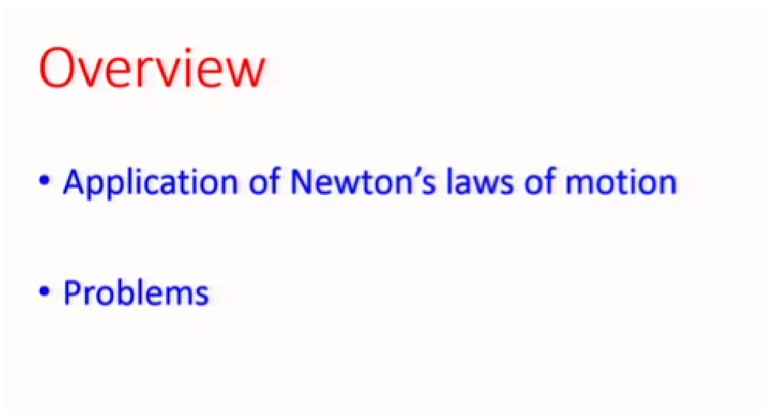
Advanced Dynamics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

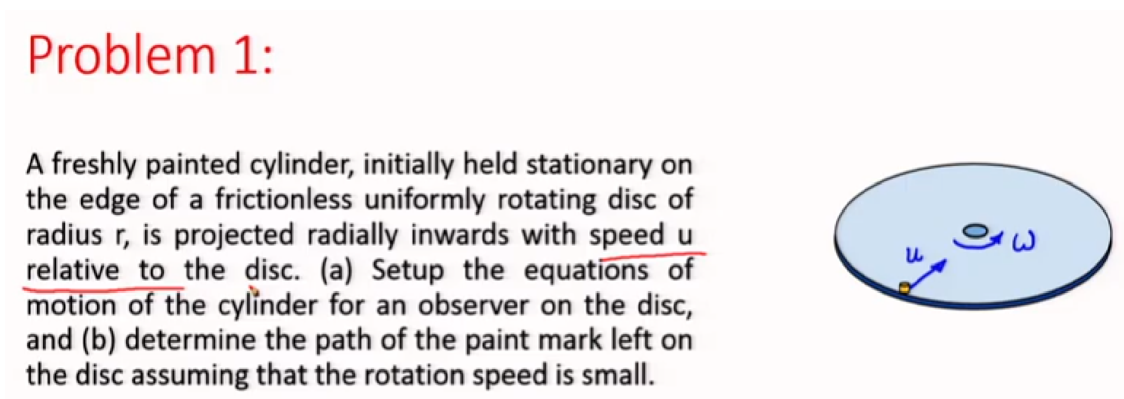
Module No # 02
Lecture No # 09
Particle Kinetics – IV

We will continue our discussions on kinetics of particles, and in this lecture I am going to show you how our analysis can be used to understand certain natural phenomena.

(Refer Slide Time: 00:34)

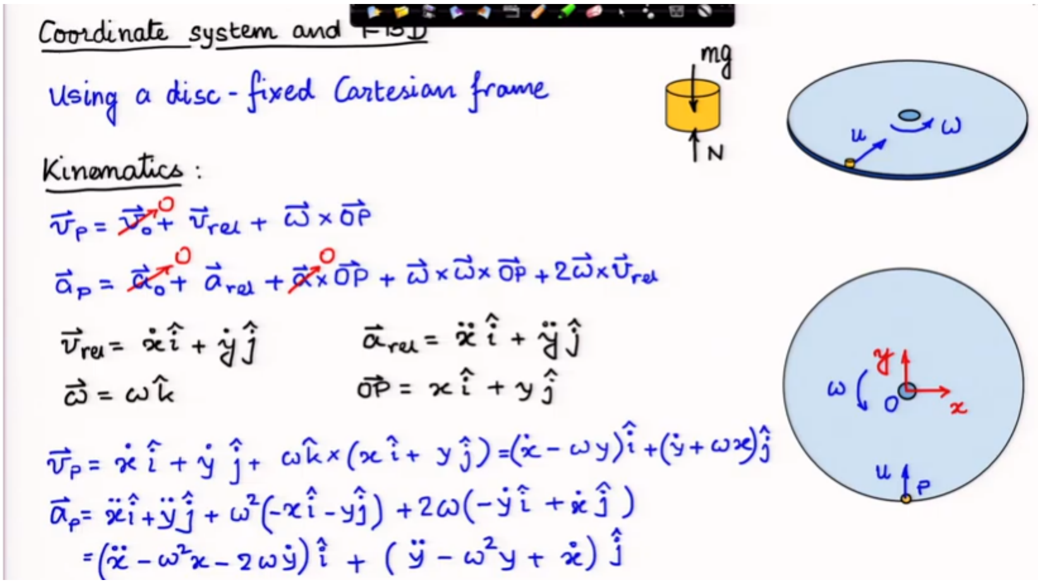


(Refer Slide Time: 00:42)



We start with the above problem. Later, I am going to discuss a natural phenomenon that can be understood from this problem.

(Refer Slide Time: 01:49)



The first step is to setup the coordinate system. I am going to use the Cartesian coordinate system which I will fix to the disk, as shown above. The disc is rotating in a counter clock wise direction as seen from the top. For simplicity, I am considering that the particle P is initially on the negative y axis at y=-r, and projected along the positive y axis direction towards the center.

The velocity and acceleration vectors of the particle expressed in the chosen (rotating) frame are obtained as follows

$$\vec{v}_P = \cancel{\vec{v}_O}^{\,0} + \vec{v}_{rel} + \vec{\omega} \times \vec{OP}$$

$$\vec{a}_P = \cancel{\vec{a}_O}^{\,0} + \vec{a}_{rel} + \cancel{\vec{\alpha}}^{\,0} \times \vec{OP} + \vec{\omega} \times \vec{\omega} \times \vec{OP} + 2\vec{\omega} \times \vec{v}_{rel}$$

$$\vec{v}_{rel} = \dot{x}\hat{i} + \dot{y}\hat{j} \qquad \vec{a}_{rel} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

$$\vec{\omega} = \omega\hat{k} \qquad \vec{OP} = x\hat{i} + y\hat{j}$$

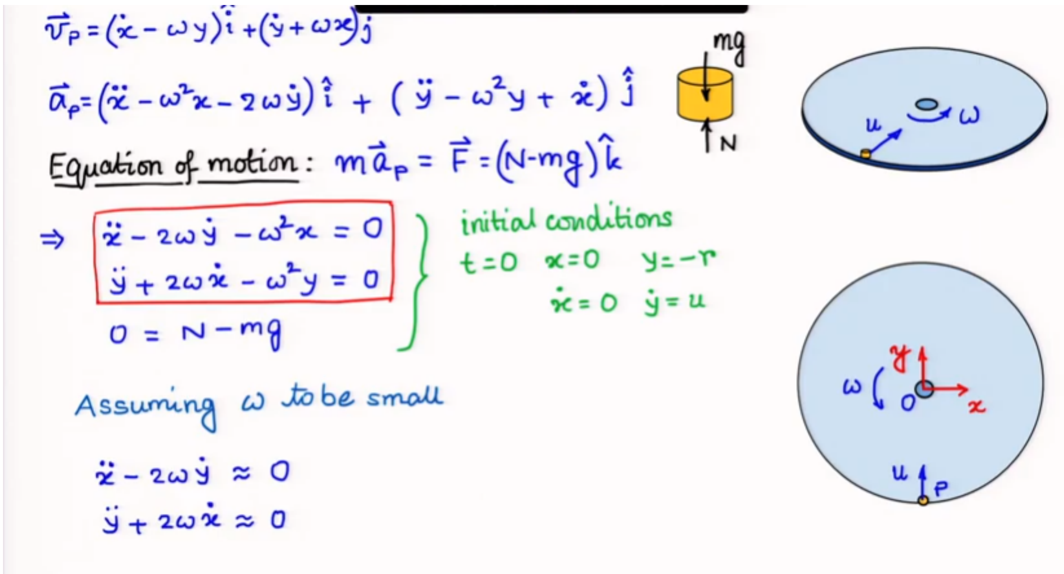
$$\vec{v}_P = \dot{x}\hat{i} + \dot{y}\hat{j} + \omega\hat{k} \times (x\hat{i} + y\hat{j}) = (\dot{x} - \omega y)\hat{i} + (\dot{y} + \omega x)\hat{j}$$

$$\vec{a}_P = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \omega^2(-x\hat{i} - y\hat{j}) + 2\omega(-\dot{y}\hat{i} + \dot{x}\hat{j})$$

$$= (\ddot{x} - \omega^2 x - 2\omega\dot{y})\hat{i} + (\ddot{y} - \omega^2 y + \dot{x})\hat{j}$$

Here, it is assumed that the disc is rotating at a constant angular speed ω.

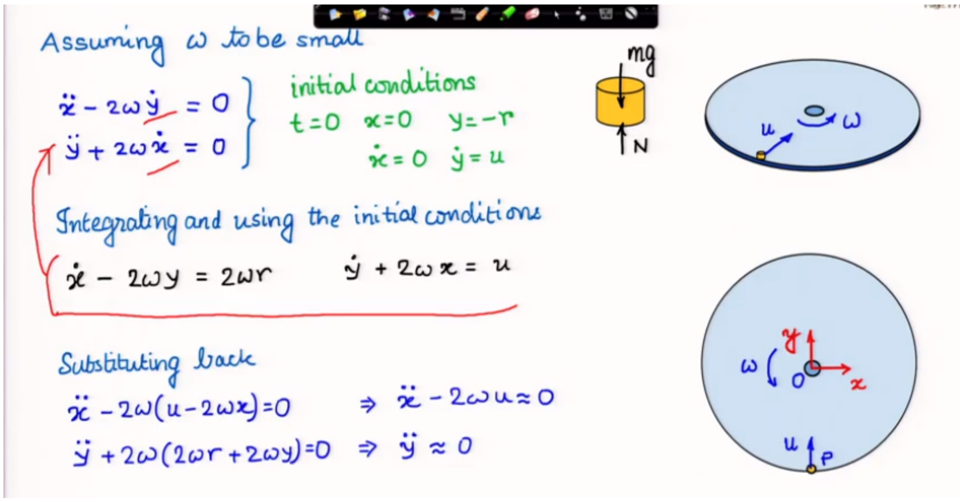
(Refer Slide Time: 05:40)



Using the expression of acceleration in Newton's 2nd law, and neglecting friction, we arrive at the equation of motion as shown above. This is a linear equation and can be solved exactly, though the solution is involved. Here, we will make an approximation in the view of our later discussions in this lecture. We consider that the contribution of the terms involving ω^2 is small, and hence drop them. This leads us to the simplified equations

$$\ddot{x} - 2\omega\dot{y} \approx 0$$
$$\ddot{y} + 2\omega\dot{x} \approx 0$$

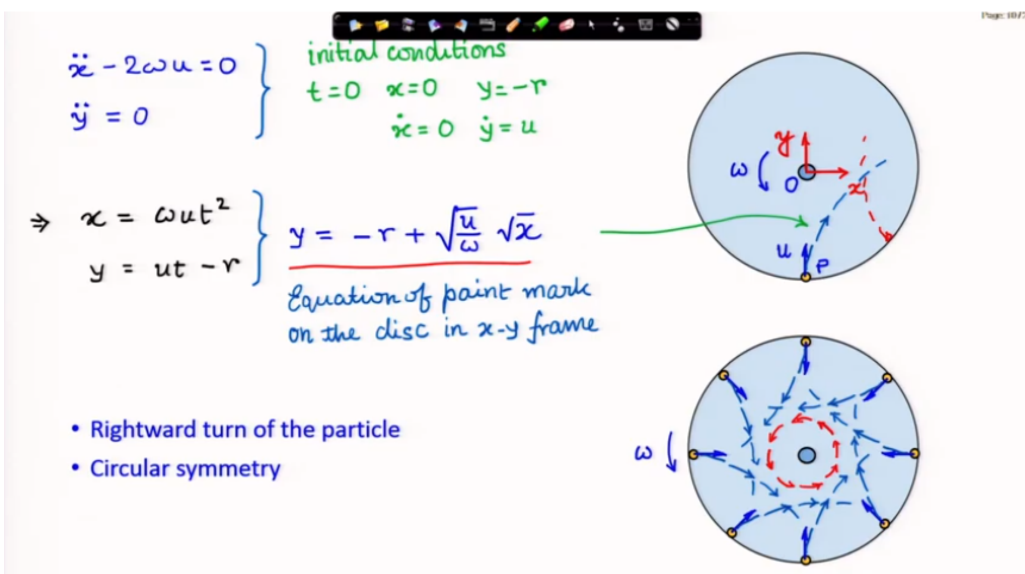
(Refer Slide Time: 08:04)



Using the initial conditions shown above, we can integrate the simplified equations of motion to obtain the velocity of the particle as seen by the disc-based observer as

$$\dot{x} - 2\omega y = 2\omega r \qquad \dot{y} + 2\omega x = u$$

(Refer Slide Time: 09:27)



Next, we differentiate the first equation and substitute the y-velocity from the second, and perform similar steps for the second equation to obtain

$$\ddot{x} - 2\omega(u - 2\omega x) = 0 \quad \Rightarrow \quad \ddot{x} - 2\omega u \approx 0$$

$$\ddot{y} + 2\omega(2\omega r + 2\omega y) = 0 \quad \Rightarrow \quad \ddot{y} \approx 0$$

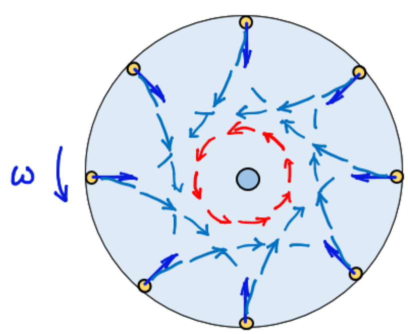
where we have once again dropped terms involving ω^2. These two equations can now be integrated easily to obtain

$$\left.\begin{aligned} x &= \omega u t^2 \\ y &= ut - r \end{aligned}\right\} \quad y = -r + \sqrt{\frac{u}{\omega}}\sqrt{x}$$

Equation of paint mark on the disc in x-y frame

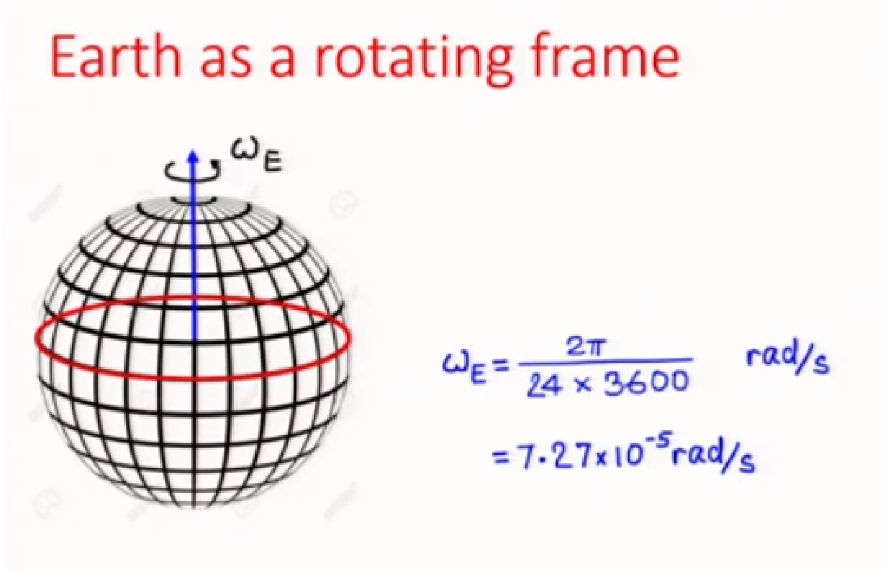
If the direction of ω is reversed, the path will curve to the left, as can be easily concluded.

If a number of particles were projected from the circumference of the disc towards its center, then all particles would have curved to the right when ω is counter clock-wise as seen from the top, as shown below.



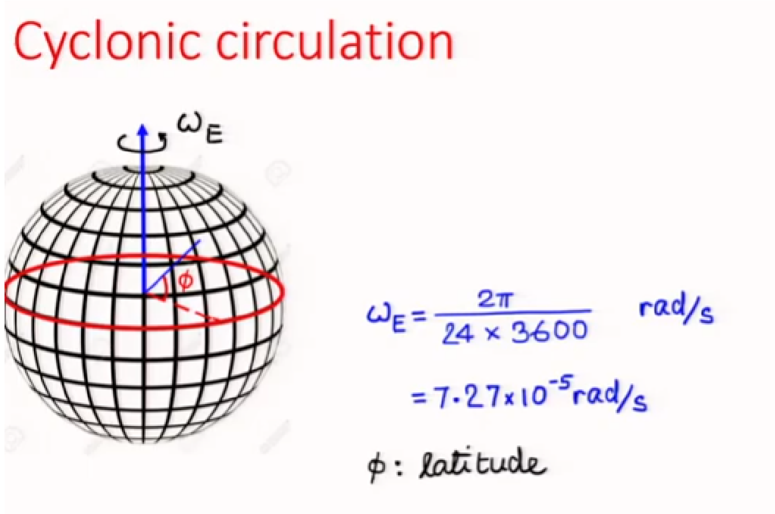
The above figure is suggestive of a cyclonic circulation, which we are going to discuss next.

(Refer Slide Time: 13:38)



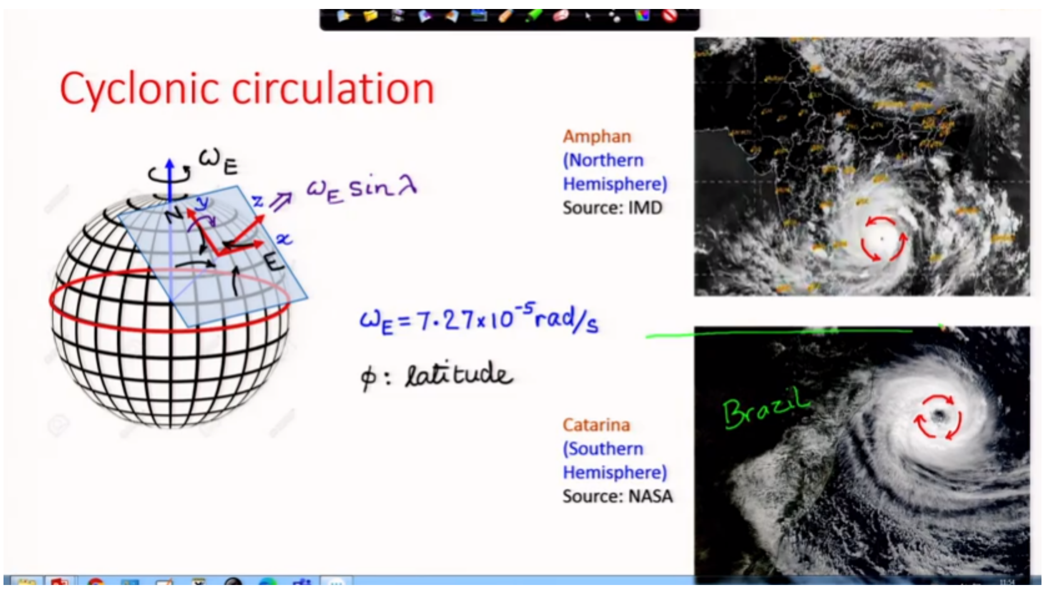
But to understand cyclonic circulation we must first consider earth as a rotating frame. I have shown here the calculation of what is the angular speed of the earth. It is $\omega=7.27\times10^{-5}$ rad/s.

(Refer Slide Time: 14:48)



Let us now look at a point on the surface of the earth at a latitude φ.

(Refer Slide Time: 15:15)

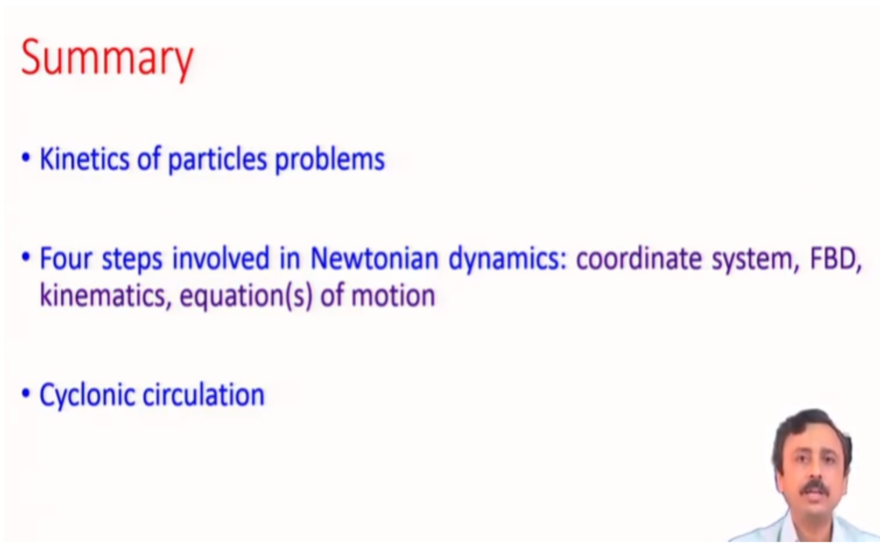


I consider the tangent plane at the point under consideration and fix up a coordinate system as shown above. The angular velocity vector component which is akin to the angular velocity of the disc example considered before is $\omega_E \sin\varphi$.

When there is a low pressure region at the location considered, air from the surrounding regions rush in towards the low pressure center. This then is similar to particles on the rotating disc projected towards its center, and the consequence is right-ward veering of the path of the air particles. This starts a cyclonic circulation.

In the Northern hemisphere where φ is positive, you have a counter clockwise rotating circulation, while in the Southern hemisphere, it is just the opposite. The slide above shows the image of the Amphan super cyclone in the Northern hemisphere off the coast of Tamil Nadu in the Bay of Bengal. The direction of circulation it is counter clockwise since it is in the Northern hemisphere. In the Southern hemisphere, off the coast of Brazil, the image of the super cyclone Catarina is shown. One can clearly notice that the cyclonic circulation direction in the Southern hemisphere is clockwise.

(Refer Slide Time: 20:10)



To summarize we have looked at kinetics of particles and applied Newtonian dynamics to understand cyclonic circulation.