Biomechanics of Joints and Orthopaedic Implants
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Lecture 19
Fundamentals of Joint Kinematics

Good morning, everybody. Welcome to the 4th module of the NPTEL Online Certification Course on Biomechanics of Joints and Orthopedic Implants. The first lecture of module 4 is on Fundamentals of Joint Kinematics.

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In this lecture, we will discuss the concepts of rigid body dynamics, particularly the force acceleration relationships, joint kinematics, and postural stability.

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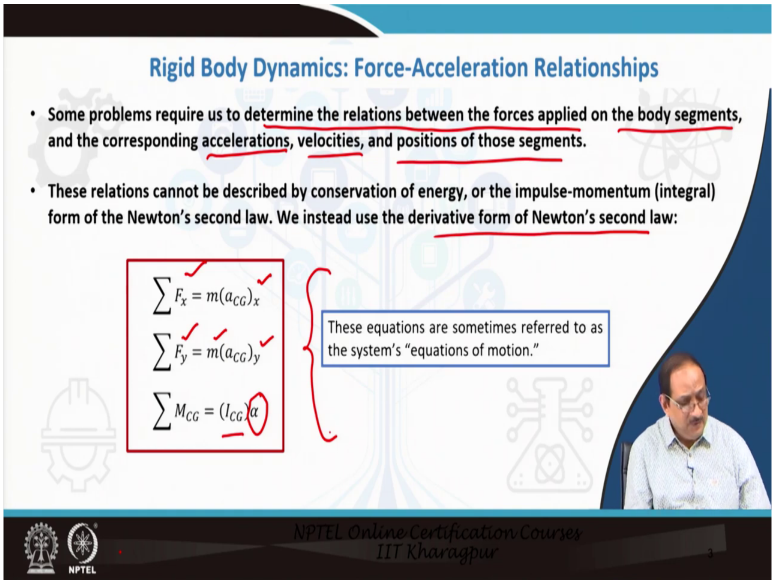

This module deals with joint kinematics, which is based on the concepts of rigid body dynamics. Some problems require us to determine the relationships between forces applied on the body segments. And the corresponding accelerations, velocities and positions of those segments. These relationships cannot be described by conservation of energy or the impulse-momentum equation of the Newton's second law.

We instead use the derivative form of Newton's second law. So, the forces applied along the x-direction, y-direction and the moment can be expressed in the form as written in the slide. So, the force along the x-direction produces an acceleration along the x-direction on the mass of the body segment m. Similarly, the force y along the y-direction has an acceleration along the y-direction for the mass m.

Now, to calculate the moment, we need the moment of inertia and the angular acceleration. So, the moment actually is equal to the moment of inertia and multiplied by the angular acceleration. So, these equations are sometimes referred to as the equations of motion of Newton's second law.

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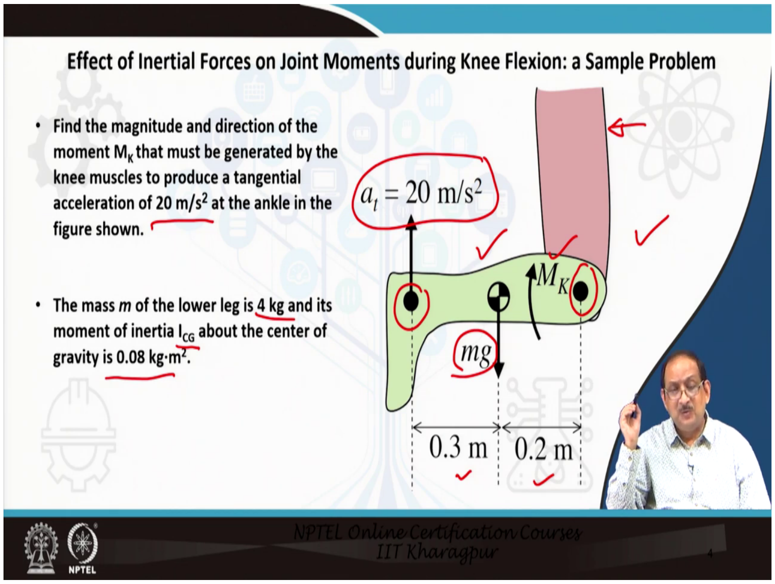

Now, let us look into a sample problem to demonstrate the effect of inertial forces on joint moments during knee flexion. The figure represents the lower extremity. Here, we have the knee joint, the ankle joint, and of course, the green part is the lower part of the leg, and the upper part of the leg thigh is indicated here. We need to find out the magnitude and direction of the moment M_k.

In the figure shown, the knee muscles must generate the moment to produce a tangential acceleration of 20 meters per second square at the ankle. So, it is a position that can be described as knee flexion, flexion of the knees. The mass m of the lower leg is given as 4kg and the moment of inertia I about the center of gravity, I_{CG} is given by the value as indicated in the slide. The dimensions of the center of gravity of the lower limb from the knee joint center and the ankle joint center is also marked here in the figure.

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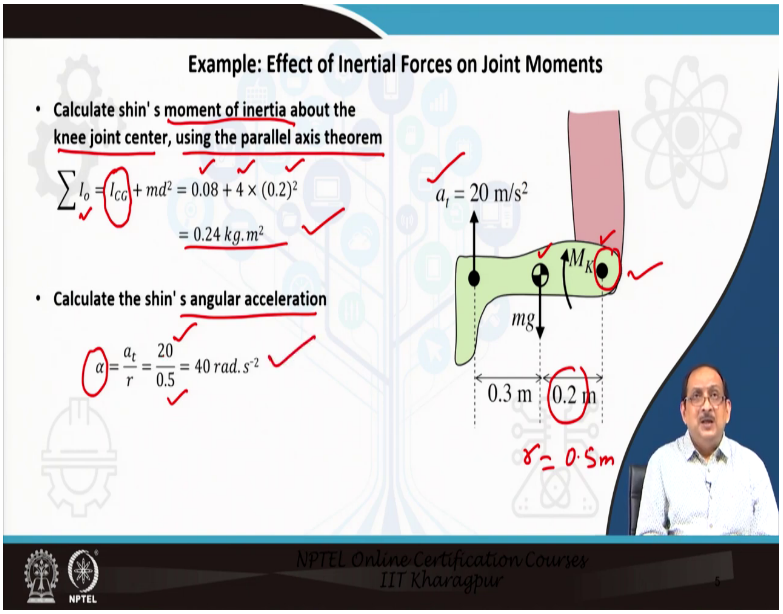

Let us now calculate the shin's moment of inertia about the knee joint center. So, first step is to calculate the moment of inertia about the knee joint center using the parallel axis theorem. So, the moment of inertia about the center of gravity or center of mass of the lower leg portion is given but we need to find out the moment of inertia about the knee joint center. So, using the parallel axis theorem;

Moment of inertia about the knee joint center, $\sum I_o = I_{CG} + md^2$

Shin's angular acceleration, $a = a_t / r$

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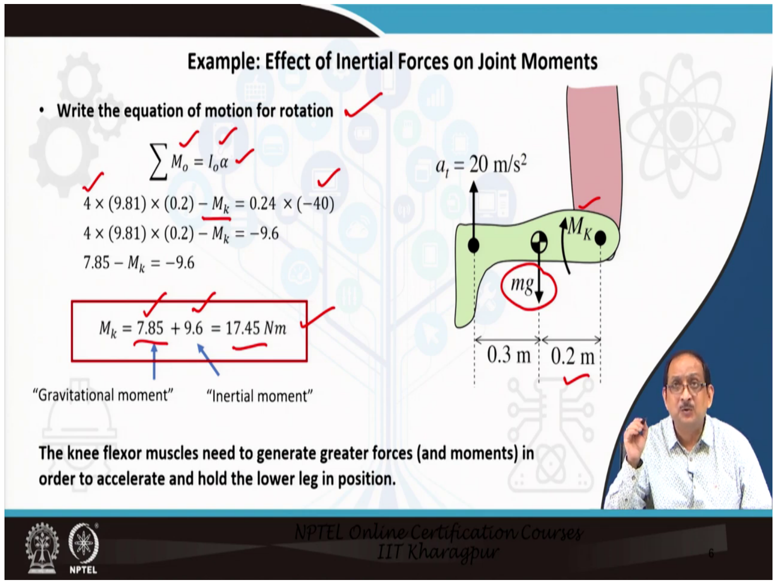

Now, let us write the equation of motion for rotation for this problem. So, summation of the moments about the knee joint center Mo is equal to the moment of inertia about that point O and the angular acceleration. So, on the left-hand side of the equation we write the moment due to the gravitational force 4kg mass acting through a distance of 0.2 will give you the gravitational moments.

And this moment is acting anti clockwise whereas the moment M_k is acting clockwise as indicated in the figure. So, these two on the left-hand side is equal to the Iα. Moment of inertia, I was calculated earlier as 0.24 and the angular acceleration as 40 rad/s^2. So, after performing the calculations, we find out the total moment required at the knee joint to be 17.45, out of which the gravitational moment is 7.85 and the inertial moment is 9.6.

$$\sum M_o = I_o \alpha$$

$$4 \times (9.81) \times (0.2) - M_k = 0.24 \times -40$$

$$M_k = 17.45 \text{ Nm}$$

So, when the knee is in a dynamic condition or the lower leg is rather in a dynamic condition due to knee flexion. We see that the total moment is more than double of the moment produced by only the gravitational force. So, the dynamic effect due to the inertial moment contributes to increasing the moment to more than double of the gravitational moment.

So, 17.45 is more than double of 7.85. So, the knee flexor muscles need to generate greater forces and moments to accelerate the lower leg and hold the lower leg in position. This is an important conclusion of this problem.

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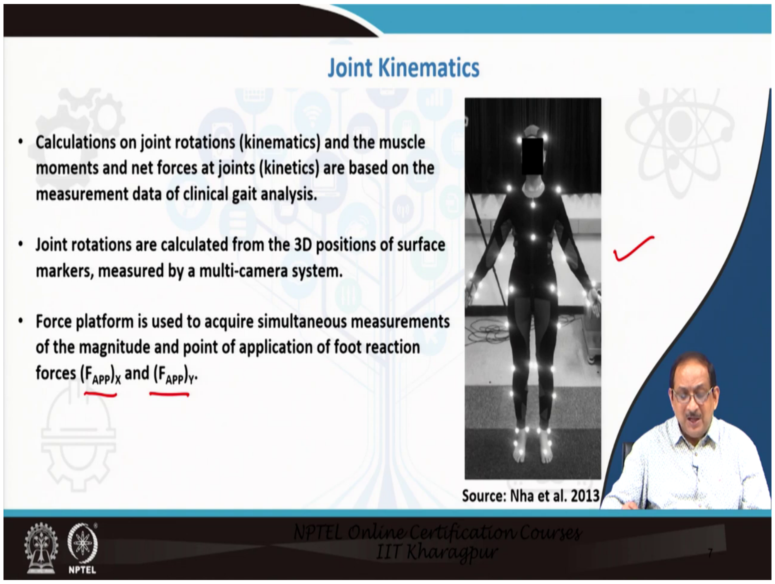

Now, let us move to the second topic that is the joint kinematics. Calculation on joint rotations, kinematics and the muscle moments and net forces at the joint. Joint forces are based on measurement data of clinical gait analysis. A figure on clinical gait analysis is indicated here in the slide.

Joint rotations are calculated from the 3D positions of the surface markers measured by a multi-camera system. We have discussed in detail the 3D motion capture system used in gait analysis earlier.

The force platform is used to acquire simultaneous measurements of the magnitude and point of application of foot reaction forces f along x-direction and y-direction if it is a 2D problem. These inputs and body segment position, velocities, and accelerations must be fed into an inverse dynamics approach method to determine joint forces and moments.

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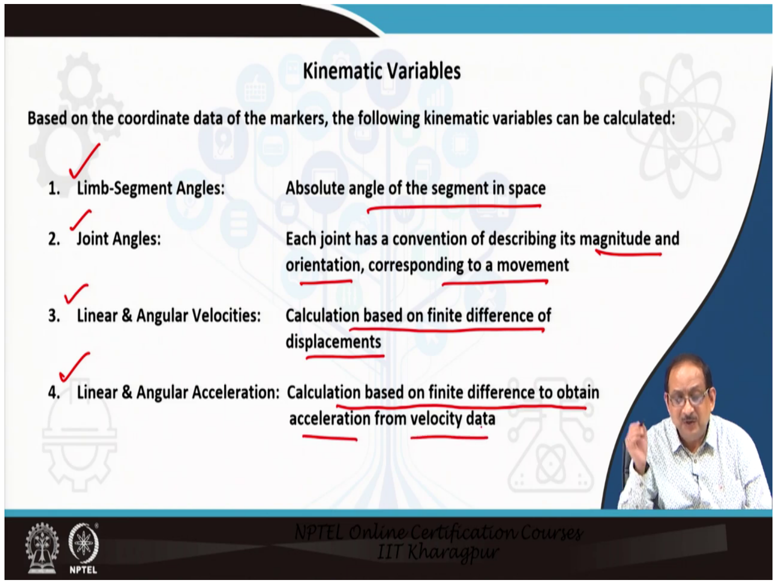

Now, based on the coordinate data of the markers as obtained from a gait analysis system, the following kinematic variables can be calculated. The link segment angle is the absolute angle of the segment in space. The joint angles so each joint has a convention of describing its magnitude and orientation corresponding to a movement. For example, if we extend the knee fully, the flexion angle or the joint angle is considered to be 0-degree flexion.

The linear and angular velocities, so these can be obtained from a calculation based on finite-difference displacements of, these can be obtained from a calculation based on finite-difference of displacements. And the 4th one, the linear and angular acceleration calculation are based on finite differences to obtain acceleration from the velocity data.

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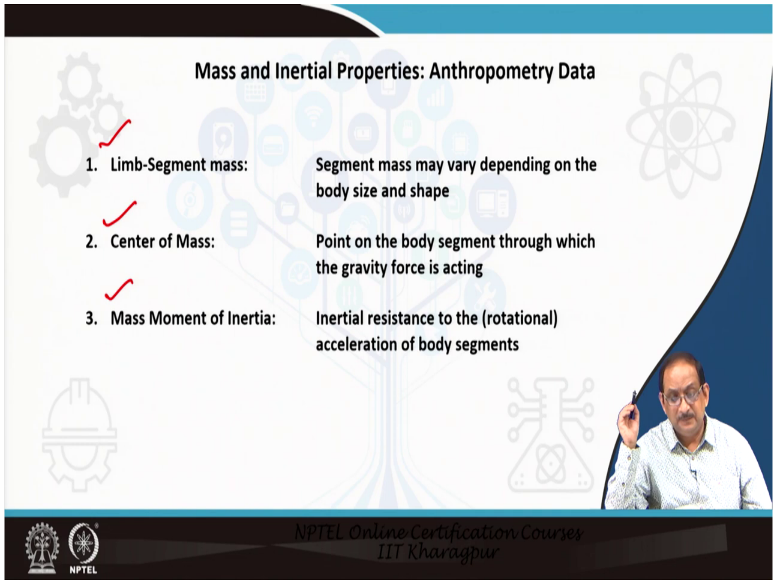

The mass and inertial properties are obtained from anthropometry data. Anthropometry data as defined earlier, also consists of the length and mass property of each body segment. So, we need the limb segment mass, the center of mass of each body segment, and the mass moment of inertia, which is the internal resistance to the rotational acceleration of the body segment.

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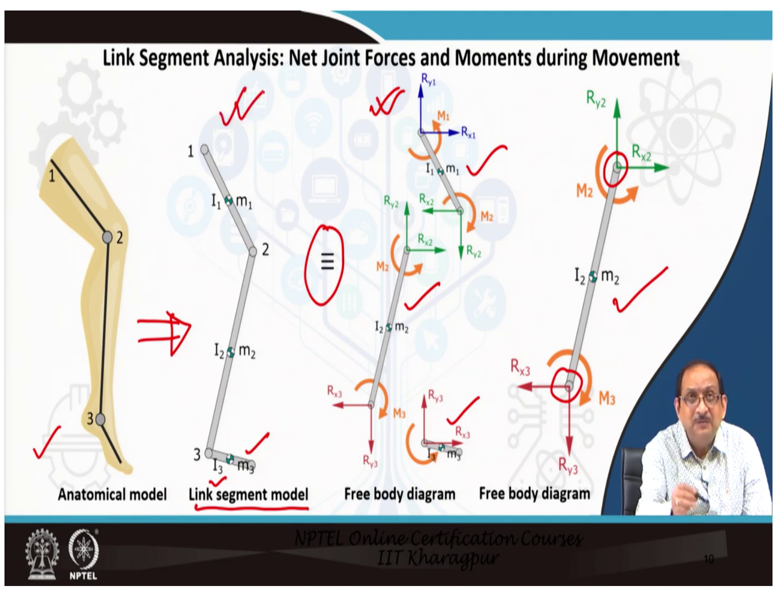

Now, let us come to a very important slide the link segment analysis to determine net joint forces and moments during a movement. In this slide on the left we see an anatomical segment of the body of leg represented with rigid elements. The mathematical model representing this anatomical model is known as the link segment model, which is very important. You can see that segment mass characterizes each segment in the whole link segment model.

Now, the link segment forces and moments could be calculated using equations of motions and an inverse dynamic approach if we consider a segment free body diagram. You can see that the segment is separated at the joints. At these joints, there would be reaction forces and moments acting at each joint as indicated in the figure for each segment.

When we assemble these three segments, we can get back the total link segment model.

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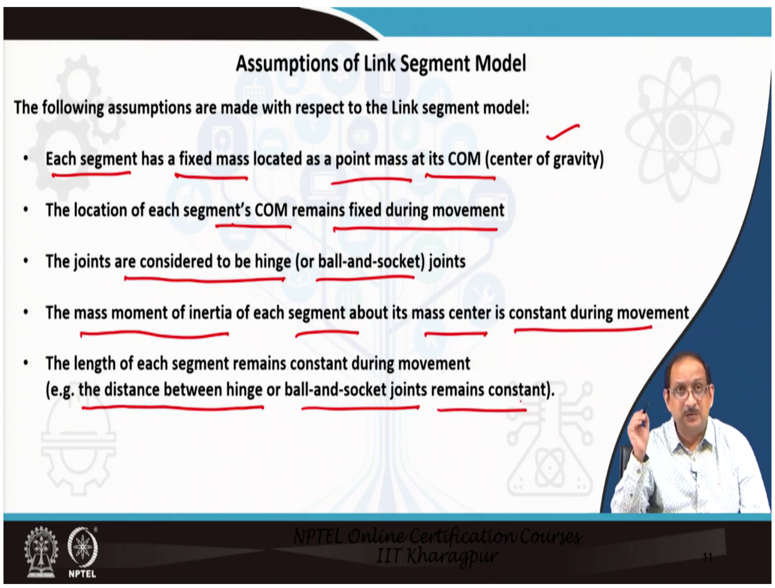

Let us list down the assumptions of the link segment model. The following assumptions are made with respect to the link segment model. The each segment has a fixed mass as a point mass located at its center of mass or center of gravity. The location of the center of each segment of mass remains fixed during movement and the joints are considered to be hinge or ball and socket joints.

The mass moment of inertia of each segment about its mass center is constant and it is also assumed to be constant during the movement. The length of each segment remains constant during movement, i.e., the distance between the hinge or ball and socket joint remains constant.

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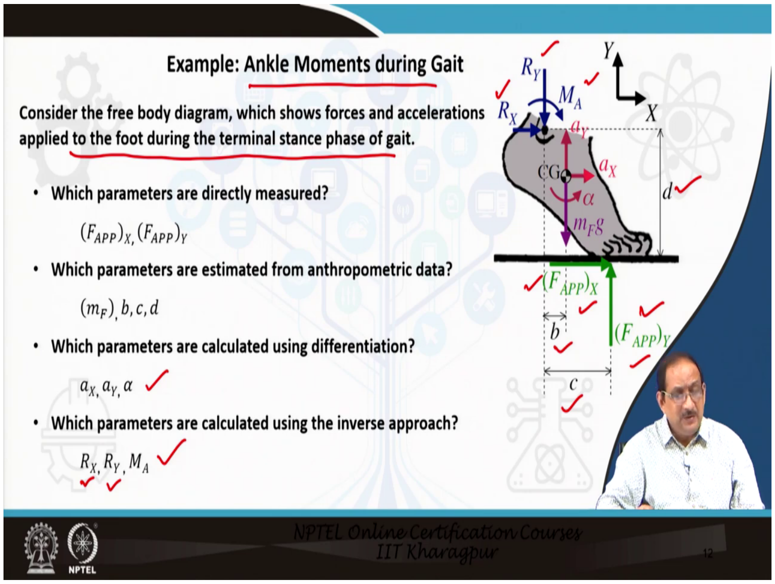

Let us consider the free body diagram, which shows forces and acceleration applied to the foot during the terminal stance phase of a gait cycle. So, we are interested finally to determine the ankle moments during this gait instant. So, we have the ground reaction forces X and Y component, we need to determine ankle joint reaction forces R_X, R_Y and the moment M_A.

But before we move on to determine the expressions for R_X, R_Y and M_A, I have a few questions to all of you. The first question is which parameters are directly measured? The answer is of course the horizontal and vertical components of the ground reaction force as indicated in the figure. The next question is which parameters are estimated from the anthropometric data: mass and inertial properties.

So, mass of the foot segment and the length b, c and d. Which parameters are calculated using differentiation? The X component of the acceleration, the Y component of the acceleration and the angular acceleration; all three can be calculated using differentiation. Note that the limb movement is measured as displacement, both linear and angular using a motion capture system.

Thereafter, the differentiation of these parameters will give you velocities and accelerations. Which parameters are calculated using the inverse approach? Those are the final joint forces and moments R_X, R_Y and M_A .

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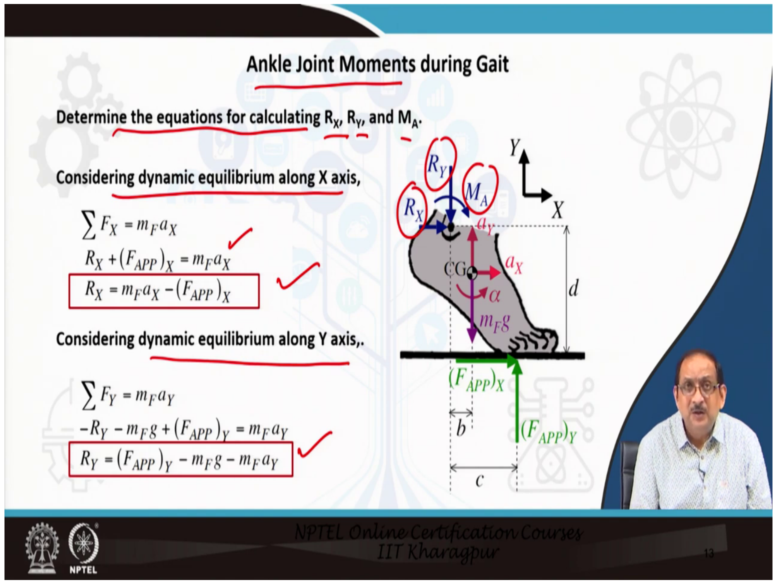

Now, let us move on with the calculations on the ankle joint forces and moments. So, determine the equations for calculating R_x, R_y and M_a as already indicated earlier considering the same free body diagram. So, if we consider dynamic equilibrium along the X-axis. We can write down the summation of the forces along the X axis equal to m multiplied by the acceleration along the X axis.

So, we can write down R_x is equal to the m_F into a_x that is the acceleration along the x axis minus the force measure ground reaction force component along X-axis. Similarly, considering the dynamic equilibrium along Y-axis, we can determine the expression for the joint reaction force along the Y-axis.

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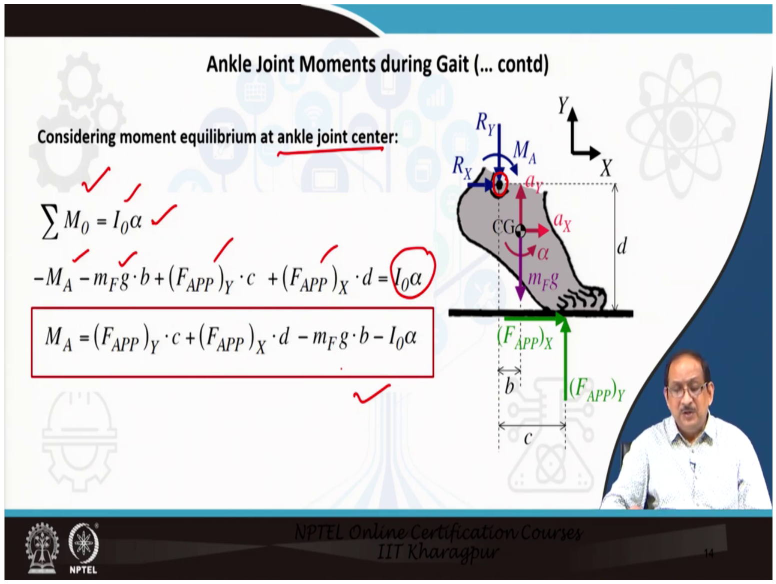

Considering moment equilibrium at the ankle joint center which is here. We can write down the moment equation considering rotation that is sum of the moments equal to Iα. So, I is the moment of inertia of the segment about the joint center of course and the angular acceleration, α.

So, if we take a moment about this joint center; the left-hand side is actually the moments due to the forces the X and Y component of the ground reaction forces the moment due to the gravitational weight of the segment and of course we have the M_A which is the moment about the joint center that we need to find out. On the right-hand side of the equation we have the $I_o\alpha$. So, we can easily find out now the M_a in terms of the ground reaction forces, the weight of the limb segment, and the moment of inertia, and the angular acceleration.

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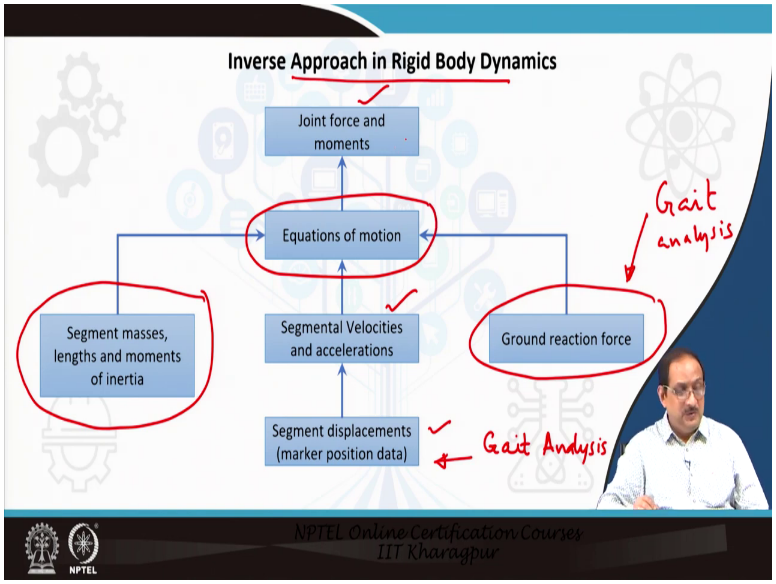

Let me summarize the inverse approach used in rigid body dynamics to find out joint forces and moments. Now, we can actually start from the segmental displacements that are obtained from the marker position data from the gait analysis. So, this is coming from gait analysis, from this segmental displacements we can calculate velocities and acceleration. And then using the equations of motion along with inputs from the segmental mass lengths and moment of inertia that is the anthropometric data.

And the ground reaction force data which is also coming from the gait analysis. So, together with the ground reaction force data and the anthropometric data on segmental mass lengths and moment of inertia, we can use the equations of motion to find out joint forces and moments.

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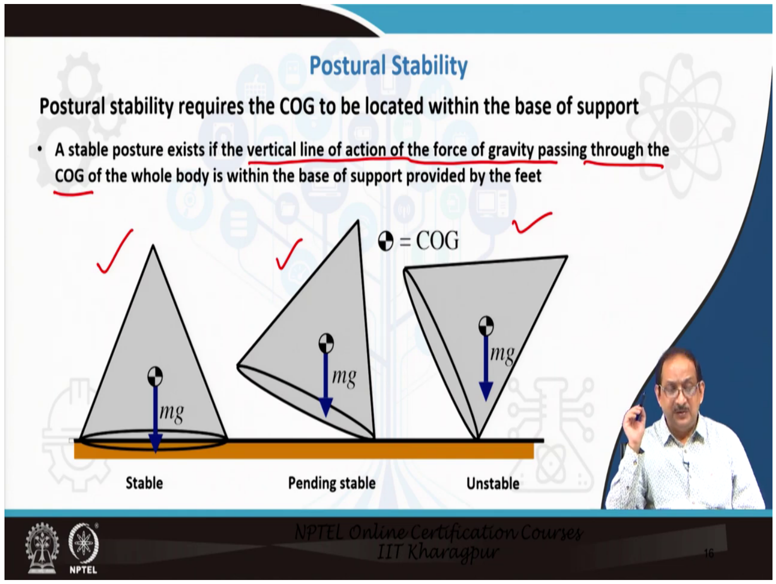

Let us come to the third topic of this lecture which is postural stability. Postural stability requires the center of gravity to be located within the base of support. So, in the figure presented here you can see the base of support of a cone is circular. The base area of support is basically circular. So, a stable posture exists if the vertical line of action of the force of gravity passing through the center of gravity of the whole body is located within the base of support provided by the feet.

So, the position of the feet is important to provide the base of support which eventually offer stability to the posture. As you can see, if we actually tilt the cone, the base of support is actually reducing. And therefore, the stability is compromised.

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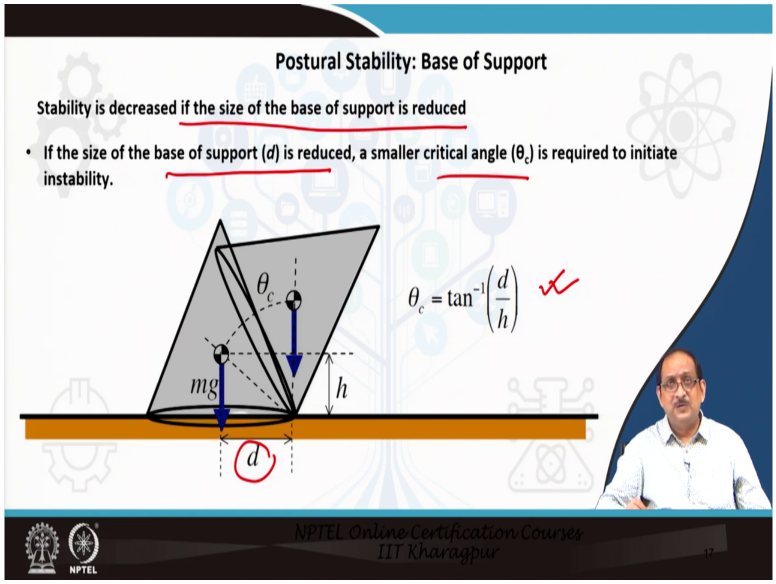

The stability is decreased if the size of the base of support is reduced. Now, we can see that there is an angle which is a critical angle. So, if the dimension of the base of support, d is reduced as seen here in the figure. A smaller critical angle will be required to initiate instability. So, this critical angle can be calculated from geometry as

$$\theta = \tan^{-1}(d/h)$$

Where h is the height of the center of gravity of the cone. So, the stability is decreased if the size of the base of support is reduced.

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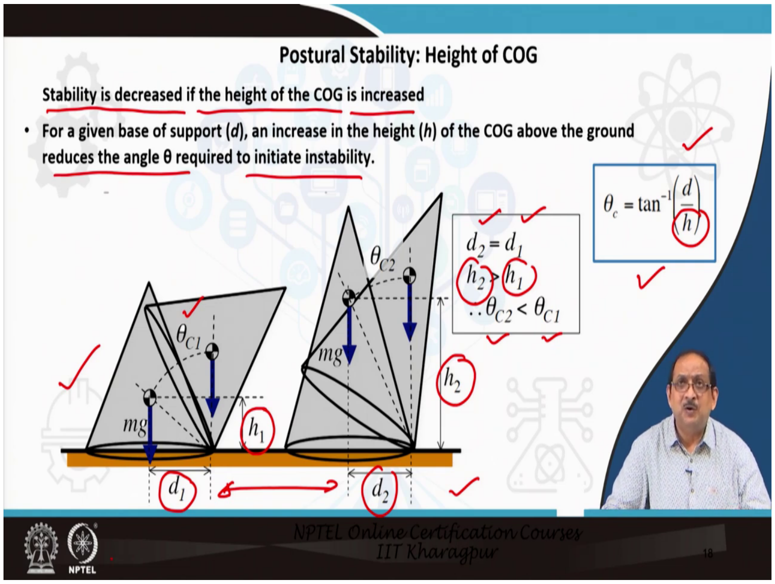

Let us discuss the influence of height of center of gravity on the postural stability. Stability is decreased if the height of the COG center of gravity is increased. On the left we consider a body with base of support d. Consider the case in which d_1 is base dimension and h_1 is the height of the COG from the ground. Now, we had earlier calculated the critical angle to initiate instability as θ_{c1}. And generally, the θ_c expression is given here as

$\theta_c = \tan^{-1}(d/h)$; where d is the base of the support and h is the height of the center of gravity.

Now, for a given base of support let us concentrate on the second figure as I indicated in the slide for a given base of support d and increase in the height from h_1 to h_2. So, the increase in the height of the COG above the ground actually reduces the angle θ required to initiate stability. So, if d_1 and d_2 are same d_1 equal to d_2 for these two cases.

But if h_2 is greater than h_1 then using the mathematical expression for the critical angle to initiate instability, we can find out that θ_{c2} will be less than θ_{c1} because h_2 is greater than h_1. Therefore, a smaller angle is necessary to initiate instability when the height of the COG of the body is increased.

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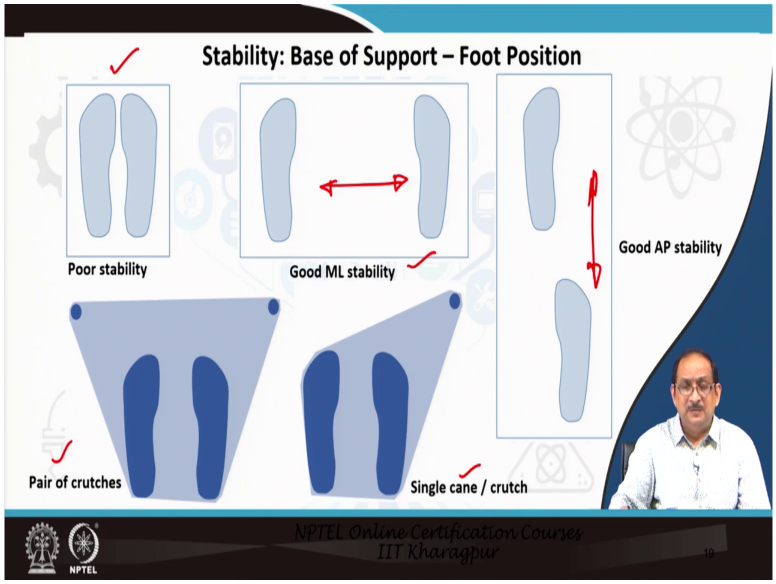

Let us now consider the foot position which provides the base of support and stability. Now, in this image the foot position is indicated looking from the top. So, it is a top view of the foot positions when the two feet are very close to each other it actually is offering poor stability. So, the first position where the two feet are close to each other is an example of a poor stability.

When the two feet are apart it actually offers good medial lateral stability. Now, if the two feet are separated along the anterior posterior direction it gives good stability along the anterior posterior direction. Now, if we take the help of other supporting structures. Say for example, a single cane or crutch or a pair of crutches we see that the area of the base of support actually increases thereby providing better stability.

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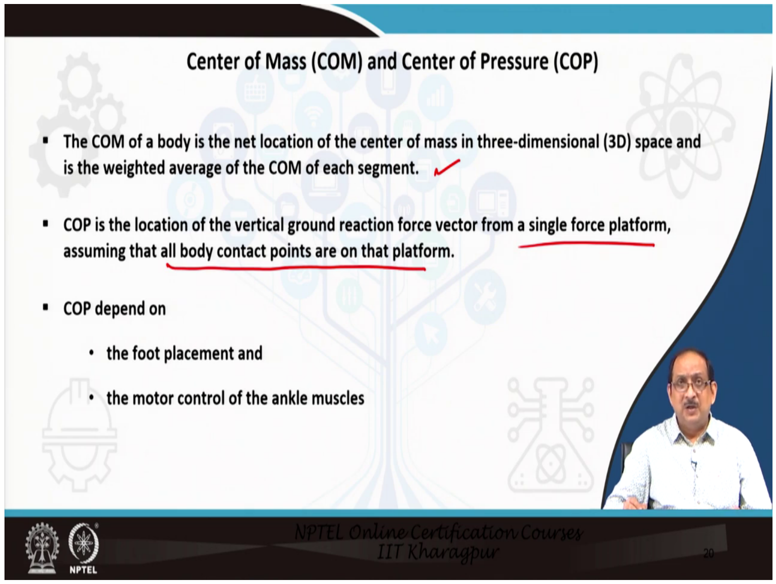

It is important to know a few features of the center of mass and center of pressure and the relationship between the two in joint kinematics and postural stability. The center of mass of a body is the net location of the center of mass in 3 dimensional space and is the weighted average of the center of mass of each segment. The center of pressure is the location of the vertical ground reaction force vector from a single force platform assuming that all body contact points are on that platform. The COP depends on the foot placement and the motor control of the ankle muscles.

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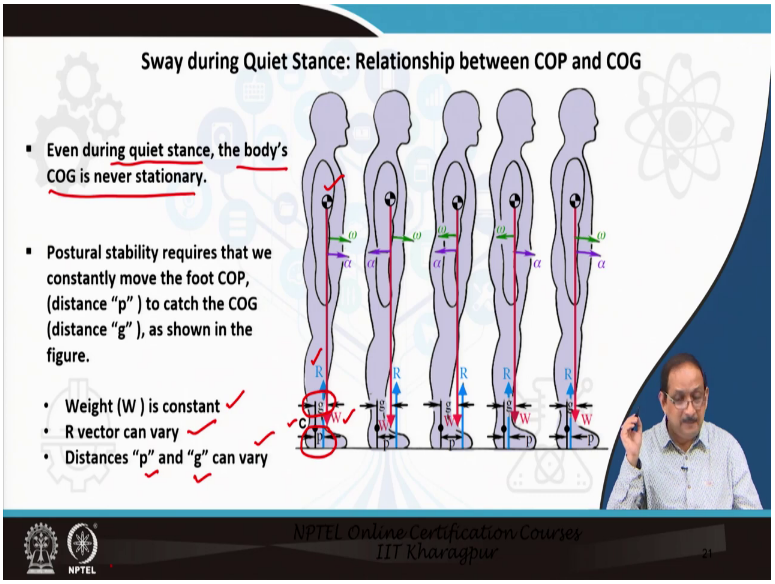

Let us discuss swaying during quiet stance and gain insight into the relationship between COP (center of pressure) and the center of gravity, COG. The figure presented here shows a subject swaying back and forth while standing on both feet on a force platform. Even during the quiet stance when we are actually standing on both feet the body's center of gravity is never stationary. So, let us look into the mechanics involved in the system.

So, first we have to focus our attention on the line of action of the body weight, W and the ground reaction force R. So, the body weight is acting through the center of gravity, COG. The ground reaction force R is acting through the center of pressure or COP. These lines of action of the forces W and R are located at a distance from the ankle joint. So, the line of action R is located at a distance or lever arm p.

Whereas the distance g as indicated in the figure is the lever arm for the weight vector about the knee ankle joint center. The ankle joint center C is indicated by the black circle in the figure. The postural stability actually requires that we constantly move the foot COP through the distance p to catch the COG acting through the distance g as shown in the figure. Please note that the weight is a constant vector whereas the R vector (the ground reaction force vector) can actually vary. And the distances p and g can also vary.

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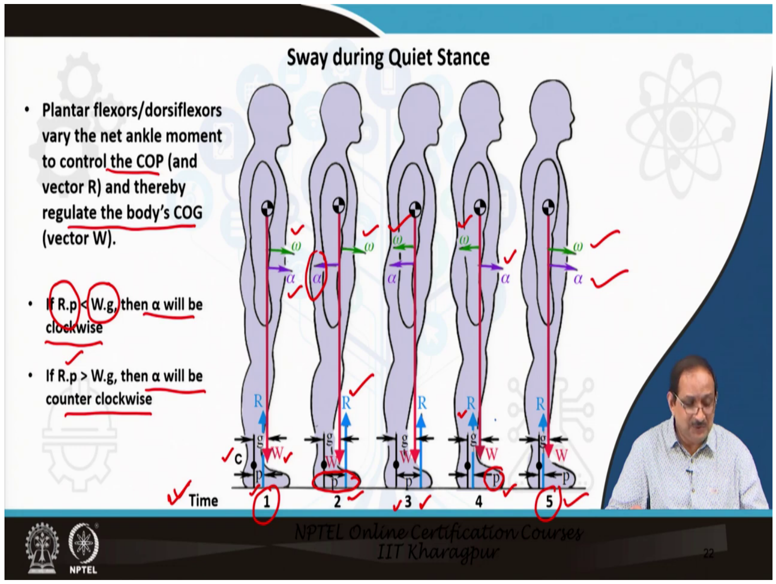

Now, swaying during quiet stance is primarily due to the activity of the plantar flexor and dorsiflexor muscles. The plantar flexor and dorsiflexor can vary the ankle moment to control the COP and of course the vector, the ground reaction vector R. And thereby regulate the body's COG through which the vector W is acting. Now, let us gain an insight into the mechanics involved in the whole swaying phenomenon.

Now, it should be noted first that there are two moments, one is the moment created by the weight that is W g. The other is the moment R p and these moments are about the ankle joint center C as indicated in the figure. Now, depending on the magnitude of W g and the magnitude of moment R p the movement will be created. Now, if W g is greater than R p then the body will tend to move forward. The body will tend to move in a clockwise direction.

So, the angular acceleration would be in the clockwise direction. Whereas if R p is greater than W g, then the angular acceleration would be counterclockwise. Now, let us consider the time instant 1, 2, 3, 4, and 5 and discuss more about the mechanics involved in the system the body's COG through which the weight vector is acting ahead of the COP.

So, the g distance is greater than the p distance and the moment W g will be greater than R p. And the body will experience a clockwise angular acceleration α which will induce angular velocity in the clockwise direction. In the second time instant in order to correct this forward imbalance. The tendency of the body to move forward as indicated in the time instant 1.

In order to correct for this forward imbalance, the subject will increase its plantar flexor muscle activity that is the ankle push down on the floor, which will actually increase the COP. So, it will increase the R vector as well as the distance p through which it acts. So, this will cause the R vector to be anterior to W as indicated in the figure time instant 2. So, R vector is anterior to W and now the moment R p will be greater than W g.

Therefore, the angular acceleration will be reversed and this will start to decrease the angular velocity until the time instant 3 after which the angular velocity will be reversed as well. So, at time instant 3 both angular velocity and angular accelerations are counterclockwise. The body at this time instant will be experiencing a backward sway as indicated in the figure time instant 3. Now, the subjects response to the backward sway is to decrease his COP.

And the R vector by reduced activation of the plantar flexor muscles thereby reducing the R vector and the lever arm p. Now, the moment at time instant 4 the moment W g will be greater than R p and the angular acceleration will reverse. And after a time period the angular velocity will decrease and finally the direction of the angular velocity will be also reversed and the body will actually return to its original condition as indicated in time instant 5.

So, if you compare time instant 5 with time instant 1 you can see that the direction of the angular velocity and angular acceleration is same. So, in summary we can say that the COP and the COG is playing a cat and mouse game.

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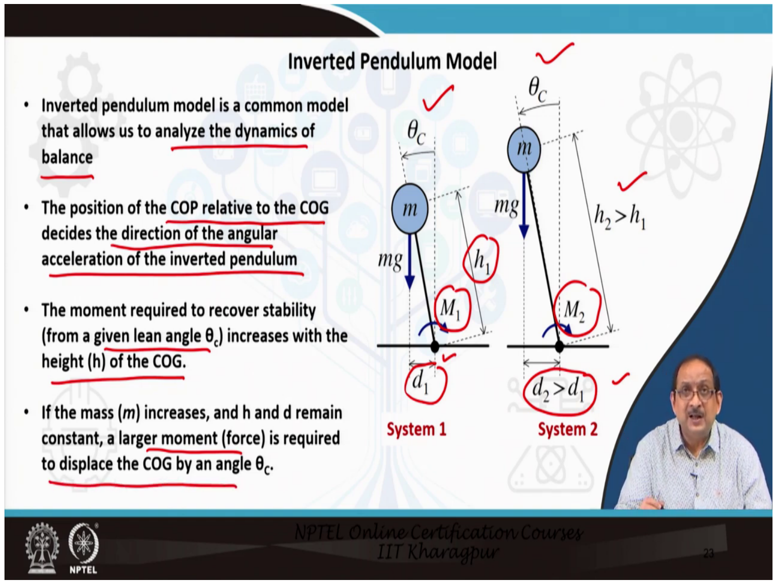

Let us consider an inverted pendulum model to analyze the dynamic of the balance. An inverted pendulum model is a common model that allow us to analyze the dynamics of balance. Now, in this slide we have presented two systems. So, one system as you can see has a mass mg and is connected to a point. The point of rotation the other mass is having the height of the COG greater than the first system h_1.

Both the masses are same and both the systems as presented here has the same θ_c in the configuration. Now, the position of the center of pressure relative to the COG the center of gravity decides the direction of the angular acceleration of the inverted pendulum. Now, in this figure presented here, the moment required to stabilize the system M_2 in system 2 is greater than the moment required to stabilize the system in 1. So, M_2 is greater than M_1. Why is that?

Because if h_2 is greater than h_1 then from the geometry on the right is for system 2 you can easily find out that d_2 will be greater than d_1. Hence the moment M_2 will be greater than M_1. Therefore, with the help of the diagram, we can understand that the moment required to recover stability from a given lean angle θ_c as stated in the configuration increases with the height of the center of gravity.

Furthermore, if we keep the height and the d constant but increase the mass m then obviously a larger moment and a larger force is required to displace the center of gravity by an angle θ_c.

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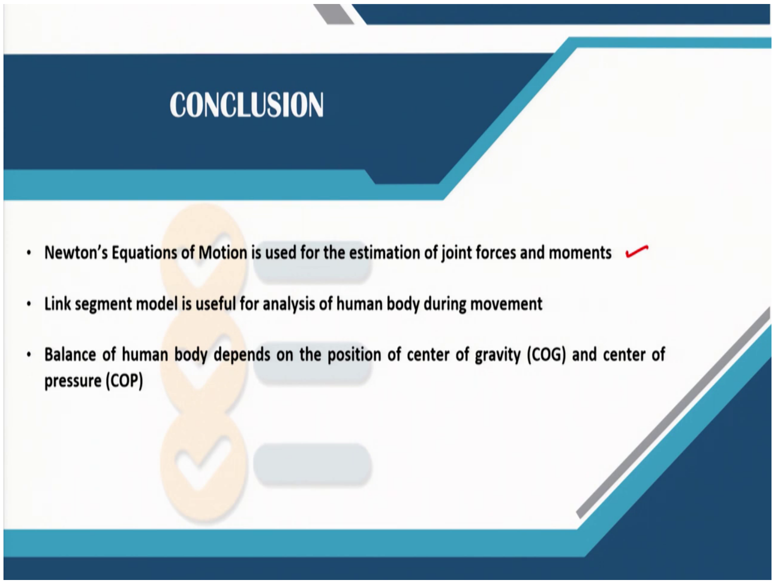

Let us come to the conclusions of this lecture. Newton's equations of motion is used for estimation of joint forces and moments. The link segment model is useful for analysis of human body during movement. Balance of human body depends on the position of center of gravity and center of pressure.

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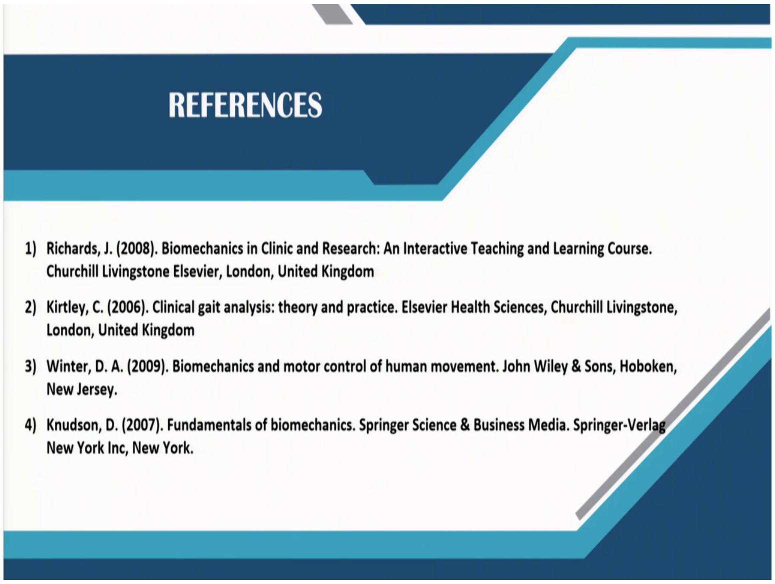

The list of references are indicated here in the slide based on which the lecture has been prepared. Thank you for listening.