

Advanced Dynamics
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Module No # 02
Lecture No # 08
Particle Kinetics – III

In this lecture we are going to continue our discussions on particle kinetics with some more problems.

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Overview

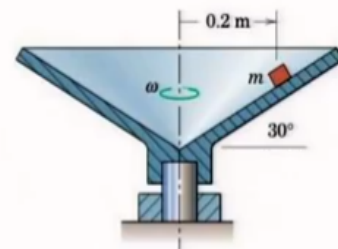
- Application of Newton's laws of motion
- Problems

We will look at some problems in which will apply Newton's law of motion and solve them.

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Problem 1:

The small object is placed on the inner surface of the conical dish at the radius shown. If the coefficient of static friction between the object and the conical surface is 0.30, for what range of angular velocities ω about the vertical axis will the block remain on the dish without slipping? Assume that speed changes are made slowly so that any angular acceleration may be neglected.



Source: Dynamics, Meriam and Kraige

The first problem is shown above.

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Coordinate system and FBD

Using the Cartesian frame as shown

Kinematics:

$$\vec{a}_A = \vec{a}_B + \vec{a}_{rel} + \vec{\omega} \times \vec{B}A + \vec{\omega} \times \vec{\omega} \times \vec{B}A + 2\vec{\omega} \times \vec{v}_{rel}$$

$$\Rightarrow \vec{a}_A = \omega(\sin 30^\circ \hat{i} + \cos 30^\circ \hat{k}) \times \omega(\sin 30^\circ \hat{i} + \cos 30^\circ \hat{k}) \times \frac{0.2}{\cos 30^\circ} \hat{i}$$

$$= 0.2\omega^2 (\sin 30^\circ \hat{k} - \cos 30^\circ \hat{i})$$

Equation of motion: $m\vec{a}_A = f_x \hat{i} + f_y \hat{j} + mg(-\sin 30^\circ \hat{i} - \cos 30^\circ \hat{k}) + N \hat{k}$

$$\Rightarrow \left. \begin{aligned} -0.2m\omega^2 \cos 30^\circ &= f_x - mg \sin 30^\circ \\ 0 &= f_y \\ 0.2m\omega^2 \sin 30^\circ &= -mg \cos 30^\circ + N \end{aligned} \right\} \frac{|f_x|}{N} \leq \mu_s \Rightarrow \frac{|g \sin 30^\circ - 0.2\omega^2 \cos 30^\circ|}{g \cos 30^\circ + 0.2\omega^2 \sin 30^\circ} \leq 0.3$$

First step is to set up the coordinate system as shown above. Notice the way I have set the coordinate system with the x-axis along the plane upwards, and z-axis perpendicular to the plane upwards, and y-axis into the plane. Once we have this coordinate system fixed then we draw the free body diagram of the block as shown. It is important to note that the friction force has two components: one along the x-axis direction, and the other along the y-axis direction in general. You must also notice very carefully that I have not shown any pseudo-forces in the free body diagram. We will not use pseudo-forces since they are out of the realm of Newtonian dynamics.

With this free body diagram next we are going to look at the kinematics of the problem. The acceleration of the block, if it sticks to the dish, is obtained as

$$\vec{a}_A = \vec{a}_B + \vec{a}_{rel} + \vec{\omega} \times \vec{B}A + \vec{\omega} \times \vec{\omega} \times \vec{B}A + 2\vec{\omega} \times \vec{v}_{rel}$$

$$\Rightarrow \vec{a}_A = \omega(\sin 30^\circ \hat{i} + \cos 30^\circ \hat{k}) \times \omega(\sin 30^\circ \hat{i} + \cos 30^\circ \hat{k}) \times \frac{0.2}{\cos 30^\circ} \hat{i}$$

$$= 0.2\omega^2 (\sin 30^\circ \hat{k} - \cos 30^\circ \hat{i})$$

The equation of motion is then obtained as

Equation of motion: $m\vec{a}_A = f_x \hat{i} + f_y \hat{j} + mg(-\sin 30^\circ \hat{i} - \cos 30^\circ \hat{k}) + N \hat{k}$

$$\Rightarrow -0.2 m \omega^2 \cos 30^\circ = f_x - mg \sin 30^\circ$$

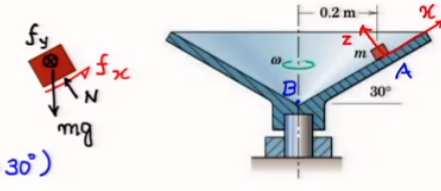
$$0 = f_y$$

$$0.2 m \omega^2 \sin 30^\circ = -mg \cos 30^\circ + N$$

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$$\frac{|g \sin 30^\circ - 0.2 \omega^2 \cos 30^\circ|}{g \cos 30^\circ + 0.2 \omega^2 \sin 30^\circ} \leq 0.3$$

$$\Rightarrow \pm (g \sin 30^\circ - 0.2 \omega^2 \cos 30^\circ) \leq 0.3 (g \cos 30^\circ + 0.2 \omega^2 \sin 30^\circ)$$

$$\Rightarrow \underline{3.4 \text{ rad/s} \leq \omega \leq 7.2 \text{ rad/s}}$$


The diagram shows a rotating dish with a vertical axis of rotation. A block of mass m is on the inner surface of the dish, which is inclined at 30 degrees. The distance from the axis to the block is 0.2 m. A free-body diagram of the block shows forces: normal force N perpendicular to the surface, weight mg acting vertically downwards, and friction forces f_x and f_y acting along the surface.

For no slip

$$\frac{|f_x|}{N} \leq \mu_s \Rightarrow \frac{|g \sin 30^\circ - 0.2 \omega^2 \cos 30^\circ|}{g \cos 30^\circ + 0.2 \omega^2 \sin 30^\circ} \leq 0.3$$

$$\Rightarrow \pm (g \sin 30^\circ - 0.2 \omega^2 \cos 30^\circ) \leq 0.3 (g \cos 30^\circ + 0.2 \omega^2 \sin 30^\circ)$$

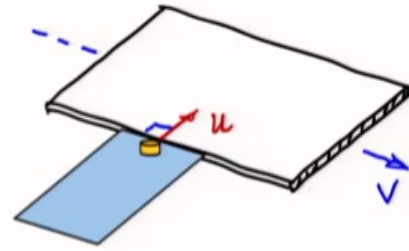
$$\Rightarrow \underline{3.4 \text{ rad/s} \leq \omega \leq 7.2 \text{ rad/s}}$$

This gives the limits on the rotation speed of the dish.

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Problem 2:

A small cylinder is projected with a speed u from an adjoining platform perpendicularly onto a long horizontal belt moving at a constant speed V . The coefficient of kinetic friction between the block and the belt is μ_k . (a) Calculate the time taken by the particle to stop on the belt, and (b) the path of the particle as seen by a ground based observer, and an observer on the belt.



The next problem is shown above. We have to calculate the time taken by the cylindrical block or particle to stop on the belt and determine the path of the particle as seen by a ground based observer and an observer on the belt.

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Coordinate system and FBD

Using a Cartesian frame fixed to the platform

Kinematics:

$$\vec{v}_c = \dot{x} \hat{i} + \dot{y} \hat{j}$$

$$\vec{v}_B = V \hat{i}$$

$$\vec{v}_{c/B} = (\dot{x} - V) \hat{i} + \dot{y} \hat{j}$$

$$\vec{f} = -\mu_k N \hat{v}_{c/B}$$

$$A_{c/B} = \frac{\vec{v}_{c/B}}{\sqrt{(\dot{x} - V)^2 + \dot{y}^2}}$$

Equation of motion: $m \vec{a}_c = \vec{F} \Rightarrow m(\ddot{x} \hat{i} + \ddot{y} \hat{j}) = N \hat{k} - mg \hat{k} - \mu_k N \hat{v}_{c/B}$

$$\Rightarrow m \ddot{x} = -\frac{\mu_k N (\dot{x} - V)}{\sqrt{(\dot{x} - V)^2 + \dot{y}^2}} \quad m \ddot{y} = -\frac{\mu_k N \dot{y}}{\sqrt{(\dot{x} - V)^2 + \dot{y}^2}}$$

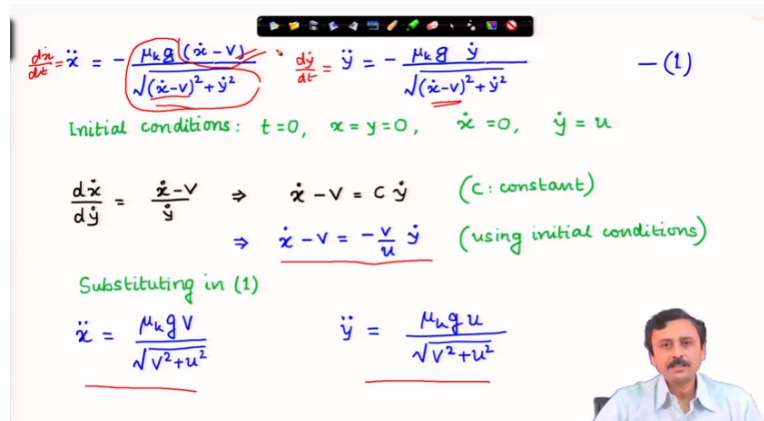
First thing is to set up the coordinate as shown above. Next, we draw the free body diagram of this cylindrical particle. The direction of the friction force on the cylinder from the belt needs to be determined from the relative velocity of the cylinder with respect to the belt as shown above. The vector equation of motion can then be written as

$$m \vec{a}_c = \vec{F} \Rightarrow m(\ddot{x} \hat{i} + \ddot{y} \hat{j}) = N \hat{k} - mg \hat{k} - \mu_k N \hat{v}_{c/B}$$

which leads to 3 scalar equations of motion

$$m\ddot{x} = -\frac{\mu_k N (\dot{x} - v)}{\sqrt{(\dot{x} - v)^2 + \dot{y}^2}} \quad m\ddot{y} = -\frac{\mu_k N \dot{y}}{\sqrt{(\dot{x} - v)^2 + \dot{y}^2}} \quad 0 = N - mg$$

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$$\frac{d\dot{x}}{dt} = \ddot{x} = -\frac{\mu_k g (\dot{x} - v)}{\sqrt{(\dot{x} - v)^2 + \dot{y}^2}} \quad \frac{d\dot{y}}{dt} = \ddot{y} = -\frac{\mu_k g \dot{y}}{\sqrt{(\dot{x} - v)^2 + \dot{y}^2}} \quad (1)$$

Initial conditions: $t=0, x=y=0, \dot{x}=0, \dot{y}=u$

$$\frac{d\dot{x}}{d\dot{y}} = \frac{\dot{x} - v}{\dot{y}} \Rightarrow \dot{x} - v = C \dot{y} \quad (C: \text{constant})$$

$$\Rightarrow \dot{x} - v = -\frac{v}{u} \dot{y} \quad (\text{using initial conditions})$$

Substituting in (1)

$$\ddot{x} = \frac{\mu_k g v}{\sqrt{v^2 + u^2}} \quad \ddot{y} = \frac{\mu_k g u}{\sqrt{v^2 + u^2}}$$

The third equation only gives the normal reaction same as the weight. We proceed with the equations of motion along x and y. Following the steps outlined in the slide above, we obtain

$$\frac{d\dot{x}}{d\dot{y}} = \frac{\dot{x} - v}{\dot{y}} \Rightarrow \dot{x} - v = C \dot{y}$$

$$\Rightarrow \dot{x} - v = -\frac{v}{u} \dot{y}$$

where C is the constant of integration, and obtained as $C = -v/u$ from the initial conditions

$$\text{Initial conditions: } t=0, x=y=0, \dot{x}=0, \dot{y}=u$$

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$$\ddot{x} = \frac{\mu_k g V}{\sqrt{V^2 + u^2}} \quad \ddot{y} = -\frac{\mu_k g u}{\sqrt{V^2 + u^2}}$$

$$\Rightarrow \dot{x} = \frac{\mu_k g V t}{\sqrt{V^2 + u^2}} \quad \dot{y} = -\frac{\mu_k g u t}{\sqrt{V^2 + u^2}} + u$$

Time to stop $\Rightarrow \dot{y}(t_s) = 0 \Rightarrow t_s = \frac{\sqrt{V^2 + u^2}}{\mu_k g}$
 (or, $\dot{x}(t_s) = V$)

$$x = \frac{1}{2} \frac{\mu_k g V t^2}{\sqrt{V^2 + u^2}} \quad y = -\frac{1}{2} \frac{\mu_k g u t^2}{\sqrt{V^2 + u^2}} + u t \quad (t \leq t_s)$$

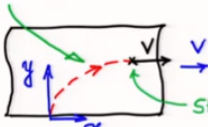
$$\Rightarrow y = -\frac{u}{V} x + \left[\frac{2\sqrt{V^2 + u^2}}{\mu_k g V} \right]^{1/2} \sqrt{x} \quad (t \leq t_s)$$

Substituting back the expression obtained above in to the 2 equations of motion, we obtain the results presented in the above slide. The time to stop t_s is also presented.

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$$y = -\frac{u}{V} x + \left[\frac{2\sqrt{V^2 + u^2}}{\mu_k g V} \right]^{1/2} \sqrt{x} \quad t \leq t_s = \frac{\sqrt{V^2 + u^2}}{\mu_k g}$$

(sliding phase only)



At $t = t_s$

$$x = \frac{1}{2} \frac{V \sqrt{V^2 + u^2}}{\mu_k g} \quad y = \frac{1}{2} \frac{u \sqrt{V^2 + u^2}}{\mu_k g}$$

For $t \geq t_s$

$$x = V t + \frac{1}{2} \frac{V \sqrt{V^2 + u^2}}{\mu_k g} \quad y = \frac{1}{2} \frac{u \sqrt{V^2 + u^2}}{\mu_k g}$$

The equation of the path of the cylinder as seen by the ground based observer is shown above, which exhibits a square-root dependence on x as long as the cylinder slides. As soon as it comes to rest on the belt, the path becomes a straight line along the x -axis direction due to the motion of the belt.

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For an observer on the belt, velocity vector

$$\vec{v}_{c/B} = (\dot{x} - v)\hat{i} + \dot{y}\hat{j} \quad \left(\dot{x} - v = -\frac{v}{u}\dot{y} \right) \quad \left(t \leq t_s = \frac{\sqrt{v^2 + u^2}}{\mu_k g} \right)$$

$$\Rightarrow \vec{v}_{c/B} = \dot{x}\hat{i}' + \dot{y}\hat{j}' = -\frac{v}{u}\dot{y}\hat{i} + \dot{y}\hat{j} \quad \text{(identical belt-fixed frame)} \\ \text{(directions } \hat{i}' = \hat{i}, \hat{j}' = \hat{j})$$

$$\Rightarrow \frac{\dot{y}}{\dot{x}} \frac{dY}{dX} = \frac{\dot{y}}{\dot{x}} = -\frac{u}{v} \quad \text{(Constant)}$$

$$\Rightarrow \underline{Y = -\frac{u}{v}X} \quad \text{Path on the belt is a straight line! } (t \leq t_s)$$

$\left(-v\frac{\sqrt{v^2 + u^2}}{\mu_k g}, \frac{1}{2}\frac{u\sqrt{v^2 + u^2}}{\mu_k g} \right)$ Sliding point $(t \geq t_s)$

Now to determine the path of the cylinder as observed by an observer on the belt, let us start with the relative velocity of the cylinder with respect to the belt, as shown above. We obtain the coordinates (X, Y) as observed by the belt based observer in the following steps.

$$\vec{v}_{c/B} = (\dot{x} - v)\hat{i} + \dot{y}\hat{j} \quad \left(\dot{x} - v = -\frac{v}{u}\dot{y} \right)$$

$$\Rightarrow \vec{v}_{c/B} = \dot{x}\hat{i}' + \dot{y}\hat{j}' = -\frac{v}{u}\dot{y}\hat{i} + \dot{y}\hat{j}$$

$$\Rightarrow \frac{dY}{dX} = \frac{\dot{y}}{\dot{x}} = -\frac{u}{v} \quad \text{(Constant)}$$

$$\Rightarrow \underline{Y = -\frac{u}{v}X}$$

It is observed that the path has a constant slope. Thus the path is a straight line as shown in the slide above.

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Summary

- Kinetics of particles problems
- Four steps involved in Newtonian dynamics: coordinate system, FBD, kinematics, equation(s) of motion

The summary of our discussions is presented above.