Advanced Dynamics
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Module No # 02
Lecture No # 06
Particle Kinetics – I

In the last past few lectures we have been looking at kinematics in various frames. Starting with this lecture we are going to discuss particle kinetics.

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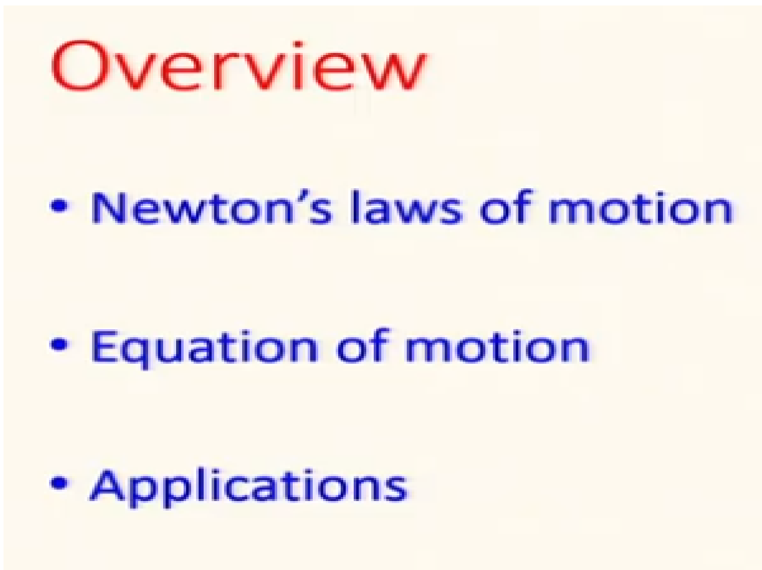

The overview of our discussion is shown above.

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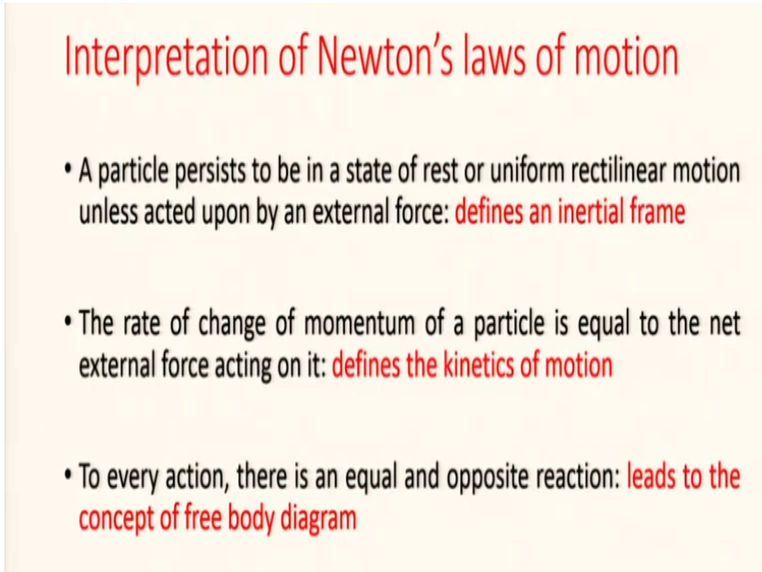

We are not going to discuss the Newton's laws of motion as written by Newton in his book Principia. We are going to interpret or understand these laws in the framework of how we understand them today. The first law says that a particle persist to be in the state of rest or uniform rectilinear motion unless acted upon by an external force. This law defines what is called an inertial frame. This is a very important point in kinetics. If I show you a video in which you see some object kept on a table and suddenly in the video you are see that the object starts moving, what will be your conclusion? The particle would be seen to be moving without force; then it defies the first law and the frame cannot be an inertial frame. This can happen when, suppose there is a camera mounted on a moving railway coach. If the train was moving at a constant speed and suddenly there is a tremendous breaking all the objects will be observed to be shifting towards the direction of the motion of the train, or opposite to the direction of acceleration. When it is breaking the acceleration is towards the backward direction. Therefore the objects move in the direction opposite to that acceleration or what is called deceleration. Therefore an accelerating frame is a non-inertial frame.

The second law gives us the equation or motion of a particle. The rate of change of momentum of a particle is equal to the net external force acting upon it. This defines the kinetics of motion. This is something like a constitutive relation between force and motion. The third law which says to every action there is an equal and opposite reaction. This leads us to the concept of the free body diagram. Of course this law the third law also has an implication for momentum conservation that we will see alter on. Therefore the third law leads us to the concept of free body diagram which is very important in Newtonian mechanics. These 3 laws form a complete recipe for deriving the equations of motion or studying dynamics of a particle. This is a complete recipe gives you the frame of reference it gives you the equation of motion and it allows you to develop the equation of motion particle-wise. Suppose the system is composed of multiple particles, we cut connections and draw free body diagrams so that we can finally write the equations of motion for individual particle.

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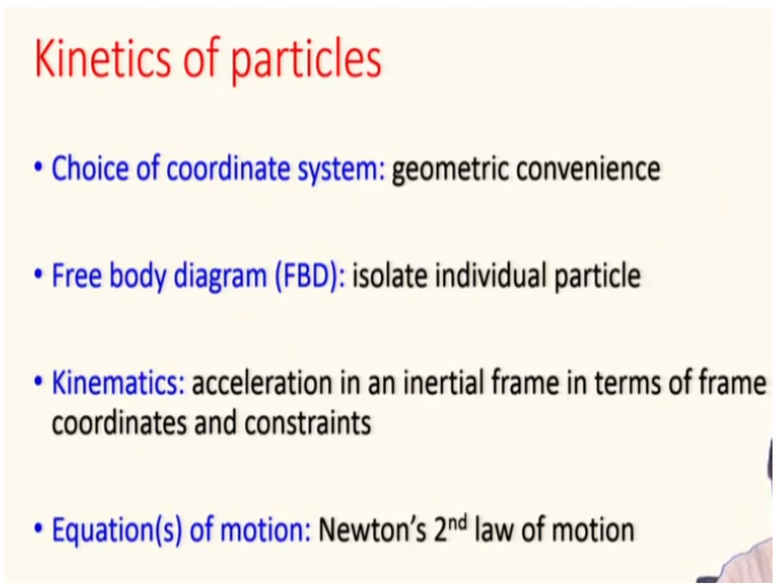

The first thing that we do in any study of mechanics is fix of a coordinate frame. Most of the time the geometrical convenience dictates the choice of frame. In principle, every problem can be solved in Cartesian coordinate frame. However, the equations might look very difficult. If a particle is moving on a circular path we use polar coordinates because it is convenient to use. One can always represent and solve it in Cartesian coordinates. The second step is to draw the free body diagrams after isolating all particle(s) using Newton's third law. The next step is to develop the kinematics: representation of acceleration as seen by inertial observer in terms of any frame that you might have chosen to solve the problem in. Finally, write down the equation(s) of motion using second law for each particle.

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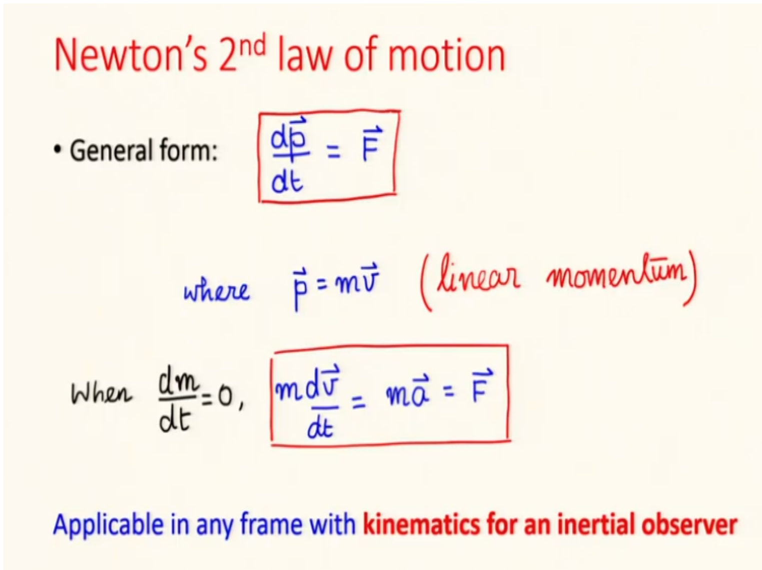

The form of Newton's second law that we typically use states that the rate of change of linear momentum is equal to the force as shown above. The origin of the left hand side of this equation is purely kinematic. It involves geometry, while the right hand side of this equation is kinetics in nature, that is, it involves force. Thus Newton's second law is kind of a constitutive model or consecutive relation for dynamics of a particle. It is a connection between the kinematics or the geometric variables and the kinetic of the force variables. When the mass is constant we have the familiar form of Newton's second law which is ma = f. Now this is the vector relation, and can be used in any frame as long as the kinematics has been represented for an inertial observer.

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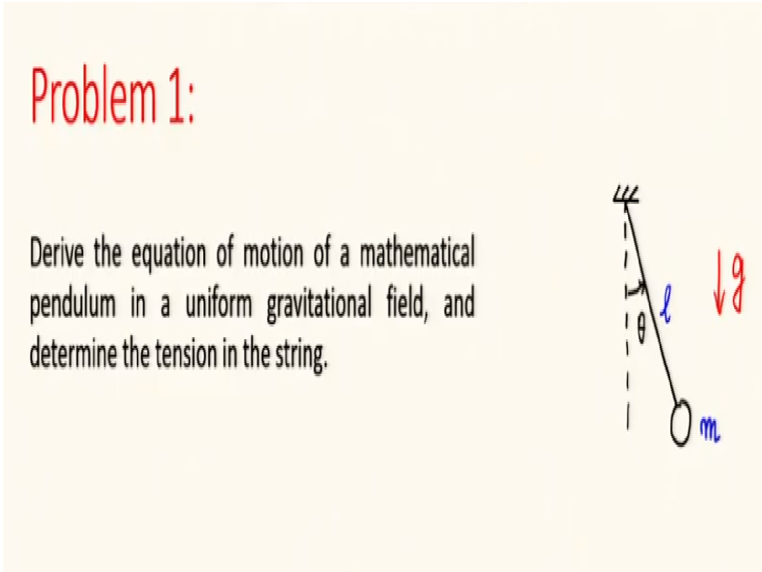

Let us start with most classical example the pendulum. We derive the equation of motion of the mathematical pendulum in a uniform gravitational field and determine the tension in the string.

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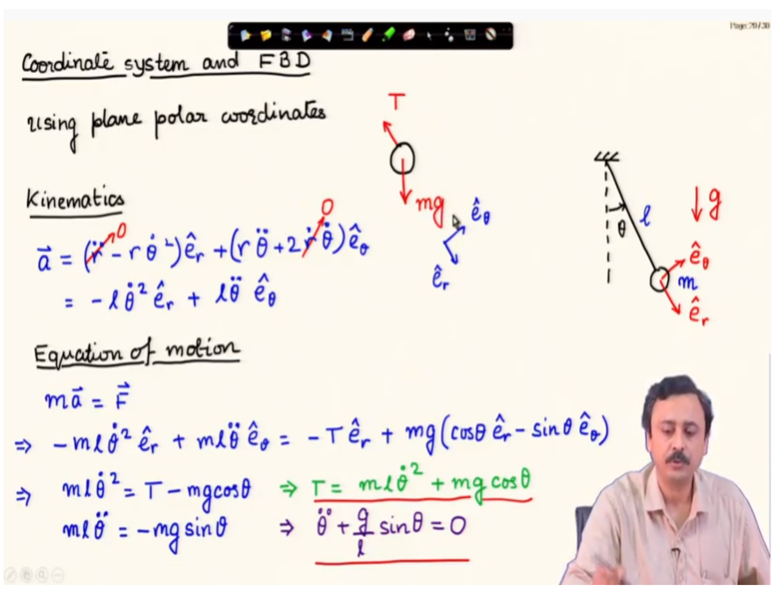

First step is to fix the coordinate system, and we choose to use the polar coordinates as shown above. The second step in the free body diagram, as shown. The acceleration vector represented in the polar coordinate system, as seen by an inertial observer, is

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta \quad (\ddot{r} = 0,\ \dot{r} = 0)$$

$$= -\ell\dot{\theta}^2\hat{e}_r + \ell\ddot{\theta}\,\hat{e}_\theta$$

Using Newton's second law

$$m\vec{a} = \vec{F}$$

$$\Rightarrow -m\ell\dot{\theta}^2\hat{e}_r + m\ell\ddot{\theta}\,\hat{e}_\theta = -T\hat{e}_r + mg(\cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta)$$

$$\Rightarrow m\ell\dot{\theta}^2 = T - mg\cos\theta \quad \Rightarrow T = m\ell\dot{\theta}^2 + mg\cos\theta$$

$$m\ell\ddot{\theta} = -mg\sin\theta \quad \Rightarrow \ddot{\theta} + \frac{g}{\ell}\sin\theta = 0$$

It may be noted that the internal force T is also part of the solution in Newtonian dynamics.

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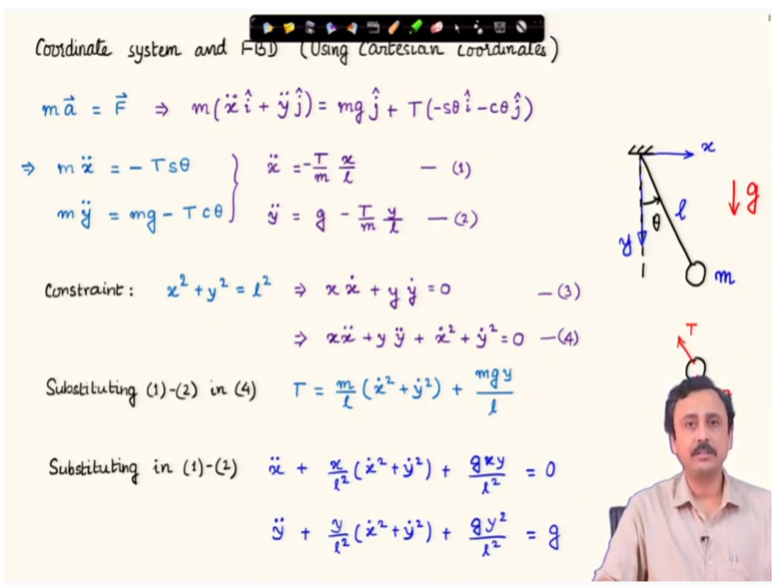
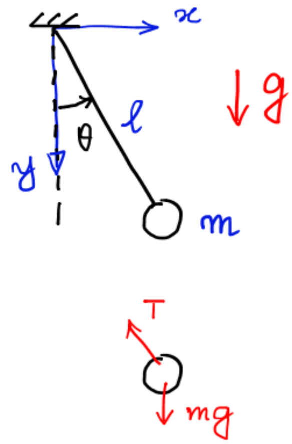


Now let us switch to a different coordinates system. The same problem of the pendulum can be represented in the Cartesian coordinate system x-y. The figure is shown above. The free body diagram is shown below



Representing the acceleration and all forces in the Cartesian coordinate system shown, we have

$$m\vec{a} = \vec{F} \Rightarrow m(\ddot{x}\hat{i} + \ddot{y}\hat{j}) = mg\hat{j} + T(-s\theta\,\hat{i} - c\theta\,\hat{j})$$

$$\Rightarrow \left.\begin{aligned} m\ddot{x} &= -Ts\theta \\ m\ddot{y} &= mg - Tc\theta \end{aligned}\right\} \quad \begin{aligned} \ddot{x} &= -\frac{T}{m}\frac{x}{\ell} \quad &-(1) \\ \ddot{y} &= g - \frac{T}{m}\frac{y}{\ell} \quad &-(2) \end{aligned}$$

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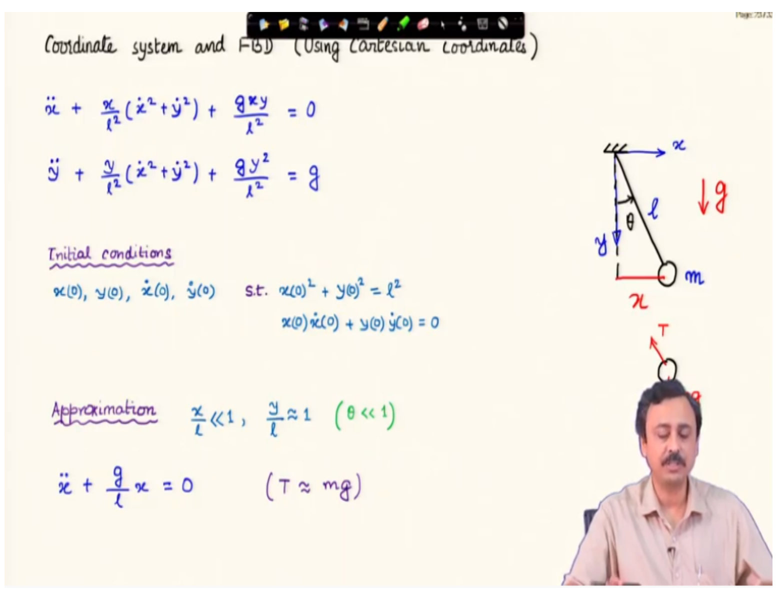

These equations are accompanied by constraints since x and y coordinates cannot vary independently. They must respect the constraint that the bob must always remain at a fixed distance l from the point of support. Thus, we have

Constraint: $x^2 + y^2 = \ell^2 \Rightarrow x\dot{x} + y\dot{y} = 0 \quad -(3)$

$$\Rightarrow x\ddot{x} + y\ddot{y} + \dot{x}^2 + \dot{y}^2 = 0 \quad -(4)$$

Using (1)-(2) in (4) we can obtain the tension T as

$$T = \frac{m}{\ell}(\dot{x}^2 + \dot{y}^2) + \frac{mgy}{\ell}$$

Substituting this expression in (1)-(2), we have the equations of motion

$$\ddot{x} + \frac{x}{\ell^2}(\dot{x}^2 + \dot{y}^2) + \frac{gxy}{\ell^2} = 0$$

$$\ddot{y} + \frac{y}{\ell^2}(\dot{x}^2 + \dot{y}^2) + \frac{gy^2}{\ell^2} = g$$

These are 2 coupled second-order ordinary differential equations of motion. The initial conditions required to integrate these two equations must be such that

Initial conditions

$x(0),\ y(0),\ \dot{x}(0),\ \dot{y}(0)$ s.t. $x(0)^2 + y(0)^2 = \ell^2$

$$x(0)\dot{x}(0) + y(0)\dot{y}(0) = 0$$

If the displacement of the bob is small (x/l << 1), then we have the approximation

Approximation $\frac{x}{\ell} \ll 1,\ \frac{y}{\ell} \approx 1 \quad (\theta \ll 1)$

$$\ddot{x} + \frac{g}{\ell}x = 0 \qquad (T \approx mg)$$

Since y is almost equal to l, the tension is approximately equal to mg.

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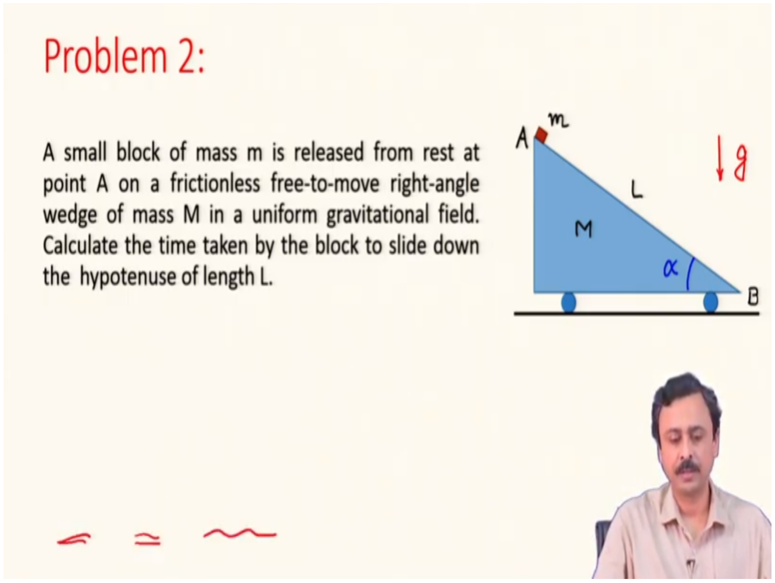

Let us look at another example as shown above. We have to find out the time taken to travel from A to B.

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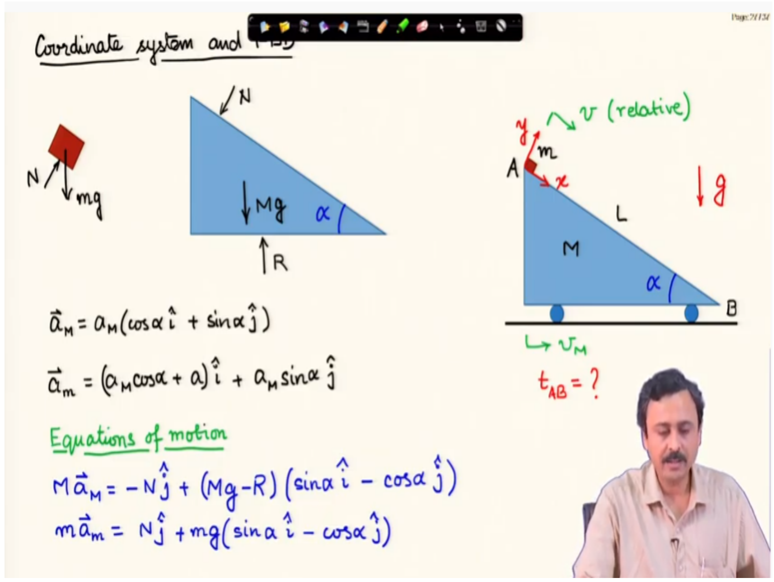

We choose a convenient coordinate system x-y which is fixed to the wedge. The second step is to draw the free body diagrams of the block and the wedge, which are shown above. The acceleration relations and the equations of motion are also shown.

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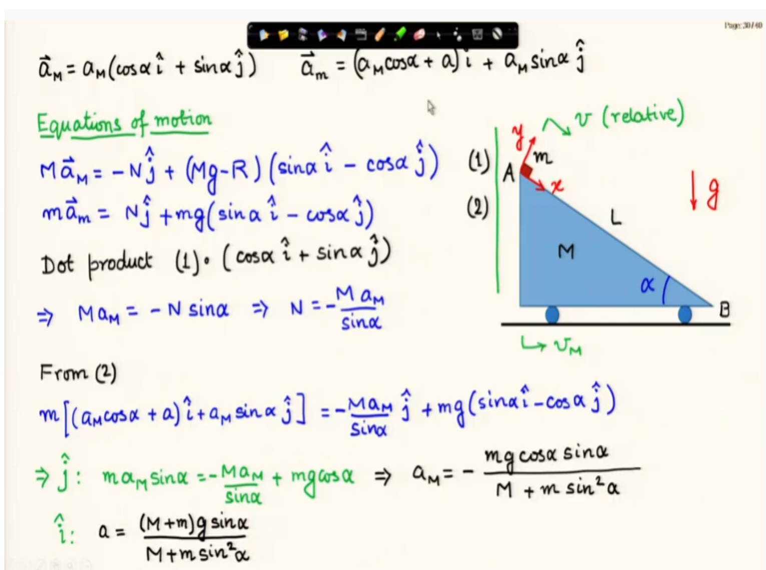

We have 4 scalar equations in 4 unknowns and we should be able to solve. The steps are shown above. Finally, we will get the acceleration of the block along the wedge and relative to it.

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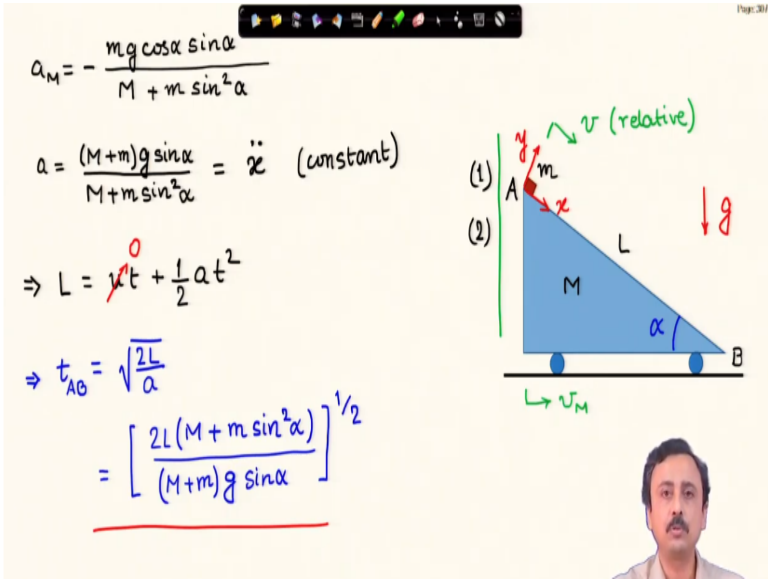

Since this acceleration is constant, we easily obtain the time taken to cover the distance L.

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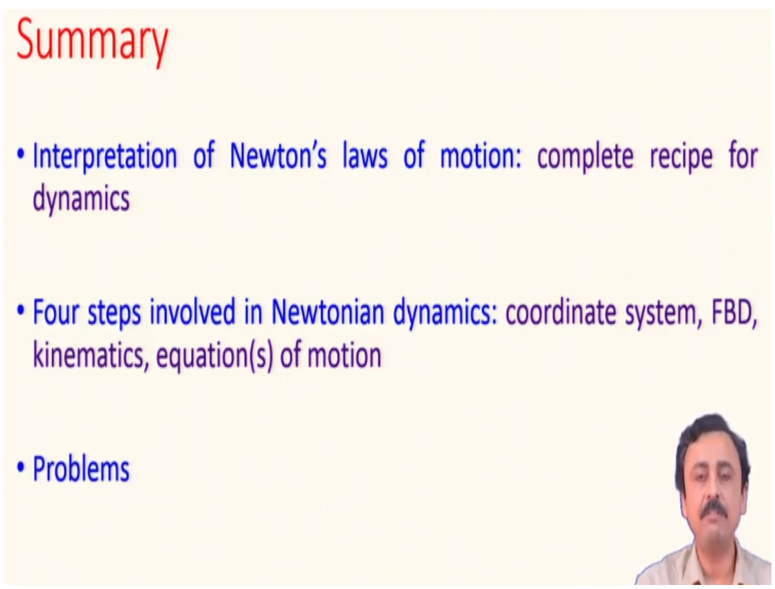

In this lecture we have introduced Newton's laws of motion which represent a complete recipe for dynamics. I have very clearly discussed the 4 step involved in deriving the equation of motion and solving a problem. The first one is setting up the coordinate system, second step is drawing the free body diagram, third step is writing out the kinematics properly as seen by inertial observer, and finally you use the Newton second law to write down the equation(s) of motion.