Biomechanics of Joints and Orthopaedic Implants
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Lecture No. 24
Concepts of Stress and Strain

Good morning everybody. In this lecture, I would like to discuss the basic concepts of stresses and strain, and the topic falls under the subject strength of materials or mechanics of deformable bodies, which deals with the relationship between internal forces deformation and external loads.

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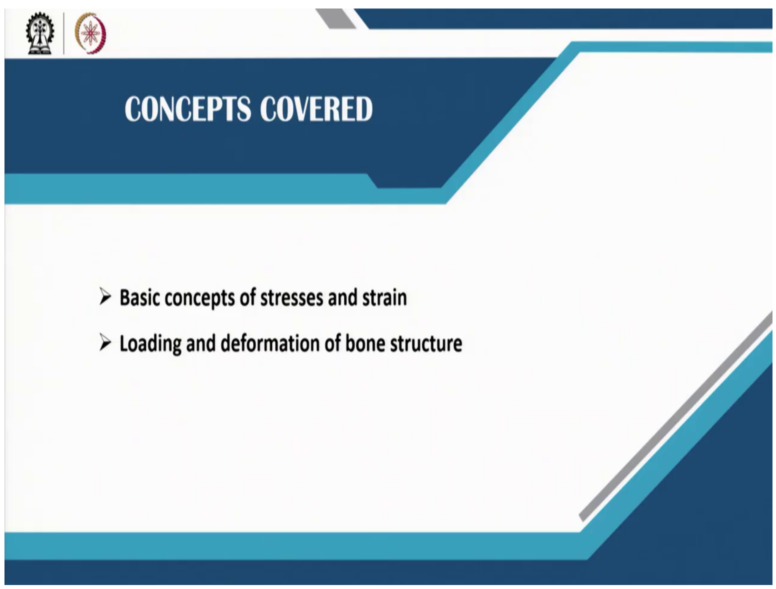

The concepts covered in this lecture are basic concepts of stresses and strain loading and deformation of bone structure.

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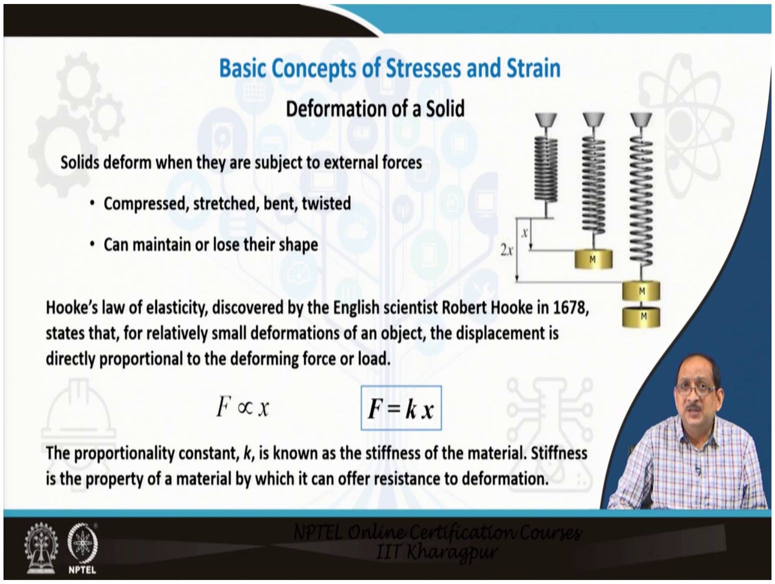

Now, let us first consider a simple spring-mass system. If the weight of mass M is hung from the end of spring, the spring undergoes an elongation of x, if the weight is doubled, the elongation increases to twice x. So, the solid bodies deform when they are subject to external forces; they are either compressed or stretched or bent or twisted. They can actually maintain their shape to some extent or lose their shape as well.

Hooke's law of elasticity states the relationship between displacement and deforming forces. For relatively small deformations of an object, the displacement is directly proportional to the deforming force, or load, which can be written in form F is proportional to x.

$$F = k\,x$$

The proportionality constant k is known as the stiffness of the material. The stiffness is a property of a material by which it can offer resistance to deformation.

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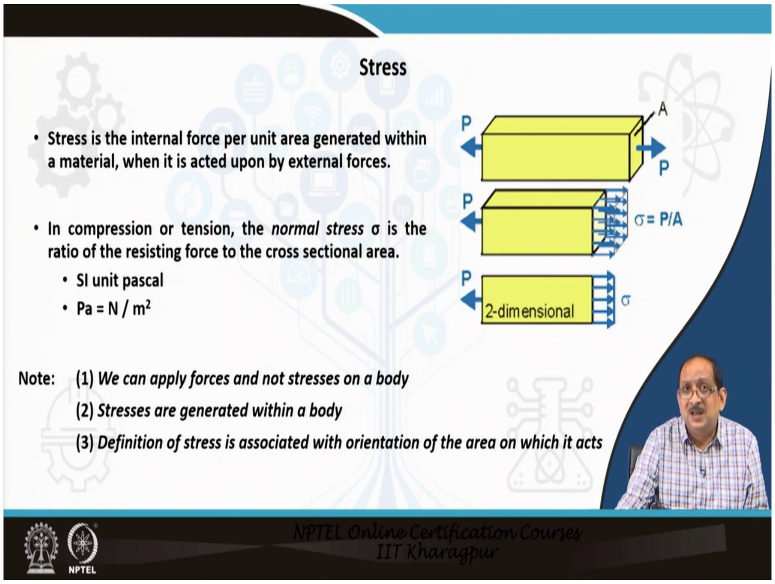

How do we define stress? Stress is the internal force per unit area generated within a material when it is acted upon by external forces in compression or tension uniaxial of force, the normal stress σ is the ratio of the resisting force to the cross-sectional area as shown in the slide. It should be noted here that we can apply forces on a body, and stresses are actually generated within a body, and most importantly, the definition of stress is associated with the orientation of the area on which it acts.

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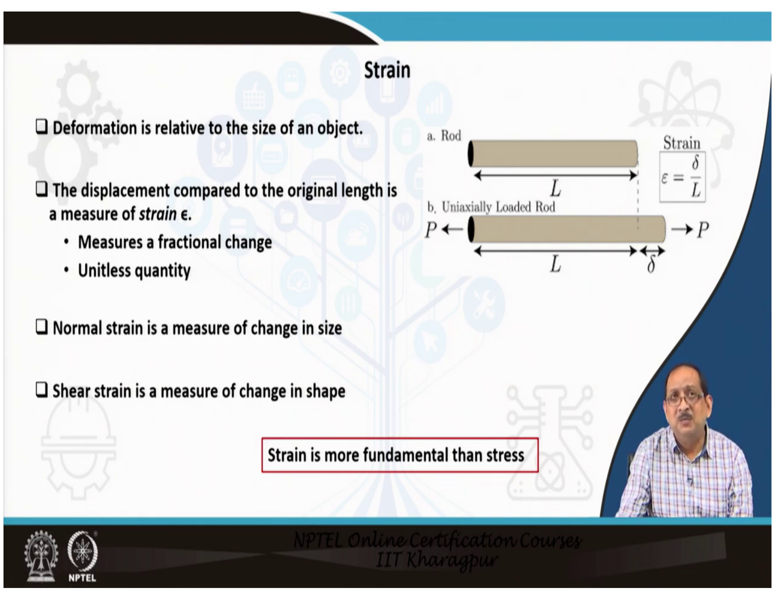

Next comes strain, when a cylindrical bar is subject to external tensile force, at one end, it may undergo a slight increase in length and decrease in diameter. The change in dimension is known as deformation. The average linear strain is the ratio of the change in length to the original length. So, strain is deformation relative to the size of an object.

The displacement compared to the original length is a measure of strain ϵ. So, it measures a fractional change, and it is a unitless quantity. Normal strain is a measure of change in size, whereas shear strain, which is an angular measurement, measures the change in shape. Strain, therefore, is more fundamental than stress because, without a strain, stresses cannot be generated within the body.

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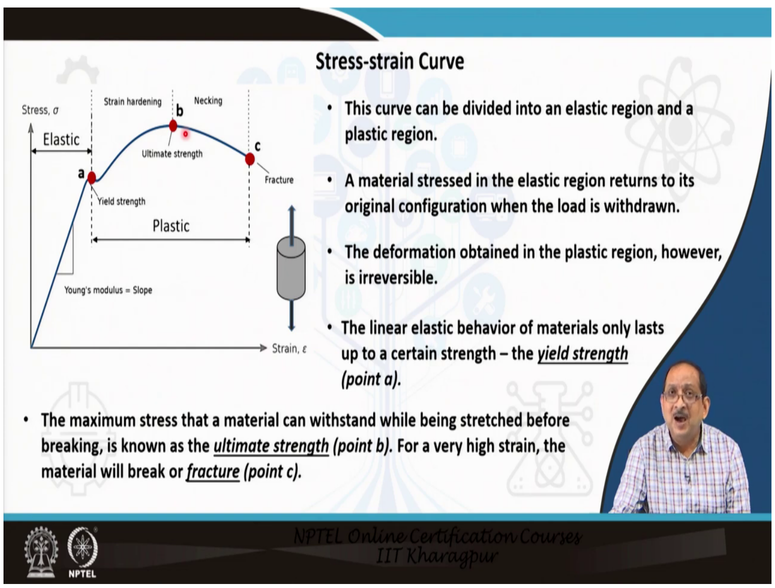

Now, let us discuss about the stress-strain curve. The stress-strain curve can be plotted from the data obtained from a tension test. So, here in we get we plot the stress-strain curve corresponding to a ductile material. So, there are a few important key points in this curve as well as two distinct regions.

The curve can be divided into an elastic region and the plastic region, the material which is stressed in the elastic region returns to its original configuration when the load is withdrawn.

However, the deformation obtained in the plastic region is irreversible. In the plastic region, the material undergoes permanent deformation.

The linear elastic behaviour of material lasts up to a limiting point corresponding to yield strength or yield stress as indicated as point 'a' in the graph. The maximum stress that a material can withstand while being stretched before breaking is known as the ultimate strength indicated by point ‘b’. The material will break or fracture at point ‘c’ as indicated in the graph for an even higher strain.

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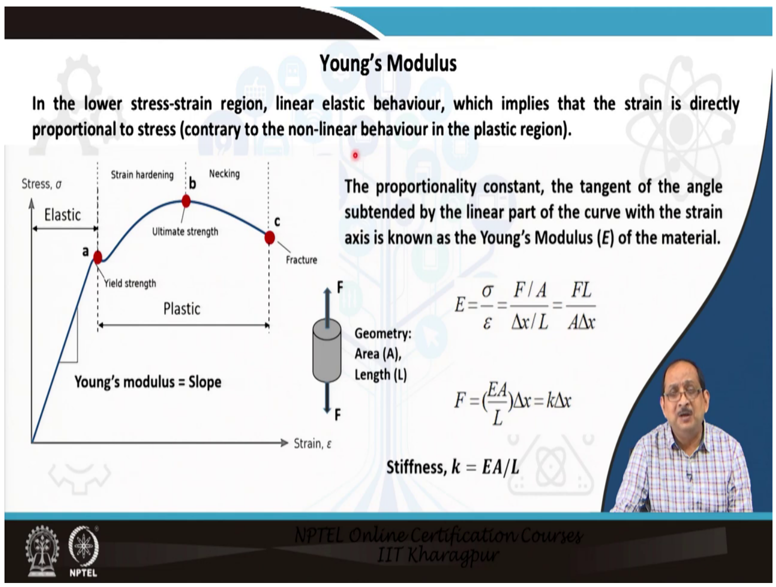

In the lower stress-strain region of the curve, linear elastic behaviour is exhibited, which implies that the strain is directly proportional to the stress contrary to the nonlinear behaviour in the plastic zone. The proportionality constant in this linear elastic zone is the tangent of the angle subtended by the linear part of the curve with the strain axis, and this proportionality constant is known as the Young's modulus of the material.

So, Young's modulus can be related to the Hooke's law and finally, the stiffness of this cylindrical specimen subject to tensile forces can be calculated as

$$K = EA/L$$

So, it is dependent on the geometry of the specimen as well as the Young's modulus.

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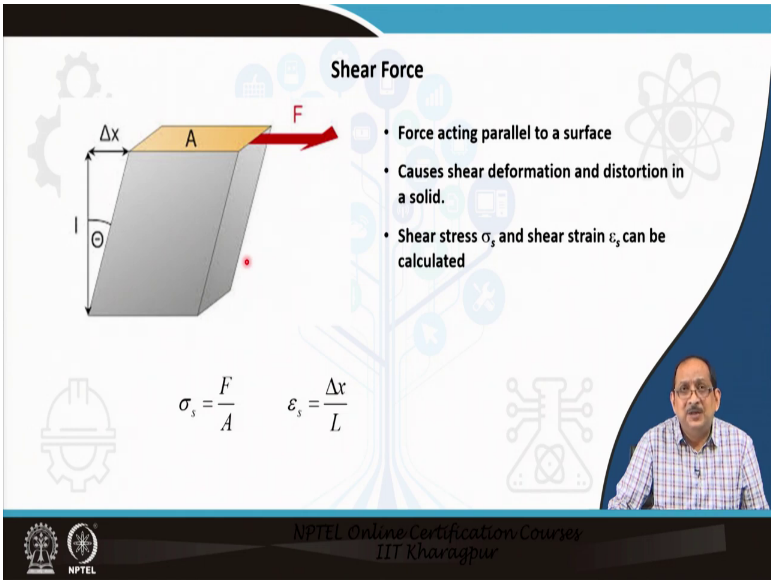

Let us define a shear force that acts parallel to a surface as shown in the figure, and it causes shear deformation and distortion in the solid object. So, this is the deformation and overall, there is a distortion in the shape of the solid object. Accordingly, shear stresses and shear strain can be calculated as indicated here in the slide.

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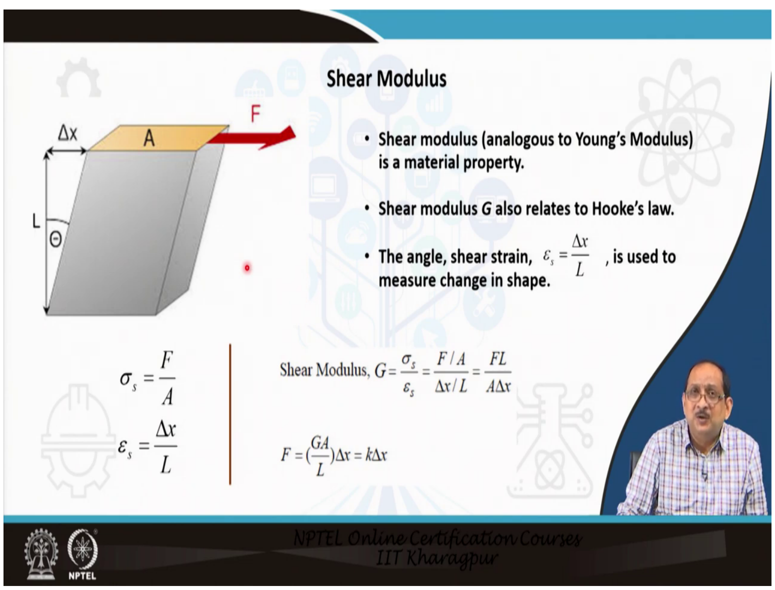

The shear modulus is another modulus analogous to the Young's modulus and it is an essential material property. The shear modulus also relates to Hooke's law; the angle known as the shear strain is used to measure the change in shape. So, the shear modulus can be associated with Hooke's law, and the shear stresses and shearing strain are indicated here in the slide.

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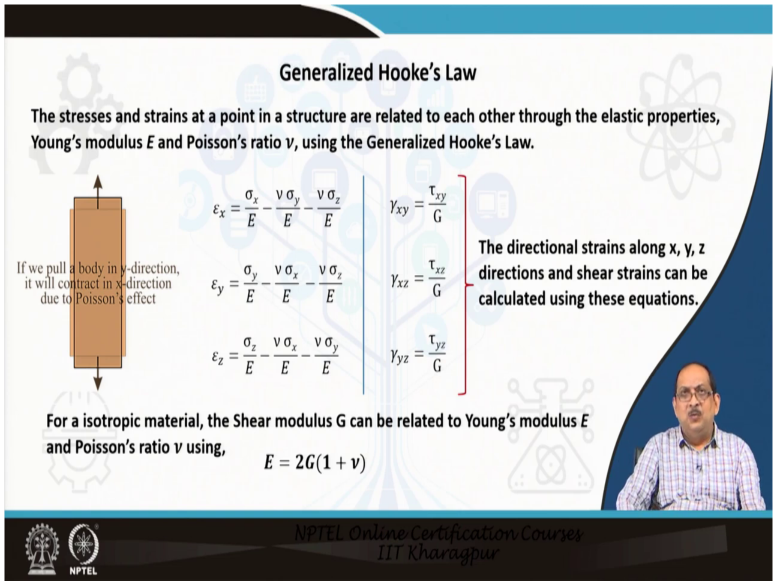

Now, let us consider a more generalized state of loading wherein it is indicated here in the figure that if we pull a body along the y-direction, it will tend to contract in the x-direction due to the poisons ratio effect. So, Poisson's ratio is the lateral contraction to the actual elongation. These stresses and strains at a point in a structure are related through the elastic properties Young's modulus and Poisson's ratio using the generalized law, generalized Hooke's law.

Actually, the strain along one particular direction is not only due to the stress along that specific direction, but also it accounts for the stresses along other orthogonal other two orthogonal directions as indicated in the generalized Hooke's law. So, the directions of the linear strains, as well as the shear strains, can be calculated using the set of 3-6 equations. It is also important to know the relationship between Young's modulus shear modulus and poisons ratio for an isotropic material.

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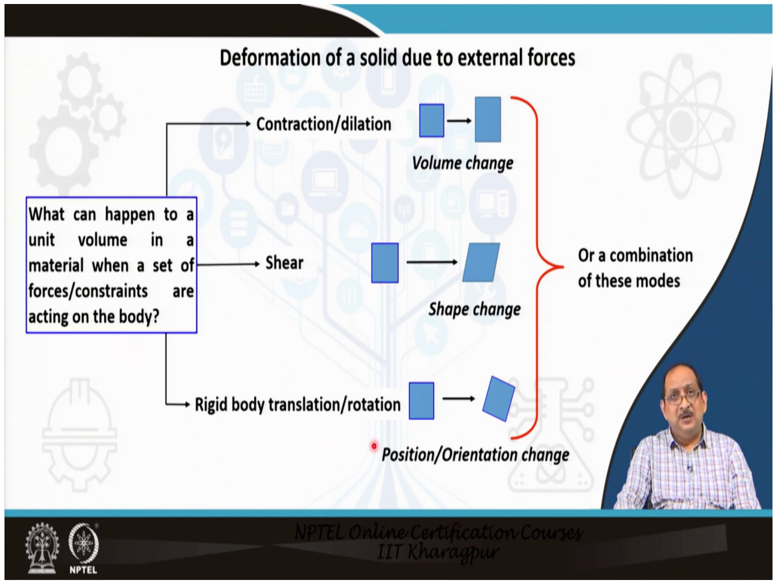

Now, what happens when a body is subjected to a set of forces and constraints in generalized form? We will consider the deformation of the solid body due to the action of the external forces and constraints, it can actually have three consequences; one is the contraction/dilation. So, it can change in its volume, other is due to shear which leads to change in shape, it can translate or rotate that means, the position and orientation can also change, but there is one option one possibility that a combination of all these modes can also happen.

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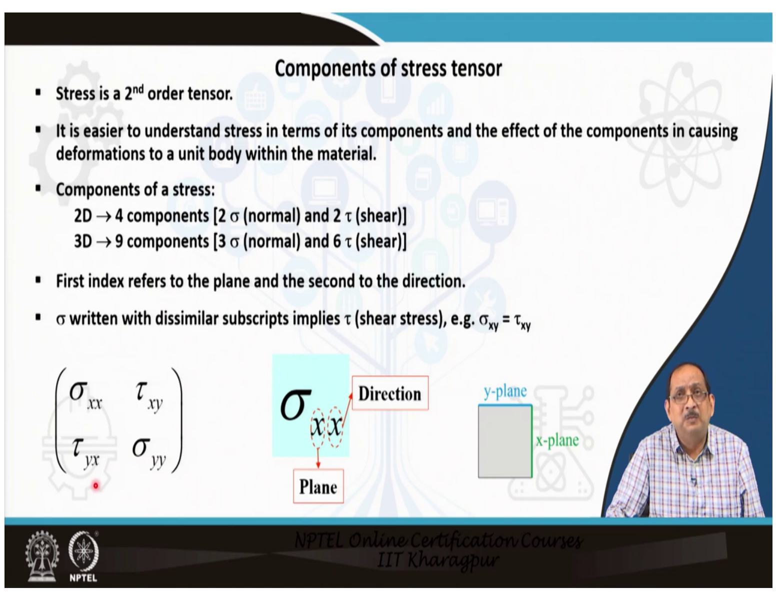

Now, let us, discuss the components of the stress tensor. Stress is a second-order tensor. A zeroth-order tensor is a scalar quantity, a first-order tensor is a vector quantity, and the second-order tensor is, for example, is stress. It is easier to understand stress in terms of its components and the effect of components in causing deformation to a unit body within a material.

For defining stress, as I have indicated earlier, we need to define the orientation of the area on which it acts. The components of stresses in 2D are 4 in total, 2 normal and 2 shear stresses. In 3D, there are nine components; 3 normal and 6 shear stresses. Let us, try to understand the components of a stress tensor in more detail.

So, here as you can see, the first index of the stress component actually indicates or refers to the plane on which it acts. So, here it is σ_x means it is acting on the plane x, and the second is the direction the second x basically indicates that it is acting along the x-direction.

If σ is written in the similar subscript, it has actually refers to shear stresses. So, σ_{xy} means τ_{xy}; because shear stresses are symbolized as τ. So, in a 2D stress component, the 2D stress components are σ_{xx} and σ_{yy} the normal stresses and shear stresses τ_{xy} and τ_{yx}, which are complementary shear stresses.

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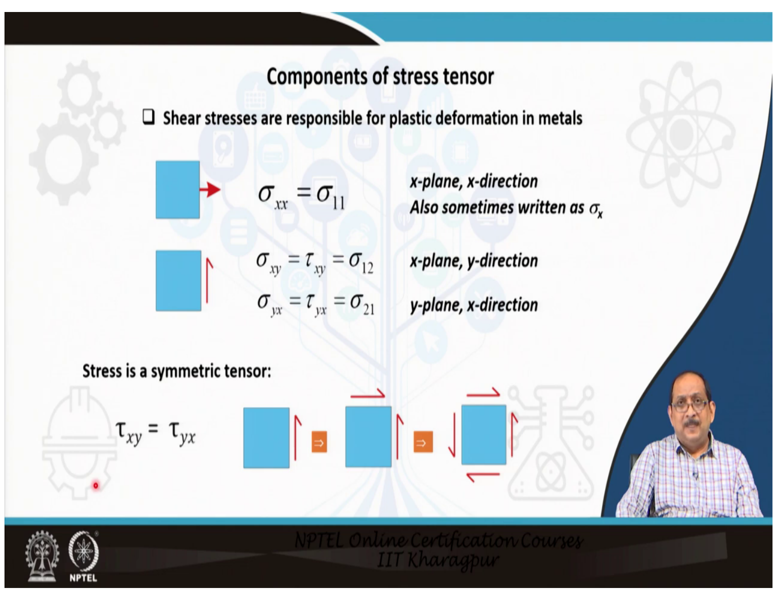

Now, shear stresses are responsible for plastic deformation in metals. Here also we can see that we have represented σ_{xx} or σ_{11}. It means it is acting on the x plane and along the x-direction whereas, τ_{xy} or σ_{12} means it is shear stress τ_{yx} is acting on the plane y but along the x-direction. Now, the stress is considered as a systematic tensor basically according to the principle of conservation of angular momentum, the stress is a systematic tensor wherein the shear stresses are complimentary. So, τ_{xy} is equal to τ_{yx}.

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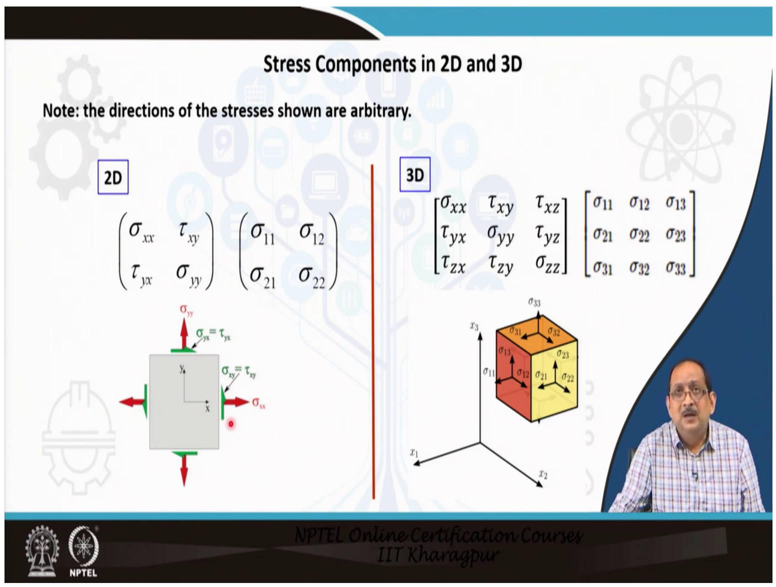

The stresses stress components in 2D and 3D systems are indicated in the slide and pictorially presented to understand each of the stress components in the tensor.

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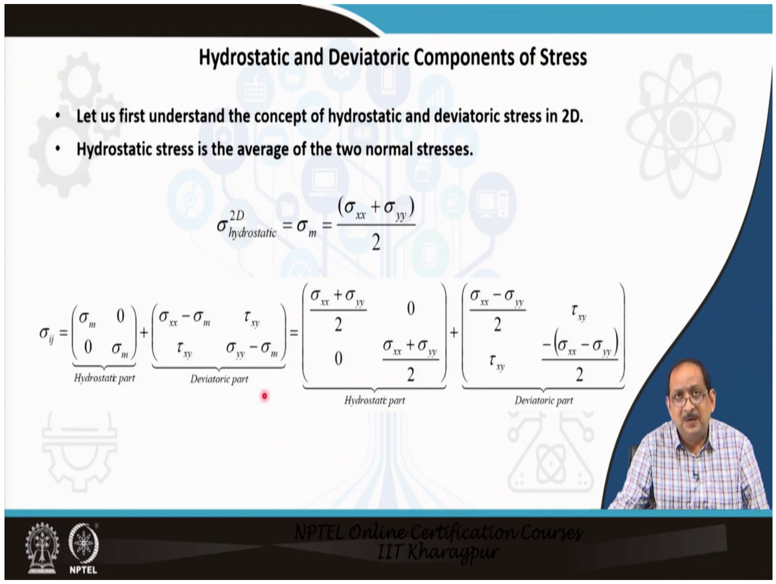

Now, the stress components can be classified as hydrostatic and deviatoric components of stress. Let us first understand the concept of hydrostatic and deviatoric stresses in a 2D system. The hydrostatic stress is basically the average of the two normal stresses as indicated here in the mathematical expression is the average of the two normal stresses. The state of stress actually can be expressed in such a way that we can obtain a hydrostatic part, and we can obtain a deviatoric part.

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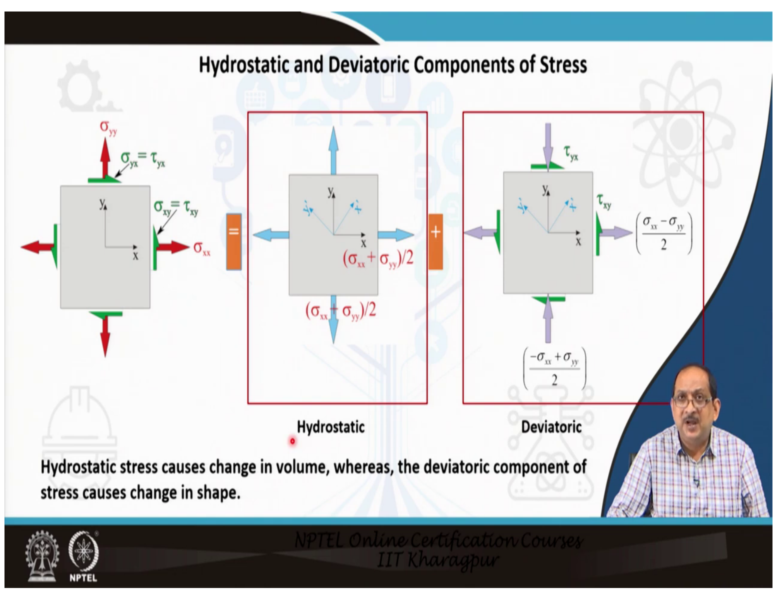

Diagrammatically, the state of stress can be represented as a hydrostatic part and a deviatoric part. The hydrostatic stress causes a change in volume, whereas the deviatoric component of stress causes a change in shape.

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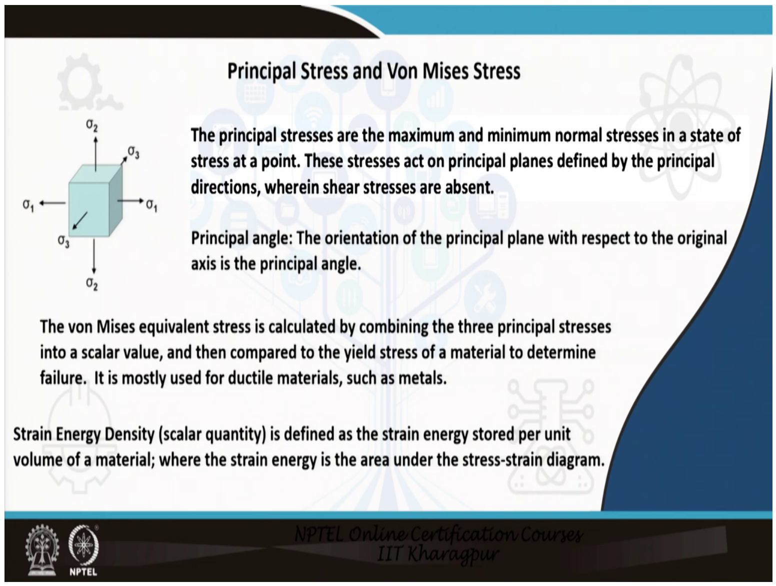

Now, let us discuss the principal stress and some other scalar quantities that are useful in biomechanics subject. The principal stress acts on the principal planes defined by the specific direction, wherein shear stresses are 0 or absent. The principal angle is the orientation of the principal plane with respect to the original axis.

The von Mises equivalent stress is a useful scalar quantity calculated by combining the three principal stresses. Thereafter, this value is compared with the yield stress of a material to determine failure. It is mostly useful for ductile material such as metal. Another important scalar quantity, strain energy density, is the strain energy stored per unit volume of a material where the strain energy is the area under the stress-strain curve.

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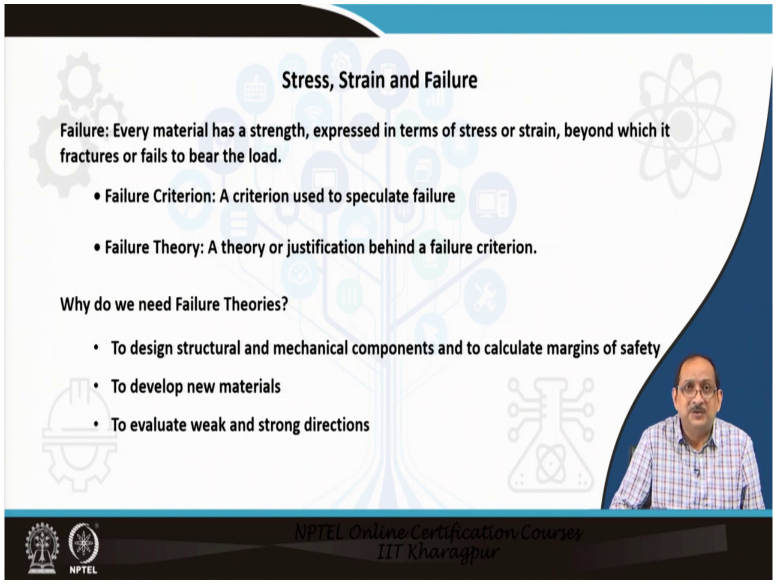

Now, what is the relation between stresses strain and failure? This needs to be explicitly defined. In every material or every material has a strength expressed in terms of stresses or strain beyond which the material fractures or fails to be at the load. So, we need actually a failure criteria to speculate failure and a theory known as failure theory to justify the failure criteria. Why do we need failure theories? To design structural and mechanical components and to calculate the margins of safety. We also need to develop new materials and to evaluate weak and strong directions.

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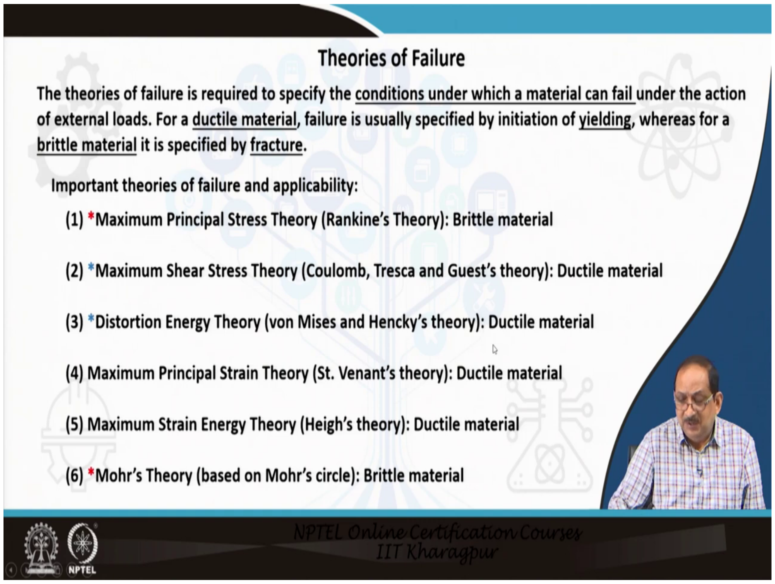

Failure theories are required to specify the conditions under which a material can fail under the action of a set of external loading conditions. For a ductile material, failure is usually defined by the initiation of yielding whereas, for a brittle material, it is specified by fracture. Some important theories of failure and their applicability is listed here.

The first is the maximum principal stress theory or Rankine's theory which is applicable for brittle material, maximum shear stress theory, popularly known as Tresca theory is suitable for ductile material, distortion energy theory or von Mises theory is applicable for ductile material, maximum principal strain theory is also useful for ductile material, maximum strain energy theory Heigh's theory is suitable for ductile material as well, Mohr's theory which is based on the Mohr circle is applicable to the brittle material.

So, within this set of 6 theories, maximum principal stress theory and Mohr's theory are popularly applicable for brittle materials and maximum shear stress theory and distortion energy theory are appropriate for ductile material.

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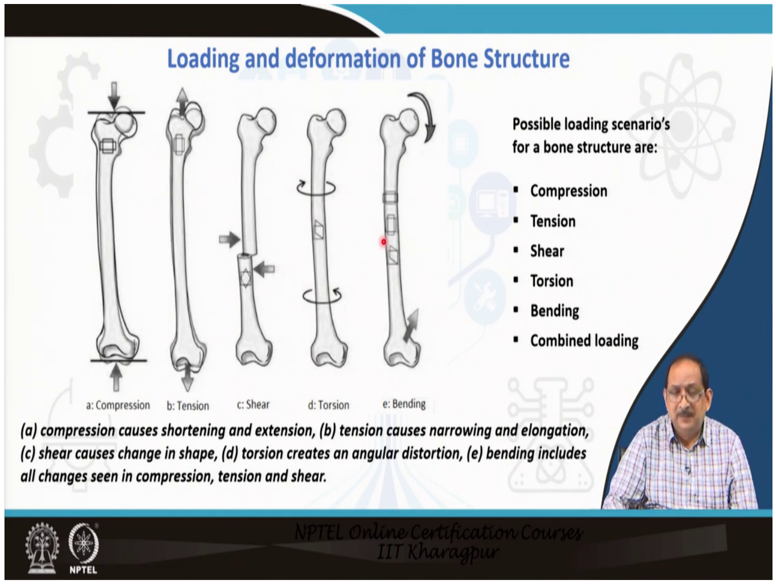

Now, let us consider the load possible loading scenarios for bone structure and the consequences of this loading scenario on the deformation of the bone structure. So, the loading probable loading scenarios are compression as indicated here tension, compression, tension, shear, torsion, bending, and finally, a combination of all of them acting on a bone structure is a common occurrence.

So, if we consider compression, you can see that we have indicated small stress elements; how the stress element is deformed is shown in this diagram. So, compression can cause shortening and extension, and tension can cause narrowing and elongation. Shear can cause a change in shape, a torsion can lead to angular distortion, and a bending mode can include all changes seen in compression, tension and shear.

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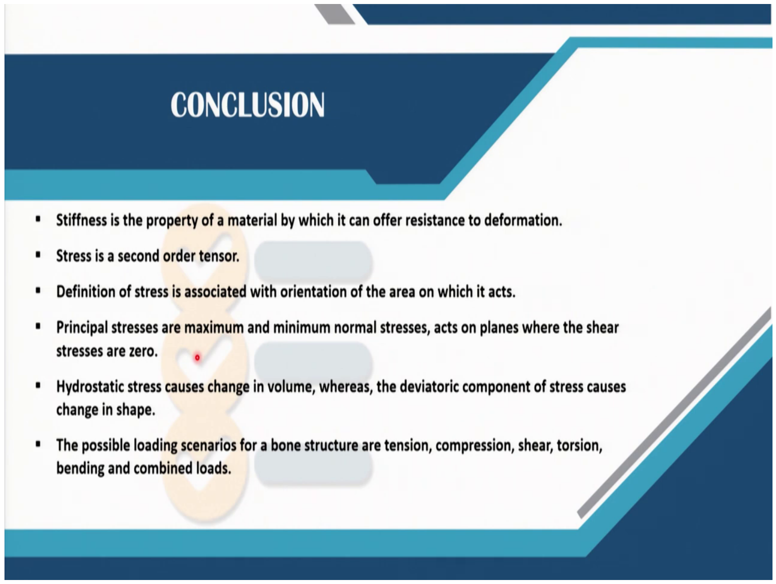

So, let us list the conclusion of this lecture. Stiffness is the property of a material by which it can offer resistance to deformation. Stress is a second-order tensor, which requires the definition of the orientation of the area on which it acts. Principal stresses are maximum and minimum normal stresses, and it acts on planes where the shear stresses as 0.

Hydrostatic stress causes a change in volume whereas, the deviatoric component of stress causes a change in shape. The possible loading scenarios for a bone structure are tension, compression, shear, torsion, bending and combined loading, considering this basic loading configuration.

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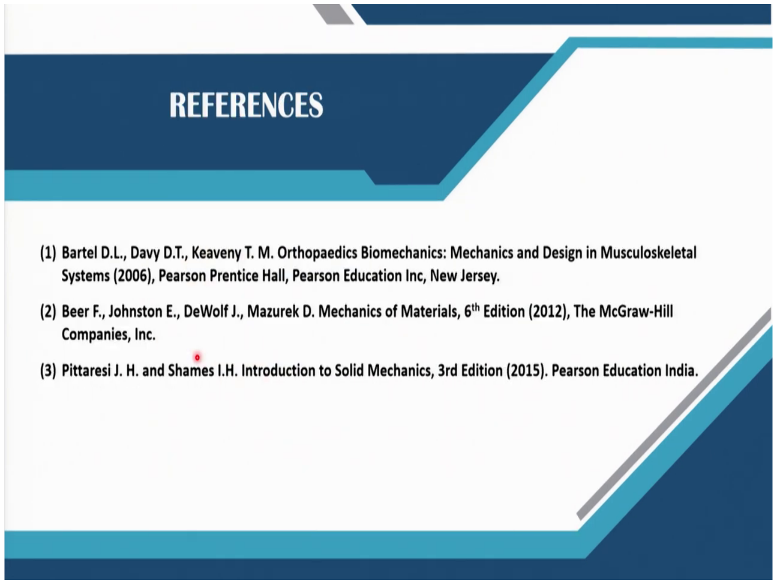

The references are listed here. These are very popular textbooks, and I like to thank you all for listening.