Advanced Dynamics
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Module No # 02
Lecture No # 07
Particle Kinetics – II

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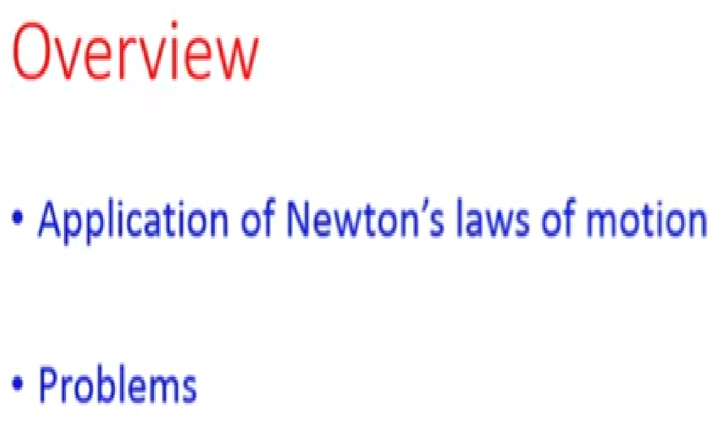

In this lecture we will continue our discussion on particle kinetics. I will show you through some examples the application of Newton's law of motion.

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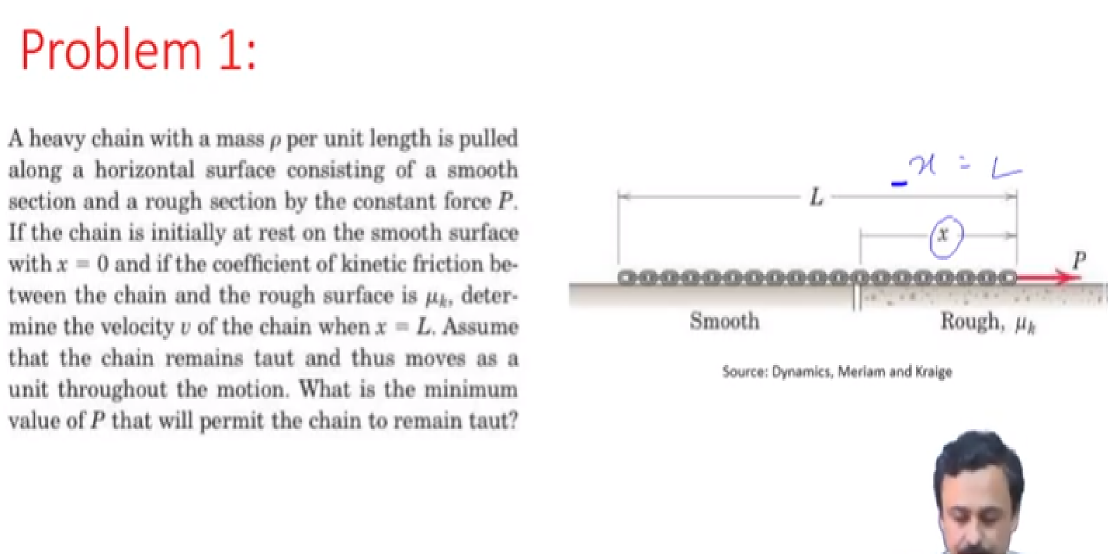

The first problem is shown above.

You have to assume that the chain remains taut and thus moves as a unit throughout the motion. This can happen only when the acceleration is non-negative.

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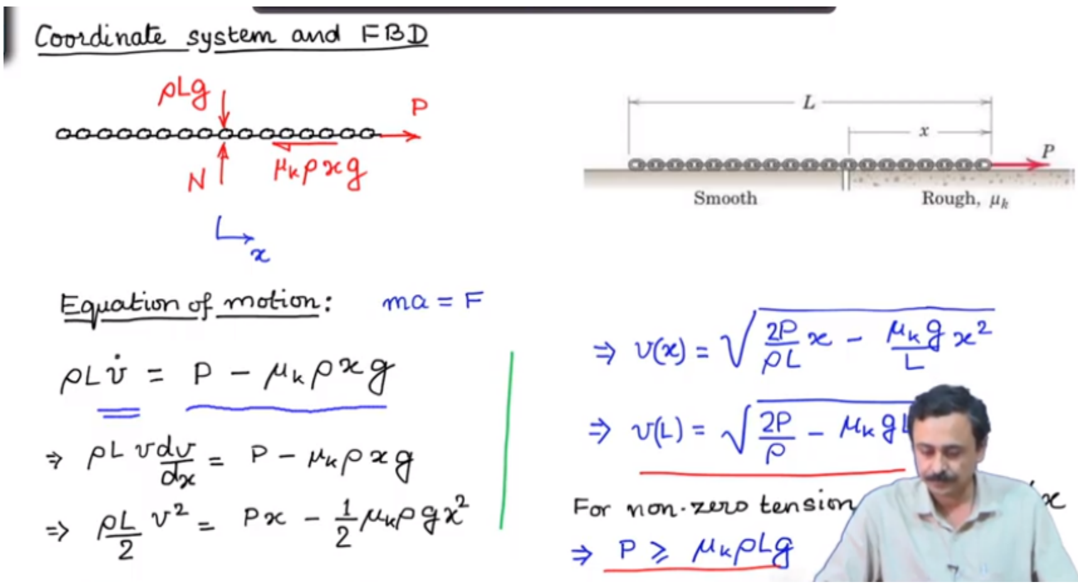

The free body diagram is shown above. The equation of motion is obtained from Newton's second law as given above. It is observed that the acceleration is a function of the displacement variable x. We follow the steps as shown below.

$$\rho L \dot{v} = P - \mu_k \rho x g$$

$$\Rightarrow \rho L v \frac{dv}{dx} = P - \mu_k \rho x g$$

$$\Rightarrow \frac{\rho L}{2} v^2 = Px - \frac{1}{2}\mu_k \rho g x^2$$

Finally, the velocity as a function of displacement is obtained as

$$v(L) = \sqrt{\frac{2P}{\rho} - \mu_k g L}$$

Furthermore, it is observed that the acceleration remains non-negative when

$$P \geq \mu_k \rho L g$$

This problem gives an idea why it is very important to apply brakes at all the wheels of a train. If only the locomotive brakes, the train will try to collapse on to the locomotive.

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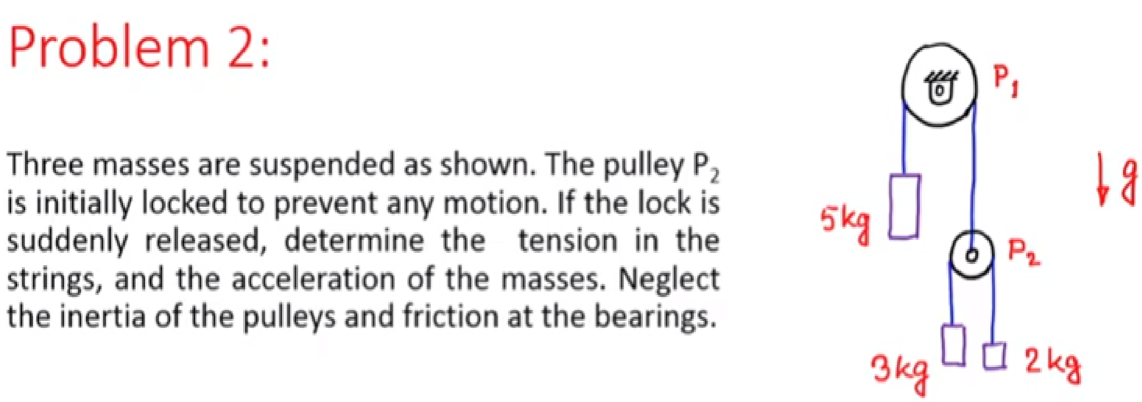

The next problem is about connected bodies as shown above.

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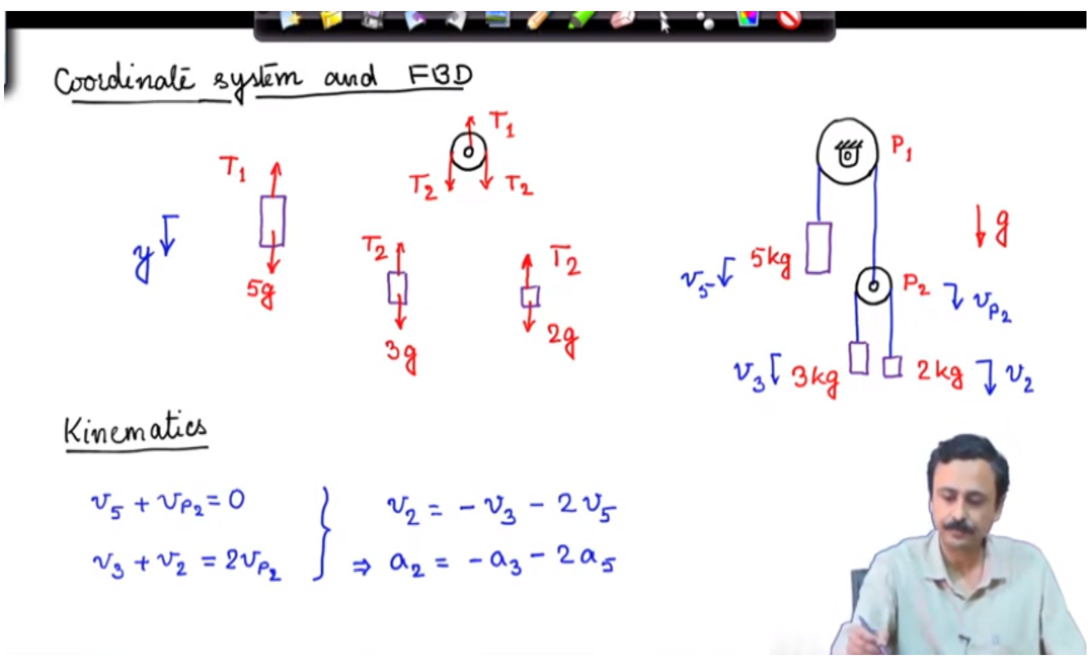

First we start with the coordinate system as usual. I have chosen the coordinate system has this y coordinate vertically downwards as positive. Then the second step is drawing free body diagrams as shown above. The kinematic relations are obtained based on our previous discussions on pulleys as

$$\left.\begin{aligned} v_5 + v_{P_2} &= 0 \\ v_3 + v_2 &= 2v_{P_2} \end{aligned}\right\} \quad \begin{aligned} &v_2 = -v_3 - 2v_5 \\ \Rightarrow\ &a_2 = -a_3 - 2a_5 \end{aligned}$$

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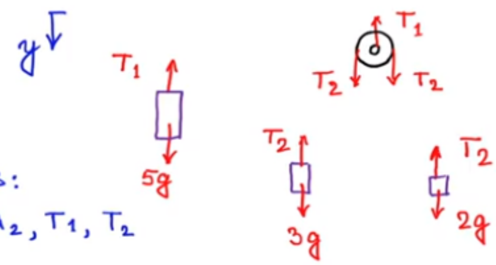
$$a_2 = -a_3 - 2a_5 \qquad (1)$$

Equations of motion : $m\vec{a} = \vec{F}$



$$\left.\begin{aligned} 5a_5 &= 5g - T_1 && (2) \\ 3a_3 &= 3g - T_2 && (3) \\ 2a_2 &= 2g - T_2 && (4) \\ 0 &= 2T_2 - T_1 && (5) \end{aligned}\right\}$$

Unknowns: a_5, a_3, a_2, T_1, T_2

Equations: 5

$$T_2 = \frac{120}{49}g \quad T_1 = \frac{240}{49}g \Rightarrow a_5 = \frac{g}{49} \quad a_3 = \frac{9}{49}g \quad a_2 = -\frac{11}{49}g$$

Using Newton's second law for each mass point and the pulley P_2 we obtain the equations as presented above. Using the kinematic relation (1) and the equations (2)-(5), we obtain the solution as shown above.

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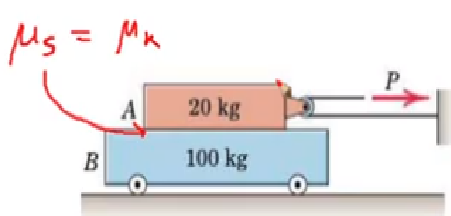
Problem 3:

If the coefficients of static and kinetic friction between the 20-kg block *A* and the 100-kg cart *B* are both essentially the same value of 0.50, determine the acceleration of each part for (*a*) *P* = 60 N and (*b*) *P* = 40 N.



Source: Dynamics, Meriam and Kraige

The next problem is shown above. This problem demonstrates how the force of friction between two bodies under forced motion may be decided by the dynamics.

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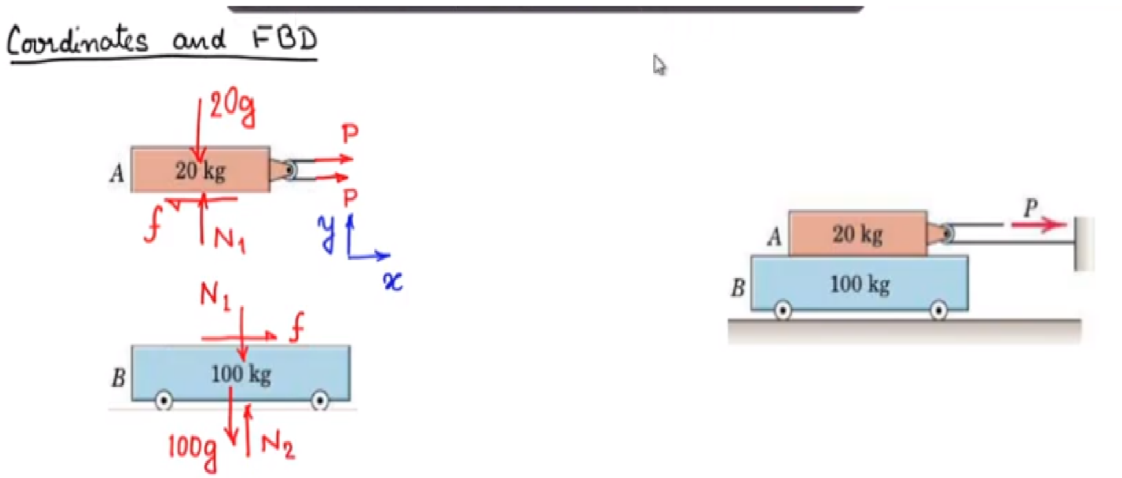

First step is to fix up a coordinate system as shown above.

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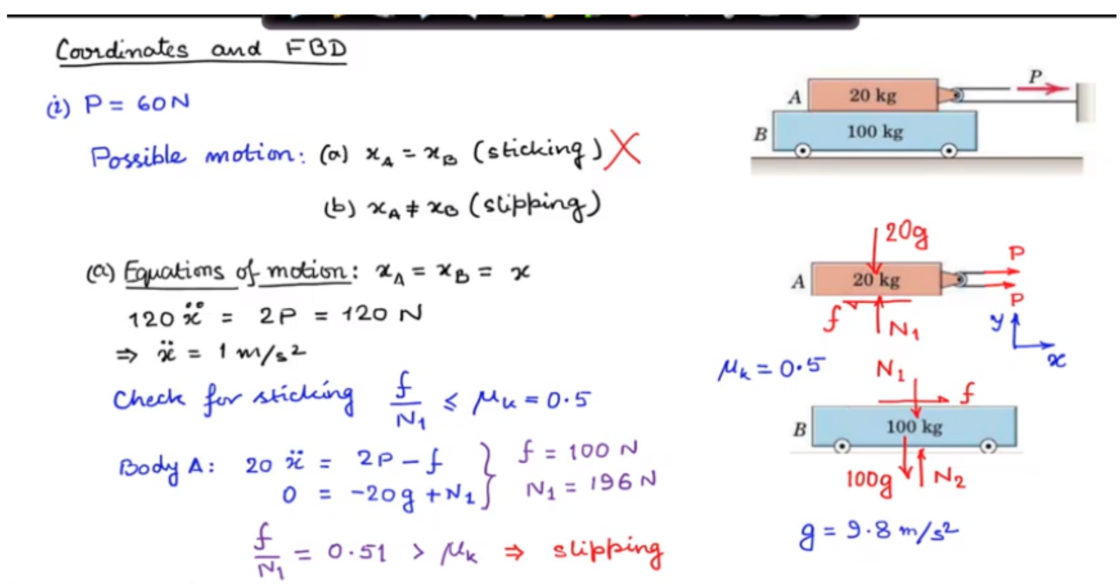

We start with the case P=60 N. There are 2 possible motions:

(a) $x_A = x_B$ (sticking)

(b) $x_A \neq x_B$ (slipping)

For case (a) the solution procedure is shown in the slide above. It may be concluded that the maximum static friction force is not enough to prevent slip. Hence, our motion possibility (a) is wrong. We move to motion possibility (b)

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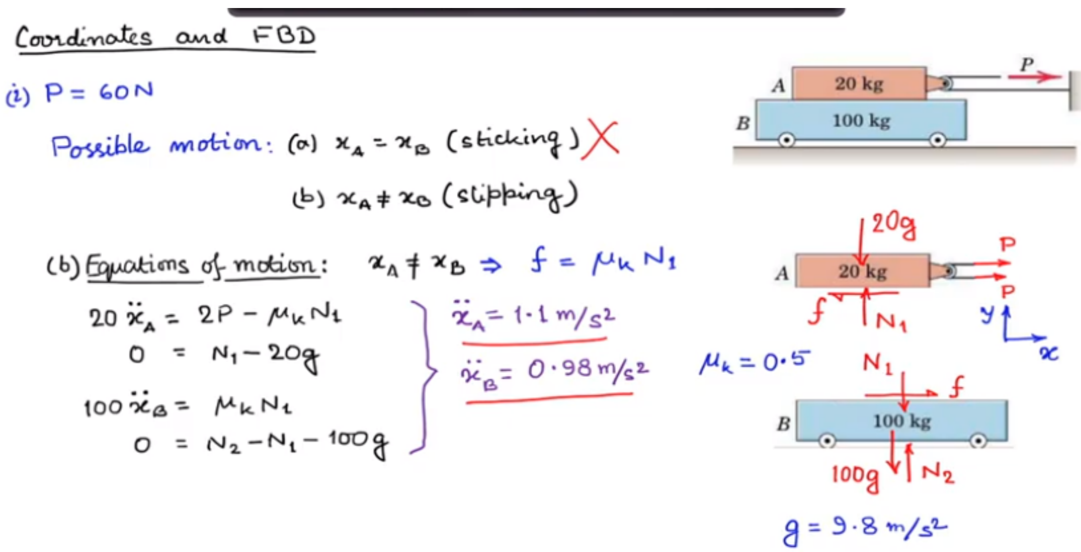

The calculations related to motion possibility (b) is shown in the slide above. Finally we calculate the accelerations of the blocks A and B as given above.

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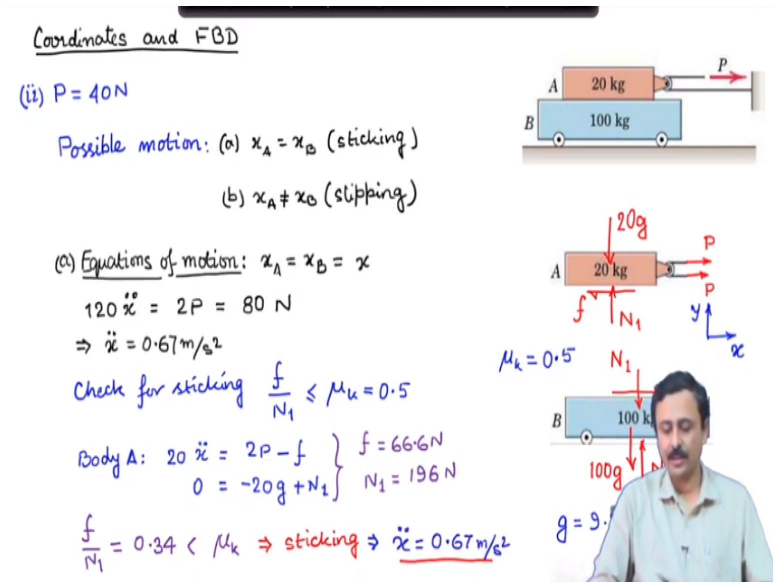

Next we go to the case with P = 40 N. In this case again we have 2 motion possibilities as shown above. We start with possibility (a). In this calculation, it is observed that the blocks can indeed stick since the force of friction required is less than the maximum static friction force limit. Hence, we need not consider possibility (b).

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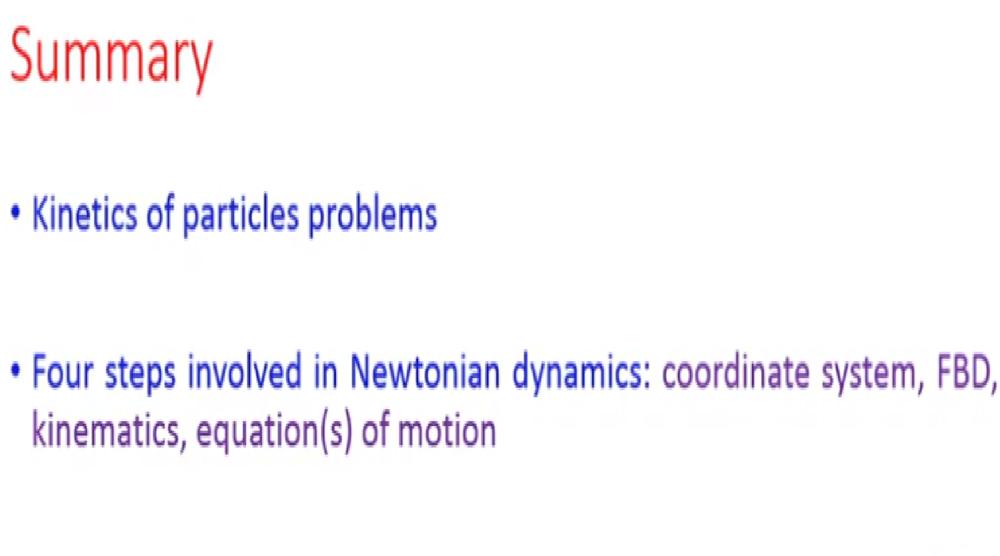

To summarize, we have discussed the kinetics of particles through some examples.