Advanced Dynamics

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Lecture - 59
Symmetries and Conservation Laws - IV

In this lecture I am going to discuss some examples of symmetries and conservation laws that how we derive these conservation laws or constants of motion and how we use them to understand the dynamics of a system.

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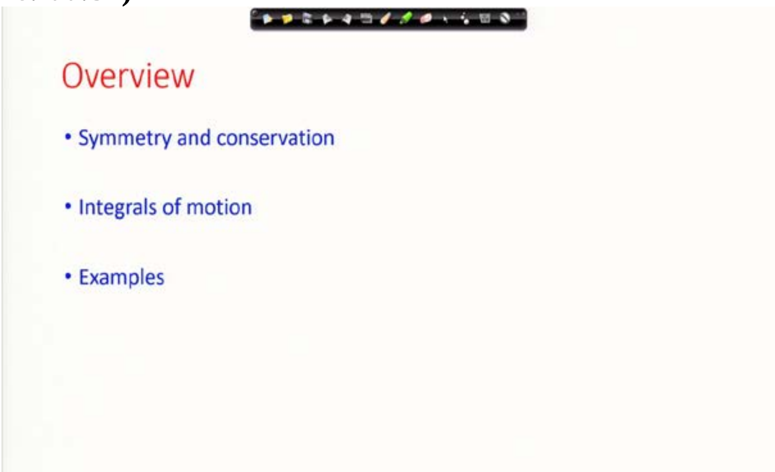

We have been discussing about symmetries and the corresponding conservation we have looked at Noether's theorem we have discussed the integrals of motion and how to find out the integrals of motion here I am going to take some examples.

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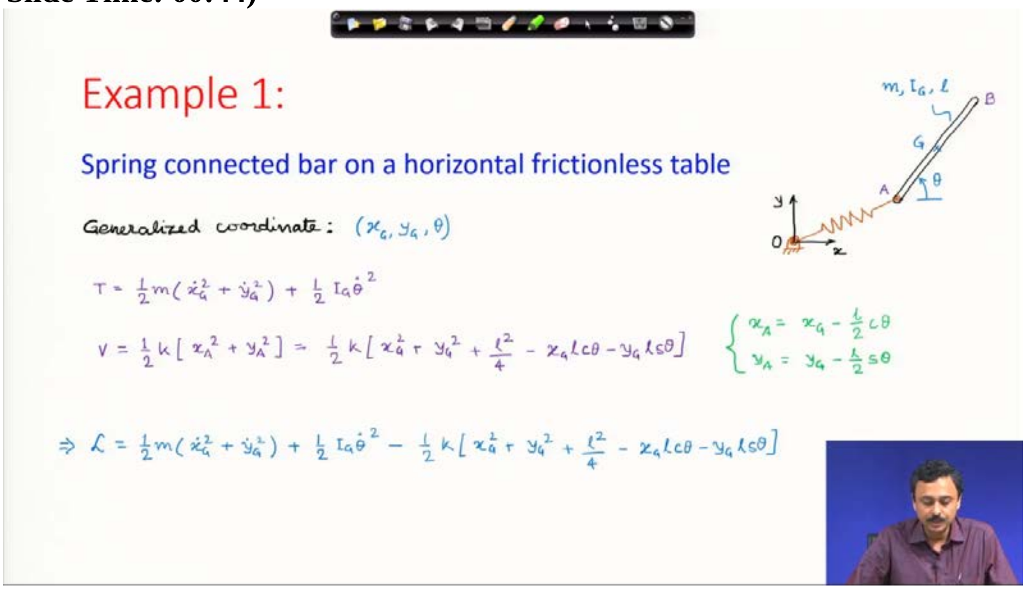

The first example is a spring connected bar on a horizontal frictionless table as shown in the slide above. The Lagrangian is also presented.

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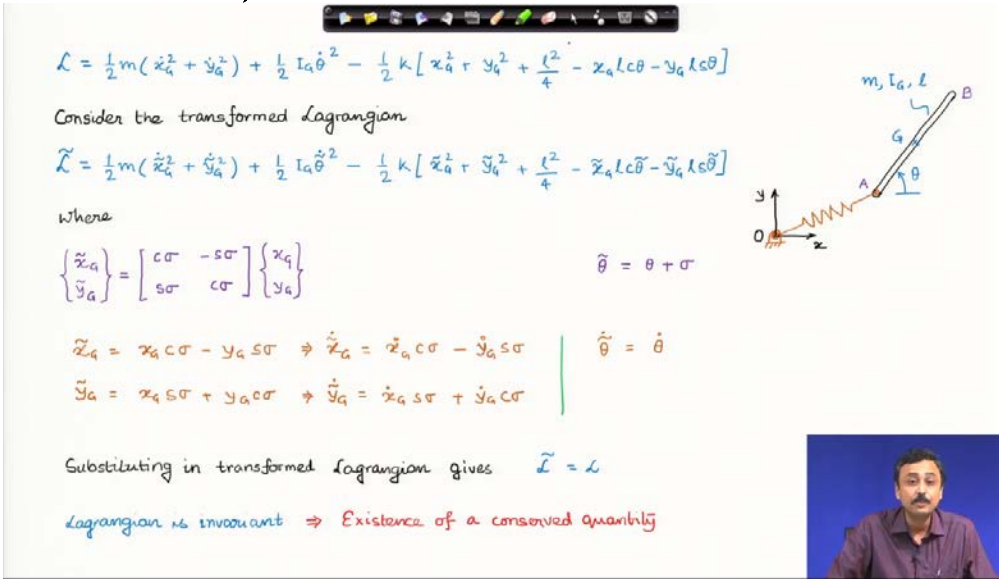

In the above slide, an infinitesimal symmetry of the Lagrangian is determined. This leads to the conclusion, based on Noether’s theorem, that there exists a conserved quantity.

Our next task is to find the conserved quantity. This is determined in the following 2 slides.

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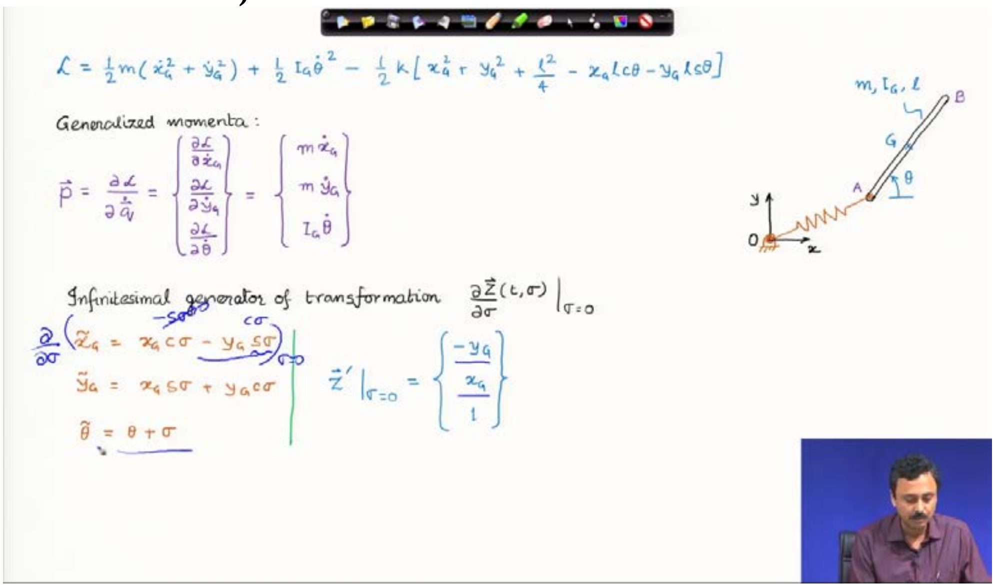

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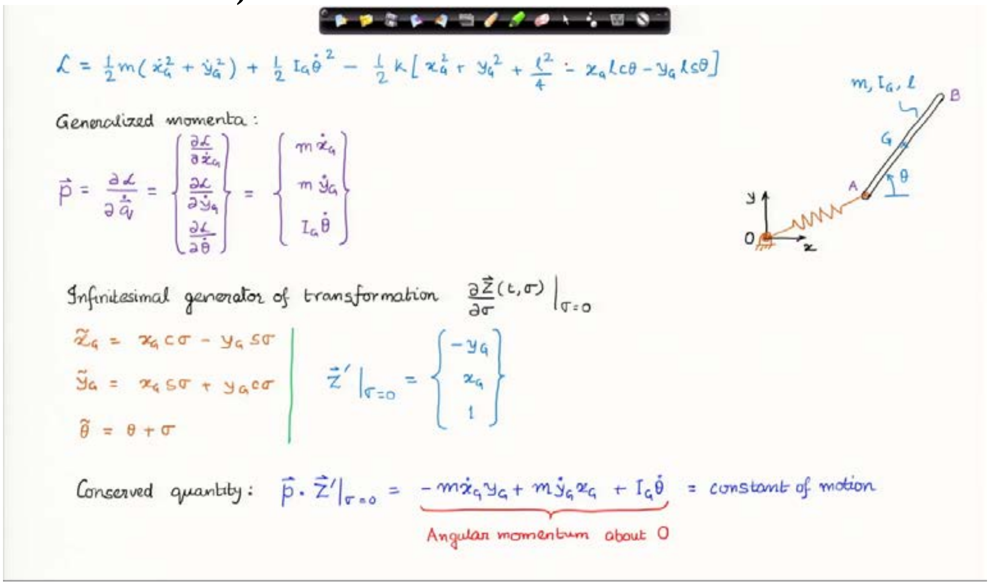

The conserved quantity is nothing but the angular momentum of the bar about the fixed point.

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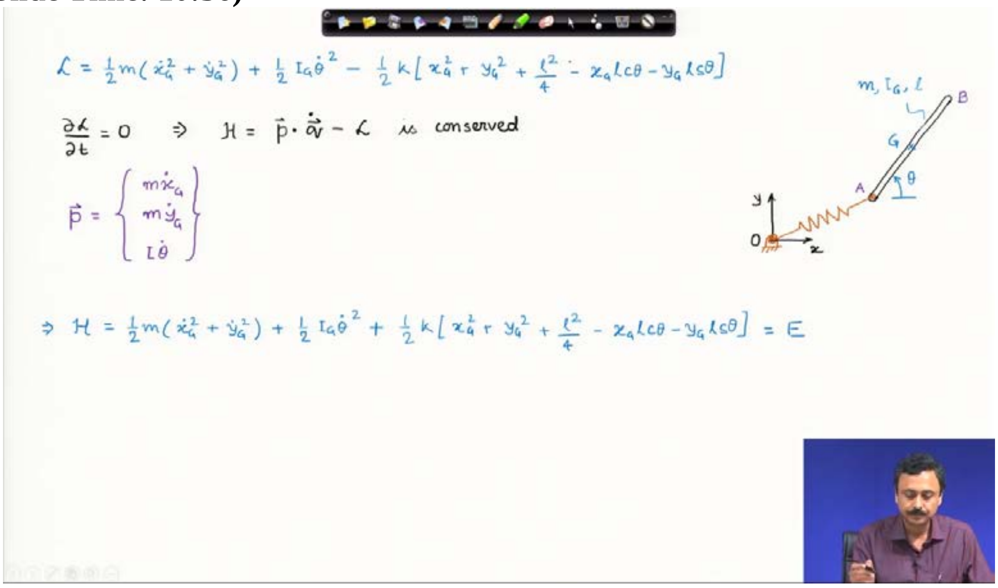

The second symmetry is obtained from the observation that the Lagrangian is not explicitly time dependent. In other words, the partial derivative of the Lagrangian with respect to time is 0, and that immediately tells us that the Hamiltonian is a conserved quantity. In this case, the Hamiltonian is the total mechanical energy of the system.

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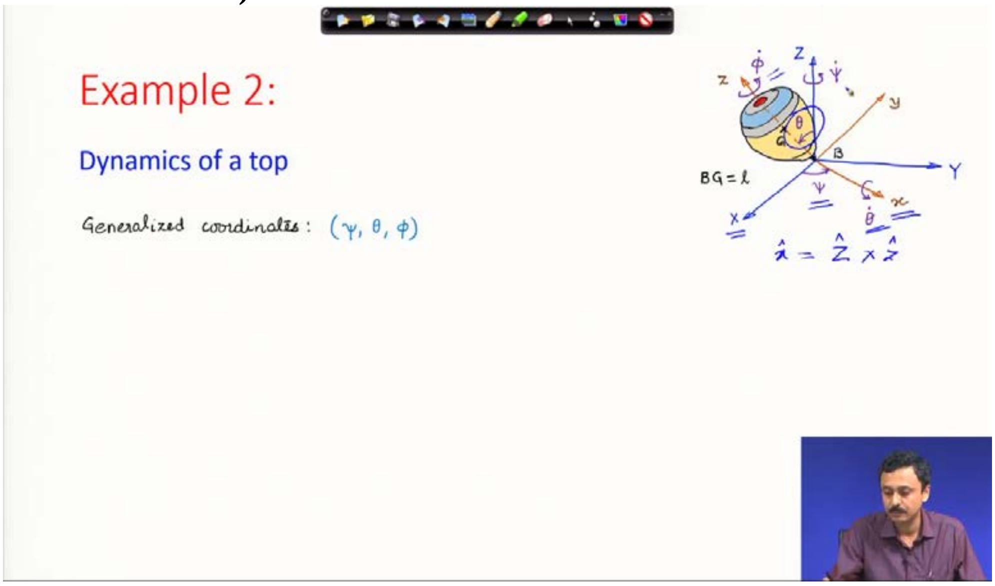

Next we look at this classic example of a spinning top. The generalized coordinates are noted in the slide above.

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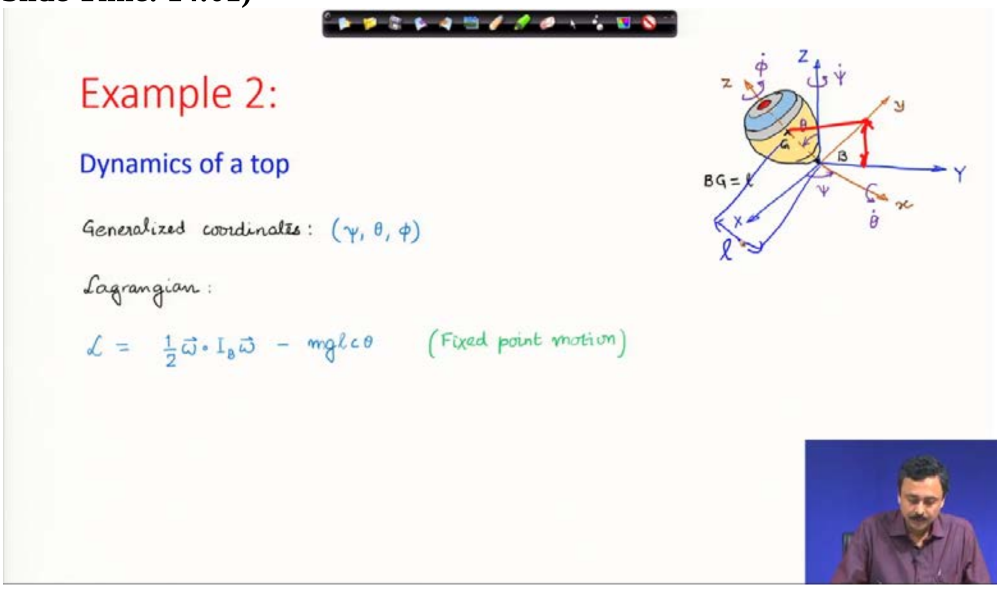

The Lagrangian can be written very easily as shown above.

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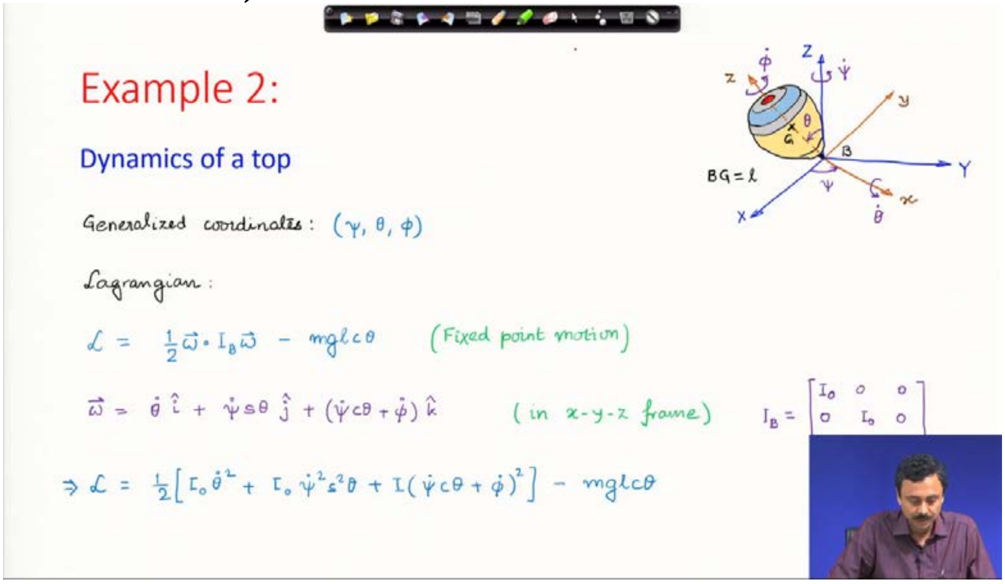

The expression of the Lagrangian in terms of the generalized coordinates is presented in the slide above.

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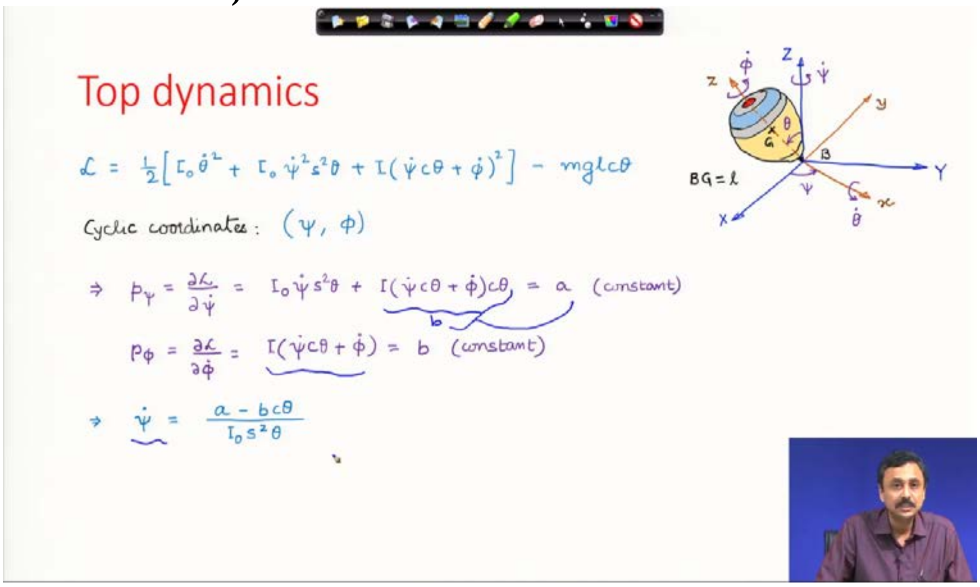

The cyclic coordinates are noted in the slide above, and the constants of motion are presented.

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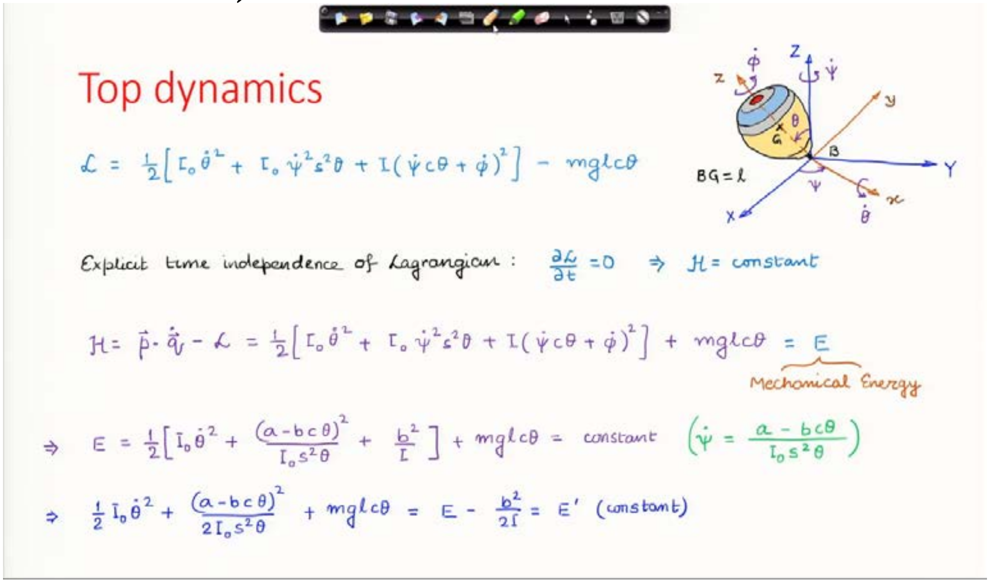

The next thing we notice is the Lagrangian is explicit time independent. This implies the Hamiltonian, which is the total mechanical energy in this case, is a conserved quantity.

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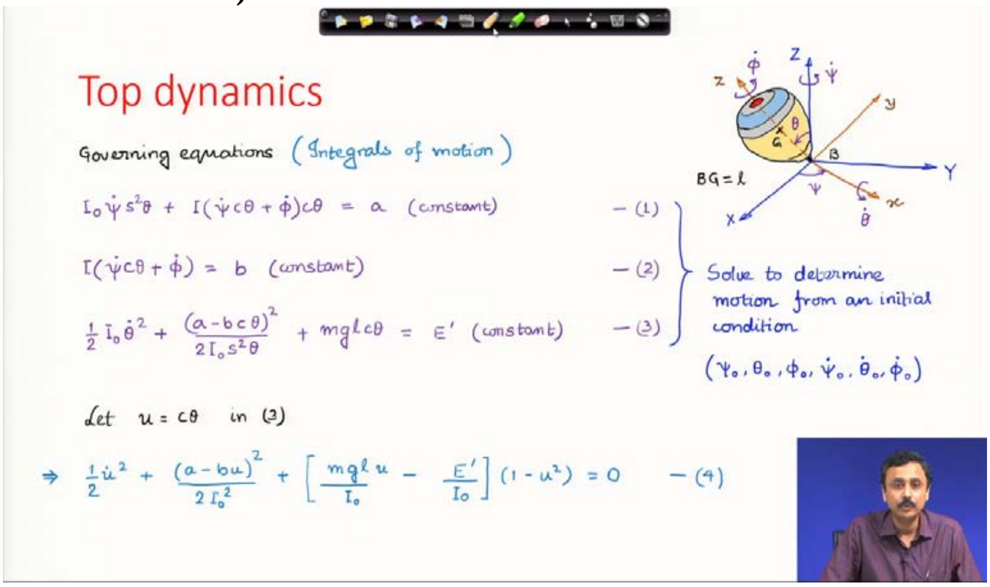

Finally the integrals of motion are noted in the above slide.

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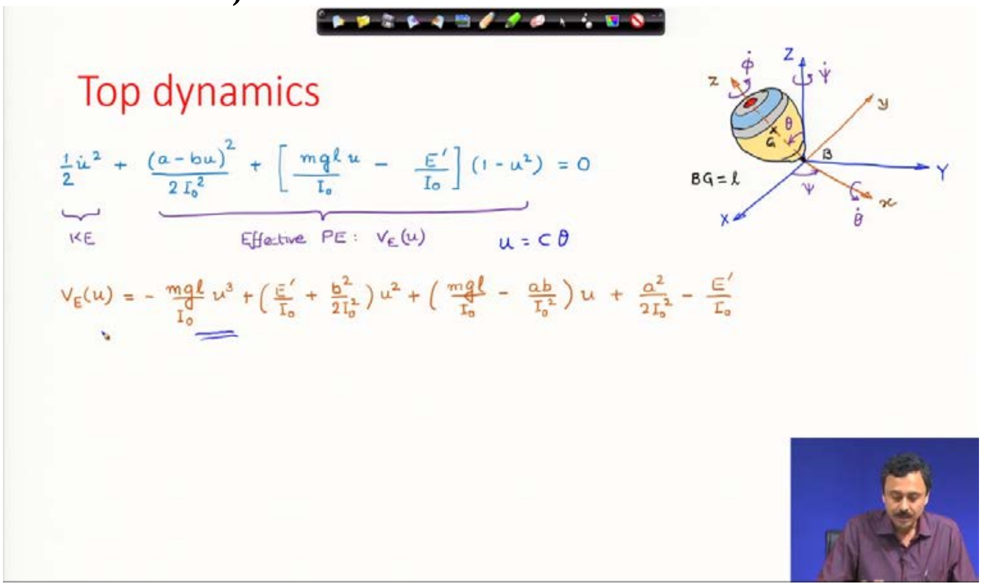

Using the integrals, one can reduce the dynamics of the top to a single dynamical equation, as shown in the slide above. Using the qualitative features of the effective potential energy expression, we study the dynamics of the top in the following 2 slides.

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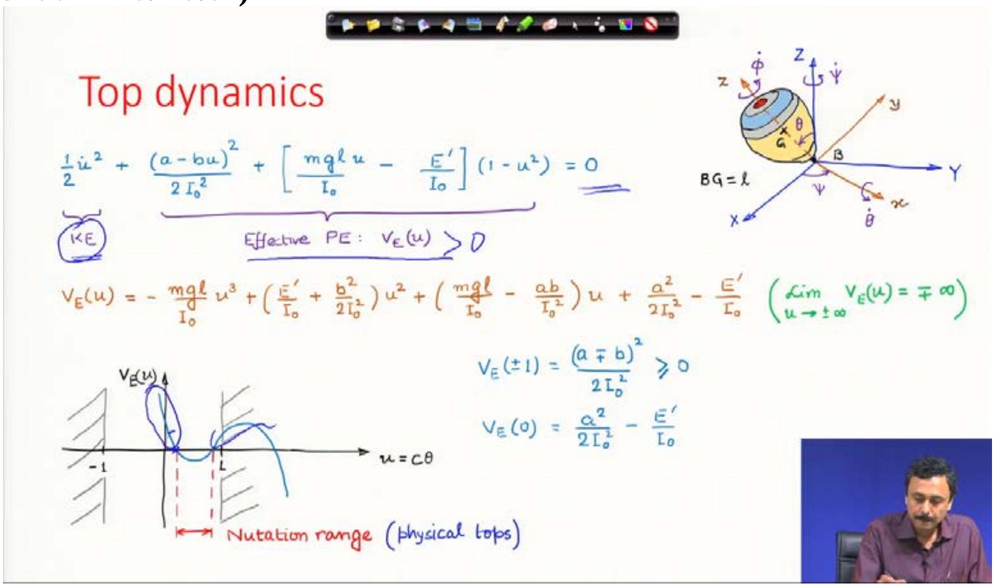

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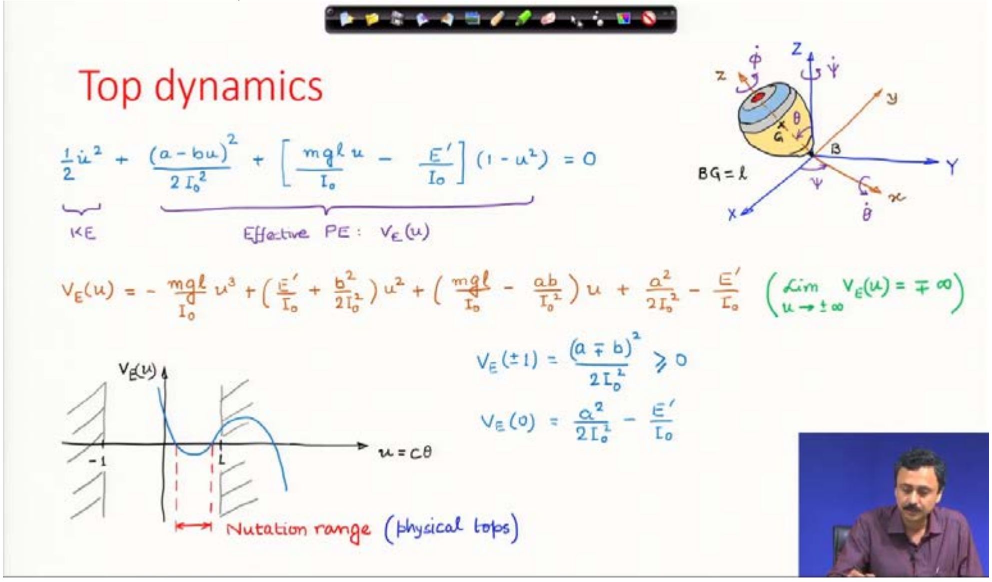

The condition of a sleeping top is presented in the following 2 slides.

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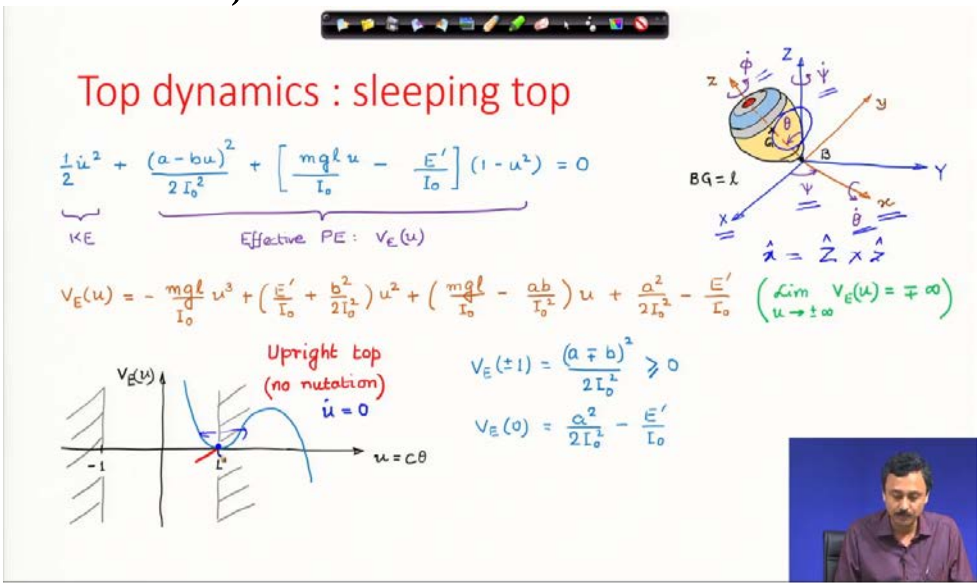

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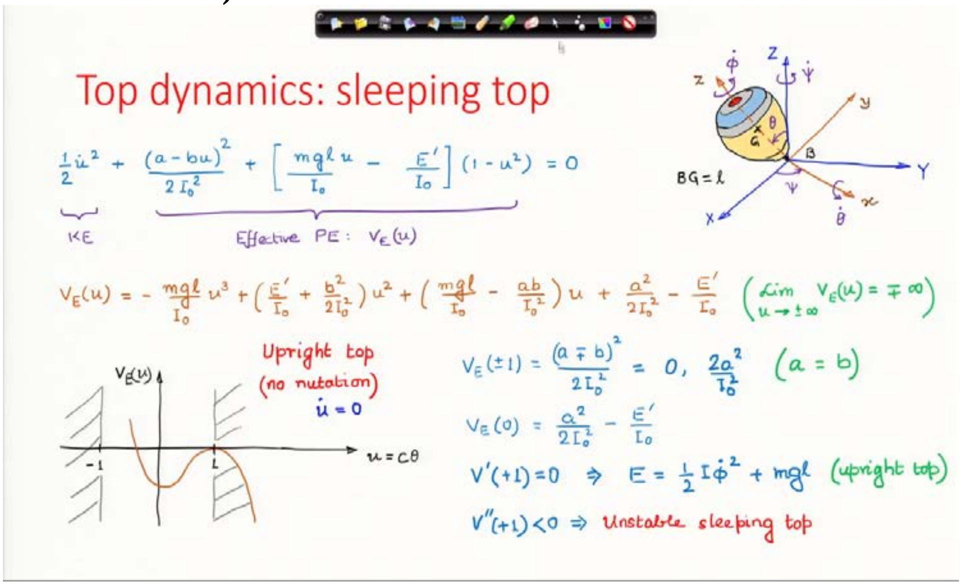

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The summary is presented in the slide above.