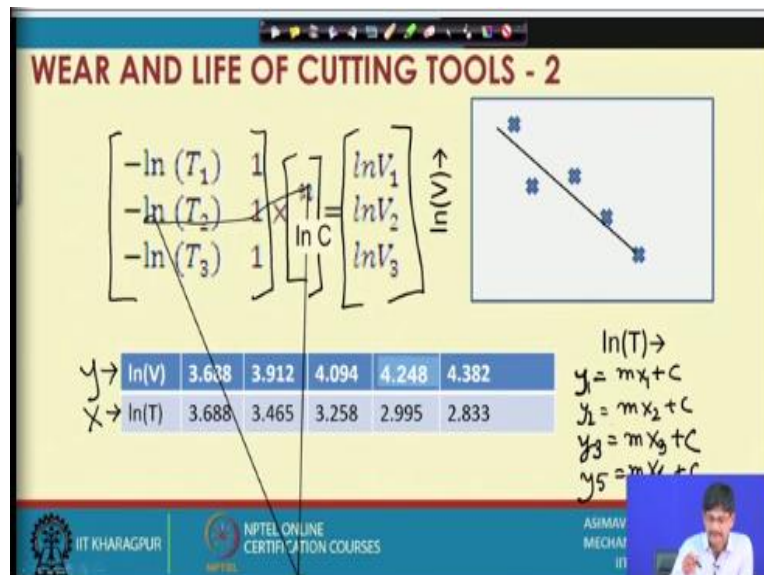
experimentally obtained points and the curve drawn by us, that error will be minimum, why not zero?

Zero will seem to be the best fit; it might not come out to be zero that is what. You can reduce this error to a minimum but it might well not come out to be zero, so that is why this concept of best fit curve has arisen. So, let us try a best fit here and get the corresponding values of n and C for that best fit. Generally best fit is obtained if you are trying out say by hand we used to do it on log-log paper.

And best fit ultimately construes through this case that these errors that we are incurring between the curve and the y coordinates. They are sum of the squares of these errors, that means $(y_1 - y)^2$. So, that we always have squares, so that they do not negate each other by being plus and minus. So, experimentally obtained value y_1, y_2, y_3, y_4, y_5 and the fitted curve, so these y values have a difference.

So, this one $\Delta y_1^2 + \Delta y_2^2 + \Delta y_3^2$ this way we sum up, sum of the squares of these errors will be equal to will be minimized. This thing can be done very elegantly by mathematical manipulation. So, let us quickly do that and obtain the answer to this question.

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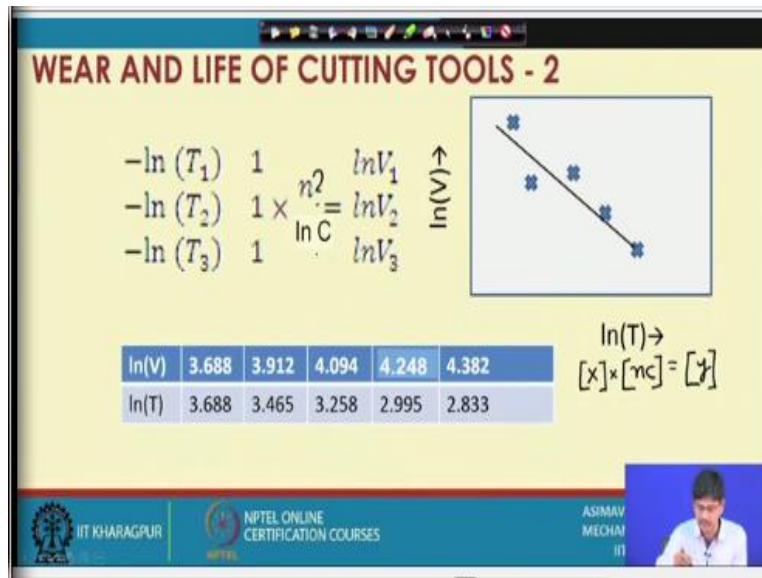
So, I hope you can read it, basically what I have done is I have expressed this in matrix form. Here I have a matrix, so what we have is? If we are having $y = mx + C$, we are actually having $y_1 = mx_1 + C$, $y_2 = mx_2 + C$, if we had two points, we would could have drawn exactly a line between these two points. But the problem is we have more data than we actually required to draw a line here, a number of such points are here.

So, m and C these are remaining constant but y_1 and x_1 they are going on changing. So, let us say here we are having 1, 2, 3, 4, 5 such values $x_4 + C$. And what are these? y_1 is nothing but these values, y values, and x is nothing but these values, it will come with a negative sign, that is why. Remember that if it is going to the other side it develops a negative sign that is why I put negative sign.

So, if we express it in matrix form taking out m and C in a separate matrix, oh! Sorry, m is the gradient and in our case it comes out to be the index of, I mean Taylor's index the index of T . So, n and C , so if we multiply this way that is $-\ln T_1 * n$ that is $\log \text{natural } T_1 * n + 1 * C = \log \text{natural } V_1$. This one into this one is equal to second one, this one so row by column multiplication yields these things.

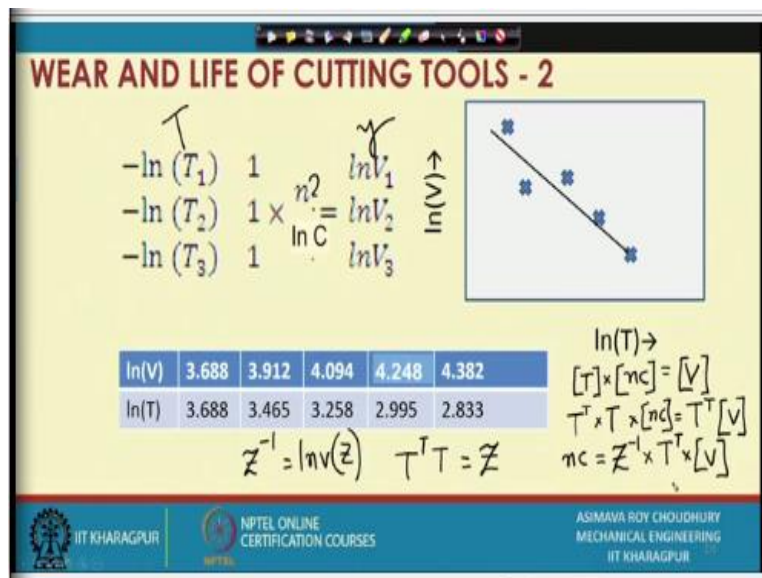
Now in these matrices it can be noticed that as the matrix is not square, it cannot be inverted therefore we cannot get a direct solution, naturally. Because if we have more equations than the number of unknowns, these are the unknowns, 2 unknowns how did have they been 2 equations, we would have been able to solve it. If there are more such equations, you cannot directly solve it. So, what we do in this case is what we do in this case I can write it here.

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That is if I have the matrix the coefficient matrix as if I write it as X multiplied by the matrix I need to find out let us write it as $nc = y$.

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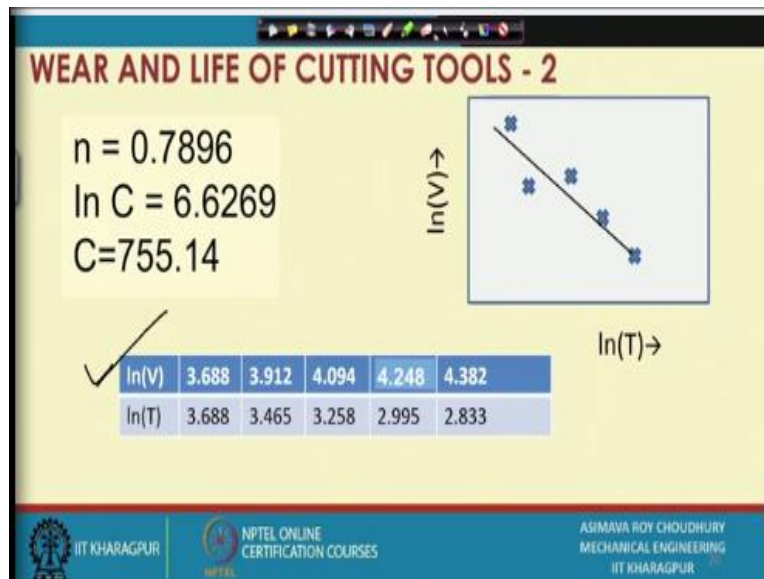
Shall we use more relevant terms T matrix let us call it, so we will easily remember. This is the T matrix and this is our y matrix and therefore we call it velocity matrix. Now all that operation of getting least square fit can be mathematically done this way. We multiply the transpose with T on two sides that is good. So, this becomes square then we invert it and send it to the other side.

So, that nc can be solved as this one, let us give a name T star T is equal to say what? Z, X we have used Z. So, we write now Z inverse let us write -1, so we also add here Z inverse means

inverse of Z . So, this nc will now be solved as, that is it, by the multiplication of these 3. Transpose of T and Z inverse which is the inverse of from here it will be clear inverse of T transpose multiplied by T , nc can be directly solved this way.

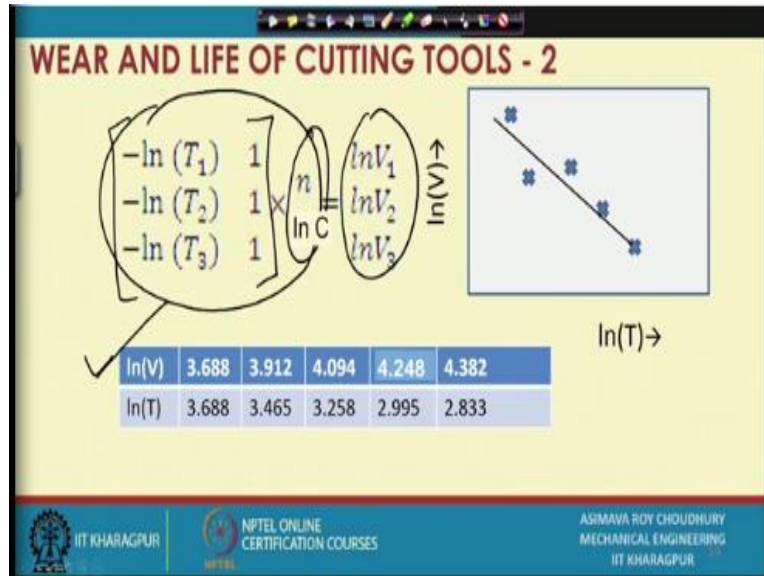
And this is the same as least square fit and it can be used in any such similar problems where the number of equations they are more than the number of unknowns, and the problem is solved in a general sense. That means the solved line does not necessarily pass through all the points but the error incurred will be the least. What I will do is, I have solved the problem, you can check it up and I will also send you a MATLAB program through which this same operation can be done. So, that it will be useful to you, so let us move on to the solution.

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So, if we are given such values of velocity and time, first what we have to do is, we have to convert them into $\ln$ values.

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Make up the matrices in what form? In this form, make up the matrices in this form where just these will be used from the table, invert this matrix that means only this one. First get its transpose multiplied on both sides then inverse the transpose multiplied by this matrix, send it to the other side and you can solve for n and C . So, with this we come to roughly the discussion on tool wear.

There are many aspects which we are leaving behind, because as you can well understand if we have metal cutting and machine tools covered in a lecture C is spanning only 10 hours. There will be many things we will be leaving by the side of our movement forward. And the only way in which we can make use of such information never discussed in the class is that by uploading from my side by uploading some textual notes with some numerical examples.

And if you can make use of those notes and ask me questions if you come across something which is difficult to understand. Only that way we can get balanced knowledge based on metal cutting and machine tools, otherwise it is extremely difficult. Otherwise we cannot cover all the aspects of all these things spanning a vast literature, so thank you very much.