Advanced Dynamics
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Module No # 02
Lecture No # 08
Particle Kinetics – III

In this lecture we are going to continue our discussions on particle kinetics with some more problems.

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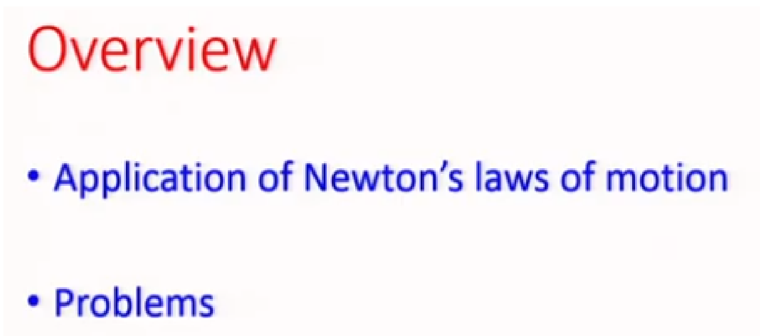

We will look at some problems in which will apply Newton's law of motion and solve them.

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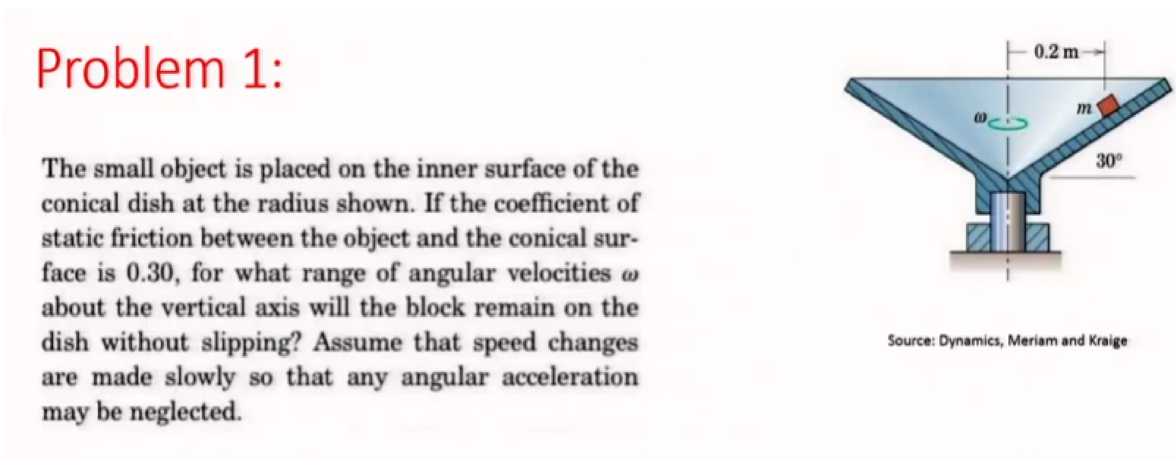

The first problem is shown above.

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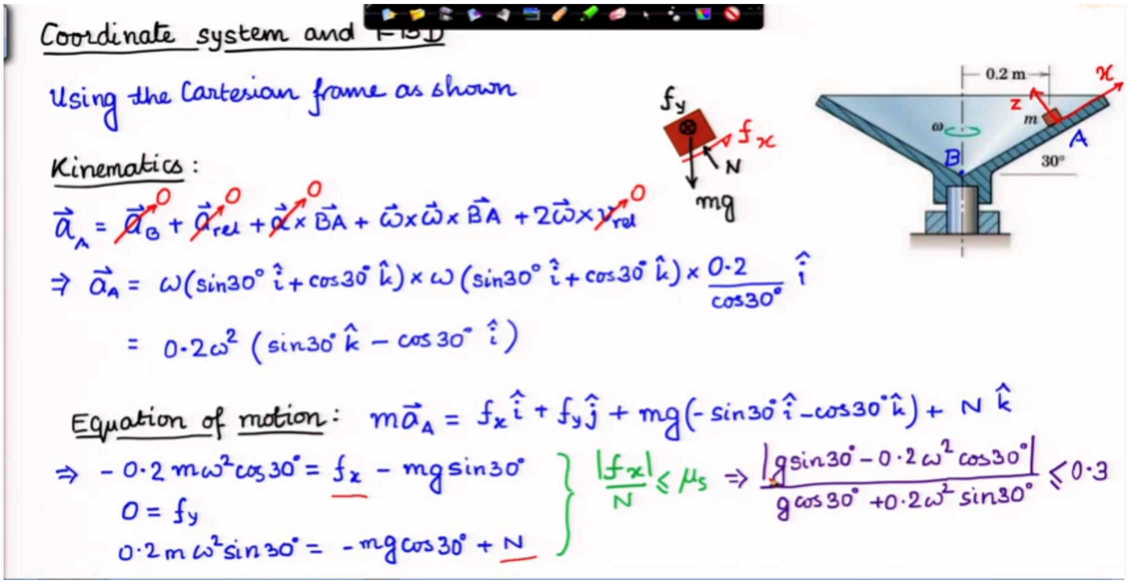

First step is to set up the coordinate system as shown above. Notice the way I have set the coordinate system with the x-axis along the plane upwards, and z-axis perpendicular to the plane upwards, and y-axis into the plane. Once we have this coordinate system fixed then we draw the free body diagram of the block as shown. It is important to note that the friction force has two components: one along the x-axis direction, and the other along the y-axis direction in general. You must also notice very carefully that I have not shown any pseudo-forces in the free body diagram. We will not use pseudo-forces since they are out of the realm of Newtonian dynamics.

With this free body diagram next we are going to look at the kinematics of the problem. The acceleration of the block, if it sticks to the dish, is obtained as

$$\vec{a}_A = \cancel{\vec{a}_B}^{0} + \cancel{\vec{a}_{rel}}^{0} + \cancel{\vec{\alpha}}^{0} \times \vec{BA} + \vec{\omega}\times\vec{\omega}\times\vec{BA} + 2\vec{\omega}\times\cancel{\vec{v}_{rel}}^{0}$$

$$\Rightarrow \vec{a}_A = \omega(\sin 30^\circ \hat{i} + \cos 30^\circ \hat{k}) \times \omega(\sin 30^\circ \hat{i} + \cos 30^\circ \hat{k}) \times \frac{0.2}{\cos 30^\circ}\hat{i}$$

$$= 0.2\omega^2(\sin 30^\circ \hat{k} - \cos 30^\circ \hat{i})$$

The equation of motion is then obtained as

Equation of motion: $m\vec{a}_A = f_x\hat{i} + f_y\hat{j} + mg(-\sin 30^\circ\hat{i} - \cos 30^\circ\hat{k}) + N\hat{k}$

$$\Rightarrow -0.2\, m\omega^2\cos 30^\circ = f_x - mg\sin 30^\circ$$
$$0 = f_y$$
$$0.2\, m\omega^2\sin 30^\circ = -mg\cos 30^\circ + N$$

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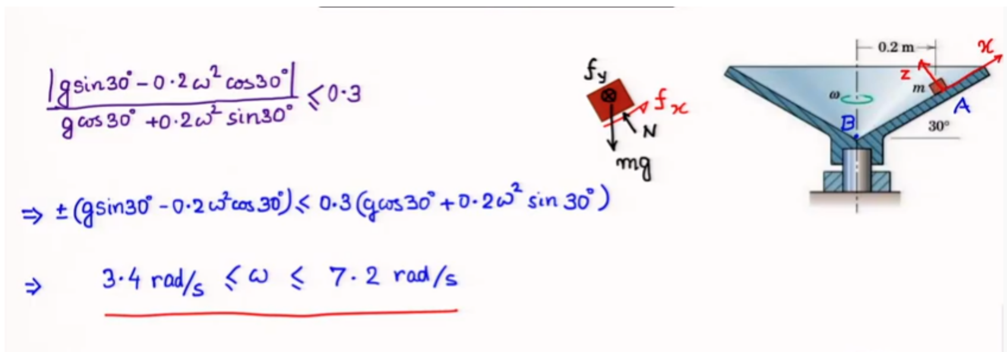

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$$\frac{|f_x|}{N} \leq \mu_s \Rightarrow \frac{|g\sin 30^\circ - 0.2\omega^2\cos 30^\circ|}{g\cos 30^\circ + 0.2\omega^2\sin 30^\circ} \leq 0.3$$

$$\Rightarrow \pm(g\sin 30^\circ - 0.2\omega^2\cos 30^\circ) \leq 0.3(g\cos 30^\circ + 0.2\omega^2\sin 30^\circ)$$

$$\Rightarrow \quad \underline{3.4 \text{ rad/s} \leq \omega \leq 7.2 \text{ rad/s}}$$

This gives the limits on the rotation speed of the dish.

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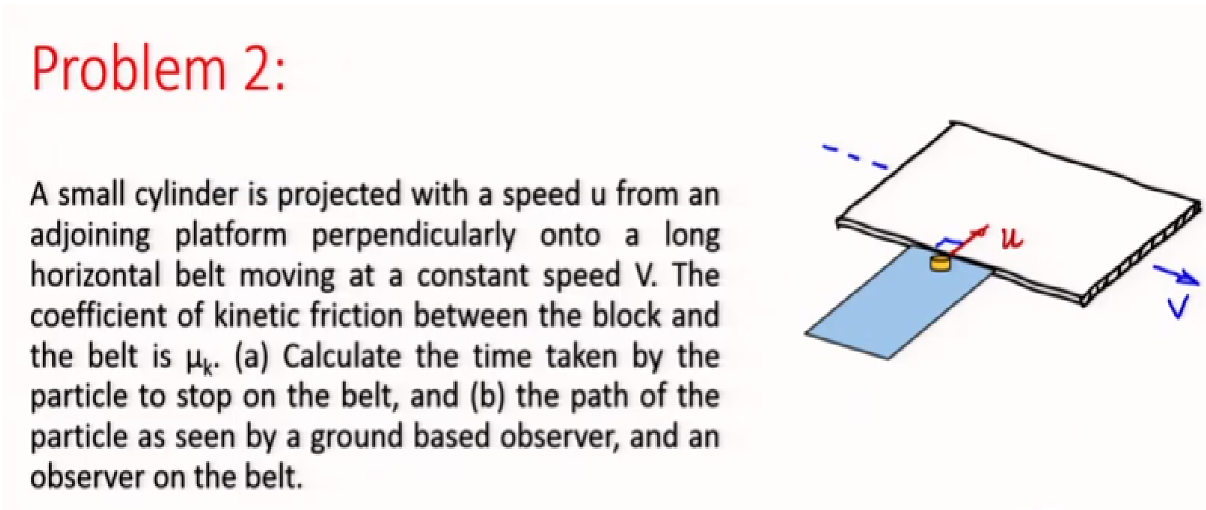

The next problem is shown above. We have to calculate the time taken by the cylindrical block or particle to stop on the belt and determine the path of the particle as seen by a ground based observer and an observer on the belt.

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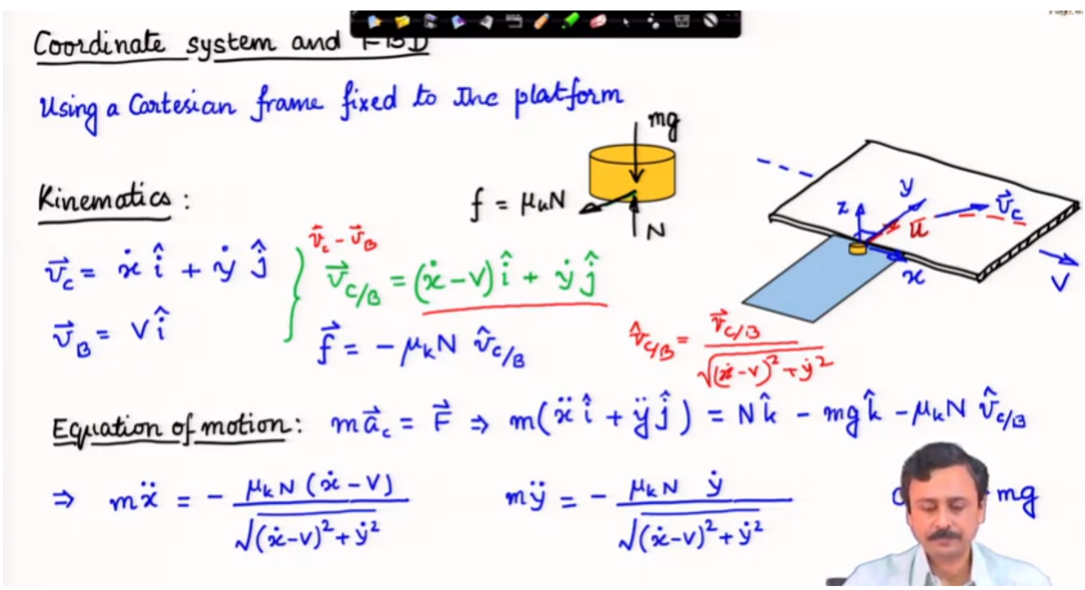

First thing is to set up the coordinate as shown above. Next, we draw the free body diagram of this cylindrical particle. The direction of the friction force on the cylinder from the belt needs to be determined from the relative velocity of the cylinder with respect to the belt as shown above. The vector equation of motion can then be written as

$$m\vec{a}_c = \vec{F} \Rightarrow m(\ddot{x}\hat{i} + \ddot{y}\hat{j}) = N\hat{k} - mg\hat{k} - \mu_k N \hat{v}_{c/B}$$

which leads to 3 scalar equations of motion

$$m\ddot{x} = -\frac{\mu_k N(\dot{x}-V)}{\sqrt{(\dot{x}-V)^2+\dot{y}^2}} \qquad m\ddot{y} = -\frac{\mu_k N\,\dot{y}}{\sqrt{(\dot{x}-V)^2+\dot{y}^2}} \qquad 0 = N - mg$$

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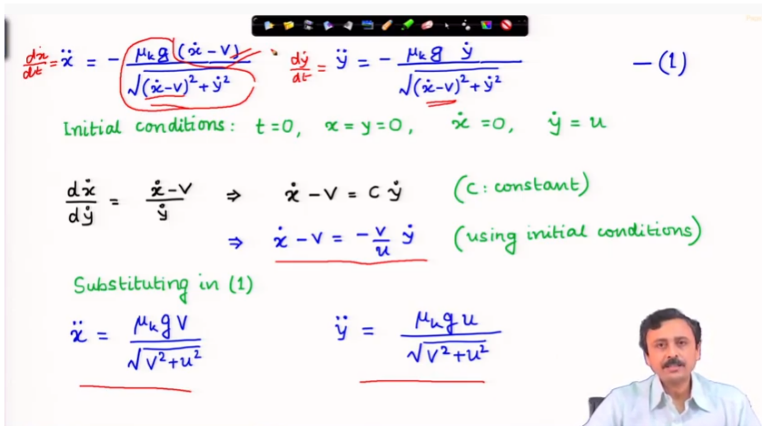

The third equation only gives the normal reaction same as the weight. We proceed with the equations of motion along x and y. Following the steps outlined in the slide above, we obtain

$$\frac{d\dot{x}}{d\dot{y}} = \frac{\dot{x}-V}{\dot{y}} \Rightarrow \dot{x} - V = C\dot{y}$$

$$\Rightarrow \dot{x} - V = -\frac{V}{u}\dot{y}$$

where C is the constant of integration, and obtained as C=-v/u from the initial conditions

Initial conditions: $t=0$, $x=y=0$, $\dot{x}=0$, $\dot{y}=u$

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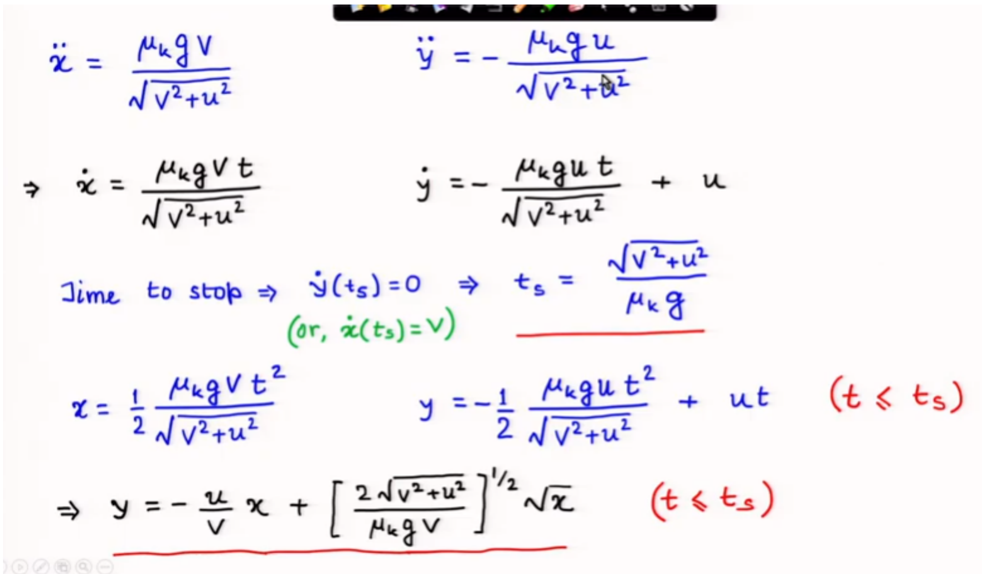

Substituting back the expression obtained above in to the 2 equations of motion, we obtain the results presented in the above slide. The time to stop t_s is also presented.

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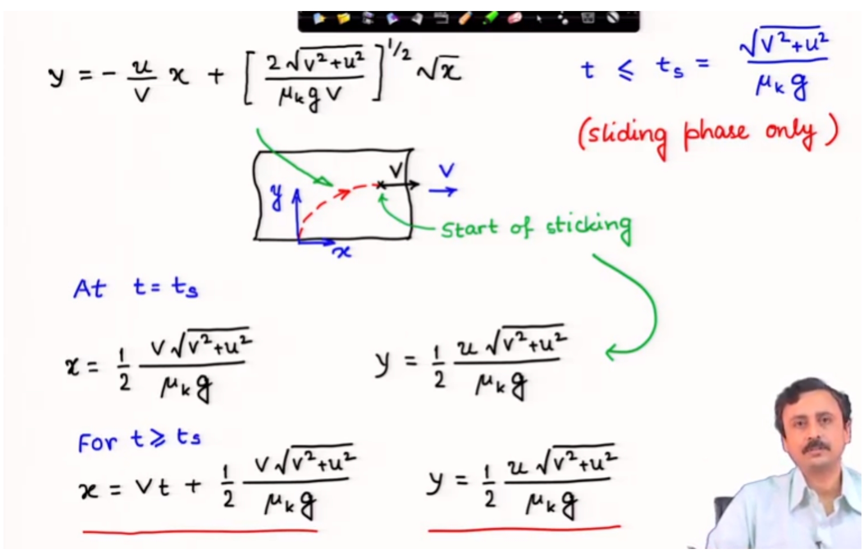

The equation of the path of the cylinder as seen by the ground based observer is shown above, which exhibits a square-root dependence on x as long as the cylinder slides. As soon as it comes to rest on the belt, the path becomes a straight line along the x-axis direction due to the motion of the belt.

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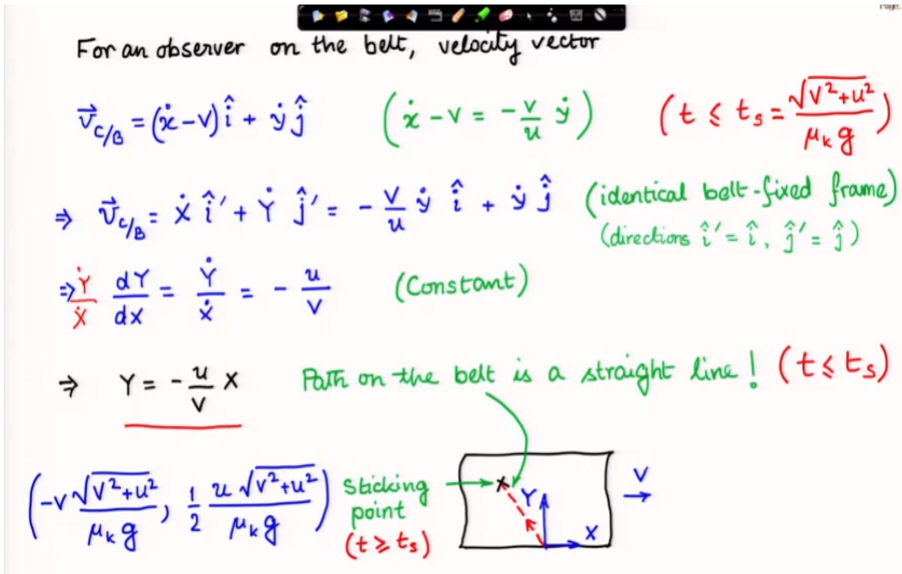

Now to determine the path of the cylinder as observed by an observer on the belt, let us start with the relative velocity of the cylinder with respect to the belt, as shown above. We obtain the coordinates (X, Y) as observed by the belt based observer in the following steps.

$$\vec{v}_{C/B} = (\dot{x}-v)\hat{i} + \dot{y}\hat{j} \qquad \left(\dot{x}-v = -\frac{v}{u}\dot{y}\right)$$

$$\Rightarrow \vec{v}_{C/B} = \dot{X}\hat{i}' + \dot{Y}\hat{j}' = -\frac{v}{u}\dot{y}\hat{i} + \dot{y}\hat{j}$$

$$\Rightarrow \frac{dY}{dX} = \frac{\dot{Y}}{\dot{X}} = -\frac{u}{v} \qquad \text{(Constant)}$$

$$\Rightarrow Y = -\frac{u}{v}X$$

It is observed that the path has a constant slope. Thus the path is a straight line as shown in the slide above.

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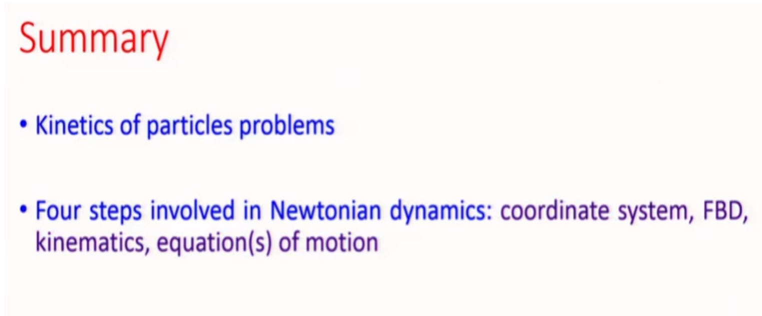

The summary of our discussions is presented above.